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LIGHT-FRONT HOLOGRAPHIC QCD*

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The relation between the hadronic short-distance constituent quark and gluon particle limit and the long-range confining domain is yet one of the most challenging aspects of particle physics due to the strong coupling nature of Quantum Chromodynamics, the fundamental theory of the strong interactions. The central question is how one can compute hadronic properties from first principles; i.e., directly from the QCD Lagrangian. The most successful theoretical approach thus far has been to quantize QCD on discrete lattices in Euclidean space-time. [1] Lattice numerical results follow from computation of frame-dependent moments of distributions in Euclidean space and dynamical observables in Minkowski space-time, such as the time-like hadronic form factors, are not amenable to Euclidean lattice computations. The Dyson-Schwinger methods have led to many important insights, such as the infrared fixed point behavior of the strong coupling constant, [2] but in practice, the analyses are limited to ladder approximation in Landau gauge. Baryon spectroscopy and the excitation dynamics of nucleon resonances encoded in the nucleon transition form factors can provide fundamental insight into the strong-coupling dynamics of QCD. New theoretical tools are thus of primary interest for the interpretation of the results expected at the new mass scale and kinematic regions accessible to the JLab 12 GeV Upgrade Project.

The AdS/CFT correspondence between gravity or string theory on a higher-dimensional anti–de Sitter (AdS) space and conformal field theories in physical space-time [3] has led to a semiclassical approximation for strongly-coupled QCD, which provides physical in-

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sights into its nonperturbative dynamics. The correspondence is holographic in the sense that it determines a duality between theories in different number of space-time dimensions. This geometric approach leads in fact to a simple analytical and phenomenologically compelling nonperturbative approximation to the full light-front QCD Hamiltonian – "Light-Front Holography". [4] Light-Front Holography is in fact one of the most remarkable features of the AdS/CFT correspondence. [3] The Hamiltonian equation of motion in the light-front (LF) is frame independent and has a structure similar to eigenmode equations in AdS space. This makes a direct connection of QCD with AdS/CFT methods possible. [4] Remarkably, the AdS equations correspond to the kinetic energy terms of the partons inside a hadron, whereas the interaction terms build confinement and correspond to the truncation of AdS space in an effective dual gravity approximation. [4]

One can also study the gauge/gravity duality starting from the bound-state structure of hadrons in QCD quantized in the light-front. The LF Lorentz-invariant Hamiltonian equation for the relativistic bound-state system is $P_\mu P^\mu |\psi(P)\rangle = (P^+P^- - \mathbf{P}_\perp^2)\,|\psi(P)\rangle = M^2|\psi(P)\rangle$, $P^\pm = P^0 \pm P^3$, where the LF time evolution operator P^- is determined canonically from the QCD Lagrangian. [5] To a first semiclassical approximation, where quantum loops and quark masses are not included, this leads to a LF Hamiltonian equation which describes the bound-state dynamics of light hadrons in terms of an invariant impact variable ζ [4] which measures the separation of the partons within the hadron at equal light-front time $\tau = x^0 + x^3$. [6] This allows us to identify the holographic variable z in AdS space with an impact variable ζ. [4] The resulting Lorentz-invariant Schrödinger equation for general spin incorporates color confinement and is systematically improvable.

Light-front holographic methods were originally introduced [7, 8] by matching the electromagnetic current matrix elements in AdS space [9] with the corresponding expression using LF theory in physical space time. It was also shown that one obtains identical holographic mapping using the matrix elements of the energy-momentum tensor [10] by perturbing the AdS metric around its static solution. [11]

A gravity dual to QCD is not known, but the mechanisms of confinement can be incorporated in the gauge/gravity correspondence by modifying the AdS geometry in the large infrared (IR) domain $z \sim 1/\Lambda_{\rm QCD}$, which also sets the scale of the strong interactions. [12] In this simplified approach we consider the propagation of hadronic modes in a fixed effective gravitational background asymptotic to AdS space, which encodes salient properties of

the QCD dual theory, such as the ultraviolet (UV) conformal limit at the AdS boundary, as well as modifications of the background geometry in the large z IR region to describe confinement. The modified theory generates the point-like hard behavior expected from QCD, [13, 14] instead of the soft behavior characteristic of extended objects. [12]

Nucleon Form Factors

In the higher dimensional gravity theory, hadronic amplitudes for the transition $A \to B$ correspond to the coupling of an external electromagnetic (EM) field $A^M(x,z)$ propagating in AdS space with a fermionic mode $\Psi_P(x,z)$ given by the left-hand side of the equation below

$$\int d^4x\, dz\, \sqrt{g}\, \bar{\Psi}_{B,P'}(x,z)\, e^A_M\, \Gamma_A\, A^M_q(x,z) \Psi_{A,P}(x,z) \sim (2\pi)^4 \delta^4\,(P' - P - q)\, \epsilon_\mu \langle \psi_B(P'), \sigma' | J^\mu | \psi_A(P), \sigma \rangle, \quad (1)$$

where the coordinates of AdS_5 are the Minkowski coordinates x^μ and z labeled $x^M = (x^\mu, z)$, with $M, N = 1, \cdots 5$, g is the determinant of the metric tensor and e^A_M is the vielbein with tangent indices $A, B = 1, \cdots, 5$. The expression on the right-hand side represents the QCD EM transition amplitude in physical space-time. It is the EM matrix element of the quark current $J^\mu = e_q \bar{q} \gamma^\mu q$, and represents a local coupling to pointlike constituents. Can the transition amplitudes be related for arbitrary values of the momentum transfer q? How can we recover hard pointlike scattering at large q from the soft collision of extended objects? [9] Although the expressions for the transition amplitudes look very different, one can show that a precise mapping of the J^+ elements can be carried out at fixed LF time, providing an exact correspondence between the holographic variable z and the LF impact variable ζ in ordinary space-time. [7]

A particularly interesting model is the "soft wall" model of Ref. [15], since it leads to linear Regge trajectories consistent with the light-quark hadron spectroscopy and avoids the ambiguities in the choice of boundary conditions at the infrared wall. In this case the effective potential takes the form of a harmonic oscillator confining potential $\kappa^4 z^2$. For a hadronic state with twist $\tau = N + L$ (N is the number of components and L the internal orbital angular momentum) the elastic form factor is expressed as a $\tau - 1$ product of poles

along the vector meson Regge radial trajectory ($Q^2 = -q^2 > 0$) [8]

$$F(Q^2) = \frac{1}{\left(1+\frac{Q^2}{M_\rho^2}\right)\left(1+\frac{Q^2}{M_{\rho'}^2}\right)\cdots\left(1+\frac{Q^2}{M_{\rho^{\tau-2}}^2}\right)}, \tag{2}$$

where $M_{\rho_n}^2 \to 4\kappa^2(n+1/2)$. For a pion, for example, the lowest Fock state – the valence state – is a twist-2 state, and thus the form factor is the well known monopole form. The remarkable analytical form of Eq. (2), expressed in terms of the ρ vector meson mass and its radial excitations, incorporates the correct scaling behavior from the constituent's hard scattering with the photon [13, 14] and the mass gap from confinement.

Computing Nucleon Form Factors in Light-Front Holographic QCD

As an illustrative example we consider in this section the spin non-flip elastic proton form factor and the form factor for the $\gamma^* p \to N(1440)P_{11}$ transition measured recently at JLab. In order to compute the separate features of the proton an neutron form factors one needs to incorporate the spin-flavor structure of the nucleons, properties which are absent in the usual models of the gauge/gravity correspondence. This can be readily included in AdS/QCD by weighting the different Fock-state components by the charges and spin-projections of the quark constituents; e.g., as given by the $SU(6)$ spin-flavor symmetry.

Using the $SU(6)$ spin-flavor symmetry the expression for the spin-non flip proton form factors for the transition $n, L \to n'L$ is [16]

$$F_{1\,n,L\to n',L}^p(Q^2) = R^4 \int \frac{dz}{z^4}\, \Psi_+^{n',L}(z) V(Q,z) \Psi_+^{n,L}(z), \tag{3}$$

where we have factored out the plane wave dependence of the AdS fields

$$\Psi_+(z) = \frac{\kappa^{2+L}}{R^2}\sqrt{\frac{2n!}{(n+L+1)!}}\, z^{7/2+L} L_n^{L+1}\left(\kappa^2 z^2\right) e^{-\kappa^2 z^2/2}. \tag{4}$$

The bulk-to-boundary propagator $V(Q,z)$ has the integral representation [17]

$$V(Q,z) = \kappa^2 z^2 \int_0^1 \frac{dx}{(1-x)^2}\, x^{\frac{Q^2}{4\kappa^2}} e^{-\kappa^2 z^2 x/(1-x)}, \tag{5}$$

with $V(Q=0,z) = V(Q,z=0) = 1$. The orthonormality of the Laguerre polynomials in (4) implies that the nucleon form factor at $Q^2 = 0$ is one if $n = n'$ and zero otherwise. Using (5) in (3) we find

$$F_1^p(Q^2) = \frac{1}{\left(1+\frac{Q^2}{M_\rho^2}\right)\left(1+\frac{Q^2}{M_{\rho'}^2}\right)}, \tag{6}$$

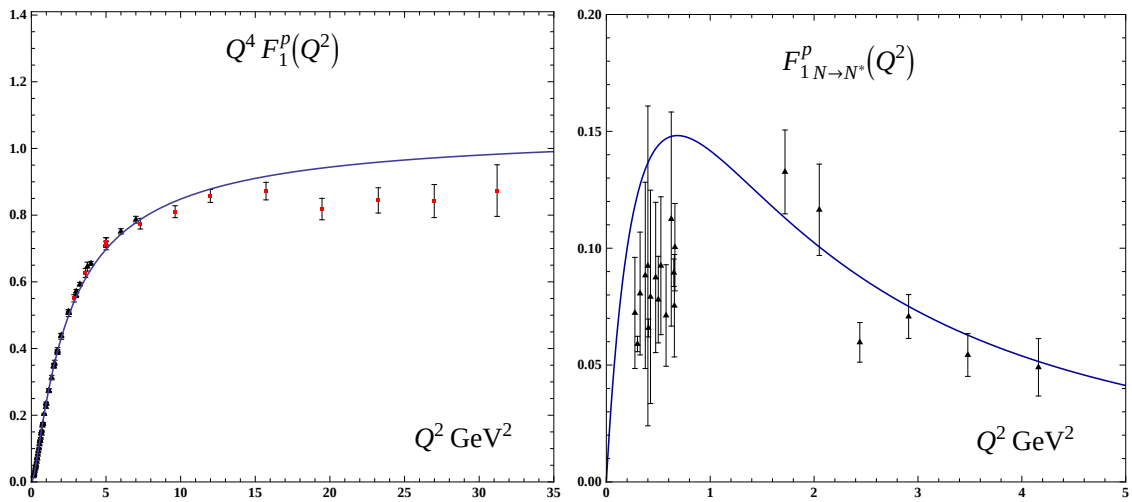

FIG. 1: Dirac proton form factors in light-front holographic QCD. Left: scaling of proton elastic form factor $Q^4 F_1^p(Q^2)$. Right: proton transition form factor $F^p_{1\,N\to N^*}(Q^2)$ for the $\gamma^* p \to N(1440)P_{11}$ transition. Data compilation from Diehl [18] (left) and CLAS π and 2π electroproduction data [19–22] (right).

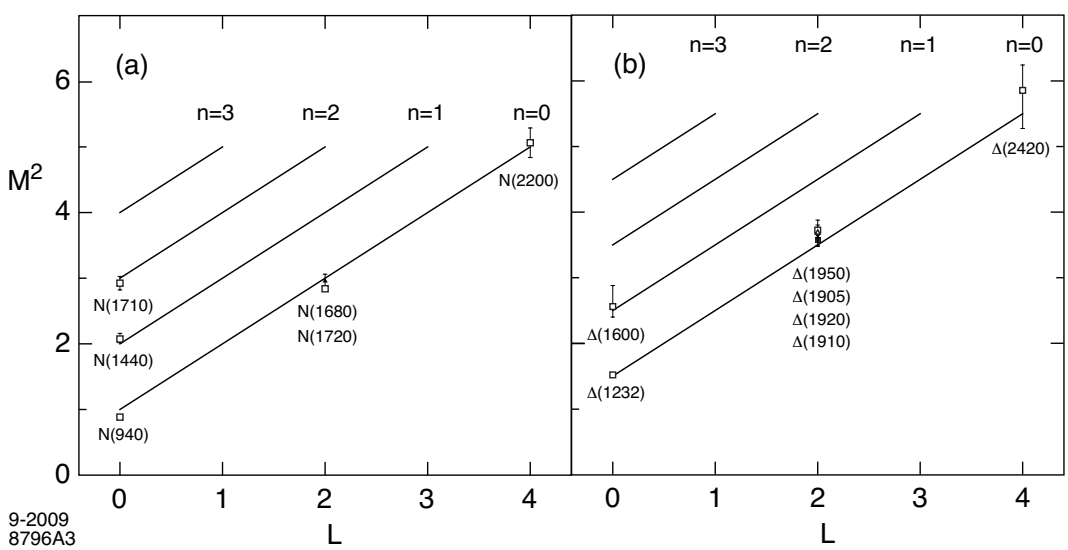

FIG. 2: Positive parity Regge trajectories for the N and Δ baryon families for $\kappa = 0.5$ GeV. Only confirmed PDG [24] states are shown.

for the elastic proton Dirac form factor and

$$F^p_{1\,N\to N^*}(Q^2) = \frac{\sqrt{2}}{3} \frac{\frac{Q^2}{M_\rho^2}}{\left(1+\frac{Q^2}{M_\rho^2}\right)\left(1+\frac{Q^2}{M_{\rho'}^2}\right)\left(1+\frac{Q^2}{M_{\rho''}^2}\right)}, \tag{7}$$

for the EM spin non-flip proton to Roper transition form factor. The results (6) and (7), compared with available data in Fig. 1, correspond to the valence approximation. The transition form factor (7) is expressed in terms of the mass of the ρ vector meson and its first two radial excited states, with no additional parameters. The results in Fig. 1 are in good agreement with experimental data. The transition form factor to the $N(1440)P_{11}$ state shown in Fig. 1 corresponds to the first radial excitation of the three-quark ground state of the nucleon. In fact, the Roper resonance $N(1440)P_{11}$ and the $N(1710)P_{11}$ are well accounted in the light-front holographic framework as the first and second radial states of the nucleon family, likewise the $\Delta(1600)P_{33}$ corresponds to the first radial excitation of the Δ family as shown in Fig. 2 for the positive-parity light-baryons. [23] In the case of massless quarks, the nucleon eigenstates have Fock components with different orbital angular momentum, $L = 0$ and $L = 1$, but with equal probability. In effect, in AdS/QCD the nucleons angular momentum is carried by quark orbital angular momentum since soft gluons do not appear as quanta in the proton.

Light-front holographic QCD methods have also been used to obtain general parton distributions (GPDs) in Ref. [25], and a study of the EM nucleon to Δ transition form factors has been carried out in the framework of the Sakai and Sugimoto model in Ref. [26]. It is certainly worth to extend the simple computations described here and perform a systematic study of the different transition form factors measured at JLab. This study will help to discriminate among models and compare with the new results expected from the JLab 12 GeV Upgrade Project, in particular at photon virtualities $Q^2 > 5$ GeV^2, which correspond to the experimental coverage of the CLAS12 detector.

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