

Spin Charge Separation in the Quantum Spin Hall State

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The quantum spin Hall state is a topologically non-trivial insulator state protected by the time reversal symmetry. We show that such a state always leads to spin-charge separation in the presence of a π flux. Our result is generally valid for any interacting system. We present a proposal to experimentally observe the phenomenon of spin-charge separation in the recently discovered quantum spin Hall system.

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Spin-charge separation is one of the deepest concepts in condensed matter physics. In the Su-Schrieffer-Heeger model of polyacetylene[1], a domain wall induces two mid-gap states, one for each spin orientation of the electron. If both states are unoccupied, or both states are occupied, the domain wall soliton has charge $\pm e$ but no spin. If only one of the state is occupied, the domain wall soliton has spin $S_z = \pm 1/2$ but no charge. In this remarkable way, the two fundamental degrees of freedom of an electron is split apart. Since then, the concept of spin-charge separation has become a corner stone in condensed matter physics. This phenomenon occurs generally in interacting quantum many-body systems in one dimension, and can be demonstrated convincingly by the bosonization techniques. The concept has also been generalized to two dimensions. In particular, it is conjectured that such a phenomenon occurs in the high temperature superconductors[2, 3]. However, this phenomenon has not yet been convincingly observed in any two dimensional systems.

Recently, a new two dimensional quantum state of matter has been theoretically proposed[4, 5, 6]. The quantum spin Hall (QSH) state is a topologically non-trivial state of matter protected by the time reversal symmetry. It has a bulk insulating gap, but has helical edge states on the sample boundary, where electron states with opposite spins counter-propagate at a given edge. This novel quantum state of matter has recently been theoretically predicted[6] and experimentally observed[7] in the *HgTe* quantum wells. The topological property of the quantum Hall (QH) state is described by an integer Chern number[8], defined over the single particle momentum space, and this integer is directly related to the experimentally observed quantum of Hall conductance. This construction can also be generalized to an interacting system, where the Chern number is defined over the space of twisted boundary conditions[9]. The topological property of the QSH state is currently described by a Z_2 topological number[10, 11, 12, 13], which is also defined over the single particle momentum space. This Z_2 classification has provided an important insight on the topological non-triviality of the QSH state. However, unlike the situation in QH systems, there are several fundamen-

tal missing links in the QSH systems. Needed is a general classification of time reversal invariant (TRI) topological insulators in two dimensions which is valid in the presence of arbitrary interactions. Such a general classification beyond the single particle band picture is especially called for since the concept of a topological Mott insulator has recently been introduced[14]. More importantly, we need to find experimentally measurable properties which directly demonstrate the topological non-triviality of the QSH state.

In this paper, we solve both problems by providing a deep connection between the concept of spin-charge separation and the QSH effect. Following Laughlin's argument for the QH effect, we consider the adiabatic insertion of a pure gauge flux in the QSH state. We show that there are four different ways of reaching the final flux of π , and that these four processes create the spin-charge separated holon, chargeon and two spinon states which are exponentially localized near the flux. We then prove two general theorems providing a Z_2 classification of TRI insulators in two dimensions. This new classification scheme is generally valid in the presence of many-body interactions, and leads to spin-charge separation as its direct physical consequence. Finally, we propose an experimental setting to observe the phenomenon of spin-charge separation in the recently discovered QSH system.

We first present an argument which is physically intuitive, but only valid when there is at least a $U_s(1)$ spin rotation symmetry. In this case, the QSH effect is simply defined as two copies of QH, with opposite Hall conductances of $\pm e^2/h$ for opposite spin orientations. Without loss of generality, we first consider a disk geometry with a gauge flux of $\phi_{\uparrow} = \phi_{\downarrow} = hc/2e$, or simply π in units of $\hbar = c = e = 1$, through a hole at the center, see Fig. 1. The gauge flux acts on both spin orientations, and the π flux preserves time reversal symmetry. We consider adiabatic processes of $\phi_{\uparrow}(t)$ and $\phi_{\downarrow}(t)$, where $\phi_{\uparrow}(t) = \phi_{\downarrow}(t) = 0$ at $t = 0$, and $\phi_{\uparrow}(t) = \phi_{\downarrow}(t) = \pm\pi$ at $t = 1$. Since the flux of π is equivalent to the flux of $-\pi$, there are four different adiabatic processes all reaching the same final flux configuration. In process (a), $\phi_{\uparrow}(t) = -\phi_{\downarrow}(t)$ and $\phi_{\uparrow}(t = 1) = \pi$. In process (b), $\phi_{\uparrow}(t) = -\phi_{\downarrow}(t)$ and $\phi_{\uparrow}(t = 1) = -\pi$. In process

(c), $\phi_\uparrow(t) = \phi_\downarrow(t)$ and $\phi_\uparrow(t=1) = \pi$. In process (d), $\phi_\uparrow(t) = \phi_\downarrow(t)$ and $\phi_\uparrow(t=1) = -\pi$. These four processes are illustrated in Fig 1. Note that process (a) and (b) preserves time reversal symmetry at all intermediate stages, while process (c) and (d) only preserves the time reversal symmetry at the final stage.

We consider a Gaussian loop surrounding the flux. As the flux $\phi_\uparrow(t)$ is turned on adiabatically, Faraday's law induction states that a tangential electric field \mathbf{E}_\uparrow is induced along the Gaussian loop. The quantized Hall conductance implies a radial current $\mathbf{j}_\uparrow = \frac{e^2}{h} \mathbf{z} \times \mathbf{E}_\uparrow$, resulting in a net charge flow ΔQ_\uparrow through the Gaussian loop:

$$\begin{aligned} \Delta Q_\uparrow &= - \int_0^1 dt \int d\mathbf{n} \cdot \mathbf{j}_\uparrow = - \frac{e^2}{h} \int_0^1 dt \int d\mathbf{l} \cdot \mathbf{E}_\uparrow \\ &= - \frac{e^2}{hc} \int_0^1 dt \frac{\partial \phi}{\partial t} = - \frac{e^2 hc}{hc 2e} = - \frac{e}{2} \end{aligned} \quad (1)$$

Identical argument applied to the down spin component shows that $\Delta Q_\downarrow = -e/2$. Therefore, this adiabatic process creates the holon state with $\Delta Q = \Delta Q_\uparrow + \Delta Q_\downarrow = -e$ and $\Delta S_z = \Delta Q_\uparrow - \Delta Q_\downarrow = 0$.

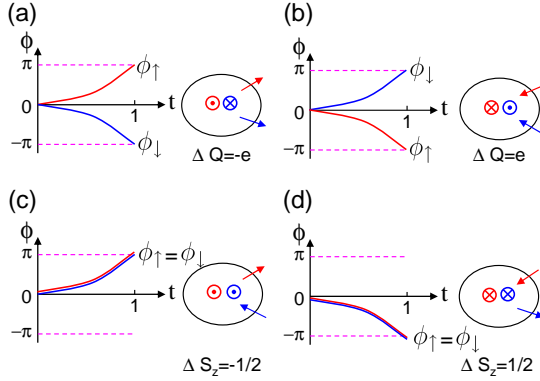


FIG. 1: Four different adiabatic processes from $\phi_\uparrow = \phi_\downarrow = 0$ to $\phi_\uparrow = \phi_\downarrow = \pm\pi$. The red (blue) curve stands for the flux $\phi_{\uparrow(\downarrow)}(t)$, respectively. The symbol “ \odot ” (“ \otimes ”) represents increasing (decreasing) fluxes, and the arrows show the current into and out of the Gaussian loop, induced by the changing flux. Charge is pumped in the processes with $\phi_\uparrow(t) = -\phi_\downarrow(t)$, while spin is pumped in those with $\phi_\uparrow(t) = \phi_\downarrow(t)$.

Applying similar arguments to process (b) gives $\Delta Q_\uparrow = \Delta Q_\downarrow = e/2$, which leads to a chargeon state with $\Delta Q = e$ and $\Delta S_z = 0$. Process (c) and (d) give $\Delta Q_\uparrow = -\Delta Q_\downarrow = e/2$ and $\Delta Q_\uparrow = -\Delta Q_\downarrow = -e/2$ respectively, we therefore obtain the spinon states with $\Delta Q = 0$ and $\Delta S_z = \pm 1/2$. The Hamiltonians $H(t)$ in the presence of the gauge flux are the same at $t = 0$ and $t = 1$, but differ in the intermediate stages of the four adiabatic processes. Assuming that the ground state is unique at $t = 0$, we obtain four final states at $t = 1$, which are the holon, chargeon and the two spinon states. Both the spin

and the charge quantum numbers are sharply defined quantum numbers[15]. The insulating state has a bulk gap Δ , and an associated coherence length $\xi = \hbar v_F / \Delta$. As long as the radius of the Gaussian loop r_G far exceeds the coherence length, *i.e.*, $r_G \gg \xi$, the spin and the charge quantum numbers are sharply defined within exponential accuracy. Recently, similar proposals of fractionalization phenomena in two-dimensions induced by topological defects have been studied in several other systems.[16, 17, 18, 19]

While the argument above is intuitive and generally valid in the presence of both interaction and disorder, it has a serious shortcoming. It relies on the $U_s(1)$ spin rotation symmetry which is not generic in the presence of spin-orbit interactions. Therefore, we first need a general definition of the concept of spin-charge separation relying only on the generic time-reversal symmetry. In the absence of spin rotational symmetry, we can still use the generic time reversal symmetry and the Kramers theorem to classify integer versus half-integer spin states. Denoting the time reversal operator as \mathcal{T} , and the charge operator as N , we give the following general definition of spin-charge separation:

- **DEFINITION I:** A generalized chargeon (or holon) state is a quantum state $|\psi_c\rangle$ satisfying $(-1)^N |\psi_c\rangle = -|\psi_c\rangle$, and $\mathcal{T}|\psi_c\rangle = |\psi_c\rangle$. A generalized spinon state is a doublet of quantum states $|\psi_s^+\rangle$ and $|\psi_s^-\rangle$, satisfying $(-1)^N |\psi_s^\pm\rangle = |\psi_s^\pm\rangle$, $\mathcal{T}|\psi_s^+\rangle = |\psi_s^-\rangle$ and $\mathcal{T}|\psi_s^-\rangle = -|\psi_s^+\rangle$.

The Kramers degeneracy is generally lifted in the presence of a magnetic field, and the resulting energy splitting of the doublet is linear in magnetic field, *i.e.* $\Delta E = g^* \mu_B |B|$. The constant of proportionality g^* can be defined as the effective g factor of the spinon. We now consider a TRI insulator without any additional spin rotational symmetry. We consider the generalizations of processes (a) and (b), by replacing the hopping matrix elements t_{ij} with $t_{ij} e^{i\theta(t)\Gamma}$ [20, 21] on all links along a string extending from the flux tube to infinity. Here Γ is a matrix in the spin space, and the intuitive discussion given above corresponds to the choice of $\Gamma = S_z$. All the following discussions are valid even if Γ is not conserved. We focus on the case with Γ an odd operator under the time reversal symmetry, *i.e.* $\mathcal{T}^{-1}\Gamma\mathcal{T} = -\Gamma$.

- **THEOREM I:** For any time-reversal odd Γ satisfying $e^{i\pi\Gamma} = -1$, an integer number of charge $N_\Gamma e$ is pumped towards the isolated flux tube during the adiabatic evolution $\theta(t) = 0 \rightarrow \pi$.

PROOF: Denote the Hamiltonian with a Γ flux as $H_\Gamma(\theta(t))$. Due to the condition $e^{i\pi\Gamma} = -1$, we know that $H_\Gamma(\pi)$ is the same as the Hamiltonian with a charge π -flux tube. Consequently, two distinct adiabatic evolutions can be defined between $H_\Gamma(0)$ and $H_\Gamma(\pi)$, that is,

the process l_Γ through spin Γ flux threading, and the process l_c through charge flux threading. The combination of them $l = l_c^{-1} \cdot l_\Gamma$ leads to a closed path in the parameter space. Given the condition that the system with no flux has a unique ground state, the charge pumped during such a process must be an integer, denoted as N_Γ . Moreover, the charge pumped during the path l_c^{-1} has to be zero, since the Hall conductivity of the system vanishes due to time-reversal symmetry. Consequently, an integer number of charge $N_\Gamma e$ is pumped towards the flux tube during the first half of the adiabatic process, l_Γ . (Such a combination of spin and charge flux threading has been introduced before in Ref.[22].) Note that the time reversal symmetry is essential for obtaining the integer charge. In the integer QH systems, a π flux generally induces a fractional charge of $e/2$ near the flux tube[16, 19].

- **THEOREM II:** A topological index, defined by $(-1)^{N_\Gamma}$, is independent of the choice of Γ , as long as $e^{i\pi\Gamma} = -1$ is satisfied.

PROOF: For two different operators Γ_1, Γ_2 satisfying $e^{i\pi\Gamma_{1,2}} = -1$, two different adiabatic paths l_1, l_2 connecting $\theta = 0$ and $\theta = \pi$ Hamiltonians are defined. Consequently, a closed path can be formed as $l = l_2^{-1} \cdot l_1$, which brings a system without flux back onto itself. If the number of charge pumped during l_1 and l_2 is N_{Γ_1} and N_{Γ_2} , respectively, the net charge pumped during such a process is given by $N_{\text{tot}} = N_{\Gamma_1} - N_{\Gamma_2}$. Since the spin Γ flux preserves time-reversal symmetry, if the initial state $|G\rangle$ is a Kramers singlet, so is the final state $|F\rangle$. This is only possible if the charge N_{tot} pumped during the closed path $l = l_2^{-1} \cdot l_1$ is an even integer, leading to the conclusion that $(-1)^{N_{\Gamma_1}} = (-1)^{N_{\Gamma_2}}$ for any two choices Γ_1 and Γ_2 .

Based these two general theorems, we see that the Z_2 topological index $(-1)^{N_\Gamma}$ is independent of the choice of Γ , which is thus a well-defined property of the TRI insulator. In this way, we obtain the following general topological classification for TRI insulators in 2D.

- **DEFINITION II:** A topologically trivial TRI insulator is defined by $(-1)^{N_\Gamma} = 1$, and a topologically non-trivial TRI insulator is defined by $(-1)^{N_\Gamma} = -1$. A topologically non-trivial TRI insulator displays the quantized spin Hall effect in the sense that a spin Γ flux of π pumps an odd number of quantized electric charges.

Since the charge pumped during such an adiabatic process is always well-defined without relying on the detail of the system, such a general Z_2 topological classification is applicable to any generic many-body system with interaction and disorder. For a nontrivial insulator with N_Γ odd, the adiabatic evolution $H_\Gamma(\theta)$, $\theta(t) = 0 \rightarrow \pi$ brings the ground state $|\psi_0\rangle$ to a state $|\psi_c\rangle$ which is a Kramers singlet but carries an odd number of electric charge $N_\Gamma e$

around the flux tube. According to the definition I, $|\psi_c\rangle$ is a chargeon or holon state. Once the flux is fixed to be π , no local perturbation can change the spin-charge separation nature of the system. In general a local operator \hat{O}_+ can be defined, which acts only around the flux tube and carries charge $-N_\Gamma e$. When N_Γ is odd, \hat{O}_+ has to form a doublet representation of time-reversal transformation \mathcal{T} together with its partner $\hat{O}_- = \mathcal{T}^{-1}\hat{O}_+\mathcal{T}$. Thus the quantum states $|\psi_s^\pm\rangle = \hat{O}_\pm |\psi_c\rangle$ carry vanishing electric charge but form a doublet representation of \mathcal{T} . According to the definition I, such a pair of state $|\psi_s^\pm\rangle$ is a spinon state.

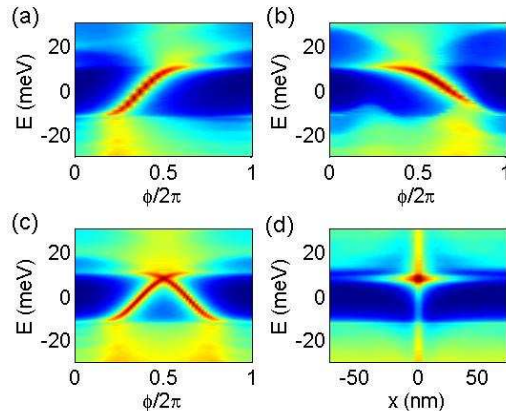


FIG. 2: Local DOS in the core of a flux tube. The dark blue color shows the bulk energy gap and the red line shows the mid-gap states. (a) and (b) show the evolution of the mid-gap state upon spin Γ -flux threadings, with $\Gamma = \sigma_z \otimes \mathbb{I}$ and $\Gamma = -(\sigma_z + \sigma_x) \otimes \mathbb{I}/\sqrt{2}$ respectively in the representation used by Ref.[6]. (c) shows the case of charge flux threading. The two mid-gap states cross at $\phi = \pi$. The spatial distribution of the mid-gap state at $\phi = \pi$ is shown in Fig. (d), in which the zero of x-axis corresponds to the position of the flux tube. Here and below, all the calculations are done for the HgTe model of $d = 70\text{\AA}$ quantum well, with a lattice constant of $a = 30\text{\AA}$.

To see more explicitly the realization of spinon and holon states through the adiabatic processes, we can study a typical model of the topologically nontrivial insulator—the four-band effective tight-binding model of HgTe quantum well proposed in Ref. 6. To study the generic case with no $U(1)$ spin symmetry, we have included the bulk-inversion asymmetry (BIA) terms in our calculation[23]. Both the spin Γ and the charge flux on a plaquette induce mid-gap states inside the gap. In Fig. 2, we show the local density of states $\rho_i(E) = \sum_n |\langle i|n\rangle|^2 \delta(E_n - E)$ in the core site i of both the charge and the spin Γ -flux tubes. As shown in Fig. 2(a) and (b), when a spin Γ -flux is threaded from 0 to 2π , the time-reversal symmetry is preserved and a Kramers pair of mid-gap states move from the valence band to the conduction band, or vice versa, depending on the choice of Γ matrix. Consequently, $\pm 2e$ units of charge are

pumped during the periodic process $\phi = 0 \rightarrow 2\pi$. The Kramers double degeneracy of the midgap states demonstrates the key statement of *Theorem II*, i.e., that even number of electric charges are pumped during any time-reversal invariant cycle. Consequently, from the proof of *Theorem II*, we know that the state at $\phi = \pi$ is a chargeon (holon) for the case of Fig. 2 (a) ((b)), respectively. On the other hand, the spinon states can be obtained by threading a charge π flux. As shown in Fig. 2 (c), a pair of mid-gap states appear around the charge flux, with a level crossing occurring at $\phi = \pi$ [24], as required by time-reversal symmetry. Consequently, only one of the mid-gap states are occupied in the final state of such an adiabatic evolution, leading to a doublet of spinon states.

We now discuss the experimental realization of spin-charge separation. Consider a hybrid structure between a type-II superconductor and the HgTe quantum well as shown in Fig. 3(a). $hc/2e$ or π flux tubes are created by a perpendicular magnetic field $H_{c1} < H < H_{c2}$, with $H_{c1,2}$ being the lower and upper critical fields of the superconductor. The superconducting flux tube has a finite size determined by the penetration depth λ . In this case, the time-reversal symmetry is broken even if the net flux is π . As shown in Fig. 3(b), the two mid-gap states are split with increasing λ . However, at a Gaussian loop with radius $r_G \gg \lambda$, there is no observable difference between such a realistic π flux and an ideal π flux threading into a plaquette, and the spinon and holon/chargeon states still exist and remain topologically distinct from a trivial many-body state of the electron system. Denoting E_1 and E_2 the energy of the two mid-gap states, and E_v and E_c the energy of valence band top and conduction band bottom, at zero temperature the ground state of the system is given by: (i) the holon state, when the chemical potential $E_v < \mu < E_1$; (ii) the spinon state with a preferred spin in the magnetic field, when $E_1 < \mu < E_2$; (iii) the chargeon state, when $E_2 < \mu < E_c$. Consequently, the spin-charge separation can be observed if we can measure the charge and spin induced by a flux tube independently. The local charge distribution in the chargeon or holon state can be probed by the recently developed scanning single-electron transistor (SET)[25], while the spin carried by the spinon state can be observed by electron spin resonance (ESR). Only when the system is in the spinon state, a transition between the two mid-gap states can occur, leading to a sharp resonance peak in the ESR signal at frequency $\omega_R = (E_2 - E_1)/\hbar$. The qualitative behavior of local charge and ESR spectrum is summarized in Fig.3 (d), as a function of the chemical potential, which can be tuned by the back gate voltage. Moreover, it can be seen from Fig.3 (b) that the energy splitting $E_2 - E_1$ is proportional to λ for small λ . Since the penetration depth of a superconductor film depends on the thickness of the film, the resonance frequency ω_R can be measured for several different λ , which should extrapolate to $\omega_R \rightarrow 0$ in the limit $\lambda \rightarrow 0$. If observed, such an asymptotic be-

havior can demonstrate the existence of a Kramers pair in the ideal case with time-reversal symmetry.

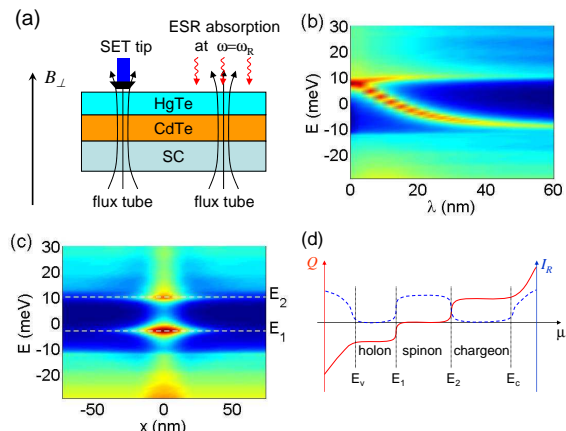


FIG. 3: (a) The superconductor-quantum well hybrid structure, with the π flux tubes generated by a perpendicular magnetic field. (b) The splitting of two mid-gap states upon increasing λ . (c) The spatial distribution of the two mid-gap states for $\lambda = 18\text{nm}$, in which the dash lines mark the energy E_1 and E_2 of the mid-gap states. (d) Illustration of the measured charge Q (red line) and intensity of ESR signal $I_R = I(\omega_R)$ at resonance frequency $\omega_R = (E_2 - E_1)/\hbar$ (blue dash line) as functions of the chemical potential.

In conclusion, we have given a Z_2 classification of TRI insulators which is generally valid in the presence of interactions and disorder. We showed that this topological property can be measured experimentally by the spin-charge separation in the presence of a π flux. We provided an experimental proposal to observe the spin-charge separation in an SC-QSH hybrid structure. Fractionalization is usually accompanied by fractional statistics. By studying the topological effective theory of the SC-QSH hybrid system, it can be shown that the spinon, holon and chargeon are all bosons, but each spinon acts as a π -flux for chargeon and holon, and vice versa. In other words, this system has nontrivial *mutual statistics* which is described by a mutual Chern-Simons theory.[26]

Such a relation between spin-charge separation and TRI topological insulators can also be generalized to higher dimensions. In a 3D TRI insulator with a π -flux ring, *Theorem I* and *II* can be generalized straightforwardly, which results in a generic Z_2 classification of 3D TRI insulators. In a three-d nontrivial insulator (which corresponds to the “strong topological insulator” defined in literature[11, 27, 28]), a closed string with π flux is a spin-charge separated extended object, satisfying our *Definition I* of spin-charge separation.

During the course of this work, we became aware of the independent work by Y. Ran, A. Vishwanath and D.-H. Lee, demonstrating spin-charge separation in similar models, which is now posted in arXiv:0801.0627 [29]. We

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