EMITTANCE ADAPTER FOR A DIFFRACTION LIMITED SYNCHROTRON RADIATION SOURCE

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Abstract

We investigate the possibility of reaching very small horizontal and vertical emittances inside an undulator in a storage ring, by means of a local exchange of the apparent horizontal and vertical emittances, performed with a combination of skew quadrupoles and one solenoid in a dedicated insertion line in the storage ring. The insertion leaves the ring parameters and its optical properties unaffected. This scheme could greatly relax the emittance requirements for a diffraction limited synchrotron light source. The lattice derivation and design is described.

INTRODUCTION

The idea of flat-to-round and round-to-flat adapters has been first introduced by Derbenev [1] and extended by him and many others [2-10]. He first envisioned applying it to a storage ring collider to form round beams at the collision point to minimize the effect of beam-beam resonances. This idea has also been considered for electron cooling [2,7]. The production of a very flat beam from a round photocathode immersed in a solenoid, followed by a round-to-flat adapter lattice, has been experimentally demonstrated [9,10].

In the electron cooler application, the beam inside the solenoid has very small angular divergences $\sigma_{x'}$ and $\sigma_{y'}$. This beam could be used for electron beam cooling because of its extremely cold temperature. We can use the same technique in a back-to-back configuration in a storage ring, for the purpose of generating a diffraction-limited X-ray synchrotron light source [5].

In order to have coherent or quasi-coherent radiation, the beam emittances ε_x and ε_y have both to be less than $\lambda/4\pi$, where λ is the X-ray wavelength. This is very difficult to achieve for ε_x, requiring complicated and demanding storage ring lattice designs, powerful damping wigglers, tight tolerances and orbit controls. The small dispersion function and the corresponding strong sextupoles also lead to small dynamic apertures.

With the proposed scheme [5], the storage ring lattice can have relaxed requirements on ε_x, as long as ε_y can be made very small, as routinely obtained in electron storage rings. In the straight section reserved for any insertion device, a flat-to-round adapter is first inserted; after which, the beam enters a solenoid that has an aperture sufficiently large to accommodate a radiator (e.g. an undulator) inside. After the solenoid, a round-to-flat adapter (mirror image of the flat-to-round adapter) restores the original coupling.

The arc ring lattice should provide a beam as flat as possible ($\varepsilon_y << \varepsilon_x$), so that $\sqrt{\varepsilon_x \varepsilon_y} < \lambda/4\pi$ for the radiation wavelength of interest.

Quantities ε_x and ε_y are the eigen-emittances of the storage ring in the canonical phase space (x, p_x, y, p_y). The adapters and the solenoid are magnets whose symplectic transport maps will not affect them [12]. It is therefore not possible to make the *canonical* momenta p_x and p_y very small simultaneously inside the solenoid. However, in the solenoid, the *mechanical* momenta x' and y' can be made small simultaneously without violating symplecticity because they are not the canonical coordinates. The adapter makes them determined solely by the small initial ε_y. Fortunately, radiation cares about the apparent emittances in the (x, x', y, y') space rather than the true emittances in the (x, p_x, y, p_y) space. In the solenoid, they are related as:

$$x' = p_x - k_s y/2; \quad y' = p_y + k_s x/2; \quad k_s = B_s/(B\rho)_0$$

with B_s the solenoid magnetic field and $(B\rho)_0$ the magnetic rigidity of the electron beam. The adapter and solenoid optics make p_x cancel with $k_s y/2$ (and p_y with $k_s x/2$), so x' (and y') will be small even when p_x (and p_y) are not.

OPTICS DESIGN

We consider a regular ring lattice that, at its junction to the flat-to-round section, has $\beta_x = \beta_y$, and $\alpha_x = \alpha_y = 0$. We also assume that the lattice has dispersion suppressors at its ends so that the dispersion vanishes in the insertion.

We describe the particle motion using the canonical coordinates $X_{can} = (x, p_x, y, p_y)$ and later transform the description to $X = (x, x', y, y')$.

The particle motion is described by [6, 12] $X_{can} = Va$:

$$V = \begin{bmatrix} \sqrt{\beta_x}\cos\varphi_x & \sqrt{\beta_x}\sin\varphi_x & 0 & 0 \\ -\dfrac{1}{\sqrt{\beta_x}}\sin\varphi_x & \dfrac{1}{\sqrt{\beta_x}}\cos\varphi_x & 0 & 0 \\ 0 & 0 & \sqrt{\beta_y}\cos\varphi_y & \sqrt{\beta_y}\sin\varphi_y \\ 0 & 0 & -\dfrac{1}{\sqrt{\beta_y}}\sin\varphi_y & \dfrac{1}{\sqrt{\beta_y}}\cos\varphi_y \end{bmatrix}$$

$$a = \begin{bmatrix} \sqrt{2\varepsilon_x}\sin\theta_x \\ \sqrt{2\varepsilon_x}\cos\theta_x \\ \sqrt{2\varepsilon_y}\sin\theta_y \\ \sqrt{2\varepsilon_y}\cos\theta_y \end{bmatrix}; \qquad X_{can} = Va = \begin{bmatrix} \sqrt{2\beta_x\varepsilon_x}\sin(\varphi_x+\theta_x) \\ \sqrt{2\varepsilon_x/\beta_x}\cos(\varphi_x+\theta_x) \\ \sqrt{2\beta_y\varepsilon_y}\sin(\varphi_y+\theta_y) \\ \sqrt{2\varepsilon_y/\beta_y}\cos(\varphi_y+\theta_y) \end{bmatrix}$$

At the entrance to the solenoid, the particle motion is described by $X_{can} = Ub$, where U is the symplectic matrix consisting of the circular mode basis vectors [6]:

$$U = \begin{bmatrix} \sqrt{\frac{1}{k_s}}\sin\varphi_+ & -\sqrt{\frac{1}{k_s}}\cos\varphi_+ & \sqrt{\frac{1}{k_s}}\sin\varphi_- & -\sqrt{\frac{1}{k_s}}\cos\varphi_- \\ \frac{\sqrt{k_s}}{2}\cos\varphi_+ & \frac{\sqrt{k_s}}{2}\sin\varphi_+ & \frac{\sqrt{k_s}}{2}\cos\varphi_- & \frac{\sqrt{k_s}}{2}\sin\varphi_- \\ \sqrt{\frac{1}{k_s}}\cos\varphi_+ & \sqrt{\frac{1}{k_s}}\sin\varphi_+ & -\sqrt{\frac{1}{k_s}}\cos\varphi_- & -\sqrt{\frac{1}{k_s}}\sin\varphi_- \\ -\frac{\sqrt{k_s}}{2}\sin\varphi_+ & \frac{\sqrt{k_s}}{2}\cos\varphi_+ & \frac{\sqrt{k_s}}{2}\sin\varphi_- & -\frac{\sqrt{k_s}}{2}\cos\varphi_- \end{bmatrix}$$

$$b = \begin{bmatrix} \sqrt{2\varepsilon_+}\sin\theta_+ \\ \sqrt{2\varepsilon_+}\cos\theta_+ \\ \sqrt{2\varepsilon_-}\sin\theta_- \\ \sqrt{2\varepsilon_-}\cos\theta_- \end{bmatrix}; \quad X_{can} = Ub = \begin{bmatrix} -\sqrt{\frac{2}{k_s}}\left(\sqrt{\varepsilon_+}\cos\Delta_+ + \sqrt{\varepsilon_-}\cos\Delta_-\right) \\ \sqrt{\frac{k_s}{2}}\left(\sqrt{\varepsilon_+}\sin\Delta_+ + \sqrt{\varepsilon_-}\sin\Delta_-\right) \\ \sqrt{\frac{2}{k_s}}\left(\sqrt{\varepsilon_+}\sin\Delta_+ - \sqrt{\varepsilon_-}\sin\Delta_-\right) \\ \sqrt{\frac{k_s}{2}}\left(\sqrt{\varepsilon_+}\cos\Delta_+ - \sqrt{\varepsilon_-}\cos\Delta_-\right) \end{bmatrix}$$

where $\Delta_+ = \varphi_+ + \theta_+$ and $\Delta_- = \varphi_- + \theta_-$. In the language of circular modes, this is equivalent to choose envelope function $\beta = 2/k_s$ and $\alpha = 0$ ($k_s > 0$).

At a distance z into the solenoid, the motion is described by $X_{can} = M_{sol}Ub$:

$$M_{sol} = \begin{bmatrix} \cos^2\frac{\varphi_s}{2} & \frac{1}{k_s}\sin\varphi_s & \frac{1}{2}\sin\varphi_s & \frac{2}{k_s}\sin^2\frac{\varphi_s}{2} \\ -\frac{k_s}{4}\sin\varphi_s & \cos^2\frac{\varphi_s}{2} & -\frac{k_s}{2}\sin^2\frac{\varphi_s}{2} & \frac{1}{2}\sin\varphi_s \\ -\frac{1}{2}\sin\varphi_s & -\frac{2}{k_s}\sin^2\frac{\varphi_s}{2} & \cos^2\frac{\varphi_s}{2} & \frac{1}{k_s}\sin\varphi_s \\ \frac{k_s}{2}\sin^2\frac{\varphi_s}{2} & -\frac{1}{2}\sin\varphi_s & -\frac{k_s}{4}\sin\varphi_s & \cos^2\frac{\varphi_s}{2} \end{bmatrix}$$

$$\varphi_s = k_s z$$

$$X_{can} = \begin{bmatrix} -\sqrt{\frac{2}{k_s}}\left(\sqrt{\varepsilon_+}\cos(\varphi_+ +\theta_+ +k_sz) + \sqrt{\varepsilon_-}\cos(\varphi_- +\theta_-)\right) \\ \sqrt{\frac{k_s}{2}}\left(\sqrt{\varepsilon_+}\sin(\varphi_+ +\theta_+ +k_sz) + \sqrt{\varepsilon_-}\sin(\varphi_- +\theta_-)\right) \\ \sqrt{\frac{2}{k_s}}\left(\sqrt{\varepsilon_+}\sin(\varphi_+ +\theta_+ +k_sz) - \sqrt{\varepsilon_-}\sin(\varphi_- +\theta_-)\right) \\ \sqrt{\frac{k_s}{2}}\left(\sqrt{\varepsilon_+}\cos(\varphi_+ +\theta_+ +k_sz) - \sqrt{\varepsilon_-}\cos(\varphi_- +\theta_-)\right) \end{bmatrix}$$

The "+" mode ("Lamor" mode) advances with the solenoid phase k_sz, but the "-" mode's phase stays still.

The adapter matches the flat optics of the ring arcs to the round optics of the solenoid. It is designed to provide a transport map $M_{adp}=UV^{-1}$, which is:

$$M_{adp} = \begin{bmatrix} \sqrt{\frac{1}{\beta_x k_s}}\sin\Delta_x - \sqrt{\frac{\beta_x}{k_s}}\cos\Delta_x & \sqrt{\frac{1}{\beta_y k_s}}\sin\Delta_y & -\sqrt{\frac{\beta_y}{k_s}}\cos\Delta_y \\ \frac{1}{2}\sqrt{\frac{k_s}{\beta_x}}\cos\Delta_x & \frac{\sqrt{\beta_x k_s}}{2}\sin\Delta_x & \frac{1}{2}\sqrt{\frac{k_s}{\beta_y}}\cos\Delta_y & \frac{\sqrt{\beta_y k_s}}{2}\sin\Delta_y \\ \sqrt{\frac{1}{\beta_x k_s}}\cos\Delta_x & \sqrt{\frac{\beta_x}{k_s}}\sin\Delta_x & -\sqrt{\frac{1}{\beta_y k_s}}\cos\Delta_y & -\sqrt{\frac{\beta_y}{k_s}}\sin\Delta_y \\ -\frac{1}{2}\sqrt{\frac{k_s}{\beta_x}}\sin\Delta_x & \frac{\sqrt{\beta_x k_s}}{2}\cos\Delta_x & \frac{1}{2}\sqrt{\frac{k_s}{\beta_y}}\sin\Delta_y & -\frac{\sqrt{\beta_y k_s}}{2}\cos\Delta_y \end{bmatrix}$$

with $\Delta_x = \varphi_+ - \varphi_x$ and $\Delta_y = \varphi_- - \varphi_y$.

After inserting this section, a particle with initial condition $X_{can} = Va$ at the end of the regular cell will be transported to an arbitrary distance z inside the solenoid:

$$X_{can} = M_{sol}M_{adp}Va = M_{sol}(UV^{-1})Va = M_{sol}Ua$$

$$X_{can} = \begin{bmatrix} -\sqrt{\frac{2}{k_s}}\left(\sqrt{\varepsilon_x}\cos(\varphi_+ +\theta_x +k_sz) + \sqrt{\varepsilon_y}\cos(\varphi_- +\theta_y)\right) \\ \sqrt{\frac{k_s}{2}}\left(\sqrt{\varepsilon_x}\sin(\varphi_+ +\theta_x +k_sz) + \sqrt{\varepsilon_y}\sin(\varphi_- +\theta_y)\right) \\ \sqrt{\frac{2}{k_s}}\left(\sqrt{\varepsilon_x}\sin(\varphi_+ +\theta_x +k_sz) - \sqrt{\varepsilon_y}\sin(\varphi_- +\theta_y)\right) \\ \sqrt{\frac{k_s}{2}}\left(\sqrt{\varepsilon_x}\cos(\varphi_+ +\theta_x +k_sz) - \sqrt{\varepsilon_y}\cos(\varphi_- +\theta_y)\right) \end{bmatrix}$$

We have simply made the emittance exchanges $\varepsilon_x \rightarrow \varepsilon_+$ and $\varepsilon_y \rightarrow \varepsilon_-$ by the flat-to-round insertion.

The map M_{adp} is chosen to satisfy the conditions: $\beta_x = \beta_y$, $\varphi_+ = \varphi_x - \pi/4$, $\varphi_- = \varphi_y + \pi/4$. It can be written as:

$$\begin{bmatrix} \frac{1}{\sqrt{2}} & 0 & -\frac{1}{\sqrt{2}} & 0 \\ 0 & \frac{1}{\sqrt{2}} & 0 & -\frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & 0 & \frac{1}{\sqrt{2}} & 0 \\ 0 & \frac{1}{\sqrt{2}} & 0 & \frac{1}{\sqrt{2}} \end{bmatrix} \begin{bmatrix} 0 & -\sqrt{\frac{2\beta_x}{k_s}} & 0 & 0 \\ \sqrt{\frac{k_s}{2\beta_x}} & 0 & 0 & 0 \\ 0 & 0 & -\sqrt{\frac{2}{k_s\beta_x}} & 0 \\ 0 & 0 & 0 & -\sqrt{\frac{k_s\beta_x}{2}} \end{bmatrix} \begin{bmatrix} \frac{1}{\sqrt{2}} & 0 & \frac{1}{\sqrt{2}} & 0 \\ 0 & \frac{1}{\sqrt{2}} & 0 & \frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{2}} & 0 & \frac{1}{\sqrt{2}} & 0 \\ 0 & -\frac{1}{\sqrt{2}} & 0 & \frac{1}{\sqrt{2}} \end{bmatrix}$$

The middle map is that of a quadrupole channel (three quadrupoles) with x phase advance of $3\pi/2$ and y phase advance of π. The two adjacent matrices mean that the triplet channel is rotated by 45° so that the quadrupoles become skew. With this insertion, a particle at the entrance of the insertion is transported inside the solenoid with $X_{can}(z) = M_{sol}Ua$. The result is:

$$X_{can}(z) = \begin{bmatrix} -\sqrt{\frac{2}{k_s}}\left(\sqrt{\varepsilon_x}\cos(\varphi_x +\frac{\pi}{4} +\theta_x +k_sz) + \sqrt{\varepsilon_y}\cos(\varphi_y +\frac{\pi}{4} +\theta_y)\right) \\ \sqrt{\frac{k_s}{2}}\left(\sqrt{\varepsilon_x}\sin(\varphi_x -\frac{\pi}{4} +\theta_x +k_sz) + \sqrt{\varepsilon_y}\sin(\varphi_y +\frac{\pi}{4} +\theta_y)\right) \\ \sqrt{\frac{2}{k_s}}\left(\sqrt{\varepsilon_x}\sin(\varphi_x -\frac{\pi}{4} +\theta_x +k_sz) - \sqrt{\varepsilon_y}\sin(\varphi_y +\frac{\pi}{4} +\theta_y)\right) \\ \sqrt{\frac{k_s}{2}}\left(\sqrt{\varepsilon_x}\cos(\varphi_x +\frac{\pi}{4} +\theta_x +k_sz) - \sqrt{\varepsilon_y}\cos(\varphi_y +\frac{\pi}{4} +\theta_y)\right) \end{bmatrix}$$

BEAM CHARACTERISTICS IN THE SOLENOID

The radiation at the radiator cares about X and not X_{can}, and X inside the solenoid is $X(z) = M_{exit}X_{can}(z)$ with:

$$M_{exit} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & -k_s/2 & 0 \\ 0 & 0 & 1 & 0 \\ k_s/2 & 0 & 0 & 1 \end{bmatrix}$$

$$X(z) = \begin{bmatrix} -\sqrt{\dfrac{2}{k_s}}\left(\sqrt{\varepsilon_x}\cos(\varphi_x + \dfrac{\pi}{4} + \theta_x + k_s z) + \sqrt{\varepsilon_y}\cos(\varphi_y + \dfrac{\pi}{4} + \theta_y)\right) \\ \sqrt{2k_s\varepsilon_y}\sin(\varphi_y + \dfrac{\pi}{4} + \theta_y) \\ \sqrt{\dfrac{2}{k_s}}\left(\sqrt{\varepsilon_x}\sin(\varphi_x - \dfrac{\pi}{4} + \theta_x + k_s z) - \sqrt{\varepsilon_y}\sin(\varphi_y + \dfrac{\pi}{4} + \theta_y)\right) \\ -\sqrt{2k_s\varepsilon_y}\cos(\varphi_y + \dfrac{\pi}{4} + \theta_y) \end{bmatrix}$$

Only the vertical emittance ε_y enters x' and y', as desired, while x and y are large, mainly determined by ε_x. The beam's appearance has a large transverse dimension but extremely small divergence. The beam is almost perfectly laminar.

We now calculate the Σ matrix of the second moments of the beam's transverse distribution. The beam's Σ_0 at the end of the regular cells is:

$$\Sigma_0 = \begin{bmatrix} \beta_x\varepsilon_x & 0 & 0 & 0 \\ 0 & \varepsilon_x/\beta_x & 0 & 0 \\ 0 & 0 & \beta_x\varepsilon_y & 0 \\ 0 & 0 & 0 & \varepsilon_y/\beta_x \end{bmatrix}$$

The total transport map from the end of regular cells to inside of solenoid is $M_{sol}M_{adp}$. With this map, the Σ_0 matrix is transformed to (in the X_{can} coordinate):

$$\Sigma_1 = \begin{bmatrix} \dfrac{\varepsilon_x + \varepsilon_y}{k_s} & 0 & 0 & -\dfrac{1}{2}(\varepsilon_x - \varepsilon_y) \\ 0 & \dfrac{k_s}{4}(\varepsilon_x + \varepsilon_y) & \dfrac{1}{2}(\varepsilon_x - \varepsilon_y) & 0 \\ 0 & \dfrac{1}{2}(\varepsilon_x - \varepsilon_y) & \dfrac{\varepsilon_x + \varepsilon_y}{k_s} & 0 \\ -\dfrac{1}{2}(\varepsilon_x - \varepsilon_y) & 0 & 0 & \dfrac{k_s}{4}(\varepsilon_x + \varepsilon_y) \end{bmatrix}$$

Note that M_{sol} depends on z but the beam distribution inside the solenoid is independent of z. This results from the optics matching by the adapter. The beam is optically "matched" into the solenoid.

Eigen-emittances can be calculated from the Σ matrices; they are just the eigen-values of the matrix $iJ\Sigma$, where J is the symplectic form. Both Σ_0 and Σ_1 have eigen-emittances equal to ε_x and ε_y.

In the X coordinates, we then transport Σ_1 by M_{exit}:

$$\Sigma_2 = M_{exit}\Sigma_1 M_{exit}^T = \begin{bmatrix} (\varepsilon_x + \varepsilon_y)/k_s & 0 & 0 & \varepsilon_y \\ 0 & \varepsilon_y k_s & -\varepsilon_y & 0 \\ 0 & -\varepsilon_y & (\varepsilon_x + \varepsilon_y)/k_s & 0 \\ \varepsilon_y & 0 & 0 & \varepsilon_y k_s \end{bmatrix}$$

From Σ_2 we obtain the beam distribution moments:

$$\sigma_x = \sigma_y = \sqrt{(\varepsilon_x + \varepsilon_y)/k_s} \text{ and } \sigma_{x'} = \sigma_{y'} = \sqrt{\varepsilon_y k_s}$$

inside the solenoid, all independent of z.

After the solenoid, the matched round beam is brought back to its flat configuration with a reversed round-to-flat adapter, so the insertion is transparent to the ring.

The two "projected-emittances" using Σ_2 are equal and found to be $\sqrt{\varepsilon_x\varepsilon_y + \varepsilon_y^2}$ which is approximately equal to $\sqrt{\varepsilon_x\varepsilon_y}$. The Diffraction limit requirement will be:

$$\sqrt{\varepsilon_x\varepsilon_y} < \lambda/4\pi$$

This is easier to achieve than $\varepsilon_x < \lambda/4\pi$ since today most storage rings light sources routinely reach $\varepsilon_y < 10^{-3}\varepsilon_x$. It should be pointed out that, since the beam is very laminar, the X-ray optics to match it will have a long Rayleigh length [13]: $L_R \approx \sigma_x/\sigma_{x'} \approx \sqrt{\varepsilon_x/\varepsilon_y}/k_s$. The transverse coherence properties of the light generated by such beam have to be further studied.

The electron motion and the light properties in the presence of the combined solenoid and undulator will require more detailed analysis. At the lowest order, the orbit distortion caused by the radiator should vanish outside of the solenoid. The first order optics remains the same as without the radiator.

A DESIGN EXAMPLE

All lattice designs, including regular cells and adapters are straightforward and should not impose technical challenges. The main design issue is the undulator inside the solenoid. Key parameters are the solenoid strength k_s, its length L and physical aperture R. L and R are only constrained by the needs of the radiator and not from optics matching.

Choice of k_s is related to the beam characteristics at the radiator. In addition one should note that the skew quadrupole strengths depend on k_s. In general, extreme values of k_s should be avoided. Typically, k_s should be in the range [0.1 m, 1.0 m].

Figure 1 shows an example of the adapter design for the SuperB case. On both ends, a dispersion suppressor cancels the arc dispersion and a triplet matches the envelope functions for the adapter. In this example we choose $\beta_x = \beta_y = 6.0$ m, $\alpha_x = \alpha_y = 0$. Subsequently there are two adapter triplets, each one generating the following transfer matrix:

$$V = \begin{bmatrix} \cos(\varphi + \dfrac{\pi}{4}) & \beta_x\sin(\varphi + \dfrac{\pi}{4}) & 0 & 0 \\ -\dfrac{1}{\beta_x}\sin(\varphi + \dfrac{\pi}{4}) & \cos(\varphi + \dfrac{\pi}{4}) & 0 & 0 \\ 0 & 0 & \cos(\varphi - \dfrac{\pi}{4}) & \beta_x\sin(\varphi - \dfrac{\pi}{4}) \\ 0 & 0 & -\dfrac{1}{\beta_x}\sin(\varphi - \dfrac{\pi}{4}) & \cos(\varphi - \dfrac{\pi}{4}) \end{bmatrix}$$

We have set $\varphi = \pi/2$ as an example, but its value is arbitrary. In general the incoming parameters β and α in the adapter section are arbitrary as well, provided that they are equal in the two planes.
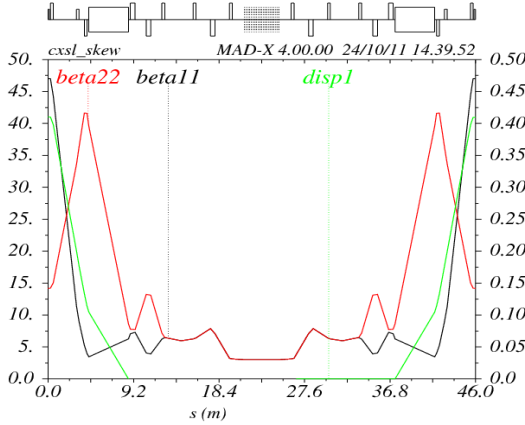
Figure 1: Coupled β-funtions through the insertion. The adapter consists of the solenoid (in the middle) and two skew triplets on its sides.

The first adapter triplet (to the left of the solenoid in Fig. 1) is tilted by 45° clockwise and the second one by 45° counter-clockwise. The 4-m solenoid, with $k_s = 2/\beta_x = 0.333/m$, is in the middle. The beam sizes inside are:

$$\sigma_x = \sigma_y \approx \sqrt{3\varepsilon_x} \text{ and } \sigma_{x'} = \sigma_{y'} = \sqrt{0.333\varepsilon_y}$$

The triplet quadrupoles are 0.5 m long and have strengths 0.906/-1.094/0.906 m⁻¹ respectively. The solenoid has a field of 2.8 Tesla for a 4 GeV beam.

The effectiveness of the scheme is shown in Fig. 2 where cosine-like and sine-like beam trajectories are tracked through the adapter. In the general case both incoming x and x' become pure x and y in the solenoid. For the chosen case $\varphi = \pi/2$, x becomes a pure y displacement and x' a pure x displacement. Incoming y and y' are the only contributions to the horizontal and vertical angles in the solenoid. Such angles are very small since we have assumed $\varepsilon_y = 10^{-2}\varepsilon_x$. This behaviour is exactly as the formulas above described. We also stress that such condition happens just in the solenoid; indeed the angles due to ε_x vanish as soon as the beam enters the solenoid. The coupling disappears after the insertion and the beam stays flat in the rest of the ring.
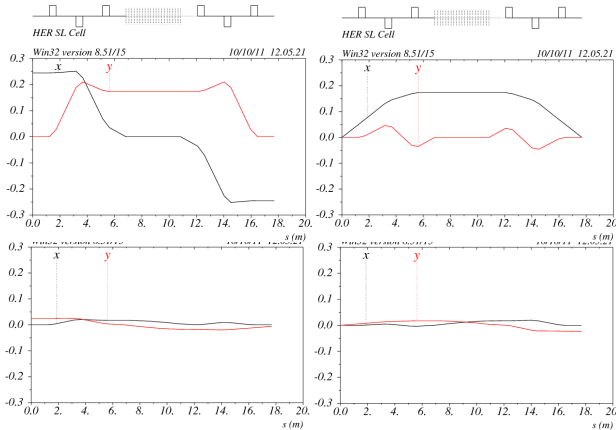

Figure 2: Tracking through the adapter for incoming beam displacements (ε_x = 10 nm, $\varepsilon_y = 10^{-2}\varepsilon_x$):

$$x = 10^{-4}\sqrt{\beta_x}; x' = 10^{-4}/\sqrt{\beta_x}; y = 10^{-5}\sqrt{\beta_x}; y' = 10^{-5}/\sqrt{\beta_x}$$

This optic is just an example and more studies are necessary to further optimize its design and flexibility (e.g. shortening of the insertion length). However it has been shown that it is not complex and can be realized with standard magnets of moderate lengths. The only element that requires careful engineering is the undulator inside a (possibly superconduting) solenoid.

SUMMARY

The emittance adapter greatly relaxes many of the technical challenges of the current approaches to achieve fully transverse coherent X-ray radiation. The storage ring could be simpler, less technically demanding and less costly than the ultra small ε_x designs ("ultimate" rings [14]), as the requirement for ε_x could be relaxed by a factor: $1/2\sqrt{\varepsilon_x/\varepsilon_y}$.

The scheme could be applied to conventional 3ʳᵈ generation storage ring facilities (already existing or new) to provide coherent soft X-rays. In an "ultimate" ring, this scheme could be used to reach coherent hard X-rays. In the case of a modest storage ring with ε_x ~10 nm, such an insertion could generate coherent light for EUV lithography at λ = 13.5 nm.

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