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Effective Lagrangian for Two-photon and Two-gluon Decays of P-wave Heavy Quarkonium $\chi_{c0,2}$ and $\chi_{b0,2}$ states

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In the traditional non-relativistic bound state calculation, the two-photon decay amplitudes of the P-wave $\chi_{c0,2}$ and $\chi_{b0,2}$ states depend on the derivative of the wave function at the origin which can only be obtained from potential models. However by neglecting the relative quark momenta, the decay amplitude can be written as the matrix element of a local heavy quark field operator which could be obtained from other processes or computed with QCD sum rules technique or lattice simulation. Following the same line as in recent work for the two-photon decays of the S-wave η_c and η_b quarkonia, we show that the effective Lagrangian for the two-photon decays of the P-wave $\chi_{c0,2}$ and $\chi_{b0,2}$ is given by the heavy quark energy-momentum tensor local operator or its trace, the $\bar{Q}Q$ scalar density and that the expression for χ_{c0} two-photon and two-gluon decay rate is given by the $f_{\chi_{c0}}$ decay constant and is similar to that of η_c which is given by f_{η_c}. From the existing QCD sum rules value for $f_{\chi_{c0}}$, we get 5 keV for the χ_{c0} two-photon width, somewhat larger than measurement, but possibly with large uncertainties.

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I. INTRODUCTION

With the recent new CLEO measurements[1, 2] of the two-photon decay rates of the even-parity, P-wave 0^{++} χ_{c0} and 2^{++} χ_{c2} states and with renewed interest in radiative decays of heavy quarkonium states, it seems appropriate to have another look at the two-photon decay of heavy quarkonium from the standpoint of an effective Lagrangian based on local operator expansion and heavy-quark spin symmetry, as done for the pseudo-scalar heavy quarkonia η_c and η_b[3, 4], for which the decay rates for the ground state and excited states could be predicted in terms of the J/ψ and Υ leptonic widths using Heavy Quark Spin Symmetry(HQSS). In the traditional non-relativisitic bound state calculation, the two-photon widths for the P-wave quarkonium state depend on the derivative at the origin of the spatial wave function which has to be extracted from potential models[5]. Though the physics of quarkonium decay seems to be better understood within the conventional framework of QCD[6], unlike the two-photon width of S-wave η_c and η_b quarkonia which can be predicted from the corresponding J/ψ and Υ leptonic widths using HQSS, there is no similar prediction for the P-wave χ_c and χ_b states and all the existing theoretical values for the decay rates are based on potential model calculations[5, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17]. To have a prediction for the two-photon width of P-wave quarkonia, one need to express the decay amplitude in terms of the matrix element of a heavy quark field local operator extracted from some known physical processes or computed in an essentially model-independent manner, such as QCD sum rules technique[18, 19] or lattice simulations[20]. In fact a value of $438 \pm 5 \pm 6, \mathrm{MeV}$ for f_{η_c} and $801 \pm 7 \pm 5, \mathrm{MeV}$ for f_{η_b} consistent with the HQSS values of 411 MeV and 836 MeV[3, 4] , respectively, have been obtained by the lattice group TWQCD Collaboration[21] recently. With similar determinations of other quarkonium decay constants, one would be able to study QCD radiative corrections and obtain the strong α_s coupling constant, for example, especially in $\chi_{b0,2}$ two-gluon decays where local operator expansion should be a better approximation than in $\chi_{c0,2}$ decays. In this paper, starting from the two-photon and two-gluon $c\bar{c} \to \gamma\gamma, gg$ and $b\bar{b} \to \gamma\gamma, gg$ amplitudes, we derive an effective Lagrangian for the two-photon and two-gluon decays for P-wave quarkonium state by neglecting the bound state relative quark momenta compared with the large outgoing photon or gluon momenta. We show that the decay amplitude is given by the heavy quark energy-momentum tensor which can be obtained from the matrix element of its trace as $< 0|\bar{c}\, c|\chi_{c0} >= m_{\chi_{c0}}\, f_{\chi_{c0}}$ and $< 0|\bar{b}\, b|\chi_{b0} >= m_{\chi_{b0}}\, f_{\chi_{b0}}$. We find that the two-photon and two-gluon decay rates of $\chi_{c0,2}$ and $\chi_{b0,2}$ are given in terms of $f_{\chi_{c0}}$ and $f_{\chi_{b0}}$, similar to the η_c and η_b two-photon decay rates given by f_{η_c} and f_{η_b}.

II. EFFECTIVE LAGRANGIAN FOR $\chi_{c0,2} \to \gamma\gamma$ AND $\chi_{b0,2} \to \gamma\gamma$

Following [22, 23], we consider the amplitude for the annihilation of a quark and an antiquark with momentum p_1 and p_2 represented by the diagrams in Fig.(1):

$$\mathcal{A} = \bar{v}(p_2)(\mathcal{O}_1 + \mathcal{O}_2)u(p_1) \tag{1}$$

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with

$$\mathcal{O}_1 = \frac{1}{i}\left[(-ie\ \not{\epsilon}_2) i \frac{(\not{p}_1 - \not{k}_1 + m_Q)}{(p_1-k_1)^2 - m_Q^2}(-ie\ \not{\epsilon}_1)\right] \quad (2)$$

$$\mathcal{O}_2 = \frac{1}{i}\left[(-ie\ \not{\epsilon}_1) i \frac{(\not{p}_1 - \not{k}_2 + m_Q)}{(p_1-k_2)^2 - m_Q^2}(-ie\ \not{\epsilon}_2)\right] \quad (3)$$

where (ϵ_1, k_1) and (ϵ_2, k_2) are the polarizations and momenta of the outgoing photons and m_Q the heavy quark mass. The total energy-momentum of the quark-antiquark system is the energy-momentum of the quarkonium bound state defined as $Q = p_1 + p_2$ and mass M.

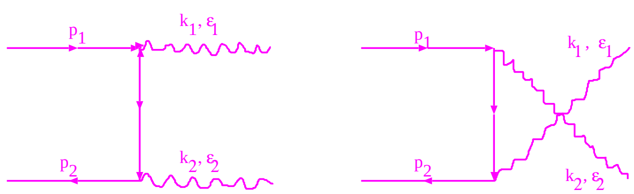

FIG. 1: Diagrams for $Q\bar{Q}$ annihilation to two photons.

Using Dirac equation and expanding $\mathcal{O}_1$ and $\mathcal{O}_2$, and putting:

$$q = p_1 - p_2, \quad Q = k_1 + k_2, \quad K = k_1 - k_2 \quad (4)$$

we have

$$\mathcal{O}_1 = -e^2 Q_{c,b}^2 \frac{(\epsilon_1\cdot\epsilon_2(\not{k}_1 - \not{k}_2) - i\epsilon(\epsilon_2, K, \epsilon_1, \sigma)\gamma_\sigma\gamma_5)/2}{[(p_1-k_1)^2 - m_Q^2]}$$

$$-e^2 Q_{c,b}^2 \frac{(-\epsilon_2\cdot(p_2 + k_1/2)\ \not{\epsilon}_1 + \epsilon_1\cdot(p_1+k_2/2)\ \not{\epsilon}_2)}{[(p_1-k_1)^2 - m_Q^2]} \quad (5)$$

$$\mathcal{O}_2 = -e^2 Q_{c,b}^2 \frac{(\epsilon_1\cdot\epsilon_2(\not{k}_2 - \not{k}_1) + i\epsilon(\epsilon_2, K, \epsilon_1, \sigma)\gamma_\sigma\gamma_5)/2}{[(p_1-k_2)^2 - m_Q^2]}$$

$$-e^2 Q_{c,b}^2 \frac{(\epsilon_2\cdot(p_1 + k_1/2)\ \not{\epsilon}_1 - \epsilon_1\cdot(p_2+k_2/2)\ \not{\epsilon}_2)}{[(p_1-k_2)^2 - m_Q^2]} \quad (6)$$

The P-wave $\chi_{c0,2}$ and $\chi_{b0,2}$ two-photon (two-gluon) decay amplitudes are given by the P-wave part of the $Q\bar{Q} \to \gamma\gamma, gg$ annihilation amplitude which is given by the $k_1\cdot q$, $\epsilon_1\cdot q$ and $\epsilon_2\cdot q$ terms in $\mathcal{O}_1, \mathcal{O}_2$. By neglecting term containing the relative quark momenta q in the quark propagator[22] we find, ($Q_{c,b}^2$ being the heavy quark charge),

$$\mathcal{M}(Q\bar{Q}\to\gamma\gamma) = -e^2 Q_{c,b}^2 \times$$

$$\bar{v}(p_2)\Big[k_1\cdot q(-2\epsilon_1\cdot k_2\ \not{\epsilon}_2 + 2\epsilon_1\cdot\epsilon_2\ \not{k}_2 + 2\epsilon_2\cdot k_1\ \not{\epsilon}_1) +$$

$$M^2(\epsilon_2\cdot q\ \not{\epsilon}_1 + \epsilon_1\cdot q\ \not{\epsilon}_2)/2\Big]u(p_1)\Big[(k_1-k_2)^2/4 - m_Q^2\Big]^{-2} \quad (7)$$

which is now reduced to the matrix element of a local operator for two-photon or two-gluon decays of P-wave quarkonia with the outgoing photon or gluon having large momenta compared to the relative quark-antiquark momenta as given by the numerator of the amplitude in Eq.(7). We have (rewriting M^2 as $2k_1\cdot k_2$ and $k_1\cdot q\epsilon_2\cdot k_1\ \not{\epsilon}_1$ as $-k_2\cdot q\epsilon_2\cdot k_1\ \not{\epsilon}_1$ in Eq.(7)):

$$\mathcal{M}(Q\bar{Q}\to\gamma\gamma) = -e^2 Q_{c,b}^2 \frac{A_{\mu\nu}\bar{v}(p_2)T_{\mu\nu}u(p_1)}{[(k_1-k_2)^2/4 - m_Q^2]^2} \quad (8)$$

with $A_{\mu\nu}$ the photon part of the amplitude and the heavy quark part $T_{\mu\nu}$ given by:

$$A_{\mu\nu} = -2\epsilon_1\cdot k_2\epsilon_{2\mu}k_{1\nu} + 2\epsilon_1\cdot\epsilon_2 k_{2\mu}k_{1\nu}$$
$$-2\epsilon_2\cdot k_1\epsilon_{1\mu}k_{2\nu} + (k_1\cdot k_2)(\epsilon_{1\mu}\epsilon_{2\nu} + \epsilon_{2\mu}\epsilon_{1\nu}) \quad (9)$$
$$T_{\mu\nu} = (q_{1\mu} - q_{2\mu})\gamma_\nu \quad (10)$$

We see that $\bar{v}(p_2)T_{\mu\nu}u(p_1)$ is the matrix element of $\theta_{Q\mu\nu} = \bar{Q}(\overrightarrow{\partial}_\mu - \overleftarrow{\partial}_\mu)\gamma_\nu Q$, the heavy quark energy-momentum tensor. The photon part can also be written in terms of the photon field operator $F_{\mu\nu}$, but for simplicity, we will keep the matrix element form given by $A_{\mu\nu}$. The effective Lagrangian for two-photon and two-gluon decay of P-wave $\chi_{c0,2}$ and $\chi_{b0,2}$ states is then given by:

$$\mathcal{L}_{\rm eff}(Q\bar{Q}\to\gamma\gamma) = -ic_1 A_{\mu\nu}\bar{Q}(\overrightarrow{\partial}_\mu - \overleftarrow{\partial}_\mu)\gamma_\nu Q \quad (11)$$
$$c_1 = -e^2 Q_{c,b}^2[(k_1-k_2)^2/4 - m_Q^2]^{-2}$$

With the matrix element of $\theta_{Q\mu\nu}$ between the vacuum and $\chi_{c0,2}$ or $\chi_{b0,2}$ given by ($Q^2 = M^2$):

$$< 0|\theta_{Q\mu\nu}|\chi_0> = T_0 M^2(-g_{\mu\nu} + Q_\mu Q_\nu/M^2),$$
$$< 0|\theta_{Q\mu\nu}|\chi_2> = -T_2 M^2\epsilon_{\mu\nu}. \quad (12)$$

where $\epsilon_{\mu\nu}$ is the polarization tensor for χ_2 state, we obtain the two-photon decay amplitude in a simple manner:

$$\mathcal{M}(\chi_0\to\gamma\gamma) = -e^2 Q_{c,b}^2 \frac{T_0 A_0}{[M^2/4 + m_Q^2]^2} \quad (13)$$

$$\mathcal{M}(\chi_2\to\gamma\gamma) = -e^2 Q_{c,b}^2 \frac{T_2 A_2}{[M^2/4 + m_Q^2]^2} \quad (14)$$

where

$$A_0 = (\frac{3}{2})M^2(M^2\epsilon_1\cdot\epsilon_2 - 2\epsilon_1\cdot k_2\epsilon_2\cdot k_1) \quad (15)$$
$$A_2 = M^2\epsilon_{\mu\nu}[M^2\epsilon_{1\mu}\epsilon_{2\nu} - 2(\epsilon_1\cdot k_2\epsilon_{2\mu}k_{1\nu} + \epsilon_2\cdot k_1\epsilon_{1\mu}k_{2\nu}$$
$$+\epsilon_1\cdot\epsilon_2 k_{1\mu}k_{2\nu})] \quad (16)$$

The above expressions agree with the well-known non-relativistic calculation of [5]. The HQSS relation $T_2 = \sqrt{3}T_0$ is obtained by [22] in a calculation of the two-photon decays of P-wave quarkonium $\chi_J, J = 0, 2$ states using the Bethe-Salpeter wave function and the relativistic spin projection operators given in this reference and in [23] which is a precursor of the recent HQSS formulation of radiative decays of heavy quarkonium[3, 4, 24]. This method allows one to compute the matrix element of derivative operator, like the energy-momentum tensor in a bound state description of P-wave heavy quarkonium state, from which HQSS relations could be obtained. However, for QCD sum rules calculation or lattice simulation, non-derivative operator is simpler to compute. Thus instead of working with the energy-momentum $\theta_{Q\mu\nu}$ operator, one could work with the trace $\theta_{Q\mu\mu}$ which, by applying Dirac equation, becomes a $\bar{Q}Q$ scalar density:

$$\theta_{Q\mu\mu} = 2m_Q\bar{Q}Q \tag{17}$$

and

$$\bar{v}(p_2)T_{\mu\mu}u(p_1) = 2m_Q\bar{v}(p_2)u(p_1) \tag{18}$$

Then the problem of computing the two-photon or two-gluon decays of P-wave quarkonium $\chi_{c0,2}$ and $\chi_{b0,2}$ states is reduced to computing the decays constants $f_{\chi_{c0}}$ or $f_{\chi_{b0}}$ states, defined as ($\chi_{0,2}$ denote here both $\chi_{c0,2}$ and $\chi_{b0,2}$ states):

$$< 0|\bar{Q}Q|\chi_0 >= m_{\chi_0}f_{\chi_0} \tag{19}$$

Comparing Eq.(12) with Eq.(19), we find:

$$T_0 = \frac{f_{\chi_0}}{3} \tag{20}$$

where we have neglected the binding energy[22] $b = 2m_Q - M$ and putting $m_Q = M/2$. This agrees with the bound state calculations of [22] and a direct calculation of $\theta_{Q\mu\nu}$ and $< 0|\bar{Q}Q|\chi_0 >$ using the expressions Eq.(24-26) of [23]. The point we would like to stress here is that the local operator expansion allows us to compute the two-photon and two-gluon decay amplitudes of $\chi_{c0,2}$ directly in terms of the $f_{\chi_{c0}}$ decay constant, without using the wave function and its derivative at the origin, as with that of η_c given in terms of f_{η_c}[3].

Another quantity of physical interest is the decay constant f_{χ_1} of the P-wave 1^{++} χ_{c1} state which enters, for example, in $B \to \chi_{c1}K$[25, 26] and $B \to \chi_{c1}\pi$[27] decays. Using expressions Eq.(24-26) in [23] for $\chi_{0,1}$ states, we find in terms of the derivative of the P-wave spatial wave function at the origin $\mathcal{R}_1'(0)$:

$$\begin{aligned} f_{\chi_0} &= 12\sqrt{\frac{3}{(8\pi m_Q)}}\left(\frac{\mathcal{R}_1'(0)}{M}\right) \\ f_{\chi_1} &= 8\sqrt{\frac{9}{(8\pi m_Q)}}\left(\frac{\mathcal{R}_1'(0)}{M}\right) \end{aligned} \tag{21}$$

which gives $f_{\chi_1} = \frac{\sqrt{3}}{2}f_{\chi_0}$. Comparing with the S-wave singlet pseudo-scalar quarkonium decay constant f_P[24] ($M \simeq 2m_Q$):

$$f_{\eta_c} = \sqrt{\frac{3}{32\,\pi\, m_Q^3}}\,\mathcal{R}_0(0)\,(4\,m_Q)\ , \tag{22}$$

we have :

$$f_{\chi_{c0}} = 12\left(\frac{\mathcal{R}_1'(0)}{\mathcal{R}_0(0)M}\right)f_{\eta_c} \tag{23}$$

where $\mathcal{R}_0(0)$ is the S-wave spatial wave function at the origin.

The two-photon decay rates of $\chi_{c0,2}$, $\chi_{b0,2}$ states can now be obtained in terms of the decay constant f_{χ_0} . We find, either by using Eq.(21) or directly Eq.(20) for T_0:

$$\Gamma_{\gamma\gamma}(\chi_{c0}) = \frac{4\pi Q_c^4\alpha_{em}^2 M_{\chi_{c0}}^3 f_{\chi_{c0}}^2}{(M_{\chi_{c0}}+b)^4}\left[1+B_0(\alpha_s/\pi)\right], \tag{24}$$

$$\Gamma_{\gamma\gamma}(\chi_{c2}) = \left(\frac{4}{15}\right)\frac{4\pi Q_c^4\alpha_{em}^2 M_{\chi_{c2}}^3 f_{\chi_{c0}}^2}{(M_{\chi_{c2}}+b)^4}\left[1+B_2(\alpha_s/\pi)\right] \tag{25}$$

where $B_0 = \pi^2/3 - 28/9$ and $B_2 = -16/3$ are NLO QCD radiative corrections[28, 29, 30]. It is interesting to note that, the expression for the χ_{c0} two-photon decay rate is similar to that for η_c[3]:

$$\Gamma_{\gamma\gamma}(\eta_c) = \frac{4\pi Q_c^4\alpha_{em}^2 M_{\eta_c} f_{\eta_c}^2}{(M_{\eta_c}+b)^2}\left[1-\frac{\alpha_s}{\pi}\frac{(20-\pi^2)}{3}\right] \tag{26}$$

In the same manner, we have , for the two-gluon decays :

$$\Gamma_{gg}(\chi_{c0}) = \left(\frac{2}{9}\right)\frac{4\pi\alpha_s^2 M_{\chi_{c0}}^3 f_{\chi_{c0}}^2}{(M_{\chi_{c0}}+b)^4}\left[1+C_0(\alpha_s/\pi)\right], \tag{27}$$

$$\Gamma_{gg}(\chi_{c2}) = \left(\frac{4}{15}\right)\left(\frac{2}{9}\right)\frac{4\pi\alpha_s^2 M_{\chi_{c2}}^3 f_{\chi_0}^2}{(M_{\chi_{c2}}+b)^4}\left[1+C_2(\alpha_s/\pi)\right] \tag{28}$$

where $C_0 = 8.77$ and $C_2 = -4.827$ are NLO QCD radiative corrections[28, 29, 30]. For comparison, the expression for $\Gamma_{gg}(\eta_c)$ is similar:

$$\Gamma_{gg}(\eta_c) = \left(\frac{2}{9}\right)\frac{4\pi\alpha_s^2 M_{\eta_c} f_{\eta_c}^2}{(M_{\eta_c}+b)^2}\left[1+4.8\frac{\alpha_s}{\pi}\right] \tag{29}$$

We have seen that, the usual expression for the decay rate $\Gamma_{\gamma\gamma}(\chi_0)$ in terms of $\mathcal{R}_1'(0)$ is now reduced to the simple form Eq.(24) by using Eq.(21). We note also that Eq.(23) shows that $f_{\chi_{c0}}$ becomes comparable to f_{η_c}, even though, in general, T_0 and $f_{\chi_{c0}}$ are of the order $O(q/M)$ compared with f_{η_c}.

In the limit of $b = 0$, the expressions for the two decay rates are exactly the same, apart from the decay constant f_{χ_0} and f_{η_c} and QCD radiative correction terms. The decay rate for χ_{c2} differs from that of χ_{c0} only by a HQSS factor. Thus by comparing the expression for χ_{c0} and η_c we could already have some estimate for the χ_{c0} two-photon and two-gluon decay rates. For $f_{\chi_{c0}}$ of $O(f_{\eta_c})$, one would expect $\Gamma_{\gamma\gamma}(\chi_{c0})$ to be in the range of a few keV and that $\Gamma_{gg}(\chi_{c0})$ is roughly of the same size as $\Gamma_{gg}(\eta_c)$ obtained with the QCD sum rules values[18] for the decay constants : $f_{\eta_c} = 374\,\mathrm{MeV}$ and $f_{\chi_{c0}} = 359\,\mathrm{MeV}$ which gives, with QCD radiative corrections (NLO value): $\Gamma_{\gamma\gamma}(\eta_c) = 4.33\,\mathrm{keV}$, $\Gamma_{\gamma\gamma}(\chi_{c0}) = 5.0\,\mathrm{keV}$ and $\Gamma_{\gamma\gamma}(\chi_{c2}) = 0.70\,\mathrm{keV}$ to be compared, with the latest CLEO result of $(2.53 \pm 0.37 \pm 0.26)\,\mathrm{keV}$ and $(0.60 \pm 0.06 \pm 0.06)\,\mathrm{keV}$; and the averages of all current measurements : $(2.31 \pm 0.10 \pm 0.12\,\mathrm{keV},\ 0.51 \pm 0.02 \pm 0.02)\,\mathrm{keV}$, respectively, for χ_{c0} and χ_{c2} two-photon width[1]. For η_c the prediction from the sum rules value

Reference	$\Gamma_{\gamma\gamma}(\chi_{c0})$(keV)	$\Gamma_{\gamma\gamma}(\chi_{c2})$(keV)	$R = \frac{\Gamma_{\gamma\gamma}(\chi_{c2})}{\Gamma_{\gamma\gamma}(\chi_{c0})}$
Barbieri[5]	3.5	0.93	0.27
Godfrey[7]	1.29	0.46	0.36
Barnes[8]	1.56	0.56	0.36
Bodwin[9]	6.70 ± 2.80	0.82 ± 0.23	$0.12^{+0.15}_{-0.06}$
Gupta[10]	6.38	0.57	0.09
Münz[11]	1.39 ± 0.16	0.44 ± 0.14	$0.32^{+0.16}_{-0.12}$
Huang[12]	3.72 ± 1.10	0.49 ± 0.16	$0.13^{+0.11}_{-0.06}$
Ebert[13]	2.90	0.50	0.17
Schuler[14]	2.50	0.28	0.11
Crater[15]	$3.34 - 3.96$	$0.43 - 0.74$	$0.13 - 0.19$
Wang[16]	3.78	–	–
Laverty[17]	$1.99 - 2.10$	$0.30 - 0.73$	$0.14 - 0.37$
This work	5.00	0.70	0.14

TABLE I: Potential model predictions for $\chi_{c0,2}$ two-photon widths compared with this work.

of f_{η_c} mentioned above is slightly less than the NLO value of 5.34 keV obtained with HQSS and is more or less in agreement with experiment. Similarly, the prediction for χ_{c0} from the sum rules value for $f_{\chi_{c0}}$ is however almost twice the CLEO value, but possibly with large theoretical uncertainties in sum rules calculation for $f_{\chi_{c0}}$, as to be expected. For comparison, we note that the Cornell potential model gives $f_{\chi_{c0}} = 338\,\mathrm{MeV}$[31]. Also a recent QCD sum rules calculation[34] gives $f_{\chi_{c0}} = 510 \pm 40\,\mathrm{MeV}$ which implies a still larger χ_{c0} two-photon decay rates. Various potential model calculations give $\Gamma_{\gamma\gamma}(\chi_{c0})$ in the range $1.2 - 6.7$ keV and $\Gamma_{\gamma\gamma}(\chi_{c2})$ in the range $0.28 - 0.93$ keV as shown in table I. From the above expressions for the decay rates, the two-photon branching ratios for χ_{c0} and and η_c would be the same in the absence of QCD radiative corrections(the two-photon χ_{c2} branching ratios is smaller by 20% with $\mathcal{B}_{\gamma J/\psi(1S)}(\chi_{c2}) = (20 \pm 1.0)\%$[2]). With QCD radiative correction, the predicted branching ratios are , for $\alpha_s = 0.28$: $\mathcal{B}_{\gamma\gamma}(\eta_c) = 2.90 \times 10^{-4}$, $\mathcal{B}_{\gamma\gamma}(\chi_{c0}) = 3.45 \times 10^{-4}$, and $\mathcal{B}_{\gamma\gamma}(\chi_{c2}) = 2.55 \times 10^{-4}$ which are very close to the measured value of $(2.4^{+1.1}_{-0.9}) \times 10^{-4}$, $(2.35 \pm 0.23) \times 10^{-4}$, and $(2.43 \pm 0.18) \times 10^{-4}$, respectively[2]. This shows that QCD radiative corrections are important in bringing the predictions close to experiments.

For the excited state $2P$ $\chi_{c0,2}$ states, there has been observation of the χ'_{c2} state above $D\bar{D}$ threshold, the $Z(3930)$ state, at $M = (3928 \pm 5 \pm 2)\,\mathrm{MeV}$ by the Belle Collaboration[32] which gives $\Gamma_{\gamma\gamma}(\chi'_{c2}) \times \mathcal{B}(D\bar{D}) = (0.18 \pm 0.05 \pm 0.03)\,\mathrm{keV}$ which implies $\Gamma_{\gamma\gamma}(\chi'_{c2}) \simeq (0.18 \pm 0.04)\,\mathrm{keV}$[33]. This would imply $\Gamma_{\gamma\gamma}(\chi'_{c0}) \simeq (1.30 \pm 0.3)\,\mathrm{keV}$, and $f_{\chi'_{c0}} \simeq 195\,\mathrm{MeV}$, comparable with the HQSS value of 279 MeV for $f_{\eta'_c}$[3]. One thus expects that $\Gamma_{gg}(\chi'_{c0})$ in the range $5 - 10\,\mathrm{MeV}$.

For $\chi_{b0,2}$ the same potential model calculation quoted in table I gives the $\chi_{b0,2}$ two-photon width about 1/10 of that for η_b which implies $f_{\chi_{b0}} \approx (1/3) f_{\eta_b}$, smaller than the value obtained from $\mathcal{R}_0(0)$ and $\mathcal{R}'_1(0)$ with the Cornell potential[31] which gives $f_{\chi_{b0}} = 0.46 f_{\eta_b}$.

III. CONCLUSION

By using local operator expansion, we show that the two-photon and two-gluon decays of the P-wave heavy quarkonium χ_{c0} and χ_{b0} state can be obtained from the heavy quark energy-momentum tensor and its trace, a $\bar{Q}Q$ scalar density. The decay rates can then be expressed in terms of $f_{\chi_{c0}}$ and $f_{\chi_{b0}}$ decay constant and are similar to that of η_c. Existing sum rules calculation for $f_{\chi_{c0}}$ however produces a χ_{c0} two-photon width about 5 keV, somewhat bigger than the CLEO measurement, but possibly with large theoretical uncertainties. It remains to be seen whether a better determination of $f_{\chi_{c0}}$ could bring the $\chi_{c0,2}$ two-photon decay rates closer to experiments or higher order QCD radiative corrections and large relativistic corrections are needed to explain the data.

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