

Nonperturbative QCD Coupling and its β -function from Light-Front Holography

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Abstract

The light-front holographic mapping of classical gravity in AdS space, modified by a positive-sign dilaton background, leads to a non-perturbative effective coupling $\alpha_s^{AdS}(Q^2)$. It agrees with hadron physics data extracted from different observables, such as the effective charge defined by the Bjorken sum rule, as well as with the predictions of models with built-in confinement and lattice simulations. It also displays a transition from perturbative to nonperturbative conformal regimes at a momentum scale ~ 1 GeV. The resulting β -function appears to capture the essential characteristics of the full β -function of QCD, thus giving further support to the application of the gauge/gravity duality to the confining dynamics of strongly coupled QCD. Commensurate scale relations relate observables to each other without scheme or scale ambiguity. In this paper we extrapolate these relations to the nonperturbative domain, thus extending the range of predictions based on $\alpha_s^{AdS}(Q^2)$.

I. INTRODUCTION

The concept of a running coupling $\alpha_s(Q^2)$ in QCD is usually restricted to the perturbative domain. However, as in QED, it is useful to define the coupling as an analytic function valid over the full space-like and time-like domains. The study of the non-Abelian QCD coupling at small momentum transfer is a complex problem because of gluonic self-coupling and color confinement. Its behavior in the non-perturbative infrared regime has been the subject of intensive study using Dyson-Schwinger equations (DSE) and Euclidean numerical lattice computation, [1] since it is a quantity of fundamental importance. We will show that the light-front holographic mapping of classical gravity in AdS space, modified by a positive-sign dilaton background $\exp(+\kappa^2 z^2)$, leads to a non-perturbative effective coupling $\alpha_s^{AdS}(Q^2)$ which is in agreement with hadron physics data extracted from different observables, as well as with the predictions of models with built-in confinement and lattice simulations.

The AdS/CFT correspondence [2] between a gravity or string theory on a higher dimensional Anti-de Sitter (AdS) space-time and conformal gauge field theories (CFT) in physical space-time has brought a new set of tools for studying the dynamics of strongly coupled quantum field theories, and it has led to new analytical insights into the confining dynamics of QCD. The AdS/CFT duality provides a gravity description in a $(d + 1)$ -dimensional AdS space-time in terms of a flat d -dimensional conformally-invariant quantum field theory defined at the AdS asymptotic boundary. [3] Thus, in principle, one can compute physical observables in a strongly coupled gauge theory in terms of a classical gravity theory.

Since the quantum field theory dual to AdS₅ space in the original correspondence [2] is conformal, the strong coupling of the dual gauge theory is constant, and its β -function is zero. Thus, one must consider a deformed AdS space in order to have a running coupling $\alpha_s^{AdS}(Q^2)$ for the gauge theory side of the correspondence. We assume a positive-sign confining dilaton background to modify AdS space, a model which gives a very good account of meson and baryon spectroscopy and form factors. We use light-front (LF) holography [4–8] to map the amplitudes corresponding to hadrons propagating in AdS space to the frame-independent LF wave functions (LFWFs) of hadrons in physical 3 + 1 space. This analysis utilizes recent developments in LF QCD which have been inspired by the AdS/CFT correspondence. [2] The resulting LFWFs provide a fundamental description of the structure and internal dynamics of hadronic states in terms of their constituent quark and gluons.

The definition of the running coupling in perturbative quantum field theory is scheme-dependent. As discussed by Grunberg, [9] an effective coupling or charge can be defined directly from physical observables. Effective charges defined from different observables can be related to each other in the leading-twist domain using commensurate scale relations (CSR). [10] A more challenging problem is to relate such observables and schemes over the full domain of momenta. An important part of this paper will be the application and test of commensurate scale relations and their tentative extension to the non-perturbative domain. Another important application is related to the potential between infinitely heavy quarks, which can be defined analytically in momentum transfer space as the product of the running coupling times the Born gluon propagator: $V(q) = -4\pi C_F \alpha_V(q)/q^2$. This effective charge defines a renormalization scheme – the α_V scheme of Appelquist, Dine, and Muzinich. [11] In fact, the holographic coupling $\alpha_s^{AdS}(Q^2)$ can be considered to be the nonperturbative extension of the α_V effective charge defined in Ref. [11].

We shall also make extensive use of the g_1 scheme, where the strong coupling $\alpha_{g_1}(Q^2)$ is determined from the Bjorken sum rule. [12] The coupling $\alpha_{g_1}(Q^2)$ has the advantage that it is the best-measured effective charge, and it can be used to extrapolate the definition of the effective coupling to large distances. [13] It has been measured at intermediate energies, and it is therefore particularly useful to study the transition from short distances, where partons are the relevant degrees of freedom, to large distances, where the hadronic degrees of freedom are present. [14]

This paper is organized as follows: after briefly reviewing in Sec. II the light-front quantization approach to the gauge/gravity correspondence, we identify a nonperturbative running coupling in Sec. III from the fifth-dimensional action of gauge fields propagating in AdS₅ space modified by a positive-sign dilaton background $\exp(+\kappa^2 z^2)$. In Sect IV we compare the results for the coupling α_s^{AdS} obtained in Sec. III with effective QCD couplings extracted from different observables and lattice results. The nonperturbative results are extended to large Q^2 by matching the holographic results to perturbative results in the transition region. In Sec. V we discuss the holographic results for the β -function in the nonperturbative domain and compare the predictions with lattice and experimental results. In Sec. VI we discuss the use of CSR to relate different effective charges, a discussion which is then extended in Sec. VIII to configuration space. Some final remarks are given in the conclusions in Sec. IX. A check on the validity of CSR is carried out in the Appendix where

the results for the g_1 , V and \overline{MS} schemes are confronted in the perturbative domain.

II. LIGHT-FRONT HOLOGRAPHY AND QCD

The basic principle underlying the AdS/CFT approach to conformal gauge theories is the isomorphism of the group of Poincaré and conformal transformations $SO(4, 2)$ to the group of isometries of AdS₅ space, the group of transformations which leave the AdS metric

$$ds^2 = \frac{R^2}{z^2} (\eta_{\mu\nu} dx^\mu dx^\nu - dz^2), \quad (1)$$

invariant (R the AdS radius). Since the metric (1) is invariant under a dilatation of all coordinates $x^\mu \rightarrow \lambda x^\mu$, $z \rightarrow \lambda z$, the variable z acts like a scaling variable in Minkowski space: different values of z correspond to different energy scales at which the hadron is examined.

In order to describe a confining theory, the conformal invariance of AdS₅ must be broken. A simple way to impose confinement and discrete normalizable modes is to truncate the regime where the string modes can propagate by introducing an infrared (IR) cutoff at a finite value $z_0 \sim 1/\Lambda_{\text{QCD}}$. Thus the “hard-wall” at z_0 breaks conformal invariance and allows the introduction of the QCD scale and a spectrum of particle states. [15] In this simplified approach the propagation of hadronic modes in a fixed effective gravitational background encodes salient properties of the QCD dual theory, such as the ultraviolet (UV) conformal limit at the AdS boundary at $z \rightarrow 0$, as well as modifications of the background geometry in the large z infrared region which are dual to confining gauge theories. As first shown by Polchinski and Strassler, [15] the AdS/CFT duality, modified to incorporate a mass scale, provides a derivation of dimensional counting rules [16] for the leading power-law fall-off of hard scattering beyond the perturbative regime. The modified theory generates the hard behavior expected from QCD, instead of the soft behavior characteristic of strings.

The conformal metric of AdS space can be modified within the AdS/QCD framework to simulate confinement forces. [17] The introduction of a dilaton profile in the AdS action can be considered equivalent to modifying the AdS metric (1) by introducing an additional warp factor $\exp(\pm\kappa^2 z^2)$ [18]

$$ds^2 = \frac{R^2}{z^2} e^{\pm\kappa^2 z^2} (\eta_{\mu\nu} dx^\mu dx^\nu - dz^2). \quad (2)$$

A dilaton profile $\exp(\pm\kappa^2 z^2)$ of either sign also leads to a two-dimensional oscillator potential $U(\zeta) \sim \kappa^4 \zeta^2$ in the relativistic LF eigenvalue equation of Ref. [4], which in turn reproduces the observed linear Regge trajectories in a Chew-Frautschi plot. Glazek and Schaden [19] have shown that in QCD a harmonic oscillator confining potential naturally arises as an effective potential between heavy quark states when higher gluonic Fock states are stochastically eliminated.

The modified metric induced by the dilaton can be interpreted in AdS space as a gravitational potential for an object of mass m in the fifth dimension: $V(z) = mc^2 \sqrt{g_{00}} = mc^2 R e^{\pm\kappa^2 z^2/2}/z$. In the case of the negative solution the potential decreases monotonically, and thus an object in AdS will fall to infinitely large values of z . For the positive solution, the potential is non-monotonic and has an absolute minimum at $z_0 = 1/\kappa$. Furthermore, for large values of z the gravitational potential increases exponentially, thus confining any object to distances $\langle z \rangle \sim 1/\kappa$. [7] We thus use the positive sign dilaton solution opposite to that of Ref. [17]. This additional warp factor leads to a well defined scale-dependent effective coupling. Introducing a positive-sign dilaton background is also relevant for describing chiral symmetry breaking, [20] since the expectation value of the scalar field associated with the quark mass and condensate does not blow-up in the far infrared region of AdS, in contrast with the original model. [17]

The soft wall model of Ref. [17] also uses the AdS/QCD framework [21, 22], where bulk fields are introduced to match the $SU(2)_L \times SU(2)_R$ chiral symmetries of QCD and its spontaneous breaking, but without an explicit connection to the internal constituent structure of hadrons. [23] Instead, axial and vector currents become the primary entities as in an effective chiral theory. In this “bottom-up” model only a limited number of operators are introduced, and consequently, only a limited number of fields are required to construct phenomenologically viable five-dimensional gravity duals.

Light-front holography provides a remarkable connection between the equations of motion in AdS space and the Hamiltonian formulation of QCD in physical space-time quantized on the light front at fixed LF time $\tau = x^+ = x^0 + x^3$, the time marked by the front of a light wave. [24] This correspondence provides a direct connection between the hadronic amplitudes $\Phi(z)$ in AdS space with LF wavefunctions $\phi(\zeta)$ describing the quark and gluon constituent structure of hadrons in physical space-time. The mapping between the LF invariant variable ζ and the fifth dimension AdS coordinate z was originally obtained by

matching the expression for electromagnetic current matrix elements in AdS space with the corresponding expression for the current matrix element, using LF theory in physical space time. [5] It has also been shown that one obtains the identical holographic mapping using the matrix elements of the energy-momentum tensor, [6] thus verifying the consistency of the holographic mapping from AdS to physical observables defined on the light front. LF holography thus provides a direct correspondence between an effective gravity theory defined in a fifth dimensional warped space and a physical description of hadrons in 3+1 space-time.

Light-front quantization is the ideal framework for describing the structure of hadrons in terms of their quark and gluon degrees of freedom. LF wave functions play the same role in hadron physics that Schrödinger wavefunctions play in atomic physics. [25] The simple structure of the LF vacuum provides an unambiguous definition of the partonic content of a hadron in QCD. A physical hadron in four-dimensional Minkowski space has four-momentum P_μ and invariant hadronic mass states, $P_\mu P^\mu = \mathcal{M}^2$, determined by the Lorentz-invariant Hamiltonian equation for the relativistic bound-state system

$$P_\mu P^\mu |\psi(P)\rangle = (P^- P^+ - \mathbf{P}_\perp^2) |\psi(P)\rangle = \mathcal{M}^2 |\psi(P)\rangle. \quad (3)$$

The hadron four-momentum generator is $P = (P^+, P^-, \mathbf{P}_\perp)$, $P^\pm = P^0 \pm P^3$, and the hadronic state $|\psi\rangle$ is an expansion in multiparticle Fock eigenstates $|n\rangle$ of the free light-front Hamiltonian: $|\psi\rangle = \sum_n \psi_n |n\rangle$. The internal partonic coordinates of the hadron are the momentum fractions $x_i = k_i^+ / P^+$ and the transverse momenta $\mathbf{k}_{\perp i}$, $i = 1, 2, \dots, n$, where n is the number of partons in a given Fock state. Momentum conservation requires $\sum_{i=1}^n x_i = 1$ and $\sum_{i=1}^n \mathbf{k}_{\perp i} = 0$. It is useful to employ a mixed representation [26] in terms of $n - 1$ independent momentum fraction variables x_j and position coordinates $\mathbf{b}_{\perp j}$, $j = 1, 2, \dots, n - 1$, so that $\sum_{i=1}^n \mathbf{b}_{\perp i} = 0$. The relative transverse variables $\mathbf{b}_{\perp i}$ are Fourier conjugates of the momentum variables $\mathbf{k}_{\perp i}$.

In AdS space the physical states are represented by normalizable modes $\Phi_P(x^\mu, z) = e^{-iP \cdot x} \Phi(z)$, with plane waves along the Poincaré coordinates and a profile function $\Phi(z)$ along the holographic coordinate z . Each LF hadronic state $|\psi(P)\rangle$ is dual to a normalizable string mode $\Phi_P(x^\mu, z)$. The hadronic mass \mathcal{M}^2 is found by solving the eigenvalue problem for the corresponding wave equation in AdS space, which, as we discuss below, is equivalent to the semiclassical approximation to the light-front bound-state Hamiltonian equation of motion in QCD. One can indeed systematically reduce the LF Hamiltonian eigenvalue equation (3)

to an effective relativistic wave equation [4] by observing that each n -particle Fock state has an essential dependence on the invariant mass of the system $M_n^2 = (\sum_{a=1}^n k_a^\mu)^2$ and thus, to a first approximation, LF dynamics depend only on M_n^2 . In impact space the relevant variable is the boost invariant transverse variable ζ which measures the separation of the quark and gluonic constituents within the hadron at the same LF time and which also allows one to separate the dynamics of quark and gluon binding from the kinematics of the constituent internal angular momentum. In the case of two constituents, $\zeta = \sqrt{x(1-x)}|\mathbf{b}_\perp|$ where $x = k^+/P^+ = (k^0 + k^3)/(P^+ + P^3)$ is the LF fraction. The result is the single-variable light-front relativistic Schrödinger equation [4]

$$\left(-\frac{d^2}{d\zeta^2} - \frac{1-4L^2}{4\zeta^2} + U(\zeta)\right)\phi(\zeta) = \mathcal{M}^2\phi(\zeta), \quad (4)$$

where $U(\zeta)$ is the effective potential, and L is the relative orbital angular momentum as defined in the LF formalism. The set of eigenvalues \mathcal{M}^2 gives the hadronic spectrum of the color-singlet states, and the corresponding eigenmodes $\phi(\zeta)$ represent the light-front wavefunctions which describe the dynamics of the hadronic constituents. This first approximation to relativistic QCD bound-state systems is equivalent to the equations of motion which describe the propagation of spin- J modes in a fixed gravitational background asymptotic to AdS space. [4] By using the correspondence between ζ in the LF theory and z in AdS space, one can identify the terms in the dual gravity AdS equations which correspond to the kinetic energy terms of the partons inside a hadron and the interaction terms that build confinement. [4] The identification of orbital angular momentum of the constituents in the light-front description is also a key element in our description of the internal structure of hadrons using holographic principles.

As we will discuss, the conformal AdS₅ metric (1) can be deformed by a warp factor $\exp(+\kappa^2 z^2)$. In the case of a two-parton relativistic bound state, the resulting effective potential in the LF equation of motion is $U(\zeta) = \kappa^4 \zeta^2 + 2\kappa^2(L + S - 1)$. [7] There is only one parameter, the mass scale $\kappa \sim 1/2$ GeV, which enters the effective confining harmonic oscillator potential. Here $S = 0, 1$ is the spin of the quark-antiquark system, L is their relative orbital angular momentum, and ζ is the Lorentz invariant coordinate defined above, which measures the distance between the quark and antiquark; it is analogous to the radial coordinate r in the Schrödinger equation. The resulting mesonic spectrum has the phenomenologically successful Regge form $\mathcal{M}^2 = 4\kappa^2(n + L + S/2)$, with equal slopes in the

orbital angular momentum and the radial quantum number n . The pion with $n = L = S = 0$ is massless for zero quark mass, consistent with chiral symmetry.

III. NONPERTURBATIVE QCD COUPLING FROM LIGHT-FRONT HOLOGRAPHY

We will show in this section how the LF holographic mapping of effective classical gravity in AdS space, modified by a positive-sign dilaton background, can be used to identify an analytically simple color-confining non-perturbative effective coupling $\alpha_s^{AdS}(Q^2)$ as a function of the space-like momentum transfer $Q^2 = -q^2$. As we shall show, this coupling incorporates confinement and agrees well with effective charge observables and lattice simulations. It also exhibits an infrared fixed point at small Q^2 and asymptotic freedom at large Q^2 . However, the fall-off of $\alpha_s^{AdS}(Q^2)$ at large Q^2 is exponential: $\alpha_s^{AdS}(Q^2) \sim e^{-Q^2/\kappa^2}$, rather than the perturbative QCD (pQCD) logarithmic fall-off. We shall show in later sections that a phenomenological extended coupling can be defined which implements the pQCD behavior.

As will be explained in Sec. V, the β -function derived from light-front holography becomes significantly negative in the non-perturbative regime $Q^2 \sim \kappa^2$, where it reaches a minimum, signaling the transition region from the infrared (IR) conformal region, characterized by hadronic degrees of freedom, to a pQCD conformal ultraviolet (UV) regime where the relevant degrees of freedom are the quark and gluon constituents. The β -function is always negative: it vanishes at large Q^2 consistent with asymptotic freedom, and it vanishes at small Q^2 consistent with an infrared fixed point. [27, 28]

Let us consider a five-dimensional gauge field G propagating in AdS₅ space in presence of a dilaton background $\varphi(z)$ which introduces the energy scale κ in the five-dimensional action. At quadratic order in the field strength the action is

$$S = -\frac{1}{4} \int d^5x \sqrt{g} e^{\varphi(z)} \frac{1}{g_5^2} G^2, \quad (5)$$

where the metric determinant of AdS₅ is $\sqrt{g} = (R/z)^5$, $\varphi = \kappa^2 z^2$ and the square of the coupling g_5 has dimensions of length. We can identify the prefactor

$$g_5^{-2}(z) = e^{\varphi(z)} g_5^{-2}, \quad (6)$$

in the AdS action (5) as the effective coupling of the theory at the length scale z . The coupling $g_5(z)$ then incorporates the non-conformal dynamics of confinement. The five-

dimensional coupling $g_5(z)$ is mapped, modulo a constant, into the Yang-Mills (YM) coupling g_{YM} of the confining theory in physical space-time using light-front holography. One identifies z with the invariant impact separation variable ζ which appears in the LF Hamiltonian: $g_5(z) \rightarrow g_{YM}(\zeta)$. Thus

$$\alpha_s^{AdS}(\zeta) = g_{YM}^2(\zeta)/4\pi \propto e^{-\kappa^2 \zeta^2}. \quad (7)$$

In contrast with the 3-dimensional radial coordinates of the non-relativistic Schrödinger theory, the natural light-front variables are the two-dimensional cylindrical coordinates (ζ, ϕ) and the light-cone fraction x . The physical coupling measured at the scale Q is the two-dimensional Fourier transform of the LF transverse coupling $\alpha_s^{AdS}(\zeta)$ (7). Integration over the azimuthal angle ϕ gives the Bessel transform

$$\alpha_s^{AdS}(Q^2) \sim \int_0^\infty \zeta d\zeta J_0(\zeta Q) \alpha_s^{AdS}(\zeta), \quad (8)$$

in the $q^+ = 0$ light-front frame where $Q^2 = -q^2 = -\mathbf{q}_\perp^2 > 0$ is the square of the space-like four-momentum transferred to the hadronic bound state. Using this ansatz we then have from Eq. (8)

$$\alpha_s^{AdS}(Q^2) = \alpha_s^{AdS}(0) e^{-Q^2/4\kappa^2}. \quad (9)$$

In contrast, the negative dilaton solution $\varphi = -\kappa^2 z^2$ leads to an integral which diverges at large ζ . The essential assumption of this paper is the identification of $\alpha_s^{AdS}(Q^2)$ with the physical QCD running coupling in its nonperturbative domain.

The flow equation (6) from the scale dependent measure for the gauge fields can be understood as a consequence of field-strength renormalization. In physical QCD we can rescale the non-Abelian gluon field $A^\mu \rightarrow \lambda A^\mu$ and field strength $G^{\mu\nu} \rightarrow \lambda G^{\mu\nu}$ in the QCD Lagrangian density \mathcal{L}_{QCD} by a compensating rescaling of the coupling strength $g \rightarrow \lambda^{-1}g$. The renormalization of the coupling $g_{phys} = Z_3^{1/2}g_0$, where g_0 is the bare coupling in the Lagrangian in the UV-regulated theory, is thus equivalent to the renormalization of the vector potential and field strength: $A_{ren}^\mu = Z_3^{-1/2}A_0^\mu$, $G_{ren}^{\mu\nu} = Z_3^{-1/2}G_0^{\mu\nu}$ with a rescaled Lagrangian density $\mathcal{L}_{\text{QCD}}^{ren} = Z_3^{-1}\mathcal{L}_{\text{QCD}}^0 = (g_{phys}/g_0)^{-2}\mathcal{L}_0$. In lattice gauge theory, the lattice spacing a serves as the UV regulator, and the renormalized QCD coupling is determined from the normalization of the gluon field strength as it appears in the gluon propagator. The inverse of the lattice size L sets the mass scale of the resulting running coupling. As in lattice gauge theory, color confinement in AdS/QCD reflects nonperturbative dynamics

at large distances. The QCD couplings defined from lattice gauge theory and the soft wall holographic model are thus similar in concept, and both schemes are expected to have similar properties in the nonperturbative domain, up to a rescaling of their respective momentum scales.

The gauge/gravity correspondence has also been used to study the running coupling of the dual field theory. Most often the AdS metric is modified to reproduce logarithmic corrections at large Q [29–35]. For example, a β -function ansatz of the boundary field theory is used as input in Refs. [31–35] to modify the AdS metrics assuming the correspondence between the AdS variable z and the energy scale E of the conformal field theory, $E \sim 1/z$, as discussed in Ref. [36]. In our paper, the effective QCD coupling is identified by using the precise mapping from z in AdS space to the transverse impact variable ζ in LF QCD.

IV. COMPARISON OF THE HOLOGRAPHIC COUPLING WITH OTHER EFFECTIVE CHARGES

The effective coupling $\alpha^{AdS}(Q^2)$ (solid line) is compared in Fig. 1 with experimental and lattice data. For this comparison to be meaningful, we have to impose the same normalization on the AdS coupling as the g_1 coupling. This defines α_s^{AdS} normalized to the g_1 scheme:

$$\alpha_{g_1}^{AdS}(Q^2 = 0) = \pi. \quad (10)$$

A more detailed discussion on the normalization at the IR fixed point will be given later.

The couplings in Fig. 1 agree well in the strong coupling regime up to $Q \sim 1$ GeV. The value $\kappa = 0.54$ GeV has been determined from the vector meson principal Regge trajectory. [7]. The lattice results shown in Fig. 1 from Ref. [37] have been scaled to match the perturbative UV domain. The effective charge α_{g_1} has been determined in Ref [38] from several experiments. Fig. 1 also displays other couplings from different observables as well as α_{g_1} which is computed from the Bjorken sum rule [12] over a large range of momentum transfer (cyan band). At $Q^2 = 0$ one has the constraint on the slope of α_{g_1} from the Gerasimov-Drell-Hearn (GDH) sum rule [39] which is also shown in the figure. The results show no sign of a phase transition, cusp, or other non-analytical behavior, a fact which allows us to extend the functional dependence of the coupling to large distances. The smooth behavior of the holographic strong coupling also allows us to extrapolate its form to the

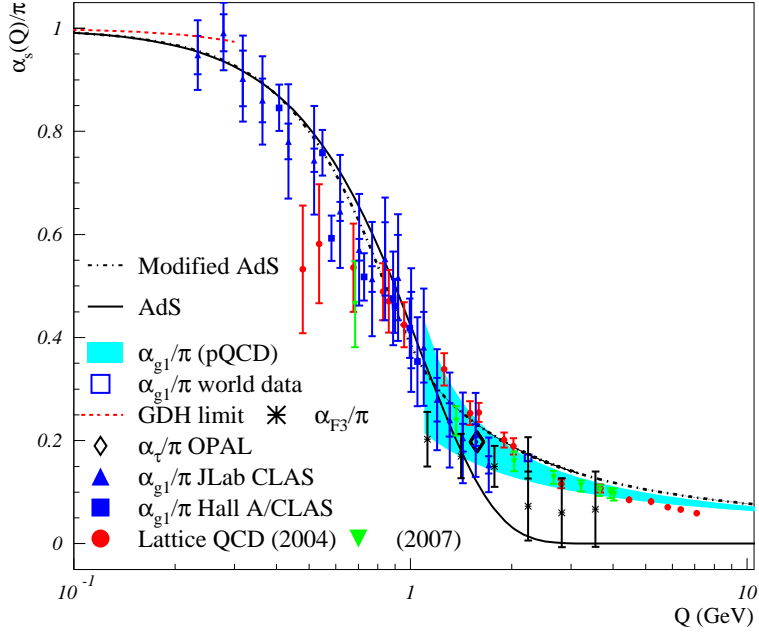


FIG. 1. The effective coupling from LF holographic mapping for $\kappa = 0.54$ GeV is compared with effective QCD couplings extracted from different observables and lattice results. Details on the comparison with other effective charges are given in Ref. [38].

perturbative domain. This is discussed further in Sec. VI.

The hadronic model obtained from the dilaton-modified AdS space provides a semi-classical first approximation to QCD. Color confinement is introduced by the harmonic oscillator potential, but effects from gluon creation and absorption are not included in this effective theory. The nonperturbative confining effects vanish exponentially at large momentum transfer (Eq. (9)), and thus the logarithmic fall-off from pQCD quantum loops will dominate in this regime.

The running coupling α_s^{AdS} given by Eq. (9) is obtained from a color-confining potential. Since the strong coupling is an analytical function of the momentum transfer at all scales, we can extend the range of applicability of α_s^{AdS} by matching to a perturbative coupling at the transition scale, $Q \sim 1$ GeV, where pQCD contributions become important. In order to have a fully analytical model, we write

$$\alpha_{Modified,g_1}^{AdS}(Q^2) = \alpha_{g_1}^{AdS}(Q^2)g_+(Q^2) + \alpha_{g_1}^{fit}(Q^2)g_-(Q^2), \quad (11)$$

where $g_{\pm}(Q^2) = 1/(1 + e^{\pm(Q^2 - Q_0^2)/\tau^2})$ are smeared step functions which match the two regimes. The parameter τ represents the width of the transition region. Here $\alpha_{g_1}^{AdS}$ is given by Eq. (9) with the normalization (10) – the plain black line in Fig. 1 – and $\alpha_{g_1}^{fit}$ in Eq. (11) is the analytical fit to the measured coupling α_{g_1} . [38] The couplings are chosen to have the same normalization at $Q^2 = 0$ given by Eq. (10). The smoothly extrapolated result (dot-dashed line) for α_s is also shown on Fig. 1. We use the parameters $Q_0^2 = 0.8 \text{ GeV}^2$ and $\tau^2 = 0.3 \text{ GeV}^2$.

V. HOLOGRAPHIC β -FUNCTION

The β -function for the nonperturbative effective coupling obtained from the LF holographic mapping in a positive dilaton modified AdS background is

$$\beta_{g_1}^{AdS}(Q^2) = \frac{d}{d \log Q^2} \alpha_{g_1}^{AdS}(Q^2) = \frac{\pi Q^2}{4\kappa^2} e^{-Q^2/(4\kappa^2)}. \quad (12)$$

The solid line in Fig. 2 corresponds to the light-front holographic result Eq. (12). Near $Q_0 \simeq 2\kappa \simeq 1 \text{ GeV}$, we can interpret the results as a transition from the nonperturbative IR domain to the quark and gluon degrees of freedom in the perturbative UV regime. The transition momentum scale Q_0 is compatible with the momentum transfer for the onset of scaling behavior in exclusive reactions where quark counting rules are observed. [16] For example, in deuteron photo-disintegration the onset of scaling corresponds to momentum transfer of 1.0 GeV to the nucleon involved. [40] Dimensional counting is built into the AdS/QCD soft and hard wall models since the AdS amplitudes $\Phi(z)$ are governed by their twist scaling behavior z^τ at short distances, $z \rightarrow 0$. [15] A similar scale for parton-hadron transition region has been observed in inclusive reactions. [41]

Also shown on Fig. 2 are the β -functions obtained from phenomenology and lattice calculations. For clarity, we present on Fig. 2 only the LF holographic predictions, the lattice results from, [37] and the experimental data supplemented by the relevant sum rules. The width of the aqua band is computed from the uncertainty of α_{g_1} in the perturbative regime. The dot-dashed curve corresponds to the extrapolated approximation given by Eq. (11). Only the point-to-point uncorrelated uncertainties of the JLab data are used to estimate the uncertainties, since a systematic shift cancels in the derivative in (12). The data have been recombined in fewer points to improve the statistical uncertainty; nevertheless,

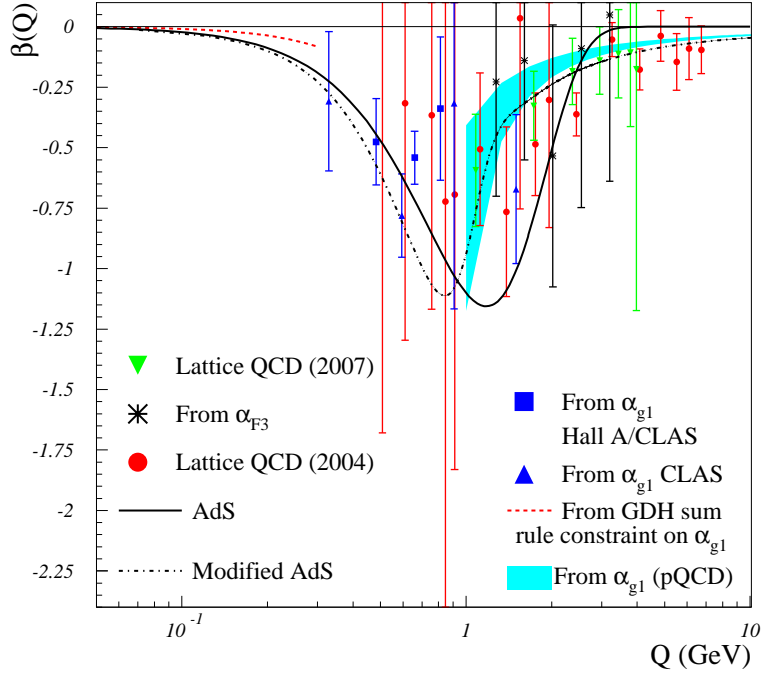


FIG. 2. Holographic model prediction for the β function compared to JLab and CCFR data, lattice simulations and results from the Bjorken sum rule.

the uncertainties are still large. Upcoming JLab Hall A and Hall B data [42] should reduce further this uncertainty. The β -function extracted from LF holography, as well as the forms obtained from the works of Cornwall, Bloch, Fisher *et al.*, [43] Burkert and Ioffe [44] and Furui and Nakajima, [37] are seen to have a similar shape and magnitude.

Judging from these results, we infer that the actual β -function of QCD will extrapolate between the non-perturbative results for $Q < 1$ GeV and the pQCD results for $Q > 1$ GeV. We also observe that the general conditions

$$\beta(Q \rightarrow 0) = \beta(Q \rightarrow \infty) = 0, \quad (13)$$

$$\beta(Q) < 0, \text{ for } Q > 0, \quad (14)$$

$$\left. \frac{d\beta}{dQ} \right|_{Q=Q_0} = 0, \quad (15)$$

$$\frac{d\beta}{dQ} < 0, \text{ for } Q < Q_0, \quad \frac{d\beta}{dQ} > 0, \text{ for } Q > Q_0. \quad (16)$$

are satisfied by our model β -function obtained from LF holography.

Eq. (13) expresses the fact that QCD approaches a conformal theory in both the far ultraviolet and deep infrared regions. In the semiclassical approximation to QCD without

particle creation or absorption, the β -function is zero and the approximate theory is scale invariant in the limit of massless quarks. [45] When quantum corrections are included, the conformal behavior is preserved at very large Q because of asymptotic freedom and near $Q \rightarrow 0$ because the theory develops a fixed point. An infrared fixed point is in fact a natural consequence of color confinement: [27] since the propagators of the colored fields have a maximum wavelength, all loop integrals in the computation of the gluon self-energy decouple at $Q^2 \rightarrow 0$. [28] Condition (14) for Q^2 large, expresses the basic anti-screening behavior of QCD where the strong coupling vanishes. The β -function in QCD is essentially negative, thus the coupling increases monotonically from the UV to the IR where it reaches its maximum value: it has a finite value for a theory with a mass gap. Equation (15) defines the transition region at Q_0 where the beta function has a minimum. Since there is only one hadronic-partonic transition, the minimum is an absolute minimum; thus the additional conditions expressed in Eq (16) follow immediately from Eqs. (13-15). The conditions given by Eqs. (13-16) describe the essential behavior of the full β -function for an effective QCD coupling whose scheme/definition is similar to that of the V -scheme.

VI. EFFECTIVE CHARGES AND COMMENSURATE SCALE RELATIONS

As noted by Grunberg, one can use observables such as heavy quark scattering or the Bjorken sum rule to define effective charges $\alpha_{\mathcal{O}}(Q^2)$ each with its own physical scale. [9] This generalizes the convention in QED where the Gell Mann-Low coupling [46] $\alpha_{QED}(Q^2)$ is defined at all scales from the scattering of infinitively heavy charged particles. Since physical quantities are involved, the relation between effective charges cannot depend on theoretical conventions such as the of the choice of an intermediate scheme. [47] This is formally the transitivity property of the renormalization group: A to B and B to C relates A to C , independent of the choice of the intermediate scheme B .

Although the perturbative β -function for every effective charge [9] is universal up to two loops at high Q^2 , each effective charge has specific characteristics which influence its behavior at small Q^2 . For example, the value and derivative of the α_{g_1} coupling at $Q^2 = 0$ are both constrained since the Bjorken sum vanishes at $Q^2 = 0$, and its derivative is given by the GDH sum rule. [38, 39]

The relations between effective charges in pQCD are given by commensurate scale rela-

tions [10]. The relative factor between the scales of the two effective charges in the CSR is set to ensure that the onset of a new quark pair in the β -function of the two couplings is synchronized. This factor can be determined by the Brodsky-Lepage-Mackenzie (BLM) procedure, [48] where all n_F and β -dependent non-conformal terms in the perturbative expansion are absorbed by the choice of the renormalization scale of the effective coupling.

This procedure also eliminates the factorial renormalon growth of perturbation theory. The commensurate scale relation between $\alpha_{g_1}(Q^2)$ and the Adler function effective charge $\alpha_D(Q^2)$ which is defined from $R_{e^+e^-}$ data is now known to four loops in PQCD [49]. The relation between observables given by the CSR is independent of the choice of the intermediate renormalization scheme. CSRs are thus precise predictions of QCD without scale or scheme ambiguity; they thus provide essential tests of the validity of QCD.

The holographic coupling $\alpha_s^{AdS}(Q^2)$ could be seen as the nonperturbative extension of the α_V effective charge defined by Appelquist *et al.*, [11] and it thus can be compared to phenomenological models for the heavy quark potential such as the Cornell potential [50] and lattice computations. Thus, an important question is how to extend the relations between observables and their effective charges to the nonperturbative domain. We can also use the CSR concept to understand the relation of $\alpha^{AdS}(Q^2)$ given by Eqs. (9) and (11) to well-measured effective charges such as the α_{g_1} coupling even in the nonperturbative domain.

VII. EXPERIMENTAL RESULTS, SCHEMES AND DATA NORMALIZATION

The effective charges α_{g_1} and α_{F_3} shown in Figs. 1 and 2 are extracted in Ref. [38] following the prescription of Grunberg. [9] Data on the spin structure function g_1 , from JLab [51] are used to form α_{g_1} . CCFR data on the structure function F_3 [52] are used to form α_{F_3} , which is then related to α_{g_1} using a CSR. The GDH and Bjorken sum rules constrain respectively the small [39] and large [12] Q^2 -behavior of the integral of g_1 and provide a description of α_{g_1} over a large domain.

We note that the works of [9] and [10] pertain to the UV domain, whereas Ref. [38] extends them to the IR region based on the analytical behavior of the coupling. The effective charge α_{g_1} is found to be approximately scale invariant in the IR domain, in agreement with an IR fixed point behavior. [28] The shape of the coupling α_{g_1} agrees with other predictions of the running coupling α_s at small Q^2 , including lattice QCD, [37] the Schwinger-Dyson

formalism, [27, 43] and the coupling of a constituent quark model which is consistent with hadron spectroscopy. [53] We point out that the essential difference between these running couplings is their value at $Q^2 = 0$: if normalized to the same point at $Q^2 = 0$, their Q^2 -dependences agree within their relative uncertainties. [38]

The aqua band in Fig. 1 for α_{g_1} is computed with the Bjorken sum rule using the relation between α_{g_1} and $\alpha_{\overline{MS}}$. [10, 38] The pQCD leading-twist expression of the Bjorken sum up to third order in $\alpha_{\overline{MS}}$ is used to estimate the Bjorken sum. The sum rule is then used to extract α_{g_1} at large Q . In the pQCD expression of the Bjorken sum rule, $\alpha_{\overline{MS}}$ is retained up to second order in β (i.e. up to β_2). The uncertainty in the band comes from the uncertainty on $\Lambda_{\overline{MS}} = 0.37_{-0.07}^{+0.04}$ and the truncation of the series. [54]

Although the effective coupling $\alpha_{g_1}(Q^2)$ has specific features of deep inelastic lepton-proton scattering, it nevertheless appears to closely mimic the shape and magnitude of the AdS/QCD coupling near the transition region $Q \simeq Q_0$. In particular, it illustrates how one can have a coupling which flows analytically from the IR strong coupling domain with an IR fixed point to the UV domain controlled by pQCD. [56]

The value of $\alpha_s^{AdS}(Q)$ at $Q = 0$ cannot be determined by our holographic approach. [57] It is also well known that even in the pQCD domain the value of running coupling is significantly scheme-dependent when the momentum transfer becomes small. It is thus reasonable to assume that such differences propagate in the IR domain and consequently the IR value of different effective charges can differ. Such differences between schemes can naturally explain the smaller IR fixed point values obtained in other computations of the strong coupling, *e.g.*, in Ref. [27], as qualitatively illustrated on Fig. 3.

Despite the different physics underlying the light-front holographic coupling $\alpha_s^{AdS}(Q^2)$ and the effective charge $\alpha_{g_1}(Q^2)$ determined empirically from measurements of the Bjorken sum rule, the shapes of the two running couplings are remarkably close in the infrared regime. The resemblance of α_s^{AdS} and α_{g_1} is understandable if we recall that α_s^{AdS} is a natural nonperturbative extension of α_V . The scale shift in the CSR between α_V and α_{g_1} is small, making them numerically very close. Furthermore every effective charge satisfies the same pQCD β -function to two loops. Thus, the extended α_s^{AdS} and α_{g_1} are also very close at high scales. The AdS and g_1 couplings share other common features: their β -functions have similar structures: zero in the IR, strongly negative in the GeV domain, and zero in the far UV. We can exploit all of these similarities to fix the normalization $\alpha_s^{AdS}(Q = 0) = \pi$

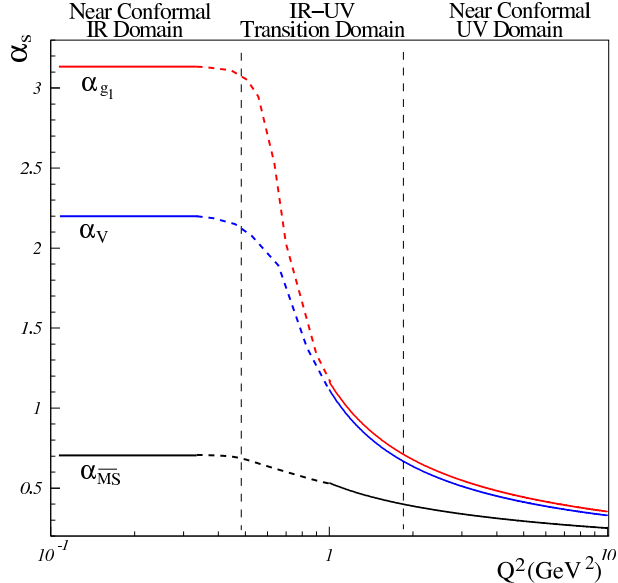


FIG. 3. How different schemes can lead to different values for the IR fixed point. The couplings are computed in the UV region. They freeze in the IR region. The interpolation between UV and IR is drawn freely and is meant to be illustrative, as are the various IR fixed point values. We note that the V and g_1 schemes are numerically close.

and to consistently extend the AdS coupling to the UV domain, consistent with pQCD.

VIII. HOLOGRAPHIC COUPLING IN CONFIGURATION SPACE

In order to obtain modifications to the instantaneous Coulomb potential in configuration space $V(r) = -C_F \alpha_V(r)/r$ from the running coupling, one must transform the coupling defined by the static quark potential $V(q) = -4\pi C_F \alpha_V(q)/q^2$ in the non-relativistic limit and extract the coefficient of $1/r$ to define the coupling $\alpha_V(r)$ in the V-scheme. The couplings are related by the Fourier transform [59]

$$\alpha_V(r) = \frac{2}{\pi} \int_0^\infty dq \alpha_V(q) \frac{\sin(qr)}{q}. \quad (17)$$

From (9) we find the expression

$$\alpha_V^{AdS}(r) = C \operatorname{erf}(\kappa r) = \frac{2}{\sqrt{\pi}} C \int_0^{\kappa r} e^{-t^2} dt, \quad (18)$$

where $C = \alpha_V(Q = 0)$. We have written explicitly the normalization at $Q = 0$ in the V-scheme since it is not expected to be equal to the normalization in the g_1 -scheme for the

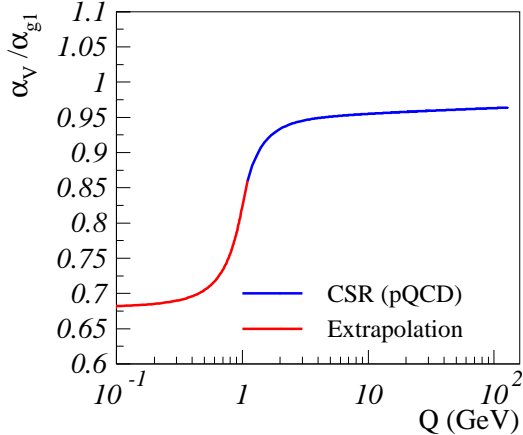


FIG. 4. Ratio $\alpha_V(Q)/\alpha_{g_1}(Q)$. The blue line represents the domain where the CSR are computed at leading twist (Eq. (19)). The red line is the extrapolation to the non-perturbative domain using the fixed point IR conformal behavior of QCD.

reasons discussed in Sec. VII.

The couplings in the V and g_1 schemes are related at leading twist by the CSR: [10]

$$\frac{\alpha_V(Q^2)}{\pi} = \frac{\alpha_{g_1}(Q^{*2})}{\pi} - 1.09 \left(\frac{\alpha_{g_1}(Q^{**2})}{\pi} \right)^2 + 25.6 \left(\frac{\alpha_{g_1}(Q^{**2})}{\pi} \right)^3 + \dots, \quad (19)$$

with $Q^* = 1.18 Q$, $Q^{**} = 2.73 Q$, and we set $Q^{***} = Q^{**}$. We have verified that this relation numerically holds at least down to $Q^2 = 0.6 \text{ GeV}^2$, as shown in the figure in the Appendix (Fig. 7). In order to transform $\alpha_{g_1}(Q^2)$ into $\alpha_V(Q^2)$ over the full Q^2 range, we extrapolate the CSR to the non-perturbative domain. For guidance, we use the fact that QCD is near-conformal at very small Q ; thus the ratio α_V/α_{g_1} is Q -independent. A model for the ratio $\alpha_V(Q)/\alpha_{g_1}(Q)$ is shown in Fig. 4. We apply this ratio to $\alpha_{Modified,g_1}^{AdS}(Q)$, Eq. (11), and then Fourier transform the result using Eq. (17) to obtain $\alpha_{Modified,V}^{AdS}(r)$. We find $C \simeq 2.2$.

A. Comparison of V and g_1 Results

The right panel of Fig. 5 displays $\alpha_V^{AdS}(r)$ (blue line) and $\alpha_V(r)$ obtained with the same procedure but applied to the JLab data (light blue band). Also shown for comparison are, on the left panel, the results in the g_1 -scheme: $\alpha_{g_1}(r)$ from JLab data (blue band), the light-front holographic result from Eqs. 11 and 18 (green line) and lattice results from [37] (red band). The same scales are used on both panels. The fact that different schemes imply

different values for the IR fixed point of α_s is exemplified in this figure in which $\alpha_s(r)$ in the V -scheme and in the g_1 -scheme freeze to the IR fixed point values of $\alpha_V(Q = 0) = 2.2$ and $\alpha_{g_1}(Q = 0) = \pi$ respectively.

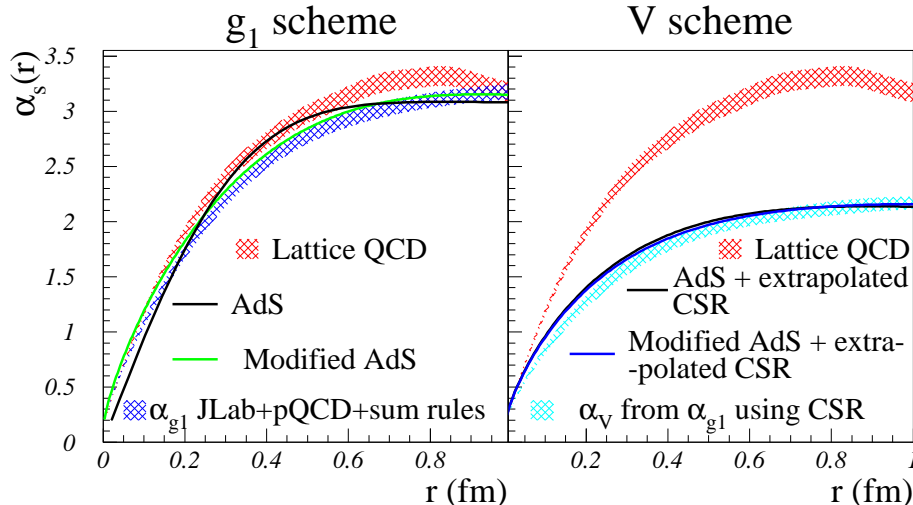


FIG. 5. Holographic model predictions for $\alpha_s(r)$ in configuration space in the g_1 scheme (left panel) and V scheme (right panel). The black line is the holographic AdS result, the green and blue lines correspond to the modified holographic results from Eq. (11) normalized respectively to α_{g_1} and α_V at $Q = 0$. The blue bands correspond to the JLab data and the red to lattice results.

The width of the light blue band is the combined uncertainty on α_V coming from: a) the uncertainty in the value Λ_{QCD} , b) the truncation of the pQCD β -series used to calculate $\alpha_{\overline{MS}}$ in the perturbative region, c) the truncation of the pQCD CSR at Q^{***} which has been estimated by using the difference between the Q^{**} and Q^{***} orders and d) the experimental uncertainties on the JLab data for α_{g_1} . The uncertainty coming from the truncation of the pQCD series for the Bjorken sum rule is negligible.

The experimental results for $\alpha_{g_1}(r)$ follow from the integrated JLab data according to Eq. (17). The contributions to the integral from the unmeasured low Q ($Q < 0.23$ GeV) and high Q ($Q > 1.71$ GeV) regions are computed using the sum rules [39] and [12] respectively. The total experimental uncertainties, as well as the uncertainty on the large Q region, are added in quadrature. This underestimates somewhat the final uncertainty. Since $\alpha_{g_1}(r)$ can be computed for any r , the experimental data and (lattice results) now appear as bands on Fig. 5 rather than a set of data points.

B. Contribution to the Instantaneous Quark-Antiquark Potential

The quark-antiquark Coulomb potential $V(r) = -4\alpha_V(r)/3r$ is shown in Fig. 6 for the running coupling computed from light-front holography and the JLab g_1 measurement. The results can be compared at large Q^2 to the phenomenological Cornell potential [50] and, in the deep UV region, to the two-loop calculation of Peter [60] as well as with the three-loop calculation of Anzai *et al.* [61]. Other recent three-loop calculations [62] are consistent with the uncertainty with the results from Ref. [61]; the central values of the three-loop parameter a_3 agree within 3 %. The uncertainty in Peter's result is mainly due to the uncertainty in $\Lambda_{\overline{MS}}$, with negligible contributions from the truncations of the pQCD $\beta_{\overline{MS}}$ series and the CSR series. The truncation uncertainties are estimated as the values of the last known order of the series. All contributions to the uncertainty are added in quadrature.

In the case of heavy quarks the light-front holographic equations reduce to a non-relativistic Schrödinger equation in configuration space with potential

$$V(r) = -\frac{4}{3} \frac{\alpha_V(r)}{r} + V_{conf}(r), \quad (20)$$

where V_{conf} for a soft-wall dilaton background is the potential for a 3-dimensional harmonic oscillator, $V_{conf} \simeq \frac{1}{2}m_{red}\omega^2 r^2$. Here m_{red} is the reduced mass of the heavy $\overline{Q}-Q$ system, $m_{red} = m_Q m_{\overline{Q}} / (m_Q + m_{\overline{Q}})$, and $\omega = \kappa^2 / (m_Q + m_{\overline{Q}})$. Remarkably, the explicit holographic confining potential V_{conf} , which is the dominant interaction for light quarks, vanishes as the inverse of the quark mass for heavy quark masses.

For finite quark masses both contributions will appear. This will bring the effective potential closer to the phenomenological Cornell potential. Thus, the comparison of the Coulomb results in Fig. 4 with the Cornell potential only holds in the limit of infinite quark masses. A detailed discussion of the confining interaction, its implication for the study of the heavy meson mass spectrum, and other aspects of the instantaneous quark-antiquark potential will be discussed elsewhere.

IX. CONCLUSIONS

We have shown that the light-front holographic mapping of effective classical gravity in AdS space, modified by a positive-sign dilaton background $\exp(+\kappa^2 z^2)$, can be used to identify a non-perturbative effective coupling $\alpha_s^{AdS}(Q)$ and its β -function. The same theory

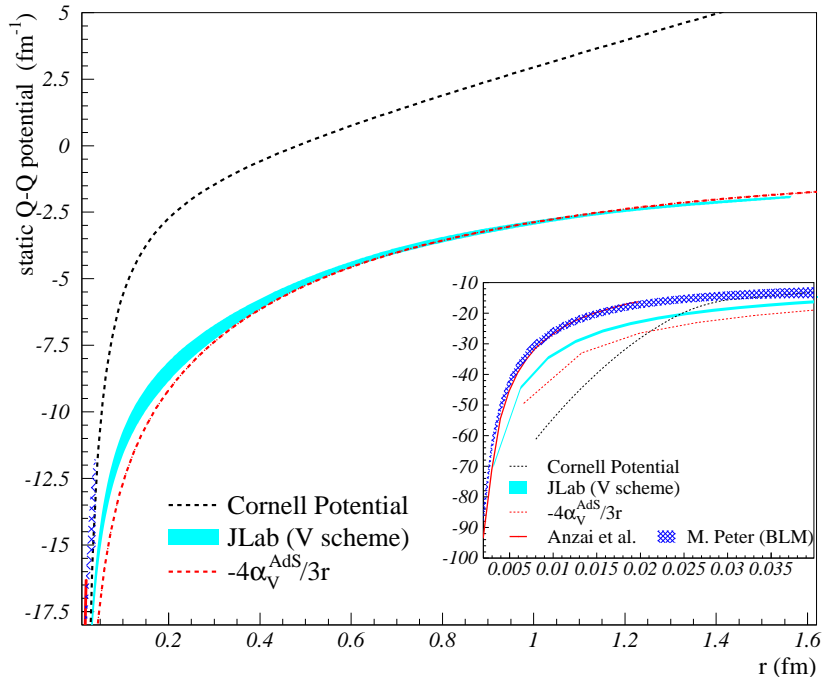


FIG. 6. Contribution of the running coupling to the quark-anti quark Coulomb potential. The red dashed line represents the result from light-front holography modified for pQCD effects using Eq. (11) and transformed to the V -scheme using the extrapolated CSR results shown in Fig. 4. The cyan band is the JLab results transformed to the V -scheme using the same CSR. The black dashed line is the Cornell potential. The inserted figure zooms into the deep UV domain. PQCD results at two loops (Peter) and three loops (Anzai *et al.*) are shown respectively by the blue band and the red line. An arbitrary offset is applied to the Anzai *et al.* results.

provides a very good description of the spectrum and form factors of light hadrons. Our analytical results for the effective holographic coupling provide new insights into the infrared dynamics and the form of the full β -function of QCD.

We also observe that the effective charge obtained from light-front holography is in very good agreement with the effective coupling α_{g_1} extracted from the Bjorken sum rule. Surprisingly, the Furui and Nakajima lattice results [37] also agrees better overall with the g_1 scheme rather than the V scheme as seen in Fig. 5. Our analysis indicates that light-front holography captures the essential dynamics of confinement, showing that it belongs to a uni-

versality class of models with built-in confinement. The holographic β -function shows the transition from nonperturbative to perturbative regimes at a momentum scale $Q \sim 1$ GeV and captures some of the essential characteristics of the full β -function of QCD, thus giving further support to the application of the gauge/gravity duality to the confining dynamics of strongly coupled QCD.

We have made extensive use of commensurate scale relations, which allows us to relate observables in different schemes and regimes. In particular, we have extrapolated the CSR to extend the relation between observables to the non-perturbative domain. In the pQCD domain, we checked that the CSR are valid. This validity provides a fundamental check of QCD since the CSR are a central pQCD prediction independent of theoretical conventions.

The normalization of the QCD coupling α_s^{AdS} at $Q^2 = 0$ appears to be considerably higher than that suggested in Ref. [1], a difference probably stemming from the different scheme choices. However, $\alpha_{g_1}(Q^2)$ has the advantage that it is the most precisely measured effective charge. As we have noted, there is a remarkable similarity of $\alpha_{g_1}(Q^2)$ to the nonperturbative strong coupling $\alpha^{AdS}(Q^2)$ obtained here except at large Q^2 where the contribution from quantum loops is dominant. To extend its utility, we have provided an analytical expression encompassing the holographic result at low Q^2 and pQCD contributions from gluon exchange at large Q^2 . The value of the confining scale of the model κ is determined from the vector meson Regge trajectory, so our small Q^2 -dependence prediction is parameter free.

There are many phenomenological applications where detailed knowledge of the QCD coupling and the renormalized gluon propagator at relatively soft momentum transfer are essential. This includes the rescattering (final-state and initial-state interactions) which create the leading-twist Sivers single-spin correlations in semi-inclusive deep inelastic scattering, [63, 64] the Boer-Mulders functions which lead to anomalous $\cos 2\phi$ contributions to the lepton pair angular distribution in the unpolarized Drell-Yan reaction, [65] and the Sommerfeld-Sakharov-Schwinger correction to heavy quark production at threshold. [66] The confining AdS/QCD coupling from light-front holography can lead to a quantitative understanding of this factorization-breaking physics. [67]

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Appendix: Consistency Check of Commensurate Scale Relations

In this appendix we verify, within the uncertainties discussed in the text, the validity of the CSR predictions in the pQCD domain.

The verification of CSR for different schemes is illustrated on Fig. 7. On the top panel of Fig. 7 we compare the full two-loop computation of $\alpha_V(Q^2)$ from Ref. [60] with the coupling $\alpha_V(Q^2)$ resulting from applying the CSR to α_{g_1} down to $Q^2 = 0.6$. The width of the bands gives the uncertainties. For the blue band (two-loop pQCD calculation), the uncertainty stems from Λ_{QCD} , the truncation of the $\alpha_{\overline{MS}}$ series to β_2 , and the truncation of the α_V series to two loops (a_2 coefficient) in [60]. All these contributions are added in quadrature. For the red band (CSR), the uncertainties come from Λ_{QCD} , the truncation of the $\alpha_{\overline{MS}}$ series to β_2 , and the truncation of the CSR series to order Q^{***} . All these contributions are again added in quadrature. The various truncation uncertainties are estimated by taking the value of the last known term of the series. The very good agreement of the results [68] allows us to check the consistency and the applicability of CSR, even into the IR-UV transition region, albeit with large uncertainties. Throughout the paper, we limit the order of our calculation to α_s^3 so that no IR terms appear.

A similar test of CSR is also shown on the bottom panel of Fig. 7. It shows α_{g_1} computed using four different methods. 1) The cyan band, corresponds to the results using the Bjorken sum rule and $\alpha_{\overline{MS}}$. 2) The green band [69]: using the CSR to obtain α_{g_1} as a function of $\alpha_{\overline{MS}}$. 3) Red line: using the CSR to obtain α_{g_1} as a function of α_V . This latter is computed from the two-loop computation of Ref. [60]. 4) Black line: using α_V from pQCD [60] as

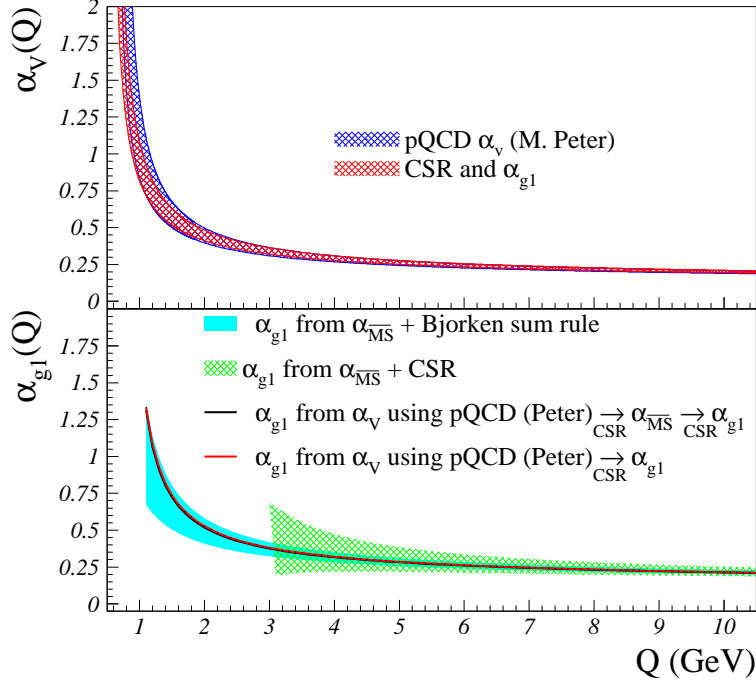


FIG. 7. Check of the CSR validity. Top panel: Comparison of $\alpha_V(Q^2)$ from the two-loop pQCD calculation of Ref. [60] and $\alpha_V(Q^2)$ obtained using the CSR with α_{g_1} as input. Bottom panel: Comparison of $\alpha_{g_1}(Q^2)$ computed using 4 different methods. The good agreement on top and bottom panels is a fundamental check of QCD.

an input to the appropriate CSR to form $\alpha_{\overline{MS}}$. This later is used as input in another CSR to form α_{g_1} . There is again excellent agreement. In addition, that the red and black lines are on top of each other verifies the transitivity property of the CSR. These agreements are non-trivial consistency checks of QCD since the CSR are central predictions of pQCD.

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In contrast, the first-order scale shift from the g_1 scheme to the V scheme is $Q^{*2} = 1.39 Q^2$, allowing an increase in the range of pQCD.

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