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# Dynamical layer decoupling in a stripe-ordered, high $T_c$ superconductor

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In the stripe-ordered state of a strongly-correlated two-dimensional electronic system, under a set of special circumstances, the superconducting condensate, like the magnetic order, can occur at a non-zero wave-vector corresponding to a spatial period double that of the charge order. In this case, the Josephson coupling between near neighbor planes, especially in a crystal with the special structure of $La_{2-x}Ba_xCuO_4$, vanishes identically. We propose that this is the underlying cause of the dynamical decoupling of the layers recently observed in transport measurements at $x = 1/8$.

High-temperature superconductivity (HTSC) was first discovered [1] in $La_{2-x}Ba_xCuO_4$. A sharp anomaly [2] in $T_c(x)$ occurs at $x = 1/8$ which is now known to be indicative [3, 4] of the existence of stripe order and of its strong interplay with HTSC. Recently, a remarkable dynamical layer decoupling has been observed [5] associated with the superconducting (SC) fluctuations below the spin-stripe ordering transition temperature, $T_{\rm spin} = 42$K.

While $T_c(x)$, as determined by the onset of a bulk Meissner effect, reaches values up to $T_c(x = 0.1) = 33$ K for $x$ somewhat smaller and larger than $x = 1/8$, $T_c(x)$ drops to the range 2–4 K for $x = 1/8$. However, in other respects, superconductivity appears to be optimized for $x = 1/8$. The d-wave gap determined by ARPES has recently been shown [6] to be largest for $x = 1/8$. Moreover, strong SC fluctuations produce an order of magnitude drop [5] in the in-plane resistivity, $\rho_{ab}$, at $T \approx T_{\rm spin}$, which is considerably higher than the highest bulk SC.

The fluctuation conductivity reveals heretofore unprecedented characteristics (as described schematically in Fig. 1): 1) $\rho_{ab}$ drops rapidly with decreasing temperature from $T_{\rm spin}$ down to $T_{KT} \approx 16$K, at which point it becomes unmeasurably small. In the range $T_{\rm spin} > T > T_{KT}$, the temperature dependence of $\rho_{ab}$ is qualitatively of the Kosterlitz-Thouless form, as if the SC fluctuations were strictly confined to a single copper-oxide plane. 2) By contrast, the resistivity perpendicular to the copper-oxide planes, $\rho_c$, increases with decreasing temperatures from $T^\star \gtrsim 300$ K, down to $T^{\star\star} \approx 35$ K. For $T < T^{\star\star}$, $\rho_c$ decreases with decreasing temperature, but it only becomes vanishingly small below $T_{3D} \approx 10$ K. Within experimental error, for $T_{KT} > T > T_{3D}$, the resistivity ratio, $\rho_c/\rho_{ab}$, is infinite! 3) The full set of usual characteristics of the SC state, the Meissner effect and perfect conductivity, $\rho_{ab} = \rho_c = 0$, is only observed below $T_c = 4K$. Thus, for $T_{3D} > T > T_c$, a peculiar form of fragile 3D superconductivity exists.

The above listed results are new, so an extrinsic explanation of some aspects of the data is possible. Here we assume that the measured properties do reflect the bulk behavior of $La_{2-x}Ba_xCuO_4$. We show that there is a straightforward way in which stripe order can lead to an enormous dynamical suppression of interplane Josephson coupling, particularly in the charge ordered low-temperature tetragonal (LTT) phase of $La_{15/8}Ba_{1/8}CuO_4$, *i.e.* $T \leq T_{\rm co} = 54$ K.

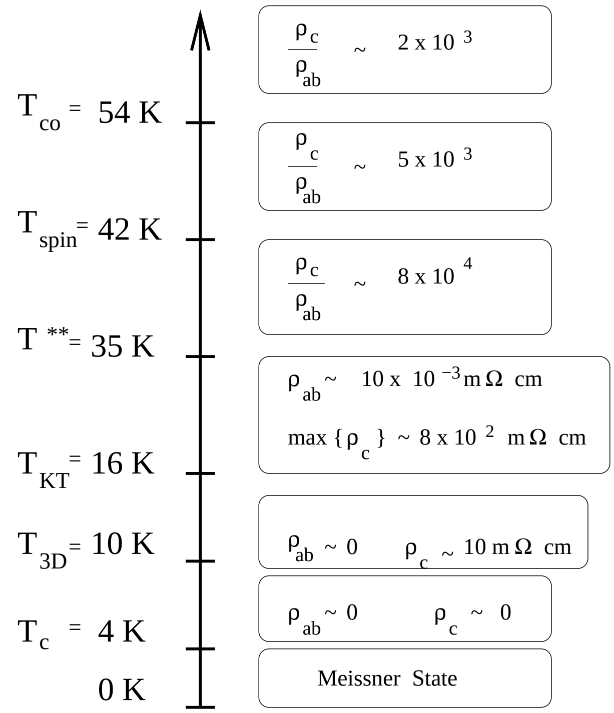


FIG. 1: Summary of the thermal phase transitions and transport regimes in $x = 1/8$ doped $La_{2-x}Ba_xCuO_4$.

The LTT structure has two planes per unit cell. In alternating planes, the charge stripes run along the x or y axes, as shown in Fig. 3. Moreover, the parallel stripes in second neighbor planes are thought to be shifted over by half a period (so as to minimize the Coulomb interactions [7]) resulting in a further doubling of the number of planes per unit cell, as seen in X-ray scattering studies. Below $T_{\rm spin}$, the spins lying between each charge stripe have antiferromagnetic (AFM) order along the stripe direction, which suffers a $\pi$ phase shift across each charge

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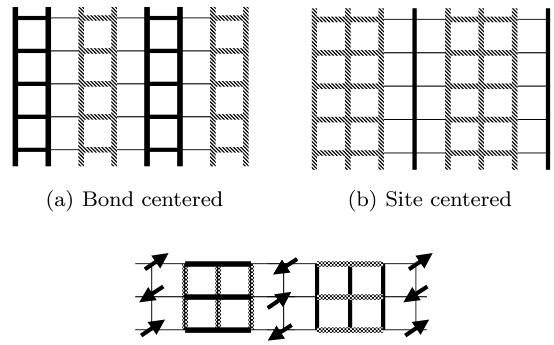
stripe, resulting in a doubling of the unit cell within the plane, see Fig. 2c. Hence, the Bragg scattering from the charge order in a given plane occurs at $(2\pi/a)\langle\pm1/4, 0\rangle$ while the spin-ordering occurs at $(2\pi/a)\langle 1/2 \pm 1/8, 1/2\rangle$.

SC order should occur most strongly within the charge stripes. Since it is strongly associated with zero center-of-mass momentum pairing, one usually expects, and typically finds in models, that the SC order on neighboring stripes has the same phase. However, as we will discuss, under special circumstances, the SC order, like the AFM order, may suffer a $\pi$ phase shift between neighboring stripes if the effective Josephson coupling between stripes is negative. Within a plane, so long as the stripe order is defect free, the fact that the SC order occurs with $k = (2\pi/a)\langle\pm1/8, 0\rangle$ has only limited observable consequences. However anti-phase SC order within a plane results in an exact cancellation of the effective Josephson coupling between first, second and third neighbor planes. This observation can explain an enormous reduction of the interplane SC correlations in a stripe-ordered phase.

Before proceeding, we remark that there is a preexisting observation, concerning the spin order, which supports the idea that interplane decoupling is a bulk feature of a stripe-ordered phase. Specifically, although the in-plane spin correlation length measured in neutron-scattering studies in particularly well prepared crystals of $La_{2-x}Ba_xCuO_4$ is $\xi_{\rm spin} \geq 40a$ [8], there are essentially no detectable magnetic correlations between neighboring planes. In typical circumstances, 3D ordering would be expected to onset when $(\xi_{\rm spin}/a)^2 J_1 \sim T$, where $J_1$ is the strength of the interplane exchange coupling. However, the same geometric frustration of the interplane couplings that we have discussed in the context of the SC order pertains to the magnetic case, as well. Thus, we propose that the same dynamical decoupling of the planes is the origin of both the extreme 2D character of the AFM and SC ordering.

We begin with a caricature of a stripe ordered state, consisting of alternating Hubbard or $t$–$J$ ladders which are weakly coupled to each other (Fig. 2). Such a caricature, which has been adopted in previous studies of superconductivity in stripe ordered systems [9, 10, 11], certainly overstates the extent to which stripe order produces quasi-1D electronic structure. However, we can learn something about the possible electronic phases and their microscopic origins, in the sense of adiabatic continuity, by analyzing the problem in this extreme limit. As shown in the figure, distinct patterns of period 4 stripes can be classified by their pattern of point group symmetry breaking as being "bond centered" or "site centered." Numerical studies of $t$–$J$ ladders [12] suggest that the difference in energy between bond and site centered stripes is small, so the balance could easily be tipped one way or another by material specific details, such as the specifics of the electron-lattice coupling.

The simplest caricature of bond centered stripes is an array of weakly coupled two-leg ladders with alternately larger and smaller doping, as illustrated in Fig. 2a. This



(a) Bond centered (b) Site centered

(c) Magnetic striped

FIG. 2: a) Pattern of a period 4 bond centered and b) site centered stripe, with nearly undoped (solid lines) and more heavily doped (hatched lines) regions. c) Sketch of the pair-field (lines) and spin (arrows) order in a period 4 site centered stripe in which both the SC and AFM order have period 8 due to an assumed $\pi$ phase shift across the intervening regions. Solid (checked) lines denote a positive (negative) pair-field.

problem was studied in Ref. 10. Because a strongly interacting electron fluid on a two-leg ladder readily develops a spin-gap,[13] *i.e.* forms a LE liquid, this structure can exhibit strong SC tendencies to high temperatures. Weak electron hopping between neighboring ladders produces Josephson coupling which can lead to a "d-wave like" SC state.[14] However, the spin-gap precludes any form of magnetic ordering, even when the ladders are weakly coupled, and there is nothing about the SC order that would prevent phase locking between neighboring planes in a 3D material. For both these reasons, this is not an attractive model for the stripe ordered state in $La_{15/8}Ba_{1/8}CuO_4$. (There is, however, evidence from STM studies on the surface of BSCCO [15] of self-organized structures suggestive of two-leg ladders.)

By contrast, a site-centered stripe is naturally related to an alternating array of weakly coupled three and one leg ladders, as shown in Fig. 2(b). Because the zero-point kinetic energy of the doped holes is generally large compared to the exchange energy, it is the three-leg ladder that we take to be the more heavily doped. The three leg ladder is known [9, 16] to develop a spin-gapped LE liquid above a rather small [16] critical doping, $x_c$ (which de-

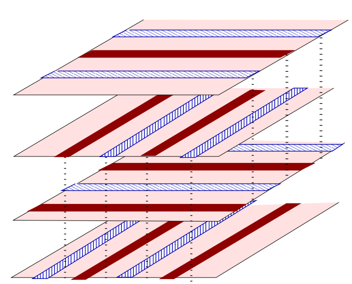


FIG. 3: Stacking of stripe planes.

pends on the interactions). An undoped or lightly doped one-leg ladder, by contrast, is better thought of as an incipient spin density wave (SDW), and has no spin-gap. Where the one-leg ladder is lightly doped it forms a Luttinger liquid with a divergent SDW susceptibility at $2k_F$. The phases of a system of alternating, weakly coupled LE and Luttinger liquids were analyzed in [11]. However, the magnetic order in $La_{15/8}Ba_{1/8}CuO_4$ produces a Bragg peak at wave-vector $(\frac{2\pi}{a})\langle\frac{1}{2}\pm\frac{1}{8},\frac{1}{2}\rangle$ in a coordinate system in which $y$ is along the stripe direction. Therefore, it is necessary to consider the case in which, in the absence of inter-ladder coupling, the one-leg ladder is initially undoped, and the three leg ladder has $x=\frac{1}{6}>x_c$.

Our model of the electronic structure of a single charge-stripe-ordered Cu-O plane is thus an alternating array of LE liquids, with a spin-gap but no charge gap, and spin-chains, with a charge gap but no spin gap. None of the obvious couplings between nearest-neighbor subsystems is relevant in the renormalization group sense, because of the distinct character of their ordering tendencies. However, certain induced second neighbor couplings, between identical systems, are strongly relevant, and, at $T=0$, lead to a broken symmetry ground-state.

The induced exchange coupling between nearest-neighbor spin-chains leads to a 2D magnetically ordered state. The issue of the sign of this coupling has been addressed previously [17, 18, 19] and found to be non-universal, as it depends on the doping level in the intervening three-leg ladder. For $x=0$, the preferred AFM order is in-phase on neighboring spin-chains, consistent with a magnetic ordering vector of $(2\pi/a)\langle 1/2, 1/2\rangle$. For large enough $x$ (probably, $x>x_c$), the ordering on neighboring spin-chains is $\pi$ phase shifted, resulting in a doubling of the unit-cell size in the direction perpendicular to the stripes, and a magnetic ordering vector $(2\pi/a)\langle 1/2\pm 1/8, 1/2\rangle$. This ordering tendency has also been found in studies of wide $t$–$J$ ladders [12].

A question that has not been addressed systematically until now is the sign of the effective Josephson coupling between neighboring LE liquids. In the case of 2-leg ladders, it was found [10, 12] that the effective Josephson coupling is positive, favoring a SC state with a spatially uniform phase. It is possible, in highly correlated systems, especially when tunneling through a magnetic impurity [20], to encounter situations in which the effective Josephson coupling is negative, therefore producing a $\pi$-junction. Zhang [21] has observed that, regardless the microscopic origin of the anti-phase character of the magnetic ordering in the striped state, if there is an approximate SO(5) symmetry relating the antiferromagnetism to the superconductivity, one should expect an anti-phase ordering of the superconductivity in a striped state. The example of tunneling through decoupled magnetic impurities [20] is a proof in principle that such behavior *can* occur. However, interplane decoupling associated with the onset of superconductivity is not seen in experiments in other cuprates, and states with periodic $\pi$ phase shifts of the SC order parameter have not yet surfaced in numerical studies of microscopic models [12]; this suggests anti-phase striped SC order is rare.

The new proposal in the present paper is that, for the reasons outlined above, the SC striped phase of $La_{15/8}Ba_{1/8}CuO_4$ has anti-phase SC and anti-phase AFM order, whose consequences we now outline. We can express the most important possible interplane Josephson-like coupling terms compactly as

$$H_{\rm inter} = \sum_j \int d\vec{r} \sum_{n,m} \mathcal{J}_{n,m} \left[ \left( \Delta_j^\star \Delta_{j+m} \right)^n + {\rm h.c.} \right] \quad (1)$$

where $\Delta_j$ is the $j$-th plane SC order parameter. The term proportional to the usual (lowest order) Josephson coupling, $\mathcal{J}_{1,1}$, and indeed, $\mathcal{J}_{1,2}$ and $\mathcal{J}_{1,3}$ all vanish by symmetry. The most strongly relevant residual interaction is the Josephson coupling between fourth-neighbor planes, $\mathcal{J}_{1,4}$. Double-pair tunnelling between nearest-neighbor planes, $\mathcal{J}_{2,1}$, is more weakly relevant, but it probably has a larger bare value since it involves half as many powers of the single-particle interplane matrix elements than $\mathcal{J}_{1,4}$. $\mathcal{J}_{1,4}$ and $\mathcal{J}_{2,1}$ have scaling dimensions 1/4 and 1 at the (KT) critical point of decoupled plains, so both are relevant. Thus, they become important when the in-plane SC correlation length $\xi \sim \xi_{1,4} \sim [\mathcal{J}_o/\mathcal{J}_{1,4}]^{1/4}$ and $\xi_{2,1} \sim [\mathcal{J}_o/\mathcal{J}_{2,1}]$, where $\mathcal{J}_o$ is the in-plane SC stiffness.

We can make a crude estimate of the magnitude of the residual interplane couplings by noting that the same interplane matrix elements (although not necessarily the same energy denominators) determine the interplane exchange couplings between spins and the interplane Josephson couplings. Defining $J_m$ to be the exchange couplings between spins $m$ planes apart, this estimate suggests that $\mathcal{J}_{n,m}/\mathcal{J}_0 \sim [J_m/J_0]^n$. In undoped $La_2CuO_4$, it has been determined [22] that $J_1/J_0 \approx 10^{-5}$, which is already remarkably small.

Although in-plane translation invariance forbids direct Josephson coupling between adjacent planes, there is an allowed biquadratic inter-plane coupling involving $\mathbf{M}$ and $\Delta$, the SDW and the SC order parameters,

$$\delta H_{\rm inter} = \mathcal{J}_{1,s} \sum_j \int d\vec{r} \, \left[ \Delta_j^* \Delta_{j+1} \mathbf{M}_j \cdot \mathbf{M}_{j+1} + {\rm h.c.} \right] \quad (2)$$

Even though $\mathbf{M} \neq 0$ for $T<T_{spin}$, this term vanishes because, not only the direction of the stripes, but also the axis of quantization of the spins (due to spin-orbit coupling) rotates [23] by 90° from plane to plane, *i.e.* $\mathbf{M}_j \cdot \mathbf{M}_{j+1} = 0$. However, a magnetic field, $H \sim 6T$, induces a 1st order spin-flop transition to a fully collinear spin state [23] in which $\mathbf{M}_j \cdot \mathbf{M}_{j+1} \neq 0$.

Thus, for perfect stripe order, the anti-phase SC order would depress, by many orders of magnitude, of the interplane Josephson couplings, which explains the existence of a broad range of $T$ in which 2D physics is apparent. Accordingly, there still would be a transition to a 3D superconductor at a temperature strictly greater than $T_{KT}$, when $\xi(T) \sim \xi_{1,4}$ or $\xi_{2,1}$, whichever is smaller.

The only evidence for the growth of $\xi$ comes indirectly from the measurement of $\rho_{ab}$; by the time $\rho_{ab}$ is "unmeasurably small," it has dropped by about 2 orders of magnitude from its value just below $T_{\rm spin}$, which implies (since $\rho_{ab} \sim \xi^{-2}$) that $\xi$ has grown by about 1 order of magnitude. Thus, if some other physics cuts off the growth of in-plane SC correlations at long scales, we may be justified in neglecting the effects of $H_{\rm inter}$.

Defects in the pattern of charge stripe order have consequences for both magnetic and SC orders. A dislocation introduces frustration into the in-plane ordering, resulting in the formation of a half-SC vortex bound to it. For the single-plane problem, this means that the long-distance physics is that of an XY spin-glass. Since there is no finite $T$ glass transition in 2D, the growth of $\xi$ will be arrested at a large scale determined by the density of dislocations. The same is true of the in-plane AFM correlations. Both $\xi$ and $\xi_{\rm spin}$ should be bounded above by the charge stripe correlation length, $\xi_{\rm ch}$. From X-ray scattering studies it is estimated that $\xi_{\rm ch} \approx 70a$ [24]. This justifies the neglect of $H_{\rm inter}$. Conversely, any defect in the charge-stripe order spoils the symmetry responsible for the exact cancellation of the Josephson coupling between neighboring planes. Finite $T$ ordering of an XY spin-glass is possible in 3D. We tentatively identify the temperature at which $\rho_c \to 0$ as a 3D glass transition. A SC glass would result in the existence of equilibrium currents (spontaneous time-reversal breaking) and in glassy long-time relaxations of the magnetization or $\rho_c$.

For $x \neq 1/8$, there is a tendency to develop discommensurations in the stripe order, which, in turn, produce regions of enhanced (or depressed) SC order with relative sign depending on the number of intervening stripe periods. So long as the stripes are dilute, the energy depends weakly on their precise spacing. Thus, to gain interlayer condensation energy, the system can self-organize so that there are always an even number of intervening stripes, thus producing an interplane Josephson coupling $\mathcal{J}_{1,1} \sim |x - 1/8|^2$. This, in turn, will lead to a dramatic increase of the 3D SC $T_c$. An enhancement of interplane coherence in any range of $T$ triggered by the magnetic field induced spin-flop transition would be a dramatic confirmation of the physics discussed here.

*Note added:* It was pointed out to us that the state discussed here was considered by A. Himeda *et al.*[25] They found that this is a good variational state for a $t - t' - J$ model at $x \sim 1/8$ for a narrow range of parameters.

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