SLAC-PUB-13906

Maxwell Equation for the Coupled Spin-Charge Wave Propagation

B. Andrei Bernevig, Xiaowei Yu and Shou-Cheng Zhang
Department of Physics, Stanford University, Stanford, California 94305

We show that the dissipationless spin current in the ground state of the Rashba model gives rise to a reactive coupling between the spin and charge propagation, which is formally identical to the coupling between the electric and the magnetic fields in the $2+1$ dimensional Maxwell equation. This analogy leads to a remarkable prediction that a density packet can spontaneously split into two counter propagation packets, each carrying the opposite spins. In a certain parameter regime, the coupled spin and charge wave propagates like a transverse "photon". We propose both optical and purely electronic experiments to detect this effect.

PACS numbers: 72.25.-b, 72.10.-d, 72.15. Gd

The generation and manipulation of spin current is essential to the rapidly developing field of spintronics[1]. For this purpose, the analogy between photonics and spintronics greatly helped our conceptual developments, and this analogy lead to the celebrated Das-Datta proposal for a spin-field transistor[2]. However, earlier attempts to realize this conceptual device were based on the rather incomplete analogy between the electron spin and the photon, and were plagued by many issues such as low spin injection rate and the requirement of the ballistic spin transport.

Recently, the remarkable phenomenon of the dissipationless spin current has been theoretically predicted[3]. A electric field E_k generates a spin current described by the response equation

$$j_j^i = \sigma_s \epsilon_{ijk} E_k \tag{1}$$

where j_j^i is the current of the i-th component of the spin along the direction j, ϵ_{ijk} is the totally antisymmetric tensor in three dimensions and the spin Hall conductivity σ_s does not depend on impurities. Since both the spin current and the electric field are even under the time reversal, this equation describes a reactive response which does not dissipate energy. One natural consequence of this equation is the intrinsic spin Hall effect[3,4], which has been recently observed experimentally in the hole doped systems[5]. Another consequence is the dissipationless spin current in the ground state[6]. In the above equation (1), the electric field can be either externally applied, or can be spontaneously generated in systems without inversion symmetry. In a two dimensional electron gas (2DEG), the confining potential along the z direction breaks the inversion symmetry, and leads to a internal electric field E_z in the ground state. According to equation (1), there is a spin current in the ground state, $j_j^i = j_0 \epsilon_{ij}$, where ϵ_{ij} is the antisymmetric symbol in two dimensions with $i, j = x, y$.

In this work, we shall show that the dissipationless spin current in the ground state makes the analogy between photonics and spintronics formally exact. In $2+1$ dimensions, the electric field has two components, while the magnetic field has only one component. If one identifies them with the in-plane components of the spin density and the charge density, respectively, the Boltzmann transport equation for the coupled spin and charge wave is formally the same as the Maxwell equation describing the electromagnetic fields, where the "speed of light" is given by the Rashba coupling constant. This behavior is in sharp contrast to the conventional Boltzmann equation for the decoupled spin and charge dynamics in semiconductors, where only purely diffusive, but no propagating motion is predicted[7]. The photonic analogy helps our understanding on how density gradient and time dependence can generate spin density, and leads to many novel predictions. We shall show that there is a parameter regime, reachable experimentally, where the coupled spin-charge wave propagates as a under-damped "photonic mode". A density packet will split spontaneously into two counter propagating packets, each carrying the opposite spins. This mechanism enables injection of spins and spin currents. The Boltzmann transport equations for the Rashba model have been studied previously in the diffusive region[10,11]. The coupled spin-charge wave propagation is a new result of this work.

A spin 1/2 Hamiltonian which includes spin orbit coupling can be written in the following general form:

$$H = \frac{p^2}{2m} + \lambda^i(p)\sigma^i, \quad i = x, y, z \tag{2}$$

where $\lambda^i(p)$ is a odd function of p, in order to preserve the time reversal symmetry. This includes a wide range of spin-orbit couplings, including 2D Rashba and Dresselhaus couplings and the 3D spin splitting of the conduction band in strained semiconductors[8]. The phase space density distribution function $n_F(p, r, t)$ and the energy matrix $\epsilon_F(p, r, t)$ are 2×2 matrices, and can be decomposed as:

$$\begin{aligned} n_F(p, r, t) &= n(p, r, t) + S^i(p, r, t)\sigma^i \\ \epsilon_F(p, r, t) &= \epsilon_s(p, r, t) + \epsilon_v^i(p, r, t)\sigma^i, \quad i = x, y, z \end{aligned} \tag{3}$$

In this letter we consider the system in the absence of external fields, such that $\epsilon_F(p, r, t) = p^2/2m + \lambda^i(p)\sigma^i$. The influence of electric and magnetic fields on the system is described in a future longer publication[9]. The Boltzmann equation reads:

SIMES, SLAC National Accelerator Center, 2575 Sand Hill Road, Menlo Park, CA 94309
Work supported in part by US Department of Energy contract DE-AC02-76SF00515.

$$\frac{\partial n_F(p,r,t)}{\partial t} - \frac{i}{\hbar}[\epsilon_F(p,r,t), n_F(p,r,t)] + \frac{1}{2}\{\frac{\partial \epsilon_F(p,r,t)}{\partial p_i}, \frac{\partial n_F(p,r,t)}{\partial r_i}\} - \frac{1}{2}\{\frac{\partial \epsilon_F(p,r,t)}{\partial r_i}, \frac{\partial n_F(p,r,t)}{\partial p_i}\} = \frac{n_F^{eq} - n_F(p,r,t)}{\tau} \tag{4}$$

where the right hand side is the collision term expressed in the relaxation time approximation, and n_F^{eq} is the equilibrium value of $n_F(p,r,t)$. τ is the momentum relaxation time. Although this approximation does not take into account the self-energy effects, it turns out to be qualitatively and quantitatively correct, as we see from comparison with the solution involving the self energy in some special cases of spin-orbit coupling[10,11]. We trace out the matrix dependence of the distribution function as well as that of the energy, and integrate the continuity and the current equations over the Fermi volume[9]. After linearization, we obtain[9]:

$$\frac{\partial n(r,t)}{\partial t} = D\frac{\partial^2 n(r,t)}{\partial r_i^2} - \frac{\partial \lambda^l(p)}{\partial p_i}\frac{\partial S^l(r,t)}{\partial r_i} \tag{5}$$

$$\frac{\partial S^k(r,t)}{\partial t} = D\frac{\partial^2 S^k(r,t)}{\partial r_i^2} - \frac{\partial \lambda^k(p)}{\partial p_i}\frac{\partial n(r,t)}{\partial r_i} + \frac{4mD}{\hbar}\epsilon_{ijk}\frac{\partial \lambda^i}{\partial p_r}\frac{\partial S^j(r,t)}{\partial r_r} - \left(\frac{2m}{\hbar}\right)^2 D\left(\frac{\partial \lambda^i}{\partial p_r}\frac{\partial \lambda^i}{\partial p_r}S^k(r,t) - \frac{\partial \lambda^i}{\partial p_r}\frac{\partial \lambda^k}{\partial p_r}S^i(r,t)\right) \tag{6}$$

where $D = \frac{\langle p_F^2\rangle \tau}{2m^2}$ is the diffusion constant and $\mu = \frac{e\tau}{m}$ is the mobility, and the carriers are electrons of charge $-e$. Aside from the self energy renormalizations, Eq.[6] gives the same result as[10,11] when particularized to the Rashba-spin-orbit coupling. The last term in the spin continuity equation represents the spin relaxation due to Dyakonov-Perel (DP) mechanism[12]. In the case of Rashba systems, the spin orbit coupling is $\lambda^i = \alpha\epsilon_{ijz}p_j$, α has the dimension of velocity, and the continuity equations become (from now on we use $\partial_i = \partial/\partial r_i$ and $\partial_t = \partial/\partial t$) :

$$\begin{aligned}
&\partial_t n = D\partial_i^2 n - \alpha\epsilon^{liz}\partial_i S^l \\
&\partial_t S^k = D\partial_i^2 S^k - \alpha\epsilon^{kiz}\partial_i n + \sqrt{\frac{D}{\tau_s}}(\delta_{kz}\partial_i S^i - \partial_k S^z) - \\
&-\frac{1}{\tau_s}(S^k + \delta_{kz}S^z)
\end{aligned} \tag{7}$$

where $r_i = (x,y)$ $(i = 1,2)$ since charge and spin motion is now confined entirely to the 2D plane. Within the current microscopic approximation, the DP spin relaxation time is given by $\tau_s^{-1} = (\frac{2m\alpha}{\hbar})^2 D$, however, in the subsequent discussions, we shall treat D and τ_s as independent, phenomenological parameters.

Let $S^\mu = (S^k, S^z)$, $\mu = x,y,z$, $k = x,y$. We can write the two dimensional vector $S^k(r,t)$, $k = x,y$ in the most general form as a sum of a longitudinal vector $S_L^k(r,t)$ and a transversal vector $S_T^k(r,t)$:

$$S^k = S_L^k + S_T^k; \quad \partial_k S_T^k = 0, \quad \epsilon_{ij}\partial_i S_L^j = 0 \tag{8}$$

Substituting this decomposition into Eq. (7), we find two sets of coupled equations:

$$\begin{aligned}
&\partial_t n = D\partial_i^2 n - \alpha\epsilon_{ki}\partial_i S_T^k \\
&\partial_t S_T^k = D\partial_i^2 S_T^k - \alpha\epsilon_{ki}\partial_i n - \frac{1}{\tau_s}S_T^k \\
&\partial_k S_T^k = 0
\end{aligned} \tag{9}$$

and

$$\begin{aligned}
&\partial_t S_L^k = D\partial_i^2 S_L^k - \sqrt{\frac{D}{\tau_s}}\partial_k S^z - \frac{1}{\tau_s}S_L^k \\
&\partial_t S^z = D\partial_i^2 S^z + \sqrt{\frac{D}{\tau_s}}\partial_k S_L^k - 2\frac{1}{\tau_s}S^z \\
&\epsilon_{ij}\partial_i S_L^j = 0
\end{aligned} \tag{10}$$

Hence the charge density couples only to the transverse spin component, while S^z couples only to the longitudinal spin component in a purely diffusive fashion[11]. In the spin continuity equation of Eq. (9), we see that the α term is nothing but the divergence of the dissipationless spin current in the ground state $j_i^k = \alpha\epsilon_{ki}n$. We shall see that this term plays the crucial role leading to the coupled spin-charge propagation.

At this point we come to a remarkable realization that the Boltzmann equation (9) for the coupled charge and transverse spin transport is exactly the Maxwell's equation in $2+1$ dimensions! In order to facilitate the comparison, let us first focus on the large spin-orbit coupling limit, where we neglect the D and the $\frac{1}{\tau_s}$ terms in Eq. (9). In $2+1$ dimensions, the source-free Maxwell equations are given by

$$\partial_\nu F_{\mu\nu} = j_\mu = 0 \tag{11}$$

$$\epsilon^{\mu\nu\rho}\partial_\mu F_{\nu\rho} = 0 \tag{12}$$

where $\mu = (0, x, y)$. In $2+1$ dimensions, the magnetic field has only one component, given by $B_z = F_{xy}$, and the two components of the electric field are given by $E_i = F_{0i}$. If we make the identification $n \to B_z$, $S_T^i \to E_i$ and $\alpha \to c$, we see that the three Boltzmann transport equations in Eq. (9) are exactly the three Maxwell's equations in the vacuum of $2+1$ dimensions, namely the Faraday's law of induction, the Ampere-Maxwell law, and the Gauss's law. More generally, the $\frac{1}{\tau_s} S_T^k$ term can be interpreted as light propagation in a metallic media, with the current density in the Ampere's law given by the Ohm's law, and the D terms can be interpreted as due to light diffusion in a random media.

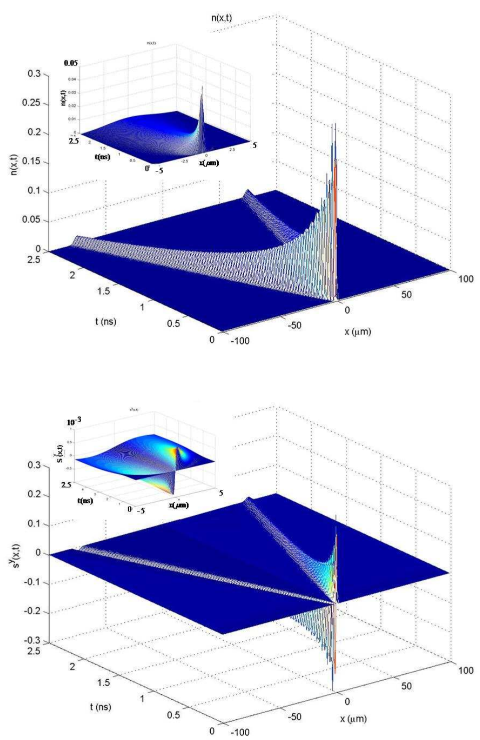

FIG. 1: Charge and spin density for $\tau_s = 1ns$, $\alpha = 3\times10^4 m/s$ and $D = 10^{-3} m^2/s$. We see propagation over distances of more than $100\mu m$. **Inset:** Charge and spin density in the diffusive regime, for small values of α ($\tau_s = 1ns$, $\alpha = 10^2 m/s$ and $D = 10^{-3} m^2/s$) has the typical Gaussian decay.

In conventional theories without spin-orbit coupling, the electron transport semiconductors is purely diffusive. However, we see that in the limit of strong spin-orbit coupling, there is a regime where a propagating, coupled spin-charge wave mode is possible. If we neglect the diffusion and the lifetime terms for the time being, we find that the most general solution to the initial condition of $n(x, y, t = 0) = f(x)$ and $S_T^i(x, y, t = 0) = 0$ is given by:

$$n(x,t) = \frac{1}{2}(f(x+\alpha t) + f(x-\alpha t))$$
$$S_T^y(x,t) = \frac{1}{2}(f(x+\alpha t) - f(x-\alpha t)) \qquad (13)$$

We see that an initial density wave packet spontaneously splits into two counter-propagating packets, each carrying the opposite spin. This phenomenon can be elegantly interpreted in the "photonic" language. In $2+1$ dimensions, the magnetic field is always pointing along the $+\hat{z}$ direction. Since the propagation vector $\mathbf{k}$, being proportional to the Poynting vector, is given by $\mathbf{k} \propto \mathbf{E} \times \mathbf{B}$, it uniquely determines the direction of the transverse electric field. Translating from the "photonic" language into the "spintronic" language, we see that the mode propagating along the $+\hat{x}$ has spins along the $+\hat{y}$ direction, while the mode propagating along the $-\hat{x}$ has spins along the $-\hat{y}$ direction. The split wave packets carries a spin current J_x^y, which is a reflection of the spin current in the ground state of the Rashba model. For a simple estimate, $\alpha = 3 \times 10^4 m/s$, and hence the mode will cross a sample of $1\mu m$ length in $30ps$. Considering that the spin coherence time in these samples can be larger than $1ns$, it means that the propagation time over $1\mu m$ distance is well shorter than 30-th part of the spin relaxation time and can hence be very useful for spin manipulation.

We now consider the more general situation including diffusion and relaxation. We suppose that a one dimensional stripe of charge density has been created, say by transient grating[13]. The initial density is given by $n(x, y; t = 0) = \delta(x)$. The solution to the full equations give $S^z(x,t) = S^y(x,t) = 0$ while $S^y(x,t)$ is generated by the spin-orbit coupling:

$$n(x,t) = \int\int \frac{1}{(2\pi)^2} \frac{i\omega + Dq^2 + \frac{1}{\tau_s}}{-(\omega-\omega_1)(\omega-\omega_2)} e^{i(\omega t - qx)} d\omega dq \qquad (14)$$

$$S^y(x,t) = \int\int \frac{1}{(2\pi)^2} \frac{i\alpha q}{(\omega-\omega_1)(\omega-\omega_2)} e^{i(\omega t - qx)} d\omega dq \qquad (15)$$

where ω_1, ω_2 are the characteristic frequencies of the system:

$$\omega_{1,2} = i(Dq^2 + \frac{1}{2\tau_s}) \pm \sqrt{\alpha^2 q^2 - \frac{1}{(2\tau_s)^2}} \qquad (16)$$

We recognize the propagating mode inside the square root. For momenta $q > 1/2\tau_s\alpha$ both characteristic frequencies contain real parts and hence describe propagating waves. However, q must not be as large as to cause damping due to the term $Dq^2 t$. The condition for this gaussian damping to be small is $Dq^2\tau_s < 1$ for $q \sim 1/\tau_s\alpha$. Therefore, the condition for the regime where a propagating mode could exist is then given by:

$$\alpha > \sqrt{\frac{D}{\tau_s}} \qquad (17)$$

This condition can be satisfied in samples where $\alpha = 3 \times 10^4 m/s$ and $D = 10^{-3} m^2/s$, with τ_s longer than $1ns$[14]. In this case, $\sqrt{\frac{D}{\tau_s}} = 10^3$, much smaller than α.

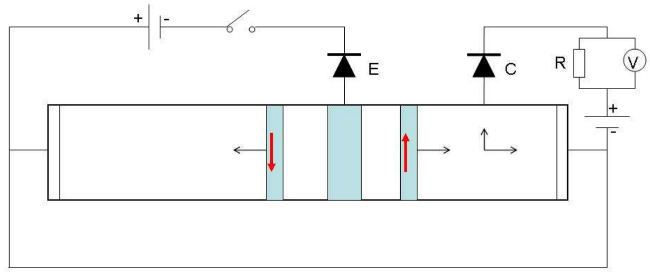

FIG. 2: A modified version of the classic Haynes-Shockley experiment. A density packet injected by the emitter spontaneously splits into two counter propagating packets with opposite spin. Unlike the settings of the Haynes-Shockley experiment, one of the two packets propagates to the collector without experiencing a sweeping electric field. The time delay between the injection pulse the the collecting pulse gives a purely electric determination of the Rashba spin-orbit coupling constant.

In the limit of very long $\tau_s \to \infty$ the integrals can be solved exactly. We give the expression of S^y in this limit:

$$S^y(x,t) = \frac{1}{4\sqrt{\pi}} \frac{1}{\sqrt{Dt}} \left[e^{-\frac{(\alpha t - x)^2}{4Dt}} - e^{-\frac{(\alpha t + x)^2}{4Dt}} \right] \qquad (18)$$

The propagating mode is $\alpha t \pm x = 0$ where either one of the damping gaussian exponentials becomes unity. The spin symmetry is odd in x, the spins propagating in the positive and negative x axis directions having opposite polarization. Note that for diminishing spin-orbit coupling $\alpha \to 0$ the spin density also vanishes, as it should. For finite τ_s in a stationary phase-type approximation the spin-density solution above gets multiplied by an exponential factor $\exp(-t/2\tau_s)$. Impressively, both spin and charge can propagate over distances well in excess of $100\mu m$ and for times well in excess of $10\tau_s$, (the full time scale is not plotted in Fig[1]).

We now propose several experiments to test the coupled spin-charge wave predicted in this work. One could inject the density packet optically, and detect the splitting of the density packet and the associated spin orientation by optical Kerr rotation. One could also detect the spin orientation through the circularly polarized luminance from the recombination with the majority carriers. Alternatively, one could detect the propagation of the density packet purely electrically, by a modified version of the classic Haynes-Shockley experiment[?] . Fig. (2) describes a narrow sample with light p-doping. Two rectifying metal-to-semiconductor point contacts are forward and reverse biased, respectively, to serve as emitter and collector electrodes. After turning on the emitter pulse, a electron density packet is injected into the sample. In conventional Haynes-Shockley setup, the electron packet would be swept to the collector electrode by a electric field. In our case, no sweeping electric field is applied, but the density packet will spontaneously split into two counter propagating packets with the opposite spin orientation, with a velocity directly given by the Rashba coupling constant α. When the right moving packet is captured by the collector electrode, a voltage pulse is registered. From the time-delay and the shape of the voltage pulse, one can determine the Rashba coupling constant and the diffusion constant by purely electric means. This experiment illustrates the fact that the injected density pulse can take advantage of the spin current in the ground state, and propagate without any applied voltage.

I. ACKNOWLEDGEMENTS

The authors want to thank D. Awschalom, H.D. Chen, Y. Kato, J. Orenstein and J. Zaanen for useful discussions. B.A.B. acknowledges support from the Stanford Graduate Fellowship Program. This work is supported by the NSF under grant numbers DMR-0342832 and the US Department of Energy, Office of Basic Energy Sciences under contract DE-AC03-76SF00515.

[1] S.A. Wolf *et al*, Science **294**, 1488 (2001).
[2] S. Datta and B. Das, Appl. Phys. Lett. **56**, 665 (1990)
[3] S. Murakami, N. Nagaosa, and S.C. Zhang, Science **301** , 1348 (2003)
[4] J. Sinova *et al*, Phys. Rev. Lett. textbf92, 126603 (2004)
[5] J. Wunderlich *et al*, cond-mat/0410295
[6] E.I. Rashba, Phys. Rev. B **68**, 241315 (2003)
[7] I. Zutic, J. Fabian, S. Das Sarma, Rev. Mod. Phys. **76**, 323 (2004)
[8] B. A. Bernevig and S.C. Zhang, cond-mat/0408442, submitted to PRB
[9] B. A. Bernevig, X. Yu, S.C. Zhang, in preparation
[10] E.G. Mishchenko, A.V. Shytov, B.I. Halperin, cond-mat/0406730
[11] A.A. Burkov, A.S. Nunez, A.H. MacDonald, Phys. Rev. B. **70**, 155308 (2004)
[12] M.I. Dyakonov and V.I. Perel, Sov. Phys. JETP **33**, 467 (1971)
[13] N. Gedik *et. al.* Science **300**, 1410 (2003)
[14] Y. Kato *et. al.*, cond-mat 0403407
[15] J. Nitta *et. al.* Phys. Rev. Lett. **78**, 1335 (1997)