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Magnetowave Induced Plasma Wakefield Acceleration for Ultra High Energy Cosmic Rays

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Magnetowave induced plasma wakefield acceleration (MPWA) in a relativistic astrophysical outflow has been proposed as a viable mechanism for the acceleration of cosmic particles to ultra high energies[1]. Here we present simulation results that clearly demonstrate the viability of this mechanism for the first time. We invoke the high frequency and high speed whistler mode for the driving pulse. The plasma wakefield so induced validates precisely the theoretical prediction. We show that under appropriate conditions, the plasma wakefield maintains very high coherence and can sustain high-gradient acceleration over a macroscopic distance. Invoking gamma ray burst (GRB) as the source, we show that MPWA production of ultra high energy cosmic rays (UHECR) beyond ZeV (10^{21}eV) is possible.

The origin of ultra high energy cosmic rays (UHECR) is a long-standing mystery in astrophysics. Thus far, the theories that attempt to explain the origin of UHECR can be broadly categorized into the "top-down" and the "bottom-up" scenarios. Each scenario faces its own theoretical and observational challenges[2]. Precision measurements[3, 4] on the yield of air-shower induced fluorescence lend support to the energy calibration of the HiRes observations[5]. Recent data from the Auger Observatory[6], also fluorescence normalized, exhibit a similar location of the "ankle" and super-GZK steepening as HiRes. The lack of a strong super-GZK flux reduces the need for top-down exotic models. If these models are indeed disfavored, then the challenge to find a viable "bottom up" mechanism to accelerate ordinary particles to and beyond 10^{20} eV becomes more acute.

Shocks, unipolar inductors and magnetic flares are the three most potent, observed, "conventional" accelerators that can be extended to account for $\sim$ ZeV($= 10^{21}$eV) energy cosmic rays[7]. Radio jet termination shocks and gamma ray bursts (GRB) have been invoked as sites for the shock acceleration, while dormant galactic center black holes and magnetars have been proposed as sites for the unipolar inductor acceleration and the flare acceleration, respectively. Each of these models, however, presents problems[7]. Evidently, novel acceleration mechanisms that can avoid the difficulties faced by these conventional models should not be overlooked.

Plasma wakefield accelerators[8, 9] are known to possess two salient features: (1) The energy gain per unit distance does not depend (inversely) on the particle's instantaneous energy or momentum. This is essential to avoid the gradual decrease of efficiency in reaching ultra high energies; (2) The acceleration is linear. Bending of the trajectory is not a necessary condition for this mechanism. This helps to minimize inherent energy loss which would be severe at ultra high energy. However, high-intensity, ultra-short photon or particle beam pulses that excite the laboratory plasma wakefields are not readily available in the astrophysical setting. It was, however, proposed[1] that large amplitude plasma wakefields can instead be excited by the astrophysically more abundant plasma "magnetowaves", whose field components are magnetic in nature ($|B| > |E|$). Protons can be accelerated beyond ZeV energy by riding on such wakefields. Though attractive, this concept has never been validated through self-consistent computer simulations. In this Letter, we report on the plasma particle-in-cell (PIC) simulation results that confirm the magnetowave-induced plasma wakefield acceleration (MPWA) concept for the first time.

Magnetized plasmas support a variety of wave modes propagating at arbitrary angles to the imposed magnetic field. For the specific case of wave modes propagating parallel to the external magnetic field, the electromagnetic waves become circularly polarized and the dispersion relation is [10]

$$\omega^2 = k^2c^2 + \frac{\omega_{ip}^2}{1 \pm \omega_{ic}/\omega} + \frac{\omega_p^2}{1 \mp \omega_c/\omega} \,, \qquad (1)$$

where the upper (lower) signs denote the right-hand (left-hand) circularly polarized waves. $\omega_p = \sqrt{4\pi e^2 n_p/m_e}$ is the electron plasma frequency, $\omega_c = eB/m_e c$ is the electron cyclotron frequency and the subscript i denotes the ion species. Each polarization has two real solutions with high and low frequency branches and both have a frequency cutoff which forms a forbidden gap for wave

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propagation. The right-hand polarized, low frequency solution is called the whistler wave which propagates at a phase velocity less than the speed of light. When the magnetic field is sufficiently strong such that $\omega_c \gg \omega_p$, the dispersion of the whistler mode becomes more linear over a wider range of wavenumbers with phase velocity approaching the speed of light (see Fig.1). The E and B components of the wave are now comparable in strength. In this regime the travelling wave pulses can maintain their shape over macroscopic distance, a condition desirable for plasma wakefield acceleration.

The ponderomotive force in a magnetized plasma has been well studied[11]. Applying the dispersion relation for the whistler wave, with the ion motion neglected, we obtain the ponderomotive force acting on an individual electron as

$$F_z = -\frac{1}{2}\frac{e^2}{m_e\omega(\omega-\omega_c)}\left[1+\frac{kv_g\omega_c}{\omega(\omega-\omega_c)}\right]\partial_\zeta E_W^2(\zeta), \quad (2)$$

where $E_W(\zeta)$ is the amplitude of the whistler wave-packet, and $\zeta \equiv z - v_g t$ the co-moving coordinate for the driving pulse. Note that E_W is perpendicular to z.

Combining this equation with the continuity equation and the Poisson equation, the longitudinal electric field in the plasma, i.e., the plasma wakefield, can be solved and it reads

$$E_z(\zeta) = -\frac{ek_pE_W^2}{m_e\omega(\omega-\omega_c)}\left[1+\frac{kv_g\omega_c}{\omega(\omega-\omega_c)}\right]\chi(\zeta), \quad (3)$$

with

$$\chi(\zeta) = \frac{k_p}{2E_W^2}\int_\zeta^\infty d\zeta' E_W^2(\zeta')\cos\left[k_p(\zeta-\zeta')\right], \quad (4)$$

where E_W is the maximum value of $E_W(\zeta)$. An expression similar to Eq. (3) has been obtained for laser-induced wakefield in a magnetized plasma[12]. For a Gaussian driving pulse with $E_W(\zeta) = E_W\exp(-\zeta^2/2\sigma^2)$, it can be shown that behind the driving pulse, i. e., $|\zeta| \gg \sigma$,

$$\chi(\zeta) = \frac{\sqrt{\pi}}{2}k_p\sigma e^{-k_p^2\sigma^2/4}\cos k_p\zeta \equiv \chi\cos k_p\zeta. \quad (5)$$

It is customary to express the plasma wakefield in terms of the Lorentz invariant “strength parameter” of the driving pulse, $a_0 \equiv eE_W/m_ec\omega$, and the “wavebreaking” field, $E_{wb} \equiv m_ec\omega_p/e$. Assuming the driving pulse frequency is centered around ω and its speed $v_g \approx \omega/k$, the maximum wakefield, or the *acceleration gradient*, attainable behind the driving pulse is then

$$G = \chi\frac{k^2c^2}{(\omega-\omega_c)^2}a_0^2E_{wb}, \quad a_0 \ll 1. \quad (6)$$

Relative to the conventional wakefields, G is enhanced by a factor $k^2c^2/(\omega-\omega_c)^2$ when ω approaches ω_c. The above expression is derived under the assumption of linear plasma perturbation, i.e., $a_0 \ll 1$. Research made over the past two decades in plasma wakefields has firmly established the generalized wakefield amplitude for all values of a_0[13], and adapting its form we obtain,

$$G = \chi\frac{k^2c^2}{(\omega-\omega_c)^2}\frac{a_0^2}{\sqrt{1+a_0^2}}E_{wb}. \quad (7)$$

Note that in the nonlinear regime ($a_0 \gg 1$) the wakefield is no longer sinusoidal in ζ but saw-tooth-like.

We have conducted computer simulations to study the MPWA process driven by a Gaussian driving whistler pulse described above. Our simulation model integrates the relativistic Newton-Lorentz equations of motion in the self-consistent electric and magnetic fields determined by the solution to Maxwell's equations[14, 15]. The 4-dimensional phase space (z, p_x, p_y, p_z) is used for the charged particle dynamics and a uniform external magnetic field, B_0, is imposed in the z-direction. In order to sustain the driving pulse shape propagating in the plasma, it is important to have modes of the pulse travelling with similar phase velocities. Therefore, we used a wavepacket with Gaussian width $\sigma = 80\Delta/\sqrt{2}$, where Δ is the cell size taken to be unity, and the wavenumber $k = 2\pi/60\Delta$. The normalized physical parameters $\omega_c/\omega_p = 6$ and $m_i/m_e = 2000$ were taken and for a uniform background plasma with electron collisionless skin depth, $c/\omega_p\Delta = 30$, this gives $\omega/\omega_p = 2.98$ and $v_g/c \simeq \omega/ck = 0.95$. Other numerical parameters used are: total number of cells in the z-direction, $L_z = 8192\Delta = 273c/\omega_p$, average number of particles per cell was 10, and the time step $\omega_p\Delta t = 0.1$. The fields were normalized by $(1/30)E_{wb}$.

We set the maximum amplitude $E_W = 10$, which gives the normalized vector potential $a_0 = eE_W/m_ec\omega =$

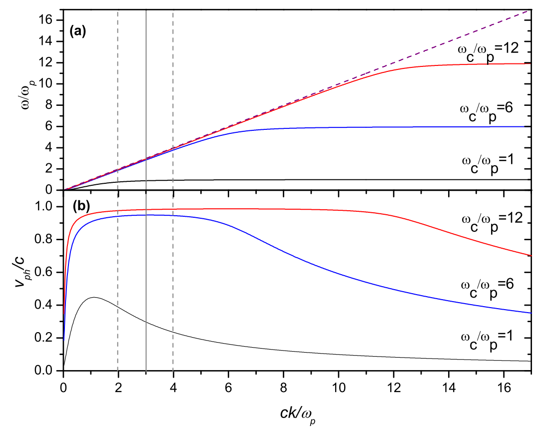

FIG. 1: (a) Frequency and (b) phase velocity versus wavenumber for different magnetic field strengths. The vertical solid line is the mean value of the pulse wavenumber that was chosen for the PIC simulation, and the dashed lines its range.

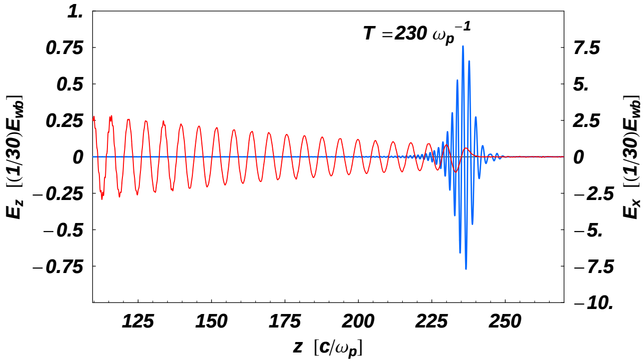

FIG. 2: A snapshot of the plasma wakefield induced by the whistler pulse. E_x is in blue and E_z in red.

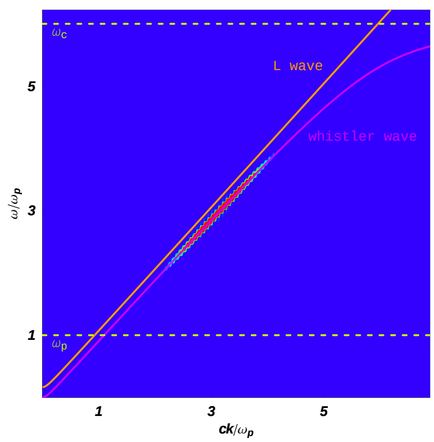

FIG. 3: The dispersion relation of the driving pulse from PIC simulation. Liner dispersion curves for the L-wave (orange) and the whistler wave (pink) are superimposed.

$0.11 \ll 1$. Thus the wakefield in our simulation is in the linear regime. The pulse was initialized at $z_0 = 500\Delta = 16.66c/\omega_p$. To avoid spurious effects, we gradually ramped up the driving pulse amplitude until $t = 100\omega_p^{-1}$, during which the plasma feedback to the driving pulse was ignored. After this time, the driving pulse-plasma interaction was tracked self-consistently. As the dispersion relation in this regime is not perfectly linear, there was a gradual spread of the pulse width. Thus χ and E_W of the driving pulse decrease accordingly. As a result, the maximum wakefield amplitude, E_z, declined in time. Even so, it agrees very well with the theoretical maximum of $E_z \sim 0.266(1/30)E_{wb}$. Fig.2 is a snapshot of E_x and E_z at $t = 230\omega_p^{-1}$. We note that while the driving pulse continues to disperse, the wakefield remains extremely coherent.

We sampled the $E_x(k)$ of the pulse, after its initialization, at every time step and analyzed it in the frequency space. Fig.3 shows the $\omega - k$ intensity generated from the

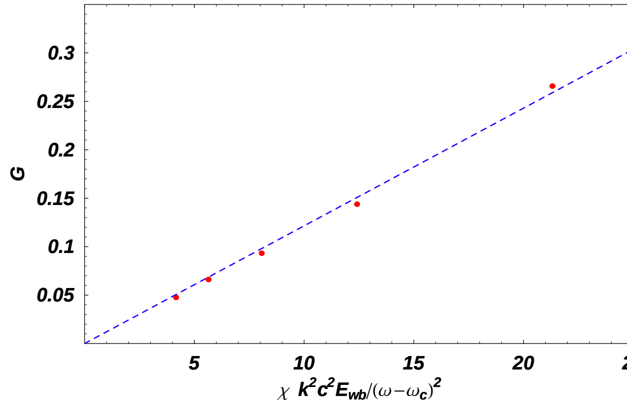

FIG. 4: Validation of the functional dependence of G in Eq.(6), where E_{wb} is in units of $(1/30)m_e c\omega_p/e$.

PIC simulation driving pulse power spectrum. It is superimposed with the theoretical curves for the left-handed circularly polarized electromagnetic wave (L-wave) and the whistler wave dispersion relations. We confirm that our driving pulse is indeed a whistler wave.

Next we validate the functional dependence of the acceleration gradient given in Eq.(6). Fig.4 plots the acceleration gradient versus $\chi k^2 c^2 E_{wb}/(\omega - \omega_c)^2$. This is performed by varying ω_c. Wavepackets with the same wavenumber $(2\pi/60\Delta)$ and the same maximum amplitude $(E_W = 10)$ are initialized with different B_0 such that $\omega_c/\omega_p = 6, 7, 8, 9, 10$. The linear fitting slope is 0.012, which agrees very well with the expected value of a_0^2. We comment that when ω approaches ω_c, i. e., when the two are in resonance, the wakefield amplitude can be dramatically increased by several orders of magnitude. In view of the successful validation by our computer simulation of the MPWA in the linear regime, we are safe to extrapolate it to the nonlinear regime according to the theoretical formula in Eq.(7).

We now relate this mechanism to the issue of UHECR acceleration. An earlier attempt has been made[1] where gamma ray bursts (GRB) were invoked as the candidate site for plasma wakefield production of UHECR. We follow the same approach here. GRBs are generally classified into long ($\tau_l \sim 10 - 100$sec) and short ($\tau_s \sim 1$sec) bursts. Within seconds (for short bursts), about $\mathcal{E}_{\rm GRB} \sim 10^{50}$erg of energy is released through gamma rays. We invoke the Neutron Star-Neutron Star coalescence as our working model for GRB. Neutron stars are known to be compact ($R_{\rm NS} \sim \mathcal{O}(10)$km) and carry intense surface magnetic fields ($B_{\rm NS} \sim 10^{13}$G). It is generally believed that when neutron stars collide, the tremendous release of energy creates a highly relativistic out-bursting fireball (jets)[16], most likely in the form of a plasma. We assume such a jet has an open-angle of $\theta_{\rm GRB} \sim 0.1$ and the initial plasma density in the jet $n_{\rm GRB} \sim 10^{26}{\rm cm}^{-3}$. We further assume that such violent collision of intense magnetic fields would create sequence of strong magnetoshocks, where whistler waves are imbedded. In the aftermath of such tremendous impact, the magnetic field-lines would be temporarily shattered and reoriented with strong poloidal field lines parallel to the axis of the jet.

From the previous discussion we see that the MPWA is most effective when the driving pulse frequency falls between the plasma frequency, ω_p, and the electron-cyclotron frequency, ω_c. Let us verify whether and where this condition can be satisfied along the GRB jet. First we note that due to the conservation of the magnetic flux, the poloidal magnetic field strength decreases as $1/r^2$ away from the epicenter of GRB. This means $\omega_c = eB/m_ec \propto (R_{\rm NS}/r)^2$. On the other hand, the continuity condition requires that the plasma density decreases as $1/r^2$ as well. Therefore, $\omega_p \propto R_{\rm NS}/r$ and the cross-over between these two parameters does exist at a distance $R \sim 100R_{\rm NS} \sim 1000$km. This is the "sweet spot" where MPWA is the most effective.

To estimate the plasma wakefield acceleration gradient at this "sweet spot", we first note the EM energy density of GRB is $E^2_{\rm GRB}/4\pi = \mathcal{E}_{\rm GRB}/c\tau_s\pi(\theta_{\rm GRB}R)^2 \sim 10^{27}$erg/cm^3. We assume that a fraction, η_a, of this outburst energy goes into the magnetoshocks. We further assume that a fraction, η_b, of the magnetoshocks energy lies in the whistler mode. This means $E^2_W \sim \eta_a\eta_b E^2_{\rm GRB}$. Based on this, the associated strength parameter a_0 is: $a_0 = \sqrt{\eta_a\eta_b}(eE_{\rm GRB})/(m_ec\omega)$. At the sweet spot where $\omega_c \sim \omega \simeq kc \sim \omega_p$, the factor $k^2c^2/(\omega^2 - \omega_c^2)$ is of the order unity. Furthermore, the extremely sharp magnetoshock fronts would render the form factor χ also of the order unity. Assume that $a_0 > 1$. Then the acceleration gradient boils down to, *cf.* Eq.(7),

$$G \sim a_0 E_{wb} = \sqrt{\eta_a\eta_b}\Big(\frac{eE_{\rm GRB}}{m_ec\omega}\Big)E_{wb}\,. \qquad (8)$$

To appreciate what this translates into physical requirements, let us assume that the range of the "sweet spot" is $\delta R \sim 0.1R \sim 10R_{\rm NS} \sim 100$km around R where the factor $k^2c^2/(\omega^2 - \omega_c^2)$ is of the order unity. Then in order for MPWA to be responsible for the production of UHECR beyond ZeV (10^{21}eV), it is necessary that $G \sim 10^{14}$eV/cm. In turn, the fractions of GRB energy imparted into the whistler mode have to be $\eta_a \sim \eta_b \sim 10^{-2}$.

As shown in Ref.[1], the stochastic encounters of the test particle with the random acceleration-deceleration phases would result in a inverse-square-law spectrum, $f(\mathcal{E}) \propto 1/\mathcal{E}^2$. The various additional energy loss mechanisms, such as few-body collision and synchrotron radiation, would degrade the power-law index to $1/\mathcal{E}^{2+\beta}$, with $0 < \beta < 1$.

Our PIC simulations have confirmed the concept of plasma wakefield excited by a magnetowave in a magnetized plasma. Different from the laser and particle beam, the magnetowaves are medium waves which cannot exist without the plasma. MPWA should thus be of interest as a fundamental phenomena in plasma physics and an alternative approach to plasma wakefield acceleration.

As a first step, we investigated MPWA in the parallel-field configuration. Since both poloidal and toroidal field components are inevitable in astro-jets, we will further investigate plasma wakefield excitation and acceleration under the cross-field configuration. In order for MPWA to be responsible for the ZeV UHECR production, the energy transfer efficiency for GRB is constrained. It would be very interesting both observationally and theoretically to test whether this constraint is valid.

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