

# Update on angles and sides of the CKM unitarity triangle from *BABAR*

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## Abstract

We report several recent updates from the *BABAR* Collaboration on the matrix elements  $|V_{cb}|$ ,  $|V_{ub}|$ , and  $|V_{td}|$  of the Cabibbo-Kobayashi-Maskawa (CKM) quark-mixing matrix, and the angles  $\beta$  and  $\alpha$  of the unitarity triangle. Most results presented here are using the full *BABAR*  $\Upsilon(4S)$  data set.

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## 1. Introduction

The Cabibbo-Kobayashi-Maskawa (CKM) quark-mixing matrix [1] elements  $V_{ij}$  represent the couplings of the charged current  $W^\pm$  between  $u_i$  and  $d_j$  quarks. In the standard model (SM) of electroweak interactions, the  $CP$  violation is a consequence of the irreducible phase in this  $3 \times 3$  unitary matrix. The main objective of the PEP-II  $B$ -factory and the *BABAR* detector [2] at SLAC is to determine the parameters in the CKM matrix at high precisions in as many ways as possible. If no single set of parameters can satisfy all measurements, one can conclude that new physics beyond the SM that contributes to  $CP$  violation exists.

The most commonly used unitarity condition of the CKM matrix is  $V_{ud}V_{ub}^* + V_{cd}V_{cb}^* + V_{td}V_{tb}^* = 0$ , which forms a triangle on the complex plane. Its angles and sides represent the more poorly known parameters of the CKM matrix. In this report, we present some recent updates on the angles ( $\alpha = \arg[-(V_{td}V_{tb}^*)/(V_{ud}V_{ub}^*)]$ ,  $\beta = \arg[-(V_{cd}V_{cb}^*)/(V_{td}V_{tb}^*)]$ )<sup>1</sup>, and the magnitudes  $|V_{cb}|$ ,  $|V_{ub}|$ , and  $|V_{td}|$ .

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<sup>1</sup> The third angle  $\gamma = \arg[-(V_{ud}V_{ub}^*)/(V_{cd}V_{cb}^*)]$  is not discussed here due to limited space.

## 2. Angles: $\beta$ and $\alpha$ from time-dependent $CP$ analyses

The angles  $\beta$  and  $\alpha$  are measured from time-dependent  $CP$  asymmetries in neutral  $B$  meson decays. The asymmetry, arising from the interference between decays with and without  $B^0$ - $\bar{B}^0$  mixing process, is proportional to  $S_f \sin(\Delta m_d \Delta t) - C_f \cos(\Delta m_d \Delta t)$ , where  $\Delta t$  is the difference between the proper decay times of the two  $B$ 's in  $\Upsilon(4S)$  decays, and  $\Delta m_d$  is the  $B^0$ - $\bar{B}^0$  mixing frequency.

Neutral  $B$  decays to  $CP$  eigenstates containing a charmonium and a  $K^{(*)0}$  mesons, which are dominated by tree-diagram processes, provide a direct measurement of  $\sin 2\beta$  [3]. To a very good approximation,  $S_f = -\eta_f \sin 2\beta$  and  $C_f = 0$  in the SM ( $\eta_f$  is the  $CP$  eigenvalue). We update the analysis of these decay modes using the full BABAR  $\Upsilon(4S)$  data set (465 million  $B\bar{B}$ ) and obtain  $-\eta_f S_f = 0.691 \pm 0.029 \pm 0.014$  and  $C_f = 0.026 \pm 0.020 \pm 0.016$  [4].

The charmless  $B$  decays dominated by  $b \rightarrow s$  penguin diagrams provide opportunities to probe new heavy particles in the loops. We update  $B \rightarrow \eta' K^0$  and  $B \rightarrow \omega K_S^0$  [5] analyses, as well as  $B \rightarrow K_S^0 K_S^0$  [6] using the full BABAR  $\Upsilon(4S)$  data set. In all cases,  $-\eta_f S_f$  is consistent with that from  $(c\bar{c})K$  final state, indicating that large new physics effects in the  $b \rightarrow s$  penguin loops are unlikely.

The angle  $\alpha$  can be measured by studying modes such as  $B \rightarrow \pi\pi, \rho\rho, \rho\pi$ , etc. Because of the sizeable contribution from penguin diagram with a different weak phase, the  $S_f$  term is modified to  $S_f = -\eta_f \sqrt{1 - C_f^2} \sin(2\alpha - 2\Delta\alpha)$ . The correction  $\Delta\alpha$  can be resolved with an isospin analysis [7]. BABAR updates the  $\pi\pi$  [8] ( $S_{\pi^+\pi^-} = -0.68 \pm 0.10 \pm 0.03$ ,  $C_{\pi^+\pi^-} = -0.25 \pm 0.08 \pm 0.02$ ) and  $\rho\rho$  [9] analyses. The branching fraction  $\mathcal{B}(B \rightarrow \rho^0 \rho^0) = (0.92 \pm 0.32 \pm 0.14) \times 10^{-6}$  leads to  $|\Delta\alpha| < 17.6^\circ$  at 90% confidence level. Combining BABAR's  $B \rightarrow \pi\pi, \rho\rho, \rho\pi$  results, we obtain  $\alpha = (81.1_{-4.9}^{+17.5})^\circ$  [10].

## 3. $|V_{cb}|$ and $|V_{ub}|$ from semi-leptonic $B$ decays

The rates of semileptonic decays of  $B$  mesons to charm and charmless final states are proportional to  $|V_{cb}|^2$  and  $|V_{ub}|^2$ , respectively. For  $b \rightarrow c\ell\nu$  we exploit the partial decay rate as a function of kinematic variables to determine the decay rate and the form factor at zero recoil point, at which the form factor suffers from the least theoretical uncertainty [11]. We update the analysis of exclusively reconstructed  $B \rightarrow D\ell\nu$  with the second  $B$  fully reconstructed in hadronic decay modes and obtain  $|V_{cb}| = (39.8 \pm 1.8 \pm 1.3 \pm 0.9) \times 10^{-3}$  [12], where the last error is due to the theoretical uncertainty in the form factor. We also perform a global fit using  $B \rightarrow D^{(0,-)} X \ell^+ \nu$  decays to simultaneously determine the branching fractions of  $B \rightarrow D^{(*)} \ell \nu$ , and form factor slopes ( $\rho_D^2, \rho_{D^*}^2$ ) in a HEQT-based parameterization. From them we obtain  $|V_{cb}| = (39.9 \pm 0.8 \pm 2.2 \pm 0.9) \times 10^{-3}$  using  $D\ell\nu$  and  $|V_{cb}| = (38.6 \pm 0.2 \pm 1.3 \pm 1.0) \times 10^{-3}$  using  $D^* \ell \nu$  [13].

For  $|V_{ub}|$ , we measure branching fractions of  $B \rightarrow \{\pi, \eta, \eta'\} \ell \nu$ , with the other  $B$  decays semileptonically, in three bins of momentum transfer  $q^2$ . We derive  $|V_{ub}|$  using several theoretical form factor calculations and find the value varies between  $(3.6\text{--}4.1) \times 10^{-3}$  with total error of about 15% [14].

The inclusive method suffers from large  $b \rightarrow c\ell\nu$  background. Experimental sensitivity comes from phase space region where  $b \rightarrow c\ell\nu$  is kinematically suppressed. However,

extracting  $|V_{ub}|$  from a limited phase space suffers from theoretical uncertainties from nonperturbative shape functions. We update the inclusive  $B \rightarrow X_u \ell \nu$  measurement with the second  $B$  fully reconstructed in hadronic decay modes.  $|V_{ub}|$  is extracted by fitting distributions of  $q^2$ , mass and momentum of the  $X_u$  system. The results range from  $(3.9\text{--}4.9) \times 10^{-3}$  [15], depending on models. Experimental uncertainty is approximate 7%.

#### 4. $|V_{td}|$ from radiative penguin processes

Because top quark is heavier than  $B$  mesons,  $V_{td}$  is only accessible through loops in  $B$  decays, either through  $B\text{--}\bar{B}$  mixing, or  $b \rightarrow d\gamma$  penguin diagrams. We measure the branching fractions of  $B \rightarrow \rho(\omega)\gamma$  using the full *BABAR* data set and find  $\mathcal{B}(B^+ \rightarrow \rho^+\gamma) = (1.20^{+0.42}_{-0.37} \pm 0.20) \times 10^{-6}$ ,  $\mathcal{B}(B^0 \rightarrow \rho^0\gamma) = (0.97^{+0.24}_{-0.22} \pm 0.06) \times 10^{-6}$ , and  $\mathcal{B}(B^0 \rightarrow \omega\gamma) = (0.50^{+0.27}_{-0.23} \pm 0.09) \times 10^{-6}$  [16]. The ratio  $|V_{td}|/|V_{ts}|$  can be extracted via  $\frac{\mathcal{B}(B \rightarrow \rho(\omega)\gamma)}{\mathcal{B}(B \rightarrow K^*\gamma)} = S|V_{td}/V_{ts}|^2 (\frac{1-m_{\rho(\omega)}^2/m_B^2}{1-m_{K^*}^2/m_B^2}) \zeta_{\rho(\omega)}^2 \times [1 + \Delta R_{\rho(\omega)}]$ . We find  $|V_{td}|/|V_{ts}| = 0.233^{+0.025+0.022}_{-0.024-0.021}$ . This value is consistent with the result from  $B_s$  mixing  $(0.208 \pm 0.002^{+0.008}_{-0.006})$  [17] from CDF and D0 Collaborations. Even though it is less precise than the results from  $B$  mixing, radiative penguin channels provide a nice independent confirmation.

#### 5. Conclusions

*BABAR* ended its  $\Upsilon(4S)$  program in late 2007. Within a year, *BABAR* has updated many measurements related to the angles and sides of the CKM unitarity triangle. There are no significant contradictions among the measurements under the CKM description of the Standard Model.

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