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A Compact Ring Design with Tunable Momentum Compaction*

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Abstract

A storage ring with tunable momentum compaction has the advantage of allowing different RMS bunch lengths to be achieved with similar RF capacity. This is potentially useful for many applications, such as a linear collider damping ring and pre-damping ring where the injected beam has a large energy spread and a large transverse emittance. A tunable bunch length also makes it easier to manipulate single bunch instabilities in commissioning and fine tuning. This paper presents a compact ring design based on a supercell that achieves momentum compaction tunability while maintaining a large dynamic aperture.

OVERVIEW

Storage rings based on alternating gradient or strong focusing are widely used, and several focusing configurations having been invented and some applied in real ring construction and operation. These include separated-function FODO cell, combined function FD-cell, double-bend achromat (DBA), triple-bend achromat (TBA) [1] and theoretical minimum emittance (TME) [2] lattices. In an electron storage ring, an electron will lose energy due to synchrotron radiation in the bending elements. Since synchrotron radiation energy loss is compensated by the RF cavities in the ring, there is a damping effect on the synchrotron oscillation with a corresponding damping time. The synchrotron radiation energy loss is in the form of randomly emitted photons, and the random quantum excitations, together with the previously mentioned synchrotron radiation damping effect, result in the single-particle equilibrium energy spread and emittance. The equilibrium transverse emittance is achieved when radiation damping is equal to the quantum excitation. The equilibrium transverse emittance in terms of the radiation integrals is given by the following expression (assuming there are only bending magnets in horizontal plane).

$$\epsilon = C_q \frac{\gamma^2 I_5}{J_u I_2} \tag{1}$$

where $C_q = 3.83 \times 10^{-13} m$, γ denotes the relativistic factor, and I_2 and I_5 are the following radiation integrals: $I_2[m^{-1}] = \oint \left(\frac{1}{\rho_x^2}\right) ds$ and $I_5[m^{-1}] = \oint \left(\frac{H_x}{\rho_x^3}\right) ds$, where $H_x = \beta_x D_x'^2 + 2\alpha_x D_x D_x' + \gamma_x D_x^2$ depends on the bending and focusing structure, ρ_x denotes the bending curvature in the horizontal plane.

From observing the three formulae above, one notes that in order to get a smaller equilibrium emittance at a given energy, ρ_x needs to be large and H_x needs to be small. A large ρ_x requirement translates into more (in number) and longer dipole magnets, while a reducing H_x depends on the quadrupole focusing scheme, such as DBA, TBA and TME.

In general, the requirement of achieving a low emittance means employment of weaker dipole (smaller dispersion) and stronger quadrupole (focusing) magnets. The betatron oscillation tune variation of off-momentum particles with respect to the on-momentum particles are characterized by the chromaticity $\Delta\nu = \xi(p)\Delta p/p_0$. The natural chromaticity from the linear optics is always negative, $\xi_0 = -\frac{1}{4\pi}\oint K\beta ds$. Sextupoles are second order magnetic elements (with respect to particle's amplitude) and can be placed at dispersive regions where the particle offsets are proportional to their momentum deviation, providing extra focusing for off-momentum particles. One observes that there is a large natural chromaticity associated with a strong focusing lattice (large K), while β tends to be smaller, as does dispersion D. All these make the chromaticity correction sextupoles stronger, which in turn decreases the dynamic aperture, which is a measure of the maximum stable phase space amplitude in the transverse planes. An adequate dynamic aperture is essential in accepting the injected beam, which usually has a large emittance and energy spread, as well as in achieving a long beam lifetime for stored beam in the ring.

There are several different approaches to evaluating and studying the mechanism of dynamic aperture. One such approach using matrix formulae was developed by K. Brown and applied to the design of second order and even higher order achromats [3]. Here we briefly review the main conclusions which are described in [3]. Any optics with n (larger than one) identical cells gives a first order achromat if the betatron phase advance equals a multipole of 2π in both transverse planes (first order transport matrix equaling unity, I). When the cell number n does not equal one or three, the second order geometric aberrations are also canceled. Another conclusion is, of all the second order chromatic aberrations, only two are independent, which can be corrected with two families of sextupoles in each transverse plane (for $n > 4$).

A general matrix notation for the transport of particles' coordinates is adopted here. For any particle with coordinates $(x, x', y, y', z, \delta_p)$ passing through a second order achromat, the final coordinates will be the same as the initial ones to second order, as R is a unity matrix and all T matrix elements equal zero. This points to a way of evaluating and improving the dynamic aperture in a storage ring for the ideal case, where one can simply design a ring op-

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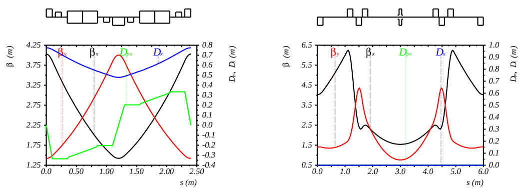

Figure 1: Dispersion and beta functions. Left: FODO arc cell; Right: straight section cell with an RF cavity.

tics which consists of sections that have no second order geometric or chromatic aberrations. W. Wan developed another approach using Lie algebra to design a general achromat to arbitrary order, taking advantage of the midplane symmetry and using multipole magnets for each order (for example, octupoles for a third order achromat) [4]. Recently Y. Cai has demonstrated, by derivations using the Lie algebraic method, that all driving terms up to fourth order resonance from chromaticity correction sextupoles can be canceled out within each arc achromat section [5].

In the following sections, a second order achromatic supercell is designed based on K. Brown's theory [3] where all second order geometric and chromatic aberrations vanish. The betatron phase advance per cell of this supercell can be tuned in a wide range, which provides a flexible way to vary the momentum compaction. This design alleviates the need to add dispersion suppressors between the arc and straight sections.

SINGLE CELL DESIGN

We consider the design of a small compact ring with a circumference around 70 m. The design goals of the arc cell are compactness, simplicity and flexibility. A standard FODO cell is picked as the basic unit, with a cell length of roughly 2.4 m. The dipole length is chosen to be 0.5 m, and a drift length of 0.25 m is used between each dipole and quadrupole magnet. A quadrupole length of 0.2 m and a sextupole length of 0.1 m are adopted, which can be made longer if necessary. There is a focusing sextupole placed on each side of the focusing quadrupole to correct the linear chromaticity (second order chromatic aberrations). For the defocusing quadrupole, the arrangement is similar. This standard FODO cell maintains a midplane symmetry, as can be observed from its first order optics shown in Fig. 1 (left). The betatron phase advance equals 60 degrees in both transverse planes. For other possible ring designs, such as a synchrotron radiation light source with ultra low emittance, the dipole length can be further increased and the dispersion will be decreased. The phase advance can be kept the same by tuning the quadrupole strength. In Fig. 1 (right), the first order optics of a straight section cell is shown, where midplane symmetry is again preserved. Four quadrupoles are used in each half to match the beta function and tune the phase advance. The straight section can be composed of such cells and the arc FODO cells discussed above with the dipole magnets being replaced by drift sections. The computer code MAD8 was used for optics matching [6].

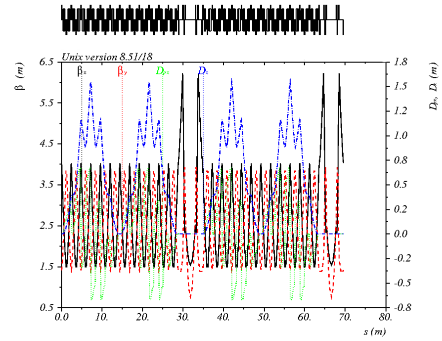

Figure 2: Dispersion and beta functions of an example ring design. The betatron phase advance of one arc FODO cell equals 60 degrees in both transverse planes.

SUPERCELL FOR THREE PHASE ADVANCES

According to Brown's theory, a supercell which consists of more than four identical cells (such as the FODO cell discussed above) can make a second order achromat, given the net betatron phase advance equals a multiple of 2π in both transverse planes. The second order geometric aberrations are canceled between these identical cells without sextupole assistance. Two groups of sextupoles in each cell correct second order chromatic aberrations and do not introduce new second order geometric aberrations. With this kind of configuration, the first order and second order transport matrix through one supercell are $\mathbf{R} = \mathbf{I}$ and $T_{ijk} = 0$. One observes that any particle's 6-D coordinates are reproduced up to second order.

Given a fixed arrangement of the dipole magnets constrained by the ring geometry, one can adjust the quadrupoles to tune the dispersion function and the momentum compaction. A supercell which contains 12 standard FODO cells is chosen here to construct one achromat module. Three different horizontal betatron phase advances are considered: 30 degrees, 60 degrees and 90 degrees. In all three cases, the total phase advance is a multiple of 2π, which ensures that the overall transport matrix is a unity matrix up to second order. There are 24 dipole magnets in each supercell. Assuming a bending angle of 7.5 degrees for each dipole magnet, the first order optics of such a supercell is constructed for the three different phase advances. The total bending angle of this supercell is 180 degrees, and its length is roughly 29 m.

COMPACT RING DESIGN

The geometry of the ring is assumed to be a racetrack shape. One supercell composes a half arc section which is

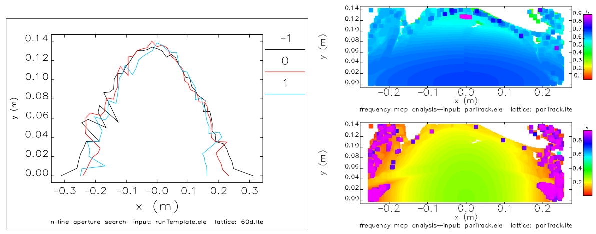

Figure 3: Left: dynamic aperture of on-momentum and off-momentum ($\pm 1\%$) particles after 1,000 turns of tracking; Right: tune footprint in x-y phase space after 1,000 turns: horizontal tune (top) and vertical tune (bottom).

nearly half of the ring. For simplicity, only two straight section cells as shown in Fig. 1 (right) are used as the matching section between the two arcs. The key point is that the phase advance in each arc section is an integer, and the second order aberrations vanish inside each arc section. The straight sections can then be used to accommodate other components, such as injection/extraction, RF cavities, damping wigglers, and insertion devices. The overall betatron tune can be controlled by tuning the phase advance with quadrupole magnets in the straight sections.

One such compact ring design is shown in Fig. 2. It consists of two arc sections and two short straight sections. Dispersion closes in each supercell and there are four periods of oscillation. After slightly tuning the arc sextupoles to cancel the chromaticity from the straights, a unity transport matrix is achieved up to second order, as also confirmed numerically in MAD8 [6]. As discussed above, in such a ring one can easily tune the phase advance of each FODO cell to be either 30 degrees or 90 degrees without moving any magnets and achieve different momentum compaction and different RMS bunch lengths. The straight sections need to be slightly tuned to match the new TWISS parameters of the arc section. One needs to note that in all cases the alpha function (derivative of the beta function) is always zero at both ends of each supercell, which essentially makes the supercell shape extendable. A comparison of different key parameters between these three operating options can be found in [7].

The dynamic aperture should be large enough to ensure efficient acceptance of an injected beam with large emittance and energy spread. The natural chromaticity from the straight sections needs to be minimized, which in turn requires weaker arc sextupoles. Usually the dynamic aperture is determined by single particle numerical tracking simulation. For electron storage rings, synchrotron radiation damping plays an important role and the number of turns one needs to track to compute the dynamic aperture depends on the damping time. In general, one damping period is thousands of turns, and it is sufficient to evaluate the dynamic aperture for particles that survive $10\% - 15\%$ of the damping period. For this compact ring running at 2 GeV, one synchrotron oscillation period is roughly 70 turns, and one damping period corresponds to 10^4 turns. The full optics was translated into the computer code Elegant [8] from MAD8 format, and the dynamic aperture was determined at 53 different angles in x-y phase space for both on-momentum and off-momentum particles. For each angle, a single particle was launched for tracking with its initial coordinate varied from small amplitude to large amplitude. Momentum offsets up to $\pm 1\%$ were considered, and the resulting 1,000 turn dynamic aperture is shown in Fig. 3 (left). One observes that particles with offsets of up to 0.2 m in the horizontal direction and 0.14 m in the vertical direction were transmitted. This is about 10^4 times the equilibrium RMS beam size ($10^4\ \sigma$) at 2 GeV. There is no obvious change in the aperture for the off-momentum particles.

The tune footprint was also investigated for 1,024 turns, with the results shown in Fig. 3 (right). One observes that there is little amplitude related detuning, with an initial transverse offset up to 0.15 m, which is much larger than the usual physical aperture. This small ring design has a relatively large emittance of 0.04 $\mu m \cdot rad$ at a beam energy of 1 GeV for the 90 degree phase advance per cell case. Due to this feature, it may be suitable for a pre-damping ring or a booster ring.

CONCLUSION AND DISCUSSION

A compact ring design based on K. Brown's achromat theory is presented. It has a tunable momentum compaction and a large dynamic aperture. The transport matrix through the arc section is always a unity matrix up to second order. The overall betatron tune is controlled in the straight sections. The dipole length and number of dipoles can be greatly increased to convert this compact ring into a low emittance ring with a circumference in the hundreds of meters or kilometer range. A 120 degree FODO cell option is being investigated, which may need fewer magnets and a shorter arc length.

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