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NLO QCD corrections to ZZ+jet production at hadron colliders

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Abstract

A fully differential calculation of the next-to-leading order QCD corrections to the production of Z-boson pairs in association with a hard jet at the Tevatron and LHC is presented. This process is an important background for Higgs particle and new physics searches at hadron colliders. We find sizable corrections for cross sections and differential distributions, particularly at the LHC. Residual scale uncertainties are typically at the 10% level and can be further reduced by applying a veto against the emission of a second hard jet. Our results confirm that NLO corrections do not simply rescale LO predictions.

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I. INTRODUCTION

Weak boson pair production at hadron colliders plays an essential part in the search for Higgs particles and for physics beyond the Standard Model (SM), since weak bosons can decay into jets, charged leptons or neutrinos and hence produce the same signatures as Higgs bosons, new coloured particles, new electroweak gauge bosons or dark matter candidates. In addition to being an important background to direct new physics searches at the Large Hadron Collider (LHC) [1], weak boson pair production also allows to search for new physics via experimental evidence for SM deviations in the form of anomalous interactions between electroweak gauge bosons [2].

Z-boson pair production has been observed at the Tevatron [3] and studied extensively by the LHC general-purpose detector collaborations [4]. Since LO predictions for hadron collider processes are affected by large QCD scale uncertainties with respect to normalisation and kinematical dependence, the inclusion of NLO QCD corrections is important when comparing predictions for cross sections and differential distributions with data. Theoretical predictions for ZZ production at leading order (LO) [5] have thus been improved by including next-to-leading order (NLO) QCD corrections without [6] and with decays [7]. More recently, Z-boson pair production at NLO has also been investigated in selected SM extensions [8]. As a window to new physics ZZ production is particularly interesting because of the absence of $ZZ\gamma$ and ZZZ couplings [9] in the SM. Probing such anomalous neutral gauge boson couplings at hadron colliders has also been studied in the literature [10].

Going beyond final states with two particles, NLO QCD corrections have been computed for the production of three weak/vector bosons [11], the production of a weak boson in association with up to three jets [12] and weak boson pair production in vector boson fusion [13]. Of particular interest is also the production of weak boson pairs with one additional jet at NLO. This process is interesting in its own right, due to the enhanced jet activity, particularly at the LHC. It also provides the real-virtual contribution to the next-to-next-to-leading order (NNLO) corrections to weak boson pair production. The production of W-boson pairs with an additional jet has thus been calculated at NLO without [14] and with [15, 16] decays. An additional contribution to the NNLO corrections that has been calculated for WW and ZZ production (at LO) including decays is the loop-induced gluon-fusion process [17]. Other building blocks for the NNLO calculation of weak boson pair

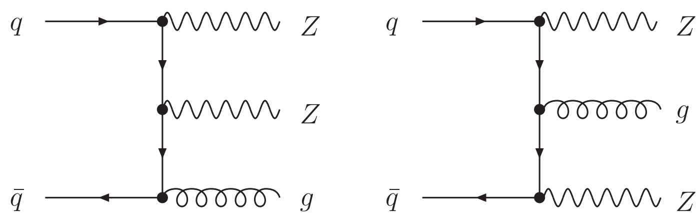

FIG. 1: Representative LO graphs for the partonic process $q\bar{q} \to ZZg$.

production have been presented in Ref. [18].

In this paper, we present a calculation of the $\mathcal{O}(\alpha_s)$ corrections to Z-boson pair production with an additional jet at hadron colliders in the SM.[1] Details of the NLO calculation are described in Sec. II. We then present numerical results in Sec. III and end with conclusions.

II. DETAILS OF THE NLO CALCULATION

At LO, all channels for ZZj production at hadron colliders are related to the amplitude $0 \to ZZq\bar{q}g$ by crossing symmetry. Therefore, the following subprocesses contribute:

$$q\bar{q} \to ZZg\,, \quad qg \to ZZq\,, \quad \bar{q}g \to ZZ\bar{q}\,,$$

where q can be either an up- or down-type quark.[2] We calculate in the 5-flavour scheme, i.e. $q = u, c, d, s, b$, and neglect all quark masses. Representative LO diagrams for the first subprocess are shown in Fig. 1. Comparing the ZZ with the corresponding WW production amplitude, key differences are that the W-boson coupling to fermions is purely left-handed, which leads to a reduced number of helicity amplitudes in that case, the produced weak bosons are distinct for WW production, but identical for ZZ production (leading to a significant increase in the number of Feynman diagrams), and graphs with triple-gauge boson vertices contribute to WW, but not to ZZ production.

[1] Partial results of our calculation have already been presented in Ref. [19].

[2] The down-type quark initiated amplitudes are obtained from the up-type quark initiated amplitudes by adjusting the chiral couplings.

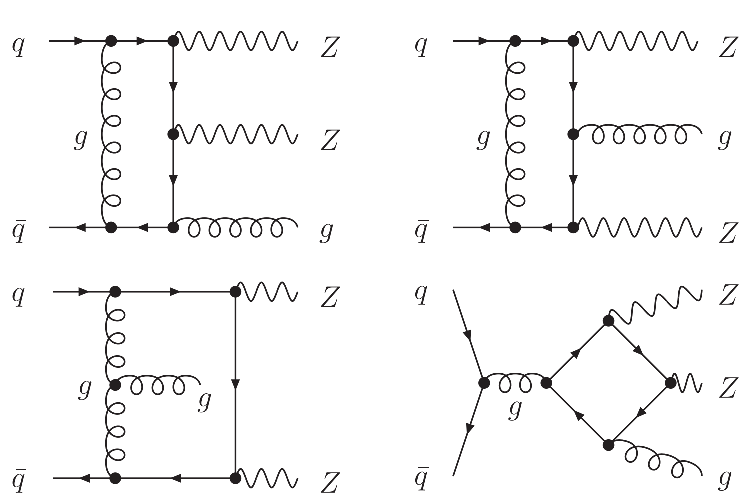

FIG. 2: Representative one-loop graphs for the partonic process $q\bar{q} \to ZZg$.

A. Virtual corrections

At $\mathcal{O}(\alpha_s)$, the most complicated loop topologies are pentagon graphs derived from the tree-level graphs via virtual gluon exchange (and crossing) and box graphs derived by closing the quark line in the tree-level graphs and attaching a $gq\bar{q}$ current. Since we calculate with $N_f = 5$ and neglect quark mass effects, graphs with Higgs boson exchange do not contribute. Triangle graphs, where three gauge bosons couple to a quark loop, also vanish in the massless quark limit. Representative one-loop graphs for the partonic process $q\bar{q} \to ZZg$ are shown in Fig. 2. Starting from the Feynman graph representation, two independent sets of amplitude expressions have been generated: one manually, the other using QGRAF [20]. Both representations employ the spinor helicity formalism of Ref. [21]. Polarisation vectors have been represented via spinor traces, i.e. kinematic invariants up to global phases. By obtaining an analytical representation for the full amplitude, we aim at promoting simplification via analytical cancellations. Especially we employ that, apart from the rank one case, all pentagon tensor integrals are reducible, i.e. can directly be written as simple combinations of box tensor integrals. For the remaining tensor integrals we employ the GOLEM-approach [22]. In this approach, the use of 6-dimensional IR finite box functions allows to isolate IR divergences in 3-point functions. We use FORM [23] and Maple to obtain tractable analytical expressions for the coefficients to the employed set of basis functions for each independent helicity amplitude, and to further simplify them. The basis functions are evaluated using the GOLEM95 implementation [24]. We note that for the reduction of box topologies one obtains the same result as with the Passarino-Veltman tensor reduction [25]. If one fully reduces all tensor integrals to a scalar integral representation, the difference between the two approaches results from the treatment of the pentagon integrals and the use of finite 6-dimensional box functions.

To treat γ_5 we employ the 't Hooft-Veltman scheme [26], where the γ^μ are split into a 4-dimensional part that anti-commutes with γ_5 and a commuting remainder.[3] As is well known, to take into account differences between the QCD corrections to axial vector and vector currents, a finite renormalisation has to be performed. To enforce the correct chiral structure of the amplitudes, a finite counterterm for the axial part is included in the used gauge boson vertex (see e.g. Ref. [28]):

$$V^\mu_{Vq\bar{q}} \sim g_v\,\gamma^\mu + Z_5\,g_a\,\gamma^\mu\gamma_5 \quad \text{with} \quad Z_5 = 1 - C_F\,\frac{\alpha_s}{\pi}\,.$$

We have verified that the relative contribution of graphs with quark loops to the integrated results is well below 1%. We therefore neglect this contribution in the following. We have compared our two independent implementations of the virtual amplitudes – both generated using the GOLEM reduction – and have found agreement of 9-16 significant digits for all contributions at two test phase space points. The discrete Bose, P and C symmetries induce relations between different helicity amplitudes that have been verified numerically as additional check. We have furthermore tested gauge invariance by confirming the Ward identity for external gluons. Furthermore, we used the same tools as in the calculation presented here to calculate numerical results for the successful comparison of virtual corrections to WWj production in Ref. [29]. That comparison therefore provides an additional check of our calculation. We have also verified that our LO amplitude implementation is correct. To calculate numerical results for the virtual contributions we employed the OmniComp-Dvegas package [30], which facilitates parallelised adaptive Monte Carlo integration and was developed in the context of Ref. [31].

[3] Note that the 't Hooft-Veltman scheme treats the observed particles in 4 dimensions, but the soft/collinear gluons in d dimensions. This guarantees that for the IR subtractions the same Catani–Seymour dipole terms as for conventional dimensional regularisation can be used [27].

B. Real corrections

The $\mathcal{O}(\alpha_s)$ real corrections channels for ZZj production at hadron colliders are related to the amplitudes $0 \to ZZq\bar{q}gg$ and $0 \to ZZq\bar{q}q'\bar{q}'$ by crossing symmetry. While all virtual corrections channels are already active at LO, new real corrections channels open up at NLO, namely the gg, qq', $q\bar{q}'$ ($q' \neq q$) and $\bar{q}\bar{q}'$ channels. Note that these new channels are effectively of LO type.

To facilitate the cancellation of soft and collinear singularities we employ the Catani-Seymour dipole subtraction method [32]. We use the SHERPA implementation [33] to calculate numerical results for the finite real corrections contribution. All amplitude and dipole contributions have been verified through comparison with results calculated with two independent implementations:[4] MadDipole/MadGraph [35] and HELAC [36]. In both comparisons 9-significant-digits agreement or better was achieved for all contributions for two test phase space points. We have also successfully compared with an in-house implementation of a set of independent dipoles.

III. NUMERICAL RESULTS

In this section, we present first NLO predictions for ZZj cross sections and differential distributions at the Tevatron ($p\bar{p}$, $\sqrt{s} = 1.96$ TeV) and LHC (pp, $\sqrt{s} = 14$ TeV) and compare them with the NLO results for WWj production given in Ref. [16].[5]

As mentioned above, our calculation employs the 5-flavour scheme and the massless quark approximation (including the b quark). We use the SM parameters[6]

$$M_Z = 91.188 \text{ GeV}\,, \quad \alpha(M_Z) = 0.00755391226\,, \quad \sin\theta_W^2 = 0.222247\,,$$

and employ CTEQ6 parton distribution function sets [37]. LO (NLO) cross sections are calculated with CTEQ6L1 (CTEQ6M) and LO (NLO) α_s running. For $\alpha_s(M_Z)$, the default LHAPDF [38] values are used: $\alpha_s(91.188 \text{ GeV}) = 0.129783$ at LO and $\alpha_s(91.70 \text{ GeV}) = 0.1179$ at NLO. In our parton-level calculation, partons are clustered into jets using the

[4] Other implementations that automate the dipole subtraction method of Ref. [32] have been reported in Ref. [34].

[5] Differences in the input parameters are minute (less than 0.1%).

[6] We provide the exact input values of our calculation in order to facilitate reproducibility.

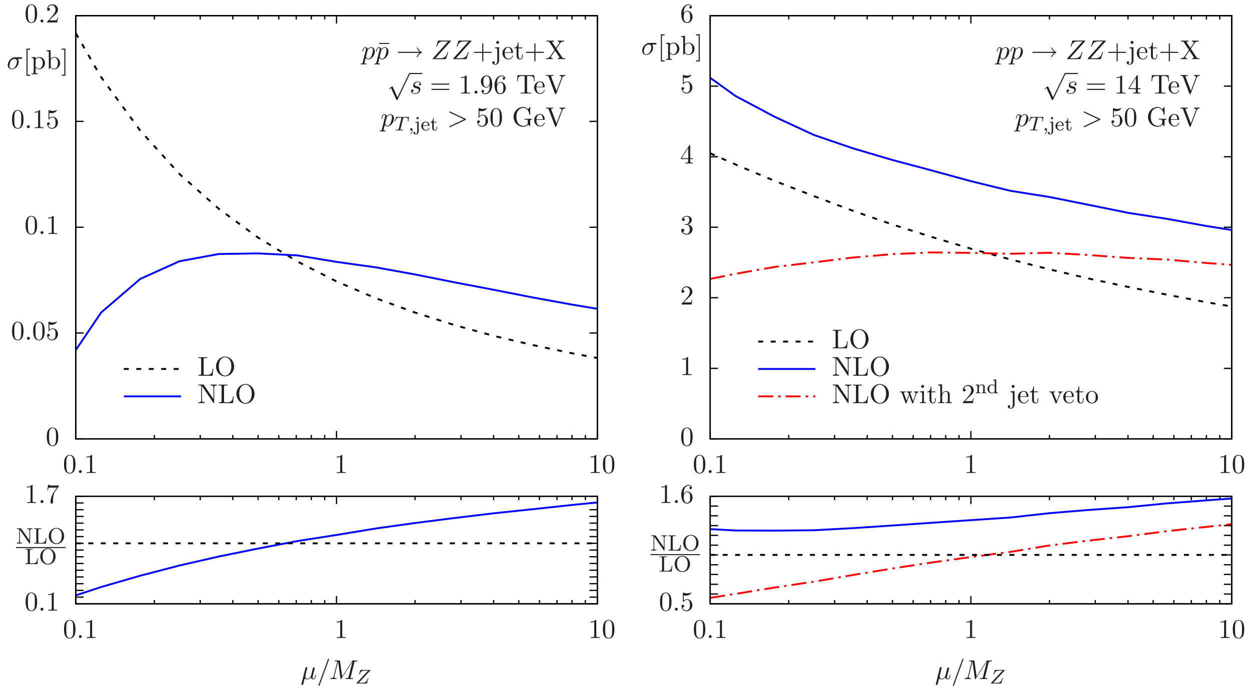

FIG. 3: Comparison of the scale dependence ($\mu_R = \mu_F = \mu$) of the ZZ+jet cross section at the Tevatron and LHC with $p_{T,\,\mathrm{jet}} > 50$ GeV for the hardest jet in LO (dotted) and NLO (solid). For the LHC the exclusive NLO cross section when a $p_{T,\mathrm{jet}} > 50$ GeV veto for additional jets is applied is also shown (dot-dashed). Input parameters are defined in the main text.

inclusive k_t algorithm [39] with $R = 0.7$. To study the scale dependence of cross section predictions we use the discrete grid $\mu = 2^{i/2} M_Z$ with $i \in \{-7, -6, \ldots, 7\}$. The renormalisation and factorisation scales are identified here ($\mu = \mu_R = \mu_F$). We apply a $p_T > 50$ GeV cut on the hardest jet unless noted otherwise.

In Fig. 3, LO and NLO predictions for ZZj production cross sections at the Tevatron and LHC are displayed as function of the QCD scale μ, which is varied by a factor 10 around the Z mass. At LO, we find a much larger scale variation at the Tevatron than at the LHC. When NLO corrections are included, the Tevatron cross section reaches its maximum at approximately $M_Z/2$ and its variation is very effectively reduced. The shape of the cross section variation at the LHC on the other hand is qualitatively unchanged when going from LO to NLO. We attribute this to new channels that become active at NLO (see Sec. II B), which have a modest impact at the Tevatron, but a sizable impact at the LHC, due to parton densities being probed in different x regions. One might be tempted to conclude

	$\Delta\sigma/\sigma(p\bar{p} \to ZZ + \text{jet})$, $\sqrt{s} = 1.96$ TeV		
	$\mu/M_Z \in [\frac{1}{2}, 2]$	$\mu/M_Z \in [\frac{1}{4}, 4]$	$\mu/M_Z \in [\frac{1}{8}, 8]$
LO	23%	44%	62%
NLO	6%	11%	19%

TABLE I: Scale uncertainty for LO and NLO cross sections for ZZ+jet production at the Tevatron as function of the scale variation. The relative scale uncertainty is defined through the envelope: $\Delta\sigma/\sigma := [\sigma_{\max}(\mu \in I) - \sigma_{\min}(\mu \in I)]/[\sigma_{\max}(\mu \in I) + \sigma_{\min}(\mu \in I)]$. Other details as in Fig. 3.

	$\Delta\sigma/\sigma(pp \to ZZ + \text{jet})$, $\sqrt{s} = 14$ TeV		
	$\mu/M_Z \in [\frac{1}{2}, 2]$	$\mu/M_Z \in [\frac{1}{4}, 4]$	$\mu/M_Z \in [\frac{1}{8}, 8]$
LO	12%	23%	34%
NLO	7%	15%	23%
NLO with 2^{nd} jet veto	0.5%	3%	6%

TABLE II: Scale uncertainty for LO and NLO cross sections for ZZ + jet production at the LHC as function of the scale variation. Other details as in Fig. 3 and Table I.

that a constant K-factor is a good approximation for the inclusive NLO cross section at the LHC. However, the K factor (also shown) varies between 1.3 and 1.6 in the displayed scale range. Following Ref. [14], we also calculate an exclusive NLO cross section for the LHC by vetoing 2-jet events with a second hardest jet with $p_T > 50$ GeV (NLO with 2^{nd} jet veto). This exclusive NLO LHC cross section decreases for scales below M_Z and has a strongly reduced scale uncertainty. Since the qualitative features of our results are similar to those found for WWj production, we quantify the ZZj LO and NLO scale uncertainties for the Tevatron and LHC in Tables I and II, respectively. When comparing our results for $\mu/M_Z \in [\frac{1}{2}, 2]$ with the corresponding WWj results extracted from Tables 4 and 1 in Ref. [16], we find deviations of less than 2 percentage points.

In Tables III and IV, we show for the Tevatron and LHC, respectively, how the ZZj LO and NLO cross sections change when the p_T cut on the hardest jet is varied. These

	$\sigma(p\bar{p} \to ZZ + \text{jet})$ [pb], $\sqrt{s} = 1.96$ TeV			
$p_{T,\text{jet}}$ cut [GeV]	20	50	100	200
LO	0.27202(3)	$0.07456(1)^{+28\%}_{-20\%}$	0.016037(2)	0.0012651(1)
NLO	0.3307(6)	$0.0836(1)^{+5\%}_{-7\%}$	0.01583(4)	0.000976(4)

TABLE III: ZZ+jet production cross section at the Tevatron with different p_T cuts for the hardest jet. The scale $\mu = M_Z$ is employed with $\mu_R = \mu_F = \mu$. The integration error is given in brackets. The minimum and maximum relative deviation from $\sigma(\mu = M_Z)$ obtained with the scale variation $\mu/M_Z \in [\frac{1}{2}, 2]$ is shown as sub- and superscript for a $p_{T,\text{jet}}$ cut of 50 GeV. Input parameters are defined in the main text.

	$\sigma(pp \to ZZ + \text{jet})$ [pb], $\sqrt{s} = 14$ TeV			
$p_{T,\text{jet}}$ cut [GeV]	20	50	100	200
LO	6.505(1)	$2.6978(4)^{+13\%}_{-11\%}$	1.0066(1)	0.22974(3)
NLO	8.01(3)	$3.653(9)^{+8\%}_{-6\%}$	1.511(4)	0.415(2)
NLO with 2$^{\text{nd}}$ jet veto		$2.637(9)^{+0.2\%}_{-1\%}$	0.755(4)	0.1005(9)

TABLE IV: ZZ+jet production cross section at the LHC with different p_T cuts for the hardest jet. For cut values above 20 GeV, we also give the NLO cross section when a $p_{T,\text{jet}} > 50$ GeV veto for additional jets is added to the selection. Other details as in Table III.

results demonstrate that all K factors are p_T cut dependent. In general, the K factor for ZZj production will have a non-negligible differential dependence. As example, we display in Fig. 4 the differential LO and NLO distributions with respect to the invariant ZZ mass and the resulting K factor at the Tevatron and LHC. The K-factor bands shown in this figure correspond to a variation of the scale μ by a factor of 2 in the NLO differential cross section only, i.e. we display $[d\sigma_{\text{NLO}}/dM_{ZZ}](\mu)/[d\sigma_{\text{LO}}/dM_{ZZ}](M_Z)$ with $\mu/M_Z \in [\frac{1}{2}, 2]$. One can distinguish the modest variation of the inclusive NLO K factor at the LHC from the strong decrease of the other K factors with increasing M_{ZZ}.

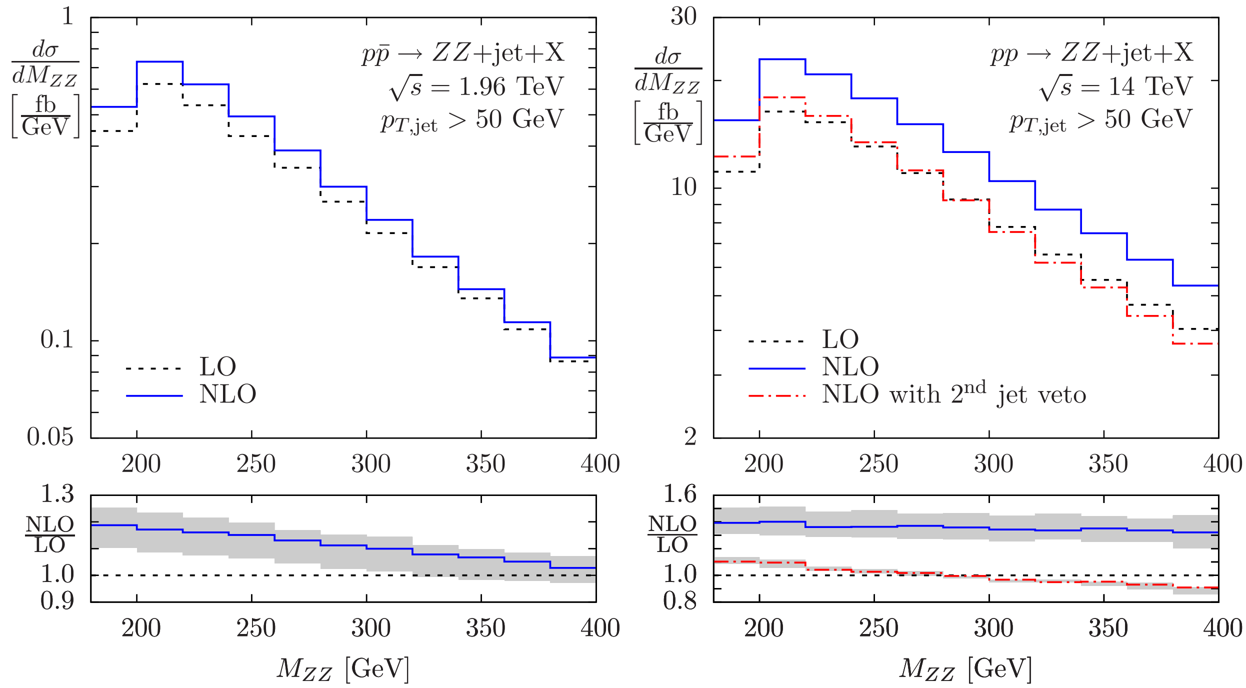

FIG. 4: ZZ invariant mass distribution for ZZ+jet production at the Tevatron and LHC with $\mu_R = \mu_F = M_Z$. The differential K factor is also shown. The K-factor bands are defined in the main text. Other details as in Fig. 3.

IV. CONCLUSIONS

In this paper we have presented first results for ZZj production at NLO QCD accuracy, obtained with fully differential parton-level Monte Carlo programs that allow to take into account realistic experimental selection cuts. For a default scale choice of $\mu = M_Z$ and a p_T cut of 50 GeV for the hardest jet we find a K factor of 1.1 and 1.35 at the Tevatron and LHC, respectively. At the Tevatron, the NLO corrections stabilise the LO prediction for cross sections considerably. The shape of the cross section variation at the LHC on the other hand is qualitatively unchanged when going from LO to NLO. Nevertheless, at the LHC stabilisation at NLO can still be achieved by applying suitable selection cuts like for example a veto against the emission of a second hard jet. Our results indicate that residual scale uncertainties are typically at the 10% level and can be further reduced to about 5% or less by applying suitable selections.

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