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Measurement of the angle α at *BABAR*

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Abstract. We present recent measurements of the CKM angle α using data collected by the *BABAR* detector at the PEP-II asymmetric-energy e^+e^- collider at the SLAC National Acce- lerator Laboratory, operating at the $\Upsilon(4S)$ resonance. We present constraints on α from $B \to \pi\pi$, $B \to \rho\rho$ and $B \to \rho\pi$ decays.

1. Introduction

The measurements of the angles α, β and γ of the Unitarity Triangle (UT) at the B-factories are providing precision tests of the Standard Model (SM) description of CP violation. This description is provided by the Cabibbo-Kobayashi-Maskawa (CKM) quark-mixing [1, 2]. We summarize the experimental constraints on the α UT angle obtained from B-meson decays to $\pi\pi$, $\rho\rho$ and $\rho\pi$ with the *BABAR* experiment at the SLAC National Accelerator Laboratory. The *BABAR* detector and the PEP-II accelerator are described elsewhere [3].

2. Analysis Method

2.1. General formula

The decay of a neutral B-meson into a pair of π or ρ mesons, $B \to hh$ ($h = \pi, \rho$), occurs via two topologies: a tree-level process and a one-loop penguin diagram. The CP parameter λ_{hh}, defined by $\lambda_{hh} = \frac{p}{q}\frac{\overline{A}}{A}$, where q and p are the complex coefficient that link the mass and the flavor eigenstates in the B system, and A ($\overline{A}$) is the B^0 ($\overline{B}^0$) decay amplitude, can be expressed in terms of α as

$$\lambda_{hh} = e^{2i\alpha}\frac{1 - (|V_{td}^* V_{tb}|/|V_{ud}^* V_{ub}|)P/Te^{-i\alpha}}{1 - (|V_{td}^* V_{tb}|/|V_{ud}^* V_{ub}|)P/Te^{i\alpha}} \ , \tag{1}$$

where T and P are complex amplitudes dominated by tree and penguin topologies, respectively.

The quantity experimentally measured is the time-dependent decay rate

$$f_{Q_{\rm tag}} = \frac{e^{-|\Delta t|/\tau}}{4\tau}\left[1 - Q_{\rm tag}C_{hh}\cos(\Delta m_d \Delta t) + Q_{\rm tag}S_{hh}\sin(\Delta m_d \Delta t)\right] \ , \tag{2}$$

where τ is the neutral B lifetime and Δm_d is the $B^0\overline{B}^0$ oscillation frequency. Δt is the proper time difference between decays of the B to hh ($B_{\rm rec}$), and the second B in the event, denoted by $B_{\rm tag}$. The $Q_{\rm tag}$ parameter is related to the flavor of the B_{tag}: $Q_{\rm tag} = +1(-1)$ if the $B_{\rm tag}$ is a B^0 ($\overline{B}^0$). The CP-violating asymmetries C_{hh} and S_{hh} are related to the λ_{hh} parameter by

$$S_{hh} = 2\mathcal{I}m(\lambda_{hh})/(1+|\lambda_{hh}|^2) \ , \qquad C_{hh} = (1-|\lambda_{hh}|^2)/(1+|\lambda_{hh}|^2) \ . \tag{3}$$

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S_{hh} reflects the CP-violation induced by the interference between the mixing and decay processes; C_{hh} is the direct CP-violating asymmetry which comes from the interference between different decay topologies. In the absence of penguin contributions ($P = 0$), C_{hh} vanishes and S_{hh} is simply related to the CKM angle α by $S_{hh} = \sin(2\alpha)$.

In the more general case of the $B^0(\overline{B}^0) \to \rho^\pm\pi^\mp$ decays, the time-dependent decay rate is given by

$$f^{\rho^\pm\pi^\mp}_{Q_{\rm tag}} = (1\pm\mathcal{A}_{\rho\pi})\frac{e^{-|\Delta t|/\tau}}{4\tau}\left[1 - Q_{\rm tag}(C_{\rho\pi} \pm \Delta C_{\rho\pi})\cos(\Delta m_d\Delta t) + Q_{\rm tag}(S_{\rho\pi} \pm \Delta S_{\rho\pi})\sin(\Delta m_d\Delta t)\right] , \quad (4)$$

where, the $\pm$ sign depends on whether the ρ meson is emitted by the W boson or comes from the spectator quark. $\mathcal{A}_{\rho\pi}$ is the direct CP violation parameter measuring the asymmetry between the $\rho^+\pi^-$ and $\rho^-\pi^+$ final states, while $\Delta S_{\rho\pi}$ and $\Delta C_{\rho\pi}$, which arise from the fact that two production modes of the ρ are possible, are dilution terms and have no CP content.

2.2. The isospin analysis

Using the strong isospin symmetry, the angle α can be extracted up to discrete ambiguities from the CP-violating asymmetries defined above [4]. The decay amplitudes of the isospin-related final states obey the pentagonal relations

$$\sqrt{2}(A^{+0}_{\rho\pi} + A^{0+}_{\rho\pi}) = 2A^{00}_{\rho\pi} + A^{+-}_{\rho\pi} + A^{-+}_{\rho\pi} , \qquad \sqrt{2}(\overline{A}^{+0}_{\rho\pi} + \overline{A}^{0+}_{\rho\pi}) = 2\overline{A}^{00}_{\rho\pi} + \overline{A}^{+-}_{\rho\pi} + \overline{A}^{-+}_{\rho\pi} ; \quad (5)$$

where $A^{ij}_{\rho\pi} = A(B^0 \text{ or } B^+ \to \rho^i\pi^j)$ and $\overline{A}^{ij}_{\rho\pi} = A(\overline{B}^0 \text{ or } B^- \to \rho^i\pi^j)$, $i, j = +, -, 0$. With the use of these relations, 12 unknowns (6 complex amplitudes with one unphysical phase, and the CKM angle α) are to be determined while 13 observables are available: $S_{\rho\pi}$, $C_{\rho\pi}$, $\Delta S_{\rho\pi}$, $\Delta C_{\rho\pi}$, $\mathcal{A}_{\rho\pi}$; four average branching fractions $\mathcal{B}(B \to \rho\pi)$; two time-dependent CP-violating asymmetries in the $B^0 \to \rho^0\pi^0$ decay ($S^{00}_{\rho\pi}$, $C^{00}_{\rho\pi}$) and two direct CP asymmetries in $B^+ \to \rho^+\pi^0$ and $B^+ \to \rho^0\pi^+$ decays.

In the case of $B \to hh$ ($h = \pi, \rho$), Eq. 5 simplify to triangular relations

$$\sqrt{2}A^{+0}_{hh} = A^{+-}_{hh} + A^{00}_{hh} , \qquad \sqrt{2}\overline{A}^{+0}_{hh} = 2\overline{A}^{+-}_{hh} + \overline{A}^{00}_{hh} . \quad (6)$$

The information counting leads then to 6 unknowns and 7 observables: 3 branching fractions $\mathcal{B}(B \to hh)$; C_{hh}, S_{hh}, C^{00}_{hh}, S^{00}_{hh}. In the $\pi\pi$ system $S^{00}_{\pi\pi}$ is impossible to measure (as the π^0 is reconstructed from two-photons decays, there is no way to measure the decay vertex), then one is left with 6 observables: α can be extracted with an 8-fold ambiguity within $[0, \pi]$ [5].

3. Experimental Results

3.1. $B \to \pi\pi$ and $B \to \rho\rho$

The various branching fractions and CP-asymmetries measured in $B \to \pi\pi$ and $B \to \rho\rho$ decays are summarized in Table 1. In the case of charged decays the charge asymmetry is defined as $\mathcal{A}_{CP}(B \to hh) = -C_{hh}$. The measurements are sufficiently well established to perform an isospin analysis.

The present measurement for the $\pi^+\pi^-$ mode excludes the absence of CP violation $(C_{\pi\pi}, S_{\pi\pi}) = (0, 0)$ at a C.L. of 6.7σ. The relatively high branching fraction of the $\pi^0\pi^0$ mode tends to separate the 8-fold ambiguities in the α extraction, which only allows a weak constraint on α to be set. With the current experimental measurements two of the eight ambiguities are nearly merged. The range $[23^o, 67^o]$ in α is excluded at the 90% C.L. [6]. The solution is in agreement with the global CKM fit [13, 14] which gives the range $[71^o, 109^o]$ at 68% C.L.

Mode	$\mathcal{B}(10^{-6})$	C	S
$\pi^+\pi^-$	$5.5 \pm 0.4 \pm 0.3$ [7]	$-0.68 \pm 0.10 \pm 0.03$ [6]	$-0.25 \pm 0.08 \pm 0.02$ [6]
$\pi^0\pi^0$	$1.83 \pm 0.21 \pm 0.13$ [6]	$-0.43 \pm 0.26 \pm 0.05$ [6]	–
$\rho^+\rho^-$	$25.5 \pm 2.1^{+3.6}_{-3.9}$ [9]	$0.01 \pm 0.15 \pm 0.06$ [9]	$-0.17 \pm 0.2^{+0.05}_{-0.06}$ [9]
$\rho^0\rho^0$	$0.92 \pm 0.32 \pm 0.14$ [10]	$0.2 \pm 0.8 \pm 0.3$ [10]	$0.3 \pm 0.7 \pm 0.2$ [10]
Mode	$\mathcal{B}(10^{-6})$	$\mathcal{A}_{\mathbf{CP}}$	
$\pi^\pm\pi^0$	$5.02 \pm 0.46 \pm 0.29$ [8]	$0.03 \pm 0.08 \pm 0.01$ [8]	
$\rho^\pm\rho^0$	$23.7 \pm 1.4 \pm 1.4$ [11]	$-0.054 \pm 0.055 \pm 0.010$ [11]	

Table 1. *Summary of BABAR measurements of $B \to \pi\pi$ and $B \to \rho\rho$ decays. The measurements for the $\rho\rho$ system corresponds to the longitudinal component of the decay rate. The errors quoted are statistical and systematic, respectively.*

The analysis of $B \to \rho\rho$ is potentially complicated due to the possible presence of three helicity states for the decay. The helicity zero state, which corresponds to longitudinal polarization of the decay, is CP-even but the helicity ±1 states are not CP eigenstates. Fortunately this complication is avoided by the experimental finding that the dominant polarization is longitudinal, $f_L(\rho^+\rho^-) = 0.992 \pm 0.024^{+0.026}_{-0.013}$ [9], $f_L(\rho^0\rho^0) = 0.75^{+0.11}_{-0.14} \pm 0.05$ [10] and $f_L(\rho^+\rho^0) = 0.950 \pm 0.015 \pm 0.006$ [11] ($f_L \equiv \Gamma_L/\Gamma$, where Γ is the total decay rate and Γ_L is the rate of the longitudinally-polarized mode). The $B^0 \to \rho^0\rho^0$ branching fraction is small compared with that of the $B^+ \to \rho^+\rho^0$ mode, which indicates that the penguin to three ratio (P/T, cf. Eq. 1) is small compared with that of the $B \to \pi\pi$ system [4]. This has the effect of merging the different ambiguities in the extraction of α. The latest $B^0 \to \rho^0\rho^0$ *BABAR* results present the first measurement of the time-dependent CP asymmetries C_L^{00} and S_L^{00}. The inclusion of these measurements has the effect of raising the 8-fold degeneracy on α: the data only favors two solutions out of eight [10, 11]. These two effects allow to set a strong constraint on α, where only two solutions are seen, corresponding to $\alpha = (92.4^{+6.0}_{-6.5})^o$ at 68% C.L. [11] for the one in agreement with the global CKM fit [13, 14].

3.2. $B \to \rho\pi$

The $B \to \rho\pi$ measurement reported here is a time-dependent amplitude analysis of $B^0 \to (\rho\pi)^0$. The interferences between the intersecting ρ resonance bands are modeled over the whole Dalitz Plot using the isobar model [15]. This allows determination of the strong phase differences from the interference pattern, which permits direct extraction of the angle α with reduced ambiguities. The Dalitz amplitudes and time-dependence are contained in the 26 coefficients of the bilinear form-factor terms occurring in the time-dependent decay rate, which are determined from a likelihood fit. The values obtained for these coefficients are converted back into the quasi-two-body CP observables (c.f. Eq. 4), which are more intuitive in their interpretation. Table 2 reports the experimental findings on these observables [12].

Observable	Value	Observable	Value
$C_{\rho\pi}$	$0.15 \pm 0.09 \pm 0.05$	$S_{\rho\pi}$	$-0.03 \pm 0.11 \pm 0.04$
$\Delta C_{\rho\pi}$	$0.39 \pm 0.09 \pm 0.09$	$\Delta S_{\rho\pi}$	$-0.01 \pm 0.14 \pm 0.06$
$C_{\rho\pi}^{00}$	$-0.10 \pm 0.40 \pm 0.53$	$S_{\rho\pi}^{00}$	$0.04 \pm 0.44 \pm 0.18$
$A_{\rho\pi}$	$-0.14 \pm 0.05 \pm 0.02$		

Table 2. *Summary of BABAR measurements from the time-dependent amplitude analysis of $B^0 \to (\rho\pi)^0$ decays. The errors quoted are statistical and systematic, respectively.*

These measurements allow the determination of the limit $\alpha = (87^{+45}_{-13})^o$ at 68% C.L., with almost no constraint at 95% C.L. This result is particularly interesting as there is an unique solution in the $[0, 180]^o$ range, which helps to break the ambiguities obtained from the $\pi\pi$ and $\rho\rho$ results. A hint of CP-violation is obtained at the level of 3σ.

4. Summary

Several analyses have been conducted in *BABAR* to extract the angle α of the UT. In the last few years the measurements of this angle have become increasingly precise. The measurements provided from the $B \to \pi\pi/\rho\rho/\rho\pi$ modes give complementary constraints on α. For the $B \to \rho\rho$ system, the inclusion of the $S^{00}_{\rho\pi}$ observable allows to favor two of the 8-fold ambiguities on α, and the relatively large $\mathcal{B}(B^+ \to \rho^+\pi^0)$, with respect to $\mathcal{B}(B^0 \to \rho^0\pi^0)$, causes the ambiguities to degenerate in two peaks, improving the precision of the constraint. The measurements from the $B^0 \to (\rho\pi)^0$ time-dependent amplitude analysis give a direct access to α, disfavoring the ambiguities. The combined constraint averaging all the $\pi\pi$, $\rho\rho$ and $\rho\pi$ measurements from *BABAR* and Belle gives $\alpha = (89.0^{+4.4}_{-4.2})^o$ at 68% C.L. (see Fig. 1), which is in good agreement with the global CKM fit [13, 14].

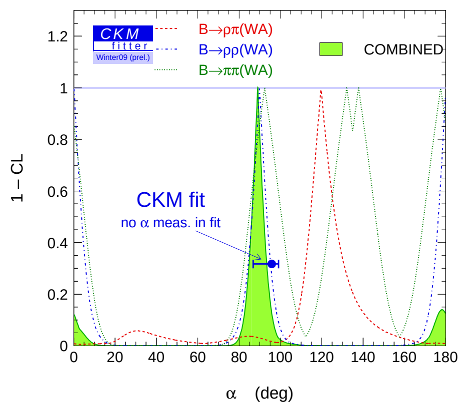

Figure 1. *Constraints on α, provided by the CKMfitter group [13], expressed as one minus the confidence level as a function of angle. The constraints are constructed averaging the BABAR and Belle measurements for the $\pi\pi$ (dotted green curve), $\rho\rho$ (dash-dotted blue curve) and $\rho\pi$ (dashed red curve) systems. The solid filled green curve represents the combined constraint using all the systems.*

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6. References

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