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A New Green's Function for the Wake Potential Calculation of the SLAC S-band Constant Gradient Accelerating Section

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Abstract

The behavior of the longitudinal wake fields excited by a very short bunch in the SLAC S-band constant gradient accelerating structures has been studied. Wake potential calculations were performed for a bunch length of 10 microns using the author's code to obtain a numerical solution of Maxwell's equations in the time domain. We have calculated six accelerating sections in the series (60-ft) to find the stationary solution. While analyzing the computational results we have found a new formula for the Green's function. Wake potentials, which are calculated using this Green's function are in amazingly good agreement with numerical results over a wide range of bunch lengths. The Green's function simplifies the wake potential calculations and can be easily incorporated into the tracking codes. This is very useful for beam dynamics studies of the linear accelerators of LCLS and FACET.

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1 Introduction

Wake fields, excited by a very short bunch in an accelerating section may play an important role in the beam dynamics of linear accelerators. They are responsible for the energy loss of a bunch, the bunch energy spread and, consequently, the adiabatic bunch emittance growth in dispersive areas. For the SLAC S-band accelerating structure we have found that there are no wake field calculations for the actual geometry of a constant gradient accelerating section. We have only found wake field calculations that have been performed for a periodic model that assumes some average geometric parameters for a cell [1] - [2]. In reality the SLAC accelerating structure is a quasi-periodic structure, which consists of a sequence of 10 ft long accelerating sections. The accelerating sections repeat each other , but the cells inside each section are different. The constant accelerating field is obtained by tapering the cross-sectional dimensions of the cells. In order to do more precise wake field calculations we have developed an azimuthal model of an accelerating section based on the dimensions of a regular 10 ft constant gradient disc-loaded section [3]. A section consists of 86 cells, formed by 85 disks and two coupler flanges. Each disk in a section has a different size hole (the difference over the etire section is about 36 %). The transverse size of a cell changes less, only by 2 %. However the coupling cells have 7 % and 4 % smaller radii.

We use the code NOVO [4] for calculating wake field potentials of a chain of accelerating sections for different bunch lengths. This is the only code, which can do reliable wake field calculations for a very short bunch. It uses the finite-difference method with improved characteristic of the dispersion curve in the region of minimum critical wave length. This code has been used for calculating the wake field potentials of the LCLS beam line elements [5]), where comparison of the measured results showed very good agreement with the numerical calculations.

It is worthwhile noting that the ratio of hole radius over the bunch length, the relevant parameter for wake field studies, is more than 1300 for the LCLS SASE FEL parameters.

2 Wake field potentials and Green's functions

Wake fields, which are excited in an accelerator structure by a relativistic bunch are usually described by the wake field potential $W(s)$, which is an integral of a longitudinal force along the bunch trajectory. An example of a wake potential of a Gaussian bunch is shown in Fig.1. Effectively the

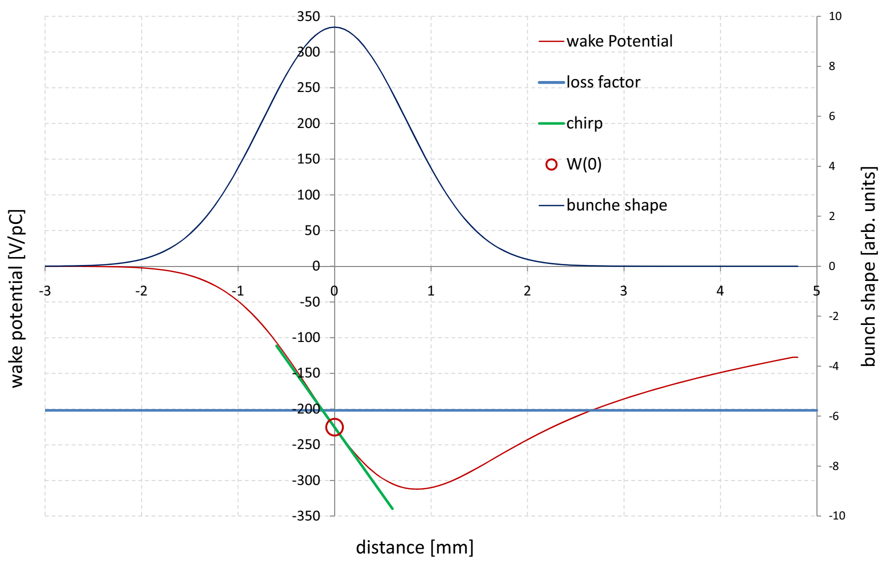

Figure 1: Wake potential of the SLAC accelerating section and the bunch shape of a 0.7 mm bunch. The blue line shows the loss factor, which is equal to the averaged value. The red circle and a green line show the value of the wake potential and it's derivative (chirp) at the center of a bunch.

wake field potential $W(s)$ describes the energy loss of a particle within a longitudinal position s in a bunch. A bunch may have an arbitrary charge density and the wake field potential may be different. However the wake potential of a point-like charge may simplify the calculation problem. Once the wake field potential $G(s)$ of a point bunch is known, then the wake field potential of an arbitrary charge $q(s)$ can be obtained from a convolution integral

$$W(s) = \frac{1}{Q}\int_{-\infty}^{s} G(s-s')q(s')ds' \tag{1}$$

For this reason, the wake field potential of a point charge is called the Green's function. In this formula, we normalize the wake potential by a total bunch charge Q

$$Q = \int_{-\infty}^{+\infty} q(s)ds \tag{2}$$

In this way we can measure wake potentials in Volts per a unit charge. Usually the unit charge is 1 pC.

The loss factor k, which represents the total energy loss of a bunch, is expressed in terms of the wake potential W(s) by

$$k = \frac{1}{Q}\int_{-\infty}^{+\infty} W(s)q(s)ds \tag{3}$$

We also normalize the loss factor by the total bunch charge Q. The loss factor also represents the averaged value of the wake potential along the bunch. This average level is shown by a blue line as a constant function at the plot of the wake potential (Fig.1).

We will also introduce a loss frequency integral $K(\omega)$, which shows the correspondent frequency distribution of the loss factor for a given bunch length

$$K(\omega) = Re\int_{0}^{\omega} W(\omega_0)q(-\omega_0)d\omega_0 \tag{4}$$

To calculate the loss frequency integral we need to Fourier transform the wake potential and the bunch charge distribution

$$W(\omega) = \frac{1}{2\pi}\int_{-\infty}^{+\infty} W(s)\exp\left(-i\frac{\omega}{c}s\right)ds \tag{5}$$

$$q(\omega) = \frac{1}{2\pi}\int_{-\infty}^{+\infty} q(s)\exp\left(-i\frac{\omega}{c}s\right)ds \tag{6}$$

Naturally that

$$K(\omega = \infty) = k \tag{7}$$

We will also calculate an energy spread due to the wake fields, which may be important if a bunch transverses a section without RF fields

$$(\delta E)^2 = \frac{1}{Q}\int_{-\infty}^{+\infty} (W(s) - k)^2 q(s)ds \tag{8}$$

Other important parameters of the wake potential are: the value of the wake potential at the center of a bunch and its derivative (slope, or chirp). These parameters are also shown in Fig.1. For a derivative it is convenient to use a parameter which is equal to the wake potential derivative multiplied by a bunch length:

$$\delta W = \frac{\partial W(0)}{\partial s} * \sigma \tag{9}$$

3 An accelerating section

The geometry of a section is shown in Fig. 2. We use Table 6-6 of reference [3] for the dimensions of a regular 10-feet constant gradient disc-loaded section. The section consists of 86 cells, formed by 85 disks and two coupler flanges. Each disk in a section has a different hole radius staring at 13.11 mm and ending with 9.62 mm (the difference is approximately 36 %). Radiuses of the input and the outgoing pipes are the same and equal to 9.55 mm . The transverse cell radius changes from 41.73 mm to 40.91 mm. The coupling cells have radiuses of 38.62 and 39.76 mm. This type of accelerating structure is $2\pi/3$ and has a period of 35 mm. The shape of an accelerating section

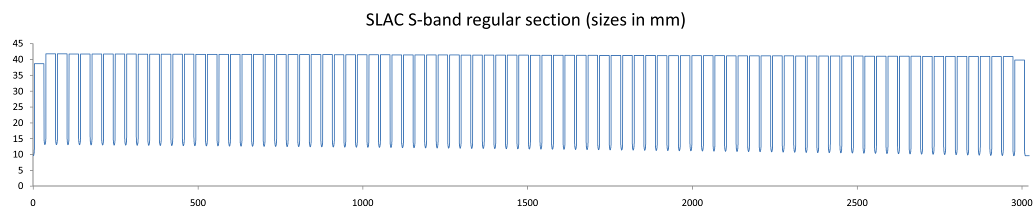

Figure 2: Geometry of the SLAC accelerating section. The cell length is equal to the one third of the RF wavelength and the total length of the section, including the coupler cells is 28 and 2/3 wavelengths (86 cells).

disk can be seen at the plot at Fig. 3, where a snapshot of the electric field lines of the wake field excited by a 0.5 mm bunch is shown. A bunch has passed the two first disks of a section. The transverse size of the first coupler cell is smaller than the next cell. The blue lines show field lines with the longitudinal projection opposite to the bunch velocity. These lines describe the decelerating forces. The green lines have collinear longitudinal projections with a bunch velocity and describe the accelerating forces. It can be seen that the structure of the wake fields is very complicated.

4 Long range wake field potentials

We start with a calculation of the long range wake field potentials to check the geometry of the model for the main resonant frequency. We have calculated a 4 m long wake potential for a 1 mm bunch. It is shown at Fig. 4. Then we calculated the spectrum using a Fourier transform of this poten-

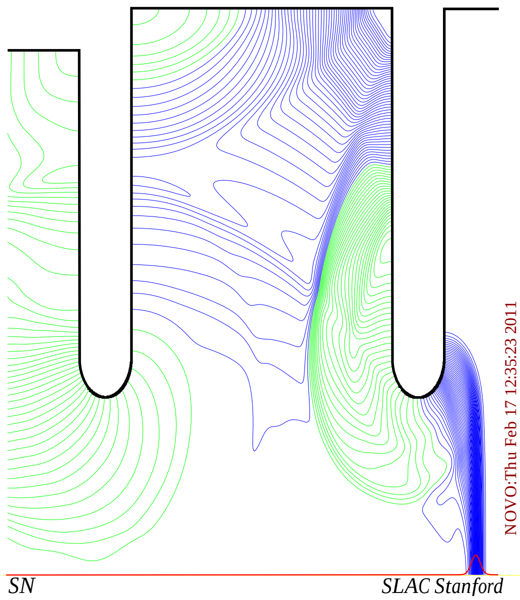

Figure 3: Electric field lines of the wake field excited by a 0.5 mm bunch in a SLAC accelerating section. A bunch has passed the two first disks of a section. The blue lines show field lines with the longitudinal projection opposite to the bunch velocity. These lines describe the deceleration forces and the green lines describe the acceleration forces.

tial. The spectrum is shown at Fig. 5. We found that the main frequency is around 2860 MHz, which corresponds well to the design value. Other resonant frequencies are: 5930 MHz, 6600 MHz and 8650 MHz. The loss frequency integral is also shown at Fig. 5. It has a staircase shape. Each step corresponds to a resonant mode. The long range wake potential can also be used to calculate the wake potential of a train of bunches. For a short length of time the attenuation of the fields can be ignored, we can then take a sum of shifted by a bunch spacing single bunch wake potentials for a wake potential of a bunch train. As an example we present a plot of the wake potential of a train of 10 bunches with a bunch spacing equal to one wavelength in Fig. 6. Using this wake potential we can calculate the energy loss of each beam or what is called call "beam loading" and the wake potential derivative (chirp). These parameters for the first ten bunches are shown in Fig. 7. The energy loss increases almost proportionally to the number of bunches and the chirp

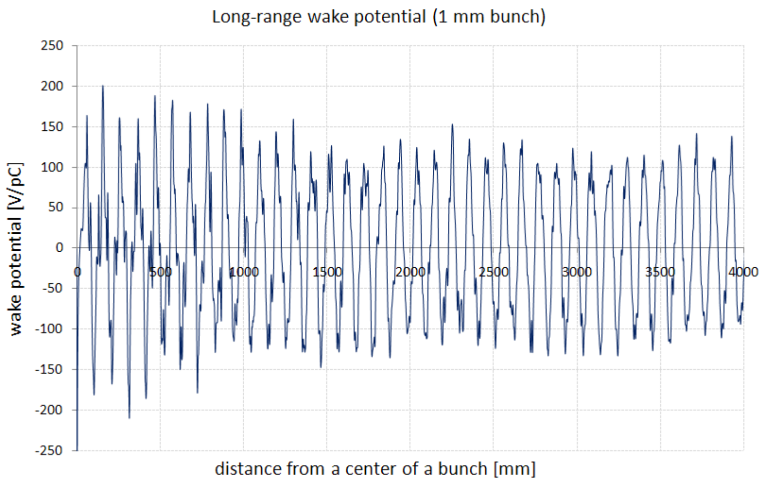

Figure 4: Long-range (4 m long) wake potential of the SLAC accelerating section, excited by a 1 mm bunch

decreases slowly with number of bunches. If the bunch charge is 250 pC then we need to compensate 400 keV energy loss for the last bunch. This can be done by shifting the RF phase for each bunch. Can we compensate the chirp variation? Probably yes. In this case we need to use at least two klystrons with different phases.

5 Wake potentials of a very short bunch

As we mentioned before, the SLAC accelerating structure is a quasi-periodic structure. Constant gradient sections are separated from each other by a cell distance of 35 mm. The diameter of a connecting pipe is 19 mm. The crucial point in the calculation of the radiation fields for any periodic or quasi-periodic structure is an accurate description of the interference of the waves produced in different cells. The interference pattern of a field (stationary solution) can be built up only if the structure is long enough. On the other hand the field, which was excited at the edge of any disks has to capture a bunch in order to take part of the kinetic energy of the bunch. The needed distance depends upon the bunch length σ. This distance can be estimated from a simple geometrical model of the radiation process. At first the bunch "head" field touches the edge of the disk at the radius a and then, at some

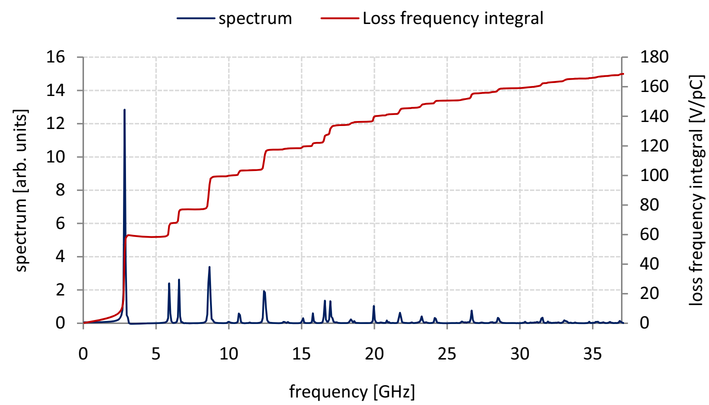

Figure 5: Spectrum of the SLAC accelerating section.

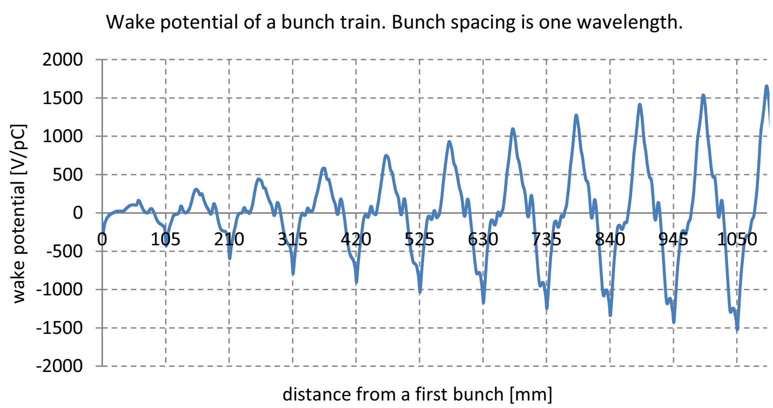

Figure 6: Wake potential of a train of bunches.

distance D, the field from the charge and current excited at of this part of the disk will catch the "tail" of the bunch

$$\sqrt{a^2 + D^2} - D = \sigma \tag{10}$$

Using the fact that $a << D$ we get

$$D \geq \frac{a^2}{2\sigma} \tag{11}$$

Shorter bunches need a longer distance. The maximum radius of a hole in a disk is 13 mm, so a 10 micron bunch will need to travel a distance of 8.5 meters to find out that it has radiated an electromagnetic field.

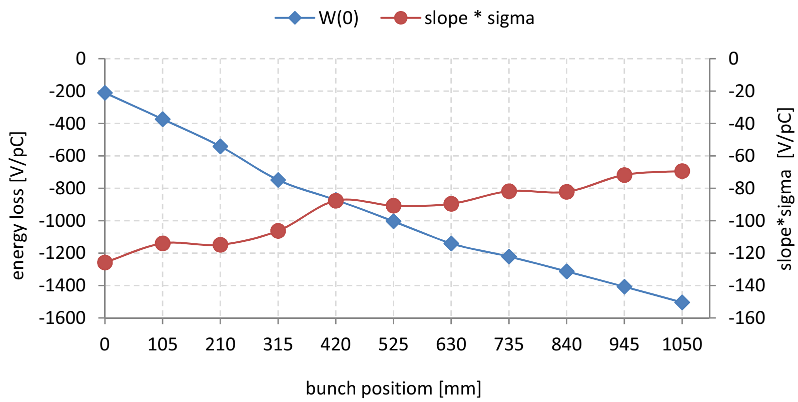

Figure 7: Wake potential and it's derivate at the center of a bunch for a train of 10 bunches with one wavelength bunch spacing.

To fulfill condition (11) we need to calculate at least 3 accelerating sections. The code gives the ability to calculate wake fields in a train of sections, which start with an infinite incoming pipe and ends with an infinite outgoing pipe. The code also allows us to calculate wake potentials in one section of a train. in order to get a stationary solution sooner, for a fewer number of sections we have calculated the wake potential gained only in the last section of a train. We have calculated trains of 1,2,3,4,5 and six sections. A strong modification of the wake fields along the train of accelerating sections is clearly seen at Fig. 8 where wake potentials for a train going through 1,2,3 and 4 sections are shown. The change in amplitude and shape with increasing number of cells can be clearly noted. The peak value of the wake is decreasing. After a larger number of cells, a new maxima appear at the tail of the bunch, which slowly moves toward the bunch center. In particular, the wakes induced by the bunch, as it proceeds down the successive sections, decrease in amplitude and become more linear around the bunch center, with a profile very close to the integral of a charge density.

Here is a simple model to explain this effect. Initially the electric field of a bunch is almost radial. After passing several disks this field will get additional fields from the charges and currents excited at the previous disks. Effectively the field at the edge of a current disk will be the field of a bunch of an "extended length". The radiation energy will be smaller than in the previous cells. After some distance the field at the disk's edge will reach a "steady state regime", as if it induced by an equivalent longer bunch. So even for a point-like bunch the radiation will be equivalent to radiation of a finite length bunch.

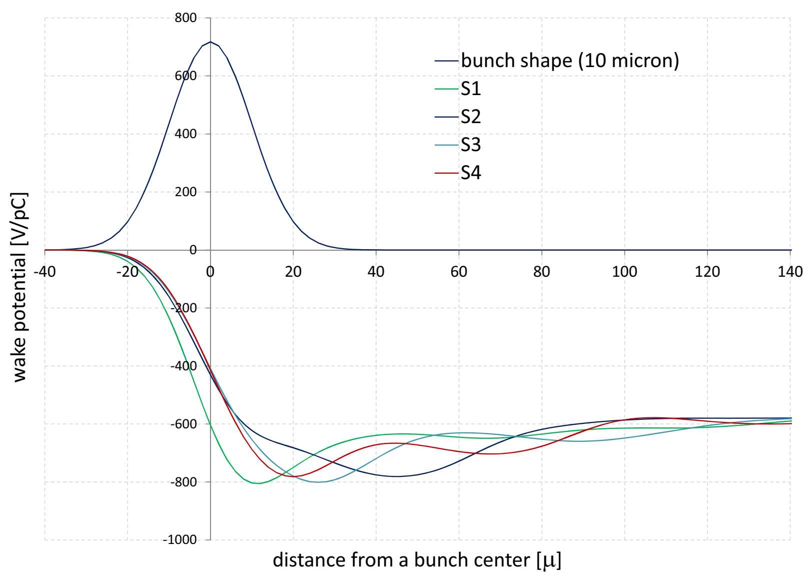

Figure 8: Consecutive wake potentials of a 10 micron in the last section of a train. The blue curve shows a bunch charge density distribution.

The loss factor, which is shown in Fig.9 as a function of the number of sections, also decreases with this number, approaching an asymptotic value after the 4th section.

We also did calculations for longer bunches. The loss factor for one section and the asymptotic loss factor as a function of the bunch length are shown in Fig. 10. There is no great difference between the loss factor for a single section and the asymptotic value for a bunch longer than 150 micron.

6 A new Green's function

The analysis of the calculated results led us think about how to make it easier to use this data. The best way is to find an appropriate Green's function $G(s)$, which can be the basis for the wake potential calculations using equation (1). Several years ago we found a Green's function for a section of TESLA cavities [6]. The TESLA accelerating structure is also a quasi-periodic structure and it has a very complicated cavity shape. We

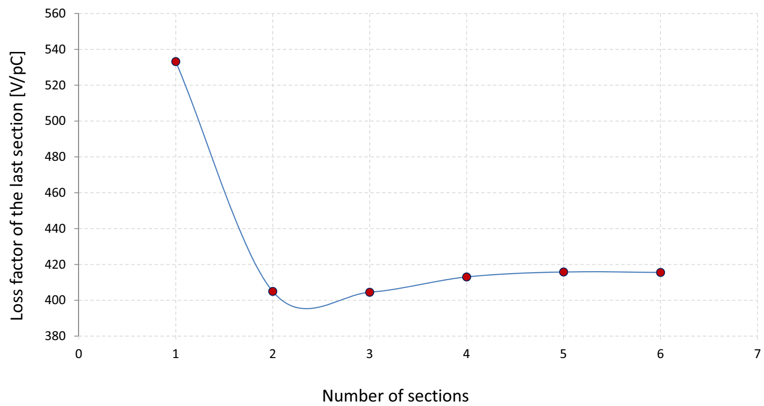

Figure 9: Loss factor of a 10 micron bunch in the last section of a train vs number of sections in a train

used the analytical frequency domain approximation given in reference [7]. The Green's function for the TESLA accelerating structure contains several parameters, which were optimized to get good agreement with numerical results over a give range of the bunch length (0.05-0.7 mm)

We tried to use same approach for the SLAC accelerating structure. We verified all parameters and also found good enough agreement with numerical calculations for the almost same bunch length. However, now we are interested in a wider range of bunch length. LCLS and FACET use bunches that range from 2 mm down to several micron, and the "TESLA" formula does not produce a good approximation in this range of the bunch length. This led us to reanalyze the computation results where we subsequently found another formula for the Green's function:

$$w_p(s) = \frac{Z_0 c}{\pi a^2}\left(1 + \frac{s}{4s_0}\right)\exp(-\sqrt{\frac{s}{s_0}}) \tag{12}$$

The main difference from the "TESLA" formula is the expression in the brackets $\left(1 + \frac{s}{4s_0}\right)$, which softens the strong exponential damping term.

In the formula (12) the wake potential function $w_p(s)$ describes the wake potential for an unit length of the accelerating structure. The optimized parameters for the SLAC structure are: a=0.01068 m, s_0=0.435 mm. The amplitude of the Green's function is then

$$\frac{Z_0 c}{\pi a^2} = \frac{120c}{a^2} = 3.154 \cdot 10^{14}\ V/C/m = 315.4\ V/pC/m \tag{13}$$

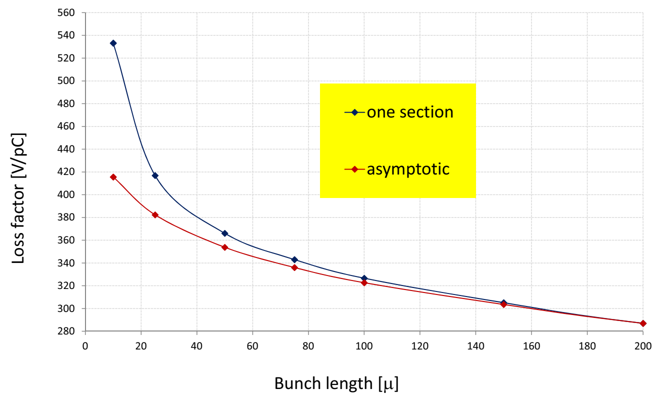

Figure 10: The loss factor for one section and the asymptotic loss factor as a function of the bunch length.

It is interesting to note that the parameter a is shorter than the average radius of the disk holes: 11.6 mm. So the maximum value of the wake potential in our calculations is higher than the results of [1] - [2] by 18 %.

The comparison of the numerically calculated loss factor and the loss factor calculated analytically using this new Green's function is shown on Fig.11. We found we have a very good agreement over the entire range of the bunch length. We also made a comparison and got good agreement for

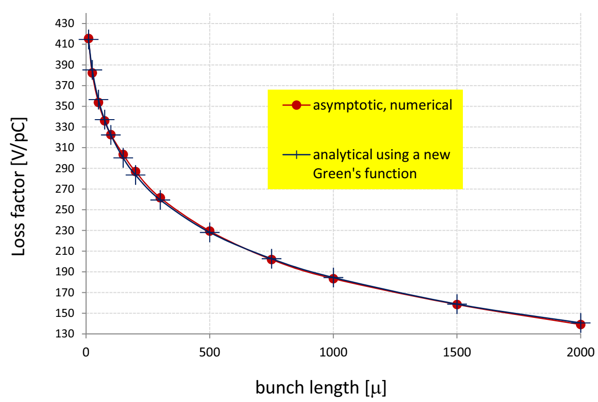

Figure 11: Comparison of the loss factor of the SLAC accelerating section calculated numerically (red circles) and analytically using the new Green's function (blue crosses).

the energy spread and the chirp. The comparison of the wake potentials (numerical and analytical using the new Green's function) for two bunch lengths is shown in Fig.12. The left plot shows the comparison for a 10 μ bunch and right plot shows comparison for a 2 mm bunch. We see that in

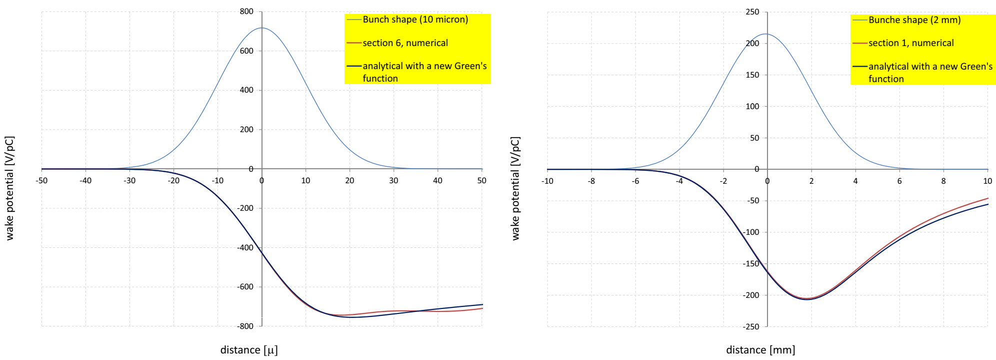

Figure 12: Comparison of the wake potentials, calculated numerically and analytically using the new Green's function for two bunch lengths. The left plot shows a comparison for a 10 μ bunch and the right plot shows the comparison for a 2 mm bunch.

the range of the length of the bunch ($\pm\sigma$) the converged parts of the wake potential are in good agreement and in the range after the bunch the wake potentials have damping oscillations around the evaluated curve. The new Green's function works very well!

7 Application to LCLS and FACET

As an application that could be useful for LCLS and FACET linacs we show the wake potential and it's derivative (see (9)) as a function of the bunch length. Fig.13 shows these values in one accelerating section.

At FACET, the bunch energy loss was measured in 1/3 of the SLAC Linac (324 sections) as a function of the bunch compressor (BC in Sector 10)) phase . For the nominal FACET parameters (Q=3.2 nC) the energy change appeared to be 120 MeV [8]. However, this energy change is not really the energy loss of a bunch after the compressor. It is the difference between the energy loss of a bunch after the bunch compressor and the energy loss of a bunch before the bunch compressor. Definitely a bunch has different bunch

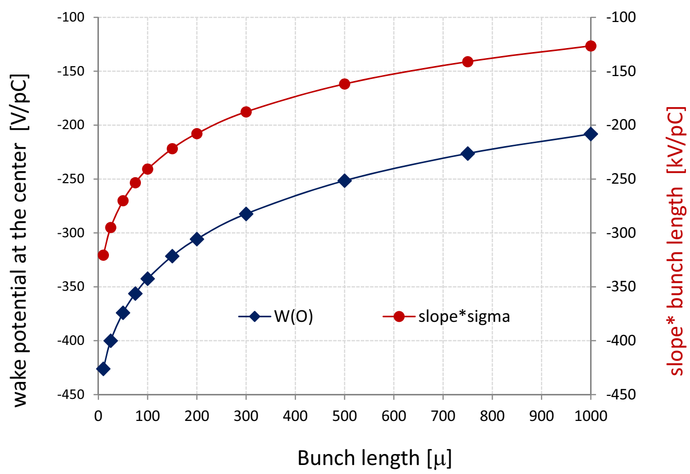

Figure 13: Wake potential at and it's derivate the center of a bunch

length before and after. Using our results for the loss factor in the SLAC accelerating section we can calculate a ratio between the bunch length of a bunch before and after the bunch compressor. This ratio is shown in Fig.14. We also calculated this ratio for an energy change of 140 MeV. According to these results a bunch length of 50 micron after the bunch compressor can be achieved only if the bunch length before the bunch compressor is approximately 0.5 mm, significantly smaller than the design value of 1.5 mm.

8 Conclusion

We have improved wake field calculations for the very short bunches in the SLAC constant gradient accelerating sections. These precise calculations shows that the maximum amplitude of the fields is 18% larger than was shown in the previous calcultions.

We have found a new Green's function, which describes very well the wake potentials of the SLAC accelerating section over a large range of short bunch lengths. This Green's function simplifies the wake potential calculations and can be easily incorporated into the tracking codes. This is important for beam dynamics study of the linear accelerators of LCLS and FACET.

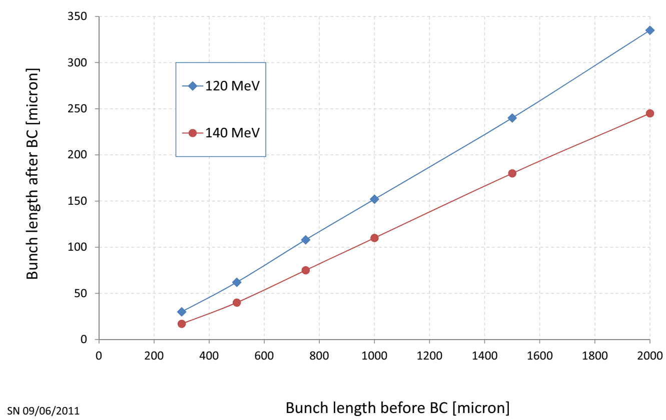

Figure 14: The length of a bunch after the bunch compressor as a function of the bunch length before the bunch compressor for the energy change of 120 and 140 MeV, based on my calculation of the wake energy loss in the SLAC accelerating section.

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