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Condensates in Quantum Chromodynamics and the Cosmological Constant

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Casher and Susskind have noted that in the light-front description, spontaneous chiral symmetry breaking in quantum chromodynamics (QCD) is a property of hadronic wavefunctions and not of the vacuum. Here we show from several physical perspectives that, because of color confinement, quark and gluon QCD condensates are associated with the internal dynamics of hadrons. We discuss condensates using condensed matter analogues, the AdS/CFT correspondence, and the Bethe-Salpeter/Dyson-Schwinger approach for bound states. Our analysis is in agreement with the Casher and Susskind model and the explicit demonstration of "in-hadron" condensates by Roberts et al., using the Bethe-Salpeter/Dyson-Schwinger formalism for QCD bound states. These results imply that QCD condensates give *zero* contribution to the cosmological constant, since all of the gravitational effects of the in-hadron condensates are already included in the normal contribution from hadron masses.

I. INTRODUCTION

Hadronic condensates play an important role in quantum chromodynamics. Two important examples are $\langle \bar{q}q \rangle \equiv \langle \sum_{a=1}^{N_c} \bar{q}_a q^a \rangle$ and $\langle G_{\mu\nu}G^{\mu\nu} \rangle \equiv \langle \sum_{a=1}^{N_c^2-1} G^a_{\mu\nu} G^{a\,\mu\nu} \rangle$, where q is a light quark (i.e., a quark with current-quark mass small compared with the QCD scale Λ_{QCD}), $G^a_{\mu\nu} = \partial_\mu A^a_\nu - \partial_\nu A^a_\mu + g_s c_{abc} A^b_\mu A^c_\nu$, a, b, c are color indices, and $N_c = 3$. (For most of the paper we focus on QCD at zero temperature and chemical potential, $T == \mu = 0$.) For QCD with N_f light quarks, the $\langle \bar{q}q \rangle = \langle \bar{q}_L q_R + \bar{q}_R q_L \rangle$ condensate spontaneously breaks

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the global chiral symmetry $\mathrm{SU}(N_f)_L \times \mathrm{SU}(N_f)_R$ down to the diagonal, vectorial subgroup $\mathrm{SU}(N_F)_{diag}$, where $N_f = 2$ (or $N_f = 3$ since s is a moderately light quark). Thus in the usual description, one identifies $\langle \bar{q}q \rangle \sim \Lambda^3_{QCD}$ and $\langle G_{\mu\nu} G^{\mu\nu} \rangle \sim \Lambda^4_{QCD}$, where $\Lambda_{QCD} \simeq 300$ MeV. These condensates are conventionally considered to be properties of the QCD vacuum and hence to be constant throughout spacetime. A consequence of the existence of such vacuum condensates is contributions to the cosmological constant from these condensates that are 10^{45} times larger than the observed value. If this disagreement were really true, it would be an extraordinary conflict between experiment and the Standard Model.

A very different perspective on QCD condensates was first presented in a seminal paper by Casher and Susskind [1] published in 1974. These authors argued that "spontaneous symmetry breaking must be attributed to the properties of the hadron's wavefunction and not to the vacuum" [1]. The Casher-Susskind argument is based on the Weinberg's infinite-momentum-frame [5] Hamiltonian formalism of QCD, which is equivalent to light-front (LF) quantization and Dirac's front form [6] rather than the usual instant form. Casher and Susskind also presented a specific model in which spontaneous chiral symmetry breaking occurs within the confines of the hadron wavefunction due to a phase change supported by the infinite number of quark and quark pairs in the light-front wavefunction. In fact, the Regge behavior of hadronic structure functions requires that LF Fock states of hadron have Fock states with an infinite number of quark and gluon partons [7, 8]. Thus, in contrast to formal discussions in statistical mechanics, infinite volume is not required for a phase transition in relativistic quantum field theories.

Spontaneous chiral symmetry breaking in QCD is often analyzed by means of an approximate solution of the Schwinger-Dyson equation for a massless quark propagator; if the running coupling $\alpha_s = g_s^2/(4\pi)$ exceeds a value of order 1, this yields a nonzero dynamical (constituent) quark mass Σ [9]. Since in the path integral, Σ is formally a source for the operator $\bar{q}q$, one associates $\Sigma \neq 0$ with a nonzero quark condensate. However, the Dyson-Schwinger equation, by itself, does not incorporate confinement and the resultant property that quarks and gluons have maximum wavelengths [10]; further, it does not actually determine where this condensate has spatial support or imply that it is a spacetime constant.

In contrast, let us consider a meson consisting of a light quark q bound to a heavy antiquark, such as a B meson. One can analyze the propagation of the light quark q in the background field of the heavy $\bar{b}$ quark. Solving the Dyson-Schwinger equation for the light quark, one obtains a nonzero dynamical mass and thus a nonzero value of the condensate $\langle \bar{q}q \rangle$. But this is not a true vacuum expectation value; instead, it is the matrix element of the operator $\bar{q}q$ in the background field of the $\bar{b}$ quark; i.e., one obtains an "in-hadron" condensate.

The concept of "in-hadron condensates" was in fact established in a series of pioneering papers by Roberts et al. [11–13] using the Bethe-Salpeter/Dyson-Schwinger analysis for bound states in QCD in conjunction with the Banks-Casher relation $-\langle\bar{q}q\rangle = \pi\rho(0)$, where $\rho(\lambda)$ denotes the density of eigenvalues $\pm i\lambda$ of the (antihermitian) Euclidean Dirac operator [14]. These authors reproduced the usual features of spontaneous chiral symmetry breaking using hadronic matrix elements of the Bethe-Salpeter eigensolution. For example, the pion matrix element $f_\pi\langle 0|\bar{\psi}\gamma^5\psi|\pi\rangle$ was shown to be finite at $m_q \to 0$; it effectively replaces the usual vacuum chiral condensate.

In this paper we show from several physical perspectives that, because of color confinement, quark and gluon QCD condensates can be regarded as being associated with the dynamics of hadron wavefunctions, rather than the vacuum itself. Thus we analyze the condensates $\langle\bar{q}q\rangle$ and $\langle G_{\mu\nu}G^{\mu\nu}\rangle$ and address the question of where they have spatial (and temporal) support. We argue, in agreement with the original work of Casher and Susskind [1], that these condensates have spatial support restricted to the interiors of hadrons, as a consequence of the fact that they are due to quark and gluon interactions, and these particles are confined within hadrons. Higher-order condensates such as $\langle(\bar{q}q)^2\rangle$, $\langle(\bar{q}q)G_{\mu\nu}G^{\mu\nu}\rangle$, etc. are also present, and our discussion implicitly also applies to these [15]. Our analysis includes consideration of condensed matter analogs, the AdS/CFT correspondence, and the Bethe-Salpeter/Dyson-Schwinger approach for bound states. Our analysis highlights the difference between chiral models where mesons are treated as elementary fields, and QCD in which all hadrons are composite systems. We note that an important consequence of the "in-hadron" nature of QCD condensates is that QCD gives *zero* contribution to the cosmological constant, since all of the gravitational effects of the in-hadron condensate are already included in the normal contribution from hadron masses.

We emphasize the subtlety in characterizing the formal quantity $\langle 0|\mathcal{O}|0\rangle$ in the usual instant form, where $\mathcal{O}$ is a product of quantum field operators, by recalling that one can render this automatically zero by normal-ordering $\mathcal{O}$. This subtlety is especially delicate in a confining theory, since the vacuum state in such a theory is not defined relative to the fields in the Lagrangian, quarks and gluons, but instead relative to the actual physical, color-singlet, states. In the front form, the analysis is simpler, since the physical vacuum is automatically trivial, up to zero modes. The light-front method provides a completely consistent formalism for quantum field theory. For example, it is straightforward to calculate the coupling of gravitons to physical particles using the light-front formalism; in particular, one can prove that the anomalous gravitational magnetic moment vanishes, Fock state by Fock state [2], in agreement with the equivalence principle [3]. Furthermore, the light-front method reproduces quantum corrections to the gravitational form factors computed in perturbation theory [4].

II. A CONDENSED MATTER ANALOGY

A formulation of quantum field theory using a Euclidean path-integral (vacuum-to-vacuum amplitude), Z, provides a precise meaning for $\langle \mathcal{O} \rangle$ as

$$\langle \mathcal{O} \rangle = \lim_{J \to 0} \frac{\delta \ln Z}{\delta J} \ , \tag{2.1}$$

where J is a source for $\mathcal{O}$. The path integral for QCD, integrated over quark fields and gauge links using the gauge-invariant lattice discretization exhibits a formal analogy with the partition function for a statistical mechanical system. In this correspondence, a condensate such as $\langle \bar{q}q \rangle$ or $\langle G_{\mu\nu} G^{\mu\nu} \rangle$ is analogous to an ensemble average in statistical mechanics. To develop a physical picture of the QCD condensates, we pursue this analogy. In a superconductor, the electron-phonon interaction produces a pairing of two electrons with opposite spins and 3-momenta at the Fermi surface, and, for $T < T_c$, an associated nonzero Cooper pair condensate $\langle ee \rangle_T$ [16], (here $\langle ... \rangle_T$ means thermal average). Since this condensate has a phase, the phenomenological Ginzburg-Landau free energy function

$$F = |\nabla \Phi|^2 + c_2(\Phi^* \Phi) + c_4(\Phi^* \Phi)^2 \tag{2.2}$$

uses a complex scalar field Φ to represent it. The formal treatment of a phase transition such as that in a superconductor begins with a partition function calculated for a finite d-dimensional lattice, and then takes the thermodynamic (infinite-volume) limit. The non-analytic behavior associated with the superconducting phase transition only occurs in this infinite-volume limit; for $T < T_c$, the (infinite-volume) system develops a nonzero value of the order parameter, namely $\langle \Phi \rangle_T$, in the phenomenological Ginzburg-Landau model, or $\langle ee \rangle_T$, in the microscopic Bardeen-Cooper-Schrieffer theory. In the formal statistical mechanics context, the minimization of the $|\nabla \Phi|^2$ term implies that the order parameter is a constant throughout the infinite spatial volume.

However, the infinite-volume limit is an idealization; in reality, superconductivity is experimentally observed to occur in finite samples of material, such as Sn, Nb, etc., and the condensate clearly has spatial support only in the volume of these samples. This is evident from either of two basic properties of a superconducting substance, namely, (i) zero-resistance flow of electric current, and (ii) the Meissner effect, that

$$|\mathbf{B}(z)| \sim |\mathbf{B}(0)| e^{-z/\lambda_L} \tag{2.3}$$

for a magnetic field $\mathbf{B}(z)$ a distance z inside the superconducting sample, where the London penetration depth λ_L is given by $\lambda_L^2 = m_e c^2/(4\pi n e^2)$, where n = electron concentration;

both of these properties clearly hold only within the sample. The same statement applies to other phase transitions such as liquid-gas or ferromagnetic; again, in the formal statistical mechanics framework, the phase transition and associated symmetry breaking by a nonzero order parameter at low T occur only in the thermodynamic limit, but experimentally, one observes the phase transition to occur effectively in a finite volume of matter, and the order parameter (e.g., magnetization M) has support only in this finite volume, rather than the infinite volume considered in the formal treatment. Similarly, the Goldstone modes that result from the spontaneous breaking of a continuous symmetry (e.g., spin waves in a Heisenberg ferromagnet) are experimentally observed in finite-volume samples. There is, of course, no conflict between the experimental measurements and the abstract theorems; the key point is that these samples are large enough for the infinite-volume limit to be a useful idealization. Finite-size scaling methods that make this connection precise are standard tools in studies of phase transitions and critical phenomena [17].

There is another important distinction between condensed matter physics and relativistic quantum field theories. The proton eigenstate in QCD is a summation over Fock states

$$|P\rangle = \sum_{n=3}^{\infty} \Psi_{n/P}(x_i, k_{\perp i}, \lambda_i)|n\rangle \tag{2.4}$$

where x_i denotes the fraction of the total proton momentum carried by the parton i, $k_{\perp,i}$ denotes the transverse momentum, λ_i denotes the helicity, and the summation extends over states with an unlimited number of gluons and sea quarks and antiquarks. In fact, the Regge behavior, $F_2(x, Q^2) \sim \sum_R \beta_R x^{1-\alpha_R}$, of hadronic structure functions [18] at small x requires that the hadronic wavefunction has Fock states $|n\rangle$ with an infinite number of quark and gluon partons. For example, Mueller [7] has shown that the BFKL (Balitsky-Fadin-Kuraev-Lipatov) behavior of the structure functions at $x \to 0$ is a result of the infinite range of gluonic Fock states. The relation between Fock states of different n is given by an infinite tower of ladder operators [8]. In the analysis by Casher and Susskind [1], spontaneous chiral symmetry breaking occurs within the confines of the hadron wavefunction due to a phase change supported by the infinite number of quark and quark pairs in the light-front wavefunction. Thus, as noted above, unlike the usual discussion in condensed matter physics, infinite volume is not required for a phase transition in relativistic quantum field theories.

III. A PICTURE OF QCD CONDENSATES

The condensed-matter physics discussion above helps to motivate our analysis for QCD. The spatial support for QCD condensates should be where the particles are whose interactions give rise to them, just as the spatial support of a magnetization M, say, is inside, not outside, of a piece of iron. The physical origin of the $\langle \bar{q}q \rangle$ condensate in QCD can be argued to be due to the reversal of helicity (chirality) of a massless quark as it moves outward and reverses its three-momentum at the boundary of a hadron due to confinement [19]. This argument suggests that the condensate has support only within the spatial extent where the quark is confined; i.e., the physical size of a hadron. Another way to motivate this is to note that in the light-front Fock state picture of hadron wavefunctions [1, 20, 21], a valence quark can flip its chirality when it interacts or interchanges with the sea quarks of multi-quark Fock states, thus providing a dynamical origin for the quark running mass. In this description, the $\langle \bar{q}q \rangle$ and $\langle G_{\mu\nu} G^{\mu\nu} \rangle$ condensates are effective quantities which originate from $q\bar{q}$ and gluon contributions to the higher Fock state light-front wavefunctions of the hadron and hence are localized within the hadron. There is a natural relation with the nucleon sigma term, $\sigma_{\pi N} = (1/2)(m_u + m_d)\langle N|\bar{q}q|N\rangle$ (where here the nucleon states are normalized as $\langle N(p')|N(p)\rangle = (2\pi)^3 \delta^3(\mathbf{p} - \mathbf{p}'))$. The in-hadron quark condensate is also connected with the pion mass according to the Gell-Mann-Oakes-Renner relation [22]

$$m_\pi^2 = -\frac{(m_u + m_d)}{f_\pi^2} \langle \bar{q}q \rangle \tag{3.1}$$

where the average is taken in the hadronic state rather than the vacuum; using the current-quark masses $m_u + m_d \simeq 12$ MeV, one has $|\langle \bar{q}q \rangle|^{1/3} \simeq 240$ MeV.

IV. CHIRAL SYMMETRY BREAKING IN THE ADS/CFT MODEL

The Anti-De Sitter/conformal field theory (AdS/CFT) correspondence between string theory in AdS space and CFT's in physical spacetime has been used to obtain an analytic, semi-classical model for strongly-coupled QCD which has scale invariance and dimensional counting at short distances and color confinement at large distances [23]. Color confinement can be imposed by introducing hard-wall boundary conditions at $z = 1/\Lambda_{QCD}$ ($z =$ AdS fifth dimension) or by modification of the AdS metric. This AdS/QCD model gives a good representation of the mass spectrum of light-quark mesons and baryons as well as the hadronic wavefunctions [24]. One can also study the propagation of a scalar field $X(z)$ as a model for the dynamical running quark mass [24]. The AdS solution has the form [25]

$$X(z) = a_1 z + a_2 z^3 \ , \tag{4.1}$$

where a_1 is proportional to the current-quark mass. The coefficient a_2 scales as Λ_{QCD}^3 and is the analog of $\langle \bar{q}q \rangle$; however, since the quark is a color nonsinglet, the propagation of $X(z)$, and thus the domain of the quark condensate, is limited to the region of color confinement. The AdS/QCD picture of effective confined condensates is in agreement with results from chiral bag models [26], which modify the original MIT bag by coupling a pion field to the surface of the bag in a chirally invariant manner. Since the effect of a_2 depends on z, the AdS picture is inconsistent with the usual picture of a constant condensate.

V. EMPIRICAL DETERMINATIONS OF THE GLUON CONDENSATE

The renormalization-invariant quantity $\langle(\alpha_s/\pi)G_{\mu\nu}G^{\mu\nu}\rangle$, where

$$G_{\mu\nu}G^{\mu\nu} = 2\sum_a(|\mathbf{B}^a|^2 - |\mathbf{E}^a|^2)) \ , \tag{5.1}$$

can be determined empirically by analyzing vacuum-to-vacuum current correlators constrained by data for $e^+e^- \to$ charmonium and hadronic τ decays [27]-[29]. Some recent values (in GeV^4) include 0.006 ± 0.012 [29](a), 0.009 ± 0.007 [29](b), and -0.015 ± 0.008 [29](c). These values show significant scatter and even differences in sign. These are consistent with the picture in which the vacuum gluon condensate vanishes; it is confined within hadrons, rather than extending throughout all of space, as would be true of a vacuum condensate.

VI. SOME OTHER FEATURES OF QCD CONDENSATES

In the picture discussed here, QCD condensates would be considered as contributing to the masses of the hadrons where they are located. This is clear, since, e.g., a proton subjected to a constant electric field will accelerate and, since the condensates move with it, they comprise part of its mass. Similarly, when a hadron decays to a non-hadronic final state, such as $\pi^0 \to \gamma\gamma$, the condensates in this hadron contribute their energy to the final-state photons. Thus, over long times, the dominant regions of support for these condensates would be within nucleons, since the proton is effectively stable (with lifetime $\tau_p >> \tau_{univ} \simeq 1.4 \times 10^{10}$ yr.), and the neutron can be stable when bound in a nucleus. In a process like $e^+e^- \to$ hadrons, the formation of the condensates occurs on the same time scale as hadronization. In accord with the Heisenberg uncertainty principle, these QCD condensates also affect virtual processes occurring over times $t \lesssim 1/\Lambda_{QCD}$.

Moreover, in our picture, condensates $\langle\bar{q}q\rangle$ in different hadrons may be chirally rotated with respect to each other, somewhat analogous to disoriented chiral condensates in heavy-ion collisions [30]. This picture of condensates can, in principle, be verified by careful lattice gauge theory measurements. Note that the lattice measurements that have inferred nonzero values of $\langle \bar{q}q \rangle$ were performed in finite volumes [31], although these were usually considered as approximations to the infinite-volume limit.

VII. APPLICATION TO OTHER ASYMPTOTICALLY FREE GAUGE THEORIES

Having discussed QCD, we next consider, as an exercise, how this approach to condensates would apply to several hypothetical asymptotically free gauge theories. We begin with a vectorial gauge theory with the gauge group SU(N_c), allowing N_c to be generalized to values $N_c \ge 3$. First, consider a theory of this type with no fermions, so that only $\langle G_{\mu\nu}G^{\mu\nu} \rangle$ need be considered. This condensate would then have support within the interior of the glueballs. Second, consider a theory with $N_f = 1$ massless or light fermion transforming according to some nonsinglet representation R of SU(N_c). The $\langle \bar{q}q \rangle$ and $\langle G_{\mu\nu}G^{\mu\nu} \rangle$ condensates in this theory would have support in the interior of the mesons, baryons, and glueballs (or mass eigenstates formed from glueballs and mesons). Here, the condensate $\langle \bar{q}q \rangle$ does not break any non-anomalous global chiral symmetry, so there would not be any Nambu-Goldstone boson (NGB). In both of these theories, the sizes of the mesons, baryons, and glueballs are $\simeq 1/\Lambda$, where Λ is the confinement scale.

We next consider asymptotically free chiral gauge theories (which are free of gauge and global anomalies) with massless fermions transforming as representations $\{R_i\}$ of the gauge group. The properties of strongly coupled theories of this type are not as well understood as those of vectorial gauge theories [32]-[34]. One possibility is that, as the energy scale decreases from large values and the associated running coupling g increases, it eventually becomes large enough to produce a (bilinear) fermion condensate, which thus breaks the initial gauge symmetry [34]. This is expected to form in the most attractive channel (MAC) $R_1 \times R_2 \to R_{cond.}$, which maximizes the quantity $\Delta C_2 = C_2(R_1) + C_2(R_2) - C_2(R_{cond.})$, where $C_2(R)$ is the quadratic Casimir invariant. Depending on the theory, several stages of self-breaking may occur [34, 35]. Let us consider an explicit model of this type, with gauge group SU(5) and massless left-handed fermion content consisting of an antisymmetric rank-2 tensor representation, ψ^{ij}_L, and a conjugate fundamental representation, $\chi_{i,L}$. This theory is asymptotically free and has a formal $\mathrm{U}(1)_\psi \times \mathrm{U}(1)_\chi$ global chiral symmetry; both U(1)'s are broken by SU(5) instantons, but the linear combination $\mathrm{U}(1)'$ generated by $Q = Q_\psi - 3Q_\chi$ is preserved. The MAC for condensation is

$$10 \times 10 \to \bar{5} \tag{7.1}$$

with $\Delta C_2 = 24/5$, and the associated condensate is

$$\langle \epsilon_{ijk\ell n} \psi_L^{jk\ T} C \psi_L^{\ell n} \rangle \ , \tag{7.2}$$

which breaks SU(5) to SU(4). Thus, as the energy scale decreases and the running $\alpha = g^2/(4\pi)$ grows, at a scale Λ at which $\alpha \Delta C_2 \sim O(1)$, this condensate is expected to form. Without loss of generality, we take $i = 1$, and note

$$\langle \epsilon_{1jk\ell n} \psi_L^{jk\ T} C \psi_L^{\ell n} \rangle \propto \langle \psi_L^{23\ T} C \psi_L^{45} - \psi_L^{24\ T} C \psi_L^{35} + \psi_L^{25\ T} C \psi_L^{34} \rangle \tag{7.3}$$

The nine gauge bosons in the coset SU(5)/SU(4) gain masses of order Λ. The six components of ψ_L^{ij} involved in the condensate (7.3) also gain dynamical masses of order Λ. These components bind to form an SU(4)-singlet meson whose wavefunction is given by the operator in (7.3). This binding involves the exchange of the various (perturbatively massless) gauge bosons of SU(4). The condensate (7.3) breaks the global U(1)$'$, but the would-be resultant NGB is absorbed by the gauge boson corresponding to the diagonal generator in SU(5)/SU(4). According to the picture discussed here, the condensate (7.3) would have spatial support in the meson with the same wavefunction. Aside from the SU(4)-singlet $\chi_{1,L}$, the remaining massless fermion content of the SU(4) theory is vectorial, consisting of a 4, ψ_L^{1j}, and a $\bar{4}$, $\chi_{j,L}$, $j = 2...4$. The formal global flavor symmetry of this effective SU(4) theory at energy scales below Λ is

$$\mathrm{U}(1)_L \times \mathrm{U}(1)_R = \mathrm{U}(1)_V \times \mathrm{U}(1)_A \ , \tag{7.4}$$

and the U(1)$_A$ is broken by SU(4) instantons. This low-energy effective field theory is asymptotically free, so that at lower energy scales, the coupling α that it inherits from the SU(5) theory continues to increase, and the theory confines and produces the condensate $\langle \psi_L^{1j\ T} C \chi_{j,L} \rangle$, which preserves the gauged SU(4) and global U(1)$_V$. We infer that $\langle \psi_L^{1j\ T} C \chi_{j,L} \rangle$ and the SU(4) gluon condensate $\langle G_{\mu\nu} G^{\mu\nu} \rangle$ have spatial support in the SU(4)-singlet baryon, meson, and glueball states of this theory.

Although the present picture associates condensates in a confining gauge theory G with G-singlet hadrons, these condensates can affect properties of G-singlet particles if they both couple to a common set of fields. For example, the $\langle \bar{F} F \rangle$ condensate and the corresponding dynamical mass Σ_F of technifermions in a technicolor (TC) theory give rise to the masses of the (TC-singlet) quarks and leptons via diagrams involving exchanges of virtual extended technicolor gauge bosons. Our analysis could also be extended to supersymmetric gauge theories, but we shall not pursue this here.

VIII. THE CASE OF AN INFRARED-FREE GAUGE THEORY

Our discussion is only intended to apply to asymptotically free gauge theories. However, we offer some remarks on the situation for a particular infrared-free theory here, namely a U(1) gauge theory with gauge coupling e and some set of fermions ψ_i with charges q_i. Here there are several important differences with respect to an asymptotically free non-Abelian gauge theory. First, while the chiral limit of QCD, i.e., quarks with zero current-quark masses, is well-defined because of quark confinement, a U(1) theory with massless charged particles is unstable, owing to the well-known fact that these would give rise to a divergent Bethe-Heitler pair production cross section. It is therefore necessary to break the chiral symmetry explicitly with bare fermion mass terms m_i. If the running coupling $\alpha_1 = e^2/(4\pi)$ at a given energy scale μ were sufficiently large, $\alpha_1(\mu) \gtrsim O(1)$, an approximate solution to the Schwinger-Dyson equation for the propagator of a fermion ψ_i with $m_i << \mu$ would suggest that this fermion gains a nonzero dynamical mass Σ_i [9] and hence, presumably, there would be an associated condensate $\langle\bar{\psi}_i\psi_i\rangle$ (no sum on i). However, in analyzing SχSB, it is important to minimize the effects of explicit chiral symmetry breaking due to the bare masses m_i. The infrared-free nature of this theory means that for any given value of α_1 at a scale μ, as one decreases m_i/μ to reduce explicit breaking of chiral symmetry, $\alpha_1(m_i)$ also decreases, approaching zero as $m_i/\mu \to 0$. Since $\alpha_1(m_i)$ should be the relevant coupling to use in the Schwinger-Dyson equation, it may in fact be impossible to realize a situation in this theory in which one has small explicit breaking of chiral symmetry and a large enough value of $\alpha_1(m_i)$ to induce spontaneous chiral symmetry breaking. A full analysis would require knowledge of the bound state spectrum of the hypothetical strongly coupled U(1) theory, but this spectrum is not reliably known.

IX. FINITE TEMPERATURE QCD

So far, we have discussed QCD and other theories at zero temperature (and chemical potential or equivalently here, baryon density). For QCD in thermal equilibrium at a finite temperature T, as T increases above the deconfinement temperature T_{dec}, both the hadrons and the associated condensates eventually disappear, although experiments at CERN and BNL-RHIC show that the situation for $T \gtrsim T_{dec}$ is more complicated than a weakly coupled quark-gluon plasma. The picture of the QCD condensates here is especially close to experiment, since, although finite-temperature QCD makes use of the formal thermodynamic, infinite-volume limit, actual heavy ion experiments and resultant transitions from confined to deconfined quarks and gluons take place in the finite volume and time interval provided by colliding heavy ions. Indeed, one of the models that has been used to analyze such experiments involves the notion of a color-glass condensate [36].

X. QCD AND THE COSMOLOGICAL CONSTANT

One of the most challenging problems in physics is that of the cosmological constant Λ (recent reviews include [38]-[40]). This enters in the Einstein gravitational field equations as [41, 42]

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R - \Lambda g_{\mu\nu} = (8\pi G_N) T_{\mu\nu}, \tag{10.1}$$

where $R_{\mu\nu}$, R, $g_{\mu\nu}$, $T_{\mu\nu}$, and G_N are the Ricci curvature tensor, the scalar curvature, the metric tensor, the stress-energy tensor, and Newton's constant. One defines

$$\rho_\Lambda = \frac{\Lambda}{8\pi G_N} \tag{10.2}$$

and

$$\Omega_\Lambda = \frac{\Lambda}{3H_0^2} = \frac{\rho_\Lambda}{\rho_c} \ , \tag{10.3}$$

where

$$\rho_c = \frac{3H_0^2}{8\pi G_N} \ , \tag{10.4}$$

and $H_0 = (\dot{a}/a)_0$ is the Hubble constant in the present era, with $a(t)$ being the Friedmann-Robertson-Walker scale parameter [41, 43]. Long before the current period of precision cosmology, it was known that Ω_Λ could not be larger than O(1). In the context of quantum field theory, this was very difficult to understand, because estimates of the contributions to ρ_Λ from (i) vacuum condensates of quark and gluon fields in quantum chromodynamics (QCD) and the vacuum expectation value of the Higgs field hypothesized in the Standard Model (SM) to be responsible for electroweak symmetry breaking, and from (ii) zero-point energies of quantum fields appear to be too large by many orders of magnitude. Observations of supernovae showed the accelerated expansion of the universe and are consistent with the hypothesis that this is due to a cosmological constant, $\Omega_\Lambda \simeq 0.76$ [44–46].

Here, using our observations concerning QCD condensates, we propose a solution to problem (i) of the contributions by these condensates to ρ_Λ, which, in the conventional approach, are much too large [50]. The QCD condensates form at times of order 10^{-5} sec. in the early universe, as the temperature T decreases below the confinement-deconfinement temperature $T_{dec} \simeq 200$ MeV. As noted above, for $T << T_{dec}$, in the conventional quantum field theory view, these condensates are considered to be constants throughout space. If one accepts this conventional view, then these condensates would contribute $(\delta\rho_\Lambda)_{QCD} \sim \Lambda_{QCD}^4$, so that $(\delta\Omega_\Lambda)_{QCD} \simeq 10^{45}$. On the contrary, if one accepts the argument that these condensates (and also higher-order ones such as $\langle(\bar{q}q)^2\rangle$ and $\langle(\bar{q}q)G_{\mu\nu}G^{\mu\nu}\rangle$) have spatial support within hadrons, not extending throughout all of space, then one makes considerable progress in solving the above problem, since the effect of these condensates on gravity is already included in the baryon term Ω_b in Ω_m and, as such, they do not contribute to Ω_Λ.

Another excessive type-(i) contribution to ρ_Λ is conventionally viewed as arising from the vacuum expectation value of the Standard-Model Higgs field, $v_{EW} = 2^{-1/4}G_F^{-1/2} = 246$ GeV, giving $(\delta\rho_\Lambda)_{EW} \sim v_{EW}^4$ and hence $(\delta\Omega_\Lambda)_{EW} \sim 10^{56}$. Similar numbers are obtained from Higgs vacuum expectation values in supersymmetric extensions of the Standard Model (recalling that the supersymmetry breaking scale is expected to be the TeV scale). However, it is possible that electroweak symmetry breaking is dynamical; for example, it may result from the formation of a bilinear condensate of fermions F (called technifermions) subject to an asymptotically free, vectorial, confining gauge interaction, commonly called technicolor (TC), that gets strong on the TeV scale [52]. In such theories there is no fundamental Higgs field. Technicolor theories are challenged by, but may be able to survive, constraints from precision electroweak data. By our arguments above, in a technicolor theory, the technifermion and technigluon condensates would have spatial support in the technihadrons and techniglueballs and would contribute to the masses of these states. We stress that, just as was true for the QCD condensates, these technifermion and technigluon condensates would not contribute to ρ_Λ. Hence, if a technicolor-type mechanism should turn out to be responsible for electroweak symmetry breaking, then there would not be any problem with a supposedly excessive contribution to ρ_Λ for a Higgs vacuum expectation value. Indeed, stable technihadrons in certain technicolor theories may be viable dark-matter candidates [55].

We next comment briefly on type-(ii) contributions. The formal expression for the energy density E/V due to zero-point energies of a quantum field corresponding to a particle of mass m is

$$E/V = \int \frac{d^3k}{(2\pi)^3}\frac{\omega(k)}{2} \ , \tag{10.5}$$

where the energy is $\omega(k) = \sqrt{\mathbf{k}^2 + m^2}$. However, first, this expression is unsatisfactory, because it is (quadratically) divergent. In modern particle physics one would tend to regard this divergence as indicating that one is using an low-energy effective field theory, and one would impose an ultraviolet cutoff M_{UV} on the momentum integration, reflecting the upper range of validity of this low-energy theory. Since neither the left- nor right-hand side of eq. (10.5) is Lorentz-invariant, this cutoff procedure is more dubious than the analogous procedure for

Feynman integrals of the form $\int d^4k\, I(k,p)$ in quantum field theory, where $I(k, p_1, ..., p_n)$ is a Lorentz-invariant integrand function depending on some set of 4-momenta $p_1, ..., p_n$. If, nevertheless, one proceeds to use such a cutoff, then, since a mass scale characterizing quantum gravity (QG) is $M_{Pl} = G_N^{-1/2} = 1.2 \times 10^{19}$ GeV, one would infer that $(\delta\rho_\Lambda)_{QG} \sim M_{Pl}^4/(16\pi^2)$, and hence $(\delta\Omega_\Lambda)_{QG} \sim 10^{120}$. With the various mass scales characterizing the electroweak symmetry breaking and particle masses in the Standard Model, one similarly would obtain $(\delta\Omega_\Lambda)_{SM} \sim 10^{56}$. Given the fact that eq. (10.5) is not Lorentz-invariant, one may well question the logic of considering it as a contribution to the Lorentz-invariant quantity ρ_Λ [56, 57]. Indeed, one could plausibly argue that, as an energy density, it should instead be part of T_{00} in the energy-momentum tensor $T_{\mu\nu}$. Phrased in a different way, if one argues that it should be associated with the $\Lambda g_{\mu\nu}$ term, then there must be a negative corresponding zero-point pressure satisfying $p = -\rho$, but the source for such a negative pressure is not evident in eq. (10.5).

The light-front quantization of the Standard Model provides another perspective. In this case, the Higgs field has the form [58] $\phi = \omega + \varphi$ where ω is a classical zero mode determined by minimizing the Yukawa potential $V(\phi)$ of the SM Lagrangian, and φ is the quantized field which creates the physical Higgs particle. The coupling of the leptons, quarks, and vector bosons to the zero mode ω give these particles their masses. The electroweak phenomenology of the LF-quantized Standard Model is, in fact, identical to the usual formulation [58]. In contrast to the conventional instant-form approach, the vacuum is trivial in the light-front formulation [59, 60], and there is no zero-point fluctuation in the light-front theory, since ω is a classical quantity. Although this eliminates any would-be type-(ii) contributions of zero-point fluctuations to the cosmological constant, the contribution to the electroweak action from the Standard Model Yukawa potential $V(\omega)$ evaluated at its minimum would, as in the conventional analysis, yield an excessively large type-(i) contribution $(\delta\Omega_\Lambda)_{EW} \sim 10^{56}$. Thus the light-front formulation of the Standard Model based on a fundamental elementary Higgs field evidently does not solve the problem with type-(i) electroweak contributions to Ω_Λ. However, as we have noted above, theories with dynamical electroweak symmetry breaking, such as technicolor, are able to solve the problem with type-(i) contributions.

XI. CONCLUDING REMARKS

We have argued from several physical perspectives that, because of color confinement, quark and gluon QCD condensates are localized within the interiors of hadrons. Our analysis is in agreement with the Casher and Susskind model and the explicit demonstration of "in-hadron condensates" by Roberts et al., using the Bethe-Salpeter/Dyson-Schwinger formalism for QCD bound states. We also discussed this physics using condensed matter analogues, the AdS/CFT correspondence, and the Bethe-Salpeter/Dyson-Schwinger approach for bound states.

In-hadron condensates provide a solution to what has hitherto commonly been regarded as an excessively large contribution to the cosmological constant by QCD condensates. We have argued that these condensates do not, in fact, contribute to Ω_Λ; instead, they have spatial support within hadrons and, as such, should really be considered as contributing to the masses of these hadrons and hence to Ω_b. We have also suggested a possible solution to what would be an excessive contribution to Ω_Λ from a hypothetical Higgs vacuum expectation value; the solution would be applicable if electroweak symmetry breaking occurs via a technicolor-type mechanism.

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