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Intrinsic Spin Hall Effect in the Two Dimensional Hole Gas

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We show that two types of spin-orbit coupling in the 2 dimensional hole gas (2DHG), with and without inversion symmetry breaking, contribute to the intrinsic spin Hall effect[1, 2]. Furthermore, the vertex correction due to impurity scattering vanishes in both cases, in sharp contrast to the case of usual Rashba coupling in the electron band. Recently, the spin Hall effect in a hole doped $GaAs$ semiconductor has been observed experimentally by Wunderlich *et al*[3]. From the fact that the life time broadening is smaller than the spin splitting, and the fact impurity vertex corrections vanish in this system, we argue that the observed spin Hall effect should be in the intrinsic regime.

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Recent theoretical work predicts dissipationless spin currents induced by an electric field in semiconductors with spin-orbit coupling[1, 2, 4]. The spin current is related to the electric field by the response equation

$$j_j^i = \sigma_s \epsilon_{ijk} E_k \tag{1}$$

where j_j^i is the current of the i-th component of the spin along the direction j and ϵ_{ijk} is the totally antisymmetric tensor in three dimensions. Because both the electric field and the spin current are even under time reversal, the spin current could be dissipationless or intrinsic, independent of the scattering rates. The response equation (1) was derived by Murakami, Nagaosa and Zhang[1] for p-doped semiconductors described by the Luttinger model of the spin-3/2 valence band. In another proposal by Sinova *et. al.* [2], the spin current is induced by a in-plane electric field in the 2-dimensional electron gas (2DEG) described by the Rashba model[2].

The spin Hall effect predicted by these recent theoretical works is fundamentally different from the extrinsic spin Hall [5, 6] effect due to Mott type of skew scattering by impurities. The intrinsic spin Hall effect arises from the spin-orbit coupling of the host semiconductor band, and has a finite value in the absence of impurities. On the other hand, the extrinsic spin Hall effect arises purely from the spin-orbit coupling to the impurity atoms, and it is not a bulk effect like the ordinary Hall effect. Because the extrinsic arises only from the impurities rather than the host atoms, its magnitude is typically many orders of magnitude smaller. The issue of impurity contributions to the spin Hall effect has been intensively investigated theoretically. Remarkably, Inoue *et. al.* [7] calculated the vertex corrections due to impurity scattering in the Rashba model of the electron band, and found that the vertex correction completely cancels the spin Hall effect. Other analytical works have obtained similar results [8] On the other hand, a number of numerical calculations have shown that the spin Hall effect is independent of the disorder in the weak disorder limit [9, 10]. Currently, this disagreement between the analytical and the numerical results is still not settled. In a insightful paper, Murakami[11] showed that the problem of the vertex correction does not occur in the Luttinger model of the hole band [11]. In fact, the vertex correction is identically zero, rendering the original prediction of Ref. [1] exact in the clean limit.

Experimental observation of the spin Hall effect has been recently reported by Kato *et. al* [12] in a electron doped sample and by Wunderlich et al in a 2DHG[3]. In this paper, we analyze the 2DHG experiment. In order to firmly establish the intrinsic spin Hall effect, one needs to establish two things. First of all, the experimental system needs to be in the clean limit, which is the case of the 2DHG experiment, as shown in the experimental paper[3]. Secondly, one needs to show that the effect is robust in the clean limit, not cancelled by the vertex corrections due to impurities. We shall show that the spin Hall effect in the 2DHG arises from two contributions, one from the Luttinger Hamiltonian describing the splitting between the light and the heavy hole bands, and one from the structural inversion symmetry breaking (SIA) of the 2DHG band, with a spin splitting scaling as k^3 [13, 14]. The later form the the spin-orbit coupling has been studied by Schliemann and Loss[14] in connection to the intrinsic spin Hall effect. This is different from the Rashba Hamiltonian of the 2DEG band, where the spin splitting scales with k. Remarkably, we find that the vertex correction due to impurity scattering vanishes for both types of spin-orbit couplings in the 2DHG band, in sharp contrast to the case of 2DEG. While the calculation details are complicated, the intuitive reason is simple: The two types of current vertices in the 2DHG have p wave and d symmetries, respectively. When these current vertices are averaged over the s wave impurity scatters, the vertex corrections vanish. These two key facts establish a firm foundation to interpret the recent experiment by Wunderlich et al in terms of the intrinsic spin Hall effect, where impurities play an unessential role.

The Hamiltonian for a 2-dimensional hole gas is a sum of both Luttinger and spin-$\vec{S} = 3/2$ SIA terms:

$$H = (\gamma_1 + \frac{5}{2}\gamma_2)\frac{k^2}{2m} - \frac{\gamma_2}{m}(\vec{k}\cdot\vec{S})^2 + \alpha(\vec{S}\times\vec{k})\cdot\hat{z} \tag{2}$$

where the confinement of the well in the z direction makes

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the momentum be quantized on this axis. The crucial difference between the SIA term for 2DHG and Rashba term for the 2DEG lies in the fact that S is a spin 3/2 matrix, describing both the light (LH) and the heavy (HH) holes. For the first heavy and light hole bands, the confinement in a well of thickness a is approximated by the relation $< k_z >= 0, \langle k_z^2 \rangle \approx (\pi\hbar/a)^2$. The energy eigenstates are:

$$E_{\pm}^{HH} = \frac{\gamma_1}{2m}k^2 \pm \frac{1}{2}\alpha k - \sqrt{\alpha^2 k^2 \pm \frac{\alpha\gamma_2}{m}k(k^2 + \langle k_z^2 \rangle) + \frac{\gamma_2^2}{m^2}(k^4 + \langle k_z^2 \rangle^2 - k^2 \langle k_z^2 \rangle)}$$
$$E_{\pm}^{LH} = \frac{\gamma_1}{2m}k^2 \pm \frac{1}{2}\alpha k + \sqrt{\alpha^2 k^2 \pm \frac{\alpha\gamma_2}{m}k(k^2 + \langle k_z^2 \rangle) + \frac{\gamma_2^2}{m^2}(k^4 + \langle k_z^2 \rangle^2 - k^2 \langle k_z^2 \rangle)} \quad (3)$$

The heavy and light hole bands are split at the Γ point by $\Delta = 2\gamma_2 \langle k_z^2 \rangle / m$ [15, 16]. Depending on the confinement scale a the Luttinger term is dominant for a not too small, while the SIA term becomes dominant for infinitely thin wells, which correspond to high junction fields.

By expanding the above formulas for small $k << \langle k_z \rangle$ it is seen that the spin splitting of the HH bands is k^3 whereas the spin splitting of the LH bands is k, in agreement with [13, 17]

$$E_{+}^{HH} - E_{-}^{HH} = \frac{3}{8}\frac{\alpha(\alpha^2 - 4\frac{\gamma_2^2}{m^2}\langle k_z^2 \rangle)}{\frac{\gamma_2^2}{m^2}\langle k_z^2 \rangle^2}k^3 + \mathcal{O}(k^5)$$
$$E_{+}^{LH} - E_{-}^{LH} = 2\alpha k + \mathcal{O}(k^3) \quad (4)$$

Figure [1] gives a typical band structure for GaAs ($\gamma_1 = 6.92,\ \gamma_2 = 2.1$) with a Γ point gap of $40meV$ and a Fermi momentum splitting of the hole band at Fermi momentum ($0.2nm^{-1}$) of $5meV$, which require a SIA splitting $\alpha \approx 10^5 m/s$.

We can expand the second term in the anisotropic Luttinger hamiltonian in terms of Clifford algebra of Dirac Γ matrices $\{\Gamma^a, \Gamma^b\} = 2\delta_{ab} I_{4\times 4}$ $(a, b = 1, ..., 5)$[4]. Since $\sqrt{\langle k_z^2 \rangle} = 0$ and $\langle k_z^2 \rangle \neq 0$ we see that the effect of confinement renders the d^a's of [4]:

$$(\vec{k} \cdot \vec{S})^2 = d_a \Gamma^a;\ \ d_1 = 0,\ d_2 = 0,\ d_3 = -\sqrt{3}k_x k_y,$$
$$d_4 = -\frac{\sqrt{3}}{2}(k_x^2 - k_y^2),\ d_5 = -\frac{1}{2}(2\langle k_z^2 \rangle - k_x^2 - k_y^2) \quad (5)$$

Since calculation with the full Hamiltonian (2) is analytically impossible, we concentrate on different situations which maintain analytic predictability. We now consider the case of of small junction field and neglect the SIA term:

$$H = \frac{\gamma_1}{2m}(k_x^2 + k_y^2 + \langle k_z^2 \rangle) + \frac{\gamma_2}{m} d_a \Gamma^a \quad (6)$$

The energies are $E_{LH,HH} = \frac{\gamma_1}{2m}(k^2 + \langle k_z^2 \rangle) \pm d$, ($d = \sqrt{d_a d_a} = \sqrt{k^4 + \langle k_z^2 \rangle^2 - \langle k_z^2 \rangle k^2}$). At $k = 0$ the heavy and light hole bands are split by a gap of $\Delta E = 2\frac{\gamma_2}{m}\langle k_z^2 \rangle \approx 2\frac{\gamma_2}{m}(\pi\hbar/a)^2$. In the experiment recently reported [3], this energy gap is of order $\Delta E = 40meV$, which corresponds to an $a = 8.3nm$ thick quantum well. The value quoted in the experiment is roughly $3 - 4nm$, making our simplistic prediction rather accurate.

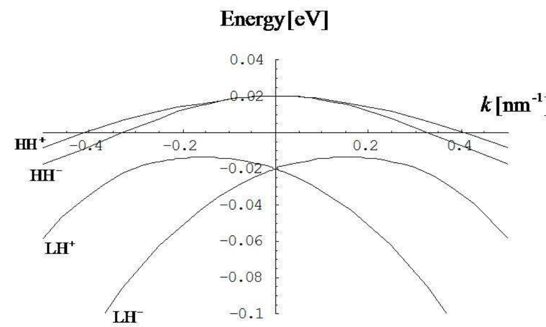

FIG. 1: Approximate band structure of the 2DHG ($\Delta = 40meV$, and the spin splitting of the heavy hole band at the k_F is around $5meV$). The confinement produces a Γ point gap between the light and heavy hole bands, whereas the SIA produces splitting in the previously degenerate Kramers doublets - the heavy and light hole bands.

We would like to compute the response of spin current $J_i^l = \frac{1}{2}\{S^l, \frac{\partial H}{\partial k_j}\}$ to an electric current $J_j = \frac{\partial H}{\partial k_j}$:

$$Q_{ij}^l(i\nu_m) = -\frac{1}{V}\int_0^\beta \langle T J_i^l(u) J_j \rangle e^{i\nu_m u} du \quad (7)$$

The spin conductance is then defined as $\sigma_{ij}^l = \lim_{\omega \to 0} \frac{Q_{ij}^l(\omega)}{-i\omega}$ and gives:

$$\sigma_{ij}^l = \frac{1}{V}\sum_k \frac{n_+^F - n_-^F}{d^3}\eta_{ab}^l \left[2\frac{m}{\gamma_2} d_b \frac{\partial d_a}{\partial k_j}\frac{\partial \varepsilon}{\partial k_i} + \epsilon_{abcde} d_e \frac{\partial d_c}{\partial k_i}\frac{\partial d_d}{\partial k_j}\right] \quad (8)$$

where η^l_{ab} is a tensor defined in [4], $n^F_\pm$ are the Fermi functions of the two bands and $\varepsilon = \frac{\gamma_1}{2m}(k_x^2 + k_y^2 + \langle k_z^2 \rangle)$ is the kinetic energy. The last term is the conserved spin conductance [4] (which represents the response of the spin projected onto the HH and LH bands [4]), whereas the first term is the contribution of the non-conserved part of the spin. Upon momentum integration, since motion along the z axis is prohibited due to confinement, the only non-zero components are $\sigma^3_{12} = -\sigma^3_{21}$ which yield:

$$\sigma^3_{12} = \frac{1}{4\pi}\left(\frac{3}{2}\frac{\gamma_1}{2\gamma_2}\left[\frac{2(k^2+\langle k_z^2\rangle)}{3\sqrt{k^4+\langle k_z^2\rangle^2-\langle k_z^2\rangle k^2}} - \ln[2\sqrt{k^4+\langle k_z^2\rangle^2-\langle k_z^2\rangle k^2}+2k^2-\langle k_z^2\rangle]\right] + \frac{2\langle k_z^2\rangle - k^2}{\sqrt{k^4+\langle k_z^2\rangle^2-\langle k_z^2\rangle k^2}}\right)\Bigg|_{k=k_{HH}}^{k=k_{LH}} \quad (9)$$

where k_{LH}, k_{HH} are the fermi momenta of the light and heavy hole bands. For the experimental data [3], the light hole band is fully occupied, so $k_{LH} = 0$ while $\sqrt{\langle k_z^2 \rangle} = 3.7 \times 10^{-26} kg\ m/s$ and $k_{HH} = 3 \times 10^{-26} kg\ m/s$. The first two terms are due to the non-conserved spin and for $GaAs$, $\sigma^{3(noncons)}_{1,2} = \frac{0.7}{8\pi}$. The last term is the conserved spin conductance $\sigma^{3(cons)}_{12} = 0.6 \times \frac{1}{4\pi}$. The total spin conductance is therefore $\sigma^3_{12} = \frac{1.9}{8\pi}$, in good agreement with the numerical estimate in [3]. In the case of infinite confinement, $\sqrt{\langle k_z^2\rangle} \to \infty$ the spin conductance from the Luttinger term vanishes, as it should, since we would then enter a SIA dominated regime.

We now investigate the effect of disorder on the Luttinger spin Hall conductance. In particular, we want to find out the vertex correction. The free Green's function in our system is defined as $G_0(\mathbf{k}, E) = [E - H]^{-1}$:

$$G_0(\mathbf{k}, i\omega_n) = \frac{i\omega_n - \epsilon(\mathbf{k}) + \frac{\gamma_2}{m} d_a \Gamma_a}{(i\omega_n - \epsilon(\mathbf{k}))^2 - \gamma_2^2 d^2/m^2} \quad (10)$$

We model the disorder as randomly distributed, spin-independent identical defects $V(\mathbf{r}) = u \sum_i \delta(\mathbf{r} - \mathbf{R}_i)$. In the Born approximation, the self-energy is related to the free Green function $\Sigma(i\omega_n) = n_{imp} u^2 \int \frac{d\mathbf{k}}{(2\pi)^2} G_0(\mathbf{k}, i\omega_n)$ Since $\int d\mathbf{k} d_3 = \int d\mathbf{k} d_4 = 0$, the self energy is an isotropic function of $\vec{k}$:

$$\Sigma(i\omega_n) = n_{imp} u^2 \int \frac{d\mathbf{k}}{(2\pi)^2} \frac{i\omega_n - \epsilon(\mathbf{k}) + \frac{\gamma_2}{m} d_5 \Gamma_5}{(i\omega_n - \epsilon(\mathbf{k}))^2 - \gamma_2^2 d^2/m^2} \quad (11)$$

where $d_5 = -\frac{1}{2}(2\langle k_z^2 \rangle - k^2)$. This is different from the bulk Luttinger case, where the $d_5(k)$ integral over $\vec{k}$ vanishes as well, but the difference is not essential. The full impurity Green function is $G(\mathbf{k}, i\omega_n) = G_0(\mathbf{k}, i\omega_n + \Sigma(i\omega_n))$. The current vertex satisfies a Bethe-Salpeter equation similar to [17]. Similar to the case of [11], since the Green's function is an even function of the in-plane total momentum, while the charge current operator $V_j = \partial H / \partial k_j$ is momentum-odd in the components k_j (because the Hamiltonian H is even in $\vec{k}$, it turns out that the free vertex cancels

$$\int \frac{d\mathbf{k}}{(2\pi)^2} G(\mathbf{k}, i\omega_n) V_j(\mathbf{k}) G(\mathbf{k}, i\omega_n - i\nu_m) = 0 \quad (12)$$

And hence the vertex correction which is an iterative function of the free vertex vanishes as well [11].

To see the effect of a very small SIA splitting ($\alpha k_F << \Delta$, $\alpha k_F << \hbar/\tau$) on the Luttinger spin Hall conductance, we treat it in perturbation theory. The calculation is long and not particularly revealing, so we just give the result. Due to the fact that the Luttinger current operator is odd in $\vec{k}$ while the SIA current operator is a constant matrix, the first order contribution in α vanishes. The first nonzero contribution is of order α^2.

We now turn to the opposite case, of strongly confined quantum wells, in which the SIA term is likely to dominate. We model the system by a Gamma point gap Δ plus a spin 3/2 Rashba term $\alpha(\vec{k} \times \vec{S})\hat{z}$ [18]. We compute the spin conductance and expand it in terms of the ratio between the SIA spin splitting and the Γ point gap, $\frac{\alpha k_f}{\Delta} < 1$. The spin conductance gets a contribution from the HH band:

$$\sigma^{3(HH)}_{12} = \frac{9}{8\pi}(1 + \frac{\alpha^2 m_{HH}}{2\Delta}) \quad (13)$$

For infinitely thin quantum wells, $\Delta \to \infty$, the HH spin conductance is $9/8\pi$ which is (besides a re-scaling factor of 2 in the spin current definition), the same as that obtained in [14] who studied the hole gas by starting with the effective HH Hamiltonian directly. The second term in Eq[13] is the first order finite thickness correction. If the Fermi level is low enough, there is also a light-hole band contribution to the spin conductance of order $\sigma^{3(LH)}_{12} = \frac{1}{8\pi}(1 + \frac{3\alpha^2 m_{LH}}{2\Delta})$. The vertex correction for a spin-3/2 Rashba-like system has been computed in $3D$ perturbatively to first order in α and was found to be finite [19]. We now compute it exactly for the $2D$ case in the heavy hole band.

Since working with spin 3/2 matrices is cumbersome and we do not need the LH states as they are fully filled [3], we now project the system onto the heavy hole states and work with the truncated Hamiltonian [13, 14]:

$$H = \frac{k^2}{2m} + \beta(k_-^3\sigma_+ - k_+^3\sigma_-) \tag{14}$$

which becomes exact in the limit of very confined quantum wells. The spin Hall conductance in the disorder-free case is $9/8\pi$, as previously obtained in [14] and as obtained above as the limit of strongly confined quantum wells. The Hamiltonian can also be expressed:

$$H = \frac{k^2}{2m} + \lambda_i(k)\sigma_i, \quad i = x, y \tag{15}$$

Where $\lambda_1 = \beta k_y(3k_x^2 - k_y^2)$ and $\lambda_2 = \beta k_x(3k_y^2 - k_x^2)$, with β a constant. Let $\lambda(k) = \sqrt{\lambda_i\lambda_i}$. The Fermi sphere is isotropic since the energy levels are $E_\pm = \frac{k^2}{2m} \pm \lambda$. The disorder-free Green function is:

$$G_0(\mathbf{k}, i\omega_n) = \frac{1}{2}\sum_{s=\pm}\frac{1 + s\hat{\lambda}_i\sigma_i}{i\omega_n - E_s} \tag{16}$$

where $\hat{\lambda}_i = \lambda_i/\lambda$. The self energy for s-wave scattering of electrons becomes a state-independent constant (not a matrix) $\Sigma(i\omega_n) = n_{imp}u^2\int\frac{d\mathbf{k}}{(2\pi)^2}G_0(\mathbf{k}, i\omega_n)$ where n_{imp} is the density of impurities while u is the impurity potential strength . Since λ_i are odd functions of the momentum components k_i, the integral $\int d\mathbf{k} f(k)\lambda_i = 0$ where $f(k)$ is any isotropic function of k. Since the spin orbit coupling small (much smaller than the Fermi energy), the density of states at zero order is a constant $D = m/2\pi\hbar^2$ while the αk^3 term in the Hamiltonian contributes to with only a first order correction. The full Green function in the presence of impurities is $G(\mathbf{k}, i\omega_n) = G_0(\mathbf{k}, i\omega_n + \Sigma(i\omega_n))$. The spin dependent part of the charge current operator $V_j(k) = \partial H/\partial k_j$ turns out to have d-wave symmetry (for example, the spin dependent part of the V_x operator reads $6\beta k_x k_y\sigma_x + 3\beta(k_y^2 - k_x^2)\sigma_y$) and it vanishes when integrated over the isotropic Fermi surface. This is the deep intuitive reason as to why the vertex correction cancels in this case, as we rigorously show below. By contrast, in the electron-band Rashba case, the spin-dependent part of the charge operator is a constant. The current vertex function $K_j(\mathbf{k}, i\omega_n, i\nu_m) = \langle G(\mathbf{k}, i\omega_n)V_j(k)G(\mathbf{k}, i\omega_n + i\nu_m)\rangle$ is a matrix function that does not commute with either the charge current operator or the Green's function. $\langle ...\rangle$ is an impurity average. It satisfies the Bethe-Salpeter equation:

$$K_j(\mathbf{k}, i\omega_n, i\nu_m) = G(\mathbf{k}, i\omega_n)V_j(\mathbf{k})G(\mathbf{k}, i\omega_n - i\nu_m) + n_{imp}u^2 G(\mathbf{k}, i\omega_n)\int\frac{d\mathbf{q}}{(2\pi)^2}K_j(\mathbf{q}, i\omega_n, i\nu_m)G(\mathbf{k}, i\omega_n - i\nu_m) \tag{17}$$

Integrating both the right and the left hand side over the momentum $\mathbf{k}$, we see that the vertex correction $\Delta V_j(i\omega_n, i\nu_m) = \int\frac{d\mathbf{q}}{(2\pi)^2}K_j(\mathbf{q}, i\omega_n, i\nu_m)$ satisfies:

$$\Delta V_j = \int\frac{d\mathbf{k}}{(2\pi)^2}G(\mathbf{k}, i\omega_n)V_j(\mathbf{k})G(\mathbf{k}, i\omega_n - i\nu_m) + \\ + n_{imp}u^2\int\frac{d\mathbf{k}}{(2\pi)^2}G(\mathbf{k}, i\omega_n)\Delta V_j G(\mathbf{k}, i\omega_n - i\nu_m) \tag{18}$$

Since the vertex correction $\Delta V_j(i\omega_n, i\nu_m)$ is a 2×2 matrix, it can be decomposed in the basis of the identity matrix and the 3 pauli matrices:

$$\Delta V_j(i\omega_n, i\nu_m) = \sum_{\mu=0}^{3}\Lambda_j^\mu(i\omega_n, i\nu_m)\sigma^\mu, \quad \mu = 0, 1, 2, 3 \tag{19}$$

where $\sigma^0 = I_{2\times2}$, the identity matrix, and $\sigma^{1,2,3}$ are the 3 Pauli matrices. The $\Lambda_j^\mu(i\omega_n, i\nu_m)$ are scalars. By introducing the decomposition in the vertex equation, multiplying to the left of both sides of the equal by a σ_α matrix and taking the trace of the above equation, we obtain:

$$2\Lambda_j^\nu = A_j^\nu(i\omega_n, i\nu_m) + \sum_{\mu=0}^{3}\Lambda_j^\mu M^{\nu\mu}(i\omega_n, i\nu_m)$$
$$M^{\nu\mu} = n_{imp}u^2\int\frac{d\mathbf{k}}{(2\pi)^2}Tr[\sigma^\nu G(\mathbf{k}, i\omega_n)\sigma^\mu G(\mathbf{k}, i\omega_n - i\nu_m)]$$
$$A_j^\nu = \int\frac{d\mathbf{k}}{(2\pi)^2}Tr[\sigma^\nu G(\mathbf{k}, i\omega_n)V_j(\mathbf{k})G(\mathbf{k}, i\omega_n - i\nu_m)] \tag{20}$$

By expanding and evaluating $M^{\nu\mu}$ (this uses the observation that $\int d\mathbf{k}\lambda^i(k)\lambda^j(k) \sim \delta_{ij}$ as well as $G_s^R G_s^A = \frac{2\pi\tau}{\hbar}\delta(\epsilon - E_s)$, where R, A stand for the retarded and advanced Green's functions, and $\tau = \hbar^3/n_{imp}u^2 m$) we observe that it is diagonal in μ, ν, that is $M^{\nu\mu} = \delta_{\nu\mu}$. Expanding the traces in Eq.(20), and since $\lambda_0(k) = \lambda_3(k) = 0$ it is easy to observe that (after azimuthal integration) $A_j^0(i\omega_n, i\nu_m) = A_j^3(i\omega_n, i\nu_m) = 0$ and hence the vertex corrections $\Lambda_j^0(i\omega_n, i\nu_m) = \Lambda_j^3(i\omega_n, i\nu_m) = 0$. We now have for the vertex correction $\Lambda_j^\nu(i\omega_n, i\nu_m) = A_j^\nu(i\omega_n, i\nu_m), \quad \nu = 1, 2$ and $j = x, y$ where after expanding the traces:

$$A_j^\nu = \sum_{s,s'=\pm} \int \frac{d\mathbf{k}}{(2\pi)^2} \frac{(s+s')\frac{k_j}{m}\hat{\lambda}_\nu + 2ss'\hat{\lambda}_\nu \frac{\partial\lambda}{\partial k_j} + (1-ss')\frac{\partial\lambda_\nu}{\partial k_j}}{2(z-E_s)(z'-E_{s'})} \tag{21}$$

with $z = i\omega_n + \Sigma(i\omega_n)$, $z' = i\omega_n - i\nu_m + \Sigma(i\omega_n - i\nu_m)$. We now compute this for $\nu = 1$, the case $\nu = 2$ being identical. Let $j = 1$ and we find for the numerator of the integrand in Eq(21):

$$\frac{k_y k_x(3k_x^2 - k_y^2)}{k^3}\left[(s+s')\frac{1}{m} + 6ss'\beta k\right] + 6\beta k_x k_y(1-ss') \tag{22}$$

Upon integration over $d\mathbf{k}$ the above expression vanishes due to the integral over the azimuthal angle and hence $A_1^1(i\omega_n, i\nu_m) = 0$. For the case $\nu = 1, j = 2$, the numerator of Eq(21) gives:

$$\frac{k_y^2(3k_x^2 - k_y^2)}{k^3}\left[(s+s')\frac{1}{m} + 6ss'\beta k\right] + 3\beta(k_x^2 - k_y^2)(1-ss') \tag{23}$$

which also vanishes upon azimuthal angle integration $A_2^1(i\omega_n, i\nu_m) = 0$. In an identical way all the components of the vertex correction tensor vanish.

We have analyzed the spin Hall transport in the case of a two-dimensional hole gas. We showed that for relative weak confinement the spin-Hall conductance is of Luttinger type and is equal to roughly $1.9e/8\pi$ for the value of parameters in [3]. For strongly confined quantum wells, the system is dominated by a structural inversion asymmetry term of spin-3/2 SIA-type. The spin conductance for this system is $9e/8\pi$ plus a correction dependent on the quantum-well size. We perform the full vertex correction and show that it vanishes for both Luttinger and SIA cases. This is in striking contrast to the k-linear Rashba case, where the vertex correction is of the same magnitude and of opposite sign to the spin orbit coupling strength. Coupled with the fact that the life time broadening is smaller than the spin splitting, we hence conclude that the spin Hall effect observed in [3] should be in the intrinsic regime. The dissipationless spin Hall conductance can be systematically determined by extrapolating the ratio of life time broadening and the spin splitting to zero.

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