SLAC-PUB-13565
WSU–HEP–0901
UH-511-1136-09
March 2009

# Relating $D^0$-$\bar{D}^0$ Mixing and $D^0 \to \ell^+\ell^-$ with New Physics

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## Abstract

We point out how, in certain models of New Physics, the same combination of couplings occurs in the amplitudes for both $D^0$-$\bar{D}^0$ mixing and the rare decays $D^0 \to \ell^+\ell^-$. If the New Physics dominates and is responsible for the observed mixing, then a very simple correlation exists between the magnitudes of each; in fact the rates for the decay $D^0 \to \ell^+\ell^-$ are completely fixed by the mixing. Observation of $D^0 \to \ell^+\ell^-$ in excess of the Standard Model prediction could identify New Physics contributions to $D^0$-$\bar{D}^0$ mixing.

*Submitted to the Physical Review D*

Work supported in part by US Department of Energy contract DE-AC02-76SF00515

## I. INTRODUCTION

Following many years of effort, there is now indisputable experimental evidence for $D^0$-$\bar{D}^0$ mixing. The current values (the HFAG 'no CPV-allowed' fit [1]) of the $D^0$ mixing parameters are

$$x_{\rm D} \equiv \frac{\Delta M_{\rm D}}{\Gamma_{\rm D}} = 0.0100^{+0.0024}_{-0.0026} \qquad \text{and} \qquad y_{\rm D} \equiv \frac{\Delta \Gamma_{\rm D}}{2\Gamma_{\rm D}} = 0.0076^{+0.0017}_{-0.0018} \ . \tag{1}$$

These show that (i) charm mixing occurs at about the percent level, (ii) $x_{\rm D}, y_{\rm D}$ are comparable in magnitude and (iii) the signs of $x_{\rm D}$ and $y_{\rm D}$ are positive (although a direct measurement of the sign of $x_{\rm D}$ is yet to be made).

While it is quite likely that the observed mixing amplitude is dominated by the Standard Model contributions, the exact predictions are quite difficult.[1] There are several reasons for this [2, 3, 4]. For example, in the "short distance" approach [5] at leading order in the Operator Product Expansion (OPE) formalism (operators of dimension $D = 6$), the individual diagrams are CKM-suppressed to the level $\mathcal{O}(\lambda^2)$ ($\lambda \simeq 0.22$ is the familiar Wolfenstein parameter), hinting that the observed charm mixing is a simple consequence of CKM structure. This is, however, not correct because severe cancellations between diagrams (even through $\mathcal{O}(\alpha_s)$) greatly reduce the $D = 6$ mixing to $\mathcal{O}(10^{-6})$ [3, 4]. As for higher ($D > 6$) orders in OPE, it is true that certain enhanced contributions have been identified [6, 7], but a definitive evaluation is lacking due to the large number of $D > 6$ operators and the inability to determine their matrix elements. A promising alternative approach which involves a hadron-level description [8] may be able to account for the observed magnitude of $y_{\rm D}$ and $x_{\rm D}$, but predicts their relative sign to be opposite. It is fair to say that this is probably not the final word on the SM analysis.

Given the uncertain status of the SM description, it would be tempting but premature [9] to attribute the observed $x_{\rm D}$ to New Physics.[2] But clearly, the possibility that NP makes a significant or even dominant contribution to the observed mixing is open. A recent comprehensive treatment of NP models [14] shows that a large number of such models can

[1] Henceforth, we will make frequent use of the abbreviations SM for Standard Model and NP for New Physics.

[2] We will focus on $x_{\rm D}$ in this paper. Not only does the SM estimate for $y_{\rm D}$ work reasonably well when long distance effects are included [10], but it has also been shown that NP effects are too small to have any significant impact [11, 12, 13].

accommodate a value of $x_{\rm D}$ at the per cent level. This encourages us to further explore the NP option. In particular, New Physics could affect charm-related processes beyond mixing, such as rare decays [15]. Of special interest are the $D^0 \to \ell^+\ell^-$ decays. At present, there are only the upper limits [16, 17, 18, 19]

$$\begin{aligned} &\mathcal{B}_{D^0\to\mu^+\mu^-} \le 1.3\times 10^{-6}, \qquad \mathcal{B}_{D^0\to e^+e^-} \le 1.2\times 10^{-6}, \qquad \text{and} \\ &\mathcal{B}_{D^0\to\mu^\pm e^\mp} \le 8.1\times 10^{-7} \ \ , \end{aligned} \tag{2}$$

all at CL=90%. Such branching fractions place bounds on possible NP couplings, which can be compared with that obtained from $D^0$-$\bar{D}^0$ mixing. In this paper we study the impact of NP on the combined system of $D^0$-$\bar{D}^0$ mixing and the rare decays $D^0 \to \ell^+\ell^-$. It should be stressed that the SM rate for the decay mode, $\mathcal{B}_{D^0\to\mu^+\mu^-} \simeq 3\times 10^{-13}$ can be estimated fairly reliably even upon accounting for the effect of long distance enhancement [20]. This smallness of the SM signal makes it easier for NP contributions to stand out. In this paper, we point out how, in certain NP models, the *same* couplings occur in the amplitudes for both $D^0$-$\bar{D}^0$ mixing and $D^0 \to \ell^+\ell^-$ decay. If the NP effects are significant in mixing, then a correlation will exist between the magnitudes of each. In fact the correlation can be very simple and striking, with the branching fraction $\mathcal{B}_{D^0\to\ell^+\ell^-}$ being proportional to the mixing parameter $x_{\rm D}$.

For each NP model considered in this paper, we shall make the simplifying assumption that the NP dominates $D^0$-$\bar{D}^0$ mixing and then derive the correlated branching fraction that is then predicted. Obviously, if NP does *not* dominate, all our results for the branching fractions become upper bounds. We shall stress the general issue of which conditions allow for such correlations and give specific examples. Finally, even if the number of parameters in a given NP model is too large to give a unique prediction for $\mathcal{B}_{D^0\to\ell^+\ell^-}$ in terms of $x_{\rm D}$ (*e.g.* $Z'$ models, *etc*), we show in Sects. III B-D how it is possible to estimate the scale of $\mathcal{B}_{D^0\to\ell^+\ell^-}$ by using the value of $x_{\rm D}$ as input.

## II. EFFECTIVE LAGRANGIANS

Heavy particles present in NP models are not produced in final states of charm quark decays. Yet, effects generated by exchanges of these new particles can be accounted for in effective operators built out of the SM degrees of freedom. That is, by integrating out degrees of freedom associated with new interactions at a heavy scale $M$, we obtain an effective hamiltonian written in the form of a series of operators of increasing dimension. Here, we restrict our attention to the leading order operators, of dimension $D = 6$. For both $D^0 - \overline{D}^0$ mixing and $D^0 \to \ell^+\ell^-$ decays, the complete basis of effective operators is known and is expressed most conveniently in terms of chiral quark fields,

$$\langle f|\mathcal{H}_{NP}|i\rangle = G \sum_{i=1} \mathrm{C}_i(\mu)\ \langle f|Q_i|i\rangle(\mu) \ \ , \tag{3}$$

where the prefactor $G$ has the dimension of inverse-squared mass, the $\mathrm{C}_i$ are dimensionless Wilson coefficients, and the $Q_i$ are the effective operators of dimension six. Throughout, our convention for defining chiral projections for a field $q(x)$ will be $q_{L,R}(x) \equiv (1 \pm \gamma_5)q(x)/2$.

For $\Delta C = 2$ processes, there are eight effective operators that can contribute [14, 21],

$$\begin{aligned}
Q_1 &= (\overline{u}_L\gamma_\mu c_L)\ (\overline{u}_L\gamma^\mu c_L)\ , & Q_5 &= (\overline{u}_R\sigma_{\mu\nu} c_L)\ (\overline{u}_R\sigma^{\mu\nu} c_L)\ , \\
Q_2 &= (\overline{u}_L\gamma_\mu c_L)\ (\overline{u}_R\gamma^\mu c_R)\ , & Q_6 &= (\overline{u}_R\gamma_\mu c_R)\ (\overline{u}_R\gamma^\mu c_R)\ , \\
Q_3 &= (\overline{u}_L c_R)\ (\overline{u}_R c_L)\ , & Q_7 &= (\overline{u}_L c_R)\ (\overline{u}_L c_R)\ , \\
Q_4 &= (\overline{u}_R c_L)\ (\overline{u}_R c_L)\ , & Q_8 &= (\overline{u}_L\sigma_{\mu\nu} c_R)\ (\overline{u}_L\sigma^{\mu\nu} c_R)\ \ .
\end{aligned} \tag{4}$$

These operators are generated at the scale $M$ where the NP is integrated out. A non-trivial operator mixing then occurs via renormalization group running of these operators between the heavy scale $M$ and the light scale $\mu$ at which hadronic matrix elements are computed.

All possible NP contributions to $c \to u\ell^+\ell^-$ can be similarly summarized. In this case, however, there are now ten operators,

$$\begin{aligned}
\widetilde{Q}_1 &= (\overline{\ell}_L\gamma_\mu \ell_L)\ (\overline{u}_L\gamma^\mu c_L)\ , & \widetilde{Q}_4 &= (\overline{\ell}_R \ell_L)\ (\overline{u}_R c_L)\ , \\
\widetilde{Q}_2 &= (\overline{\ell}_L\gamma_\mu \ell_L)\ (\overline{u}_R\gamma^\mu c_R)\ , & \widetilde{Q}_5 &= (\overline{\ell}_R\sigma_{\mu\nu} \ell_L)\ (\overline{u}_R\sigma^{\mu\nu} c_L)\ , \\
\widetilde{Q}_3 &= (\overline{\ell}_L \ell_R)\ (\overline{u}_R c_L)\ , & &
\end{aligned} \tag{5}$$

with five additional operators $\widetilde{Q}_6, \ldots, \widetilde{Q}_{10}$ being obtained respectively from those in Eq. (5) by the substitutions $L \to R$ and $R \to L$. The corresponding Wilson coefficients will be denoted as $\widetilde{C}_i(\mu)$. It is worth noting that only eight operators contribute to $D^0 \to \ell^+\ell^-$, as $\langle \ell^+\ell^-|\widetilde{Q}_5|D^0\rangle = \langle \ell^+\ell^-|\widetilde{Q}_{10}|D^0\rangle = 0$.

To obtain a general expression for $x_{\mathrm{D}}$ as implied by the effective Hamiltonian of Eq. (3), we evaluate the $D^0$-to-$\bar{D}^0$ matrix element in the modified vacuum saturation approximation of Appendix A and work at the light scale $\mu = m_c$,

$$x_{\rm D} = G\frac{f_D^2 M_D B_D}{\Gamma_D}\left[\frac{2}{3}(C_1(m_c)+C_6(m_c)) - \left[\frac{1}{2}+\frac{\eta}{3}\right] C_2(m_c) + \left[\frac{1}{12}+\frac{\eta}{2}\right] C_3(m_c) \right.$$
$$\left. -\frac{5\eta}{12}(C_4(m_c)+C_7(m_c)) + \eta(C_5(m_c)+C_8(m_c))\right] \ , \qquad (6)$$

where we have taken $N_c = 3$, we remind the reader that $\eta$ is discussed in Appendix A and the prefactor $G$ defines the scale at which NP is integrated out. To use this expression one must relate the light-scale coefficients $\{C_i(m_c)\}$ to their heavy-scale counterparts $\{C_i(M)\}$ in terms of the RG-running factors given in Appendix A.

The rare decays $D^0 \to \ell^+\ell^-$ and $D^0 \to \mu^+e^-$ are treated analogously. To the decay amplitude

$$\mathcal{M} = \bar{u}(\mathbf{p}_-, s_-)\,[A + B\gamma_5]\,v(\mathbf{p}_+, s_+) \ , \qquad (7)$$

are associated the branching fractions

$$\mathcal{B}_{D^0\to\ell^+\ell^-} = \frac{M_D}{8\pi\Gamma_{\rm D}}\sqrt{1-\frac{4m_\ell^2}{M_D^2}}\left[\left(1-\frac{4m_\ell^2}{M_D^2}\right)|A|^2 + |B|^2\right] \ ,$$
$$\mathcal{B}_{D^0\to\mu^+e^-} = \frac{M_D}{8\pi\Gamma_{\rm D}}\left(1-\frac{m_\mu^2}{M_D^2}\right)^2\left[|A|^2+|B|^2\right] \ , \qquad (8)$$

where electron mass has been neglected in the latter expression. Any NP contribution described by the operators of Eq. (5) gives for the amplitudes $A$ and $B$,

$$|A| = G\frac{f_D M_D^2}{4m_c}\left[\widetilde{C}_{3-8}+\widetilde{C}_{4-9}\right] \ ,$$
$$|B| = G\frac{f_D}{4}\left[2m_\ell\left(\widetilde{C}_{1-2}+\widetilde{C}_{6-7}\right) + \frac{M_D^2}{m_c}\left(\widetilde{C}_{4-3}+\widetilde{C}_{9-8}\right)\right] \ , \qquad (9)$$

with $\widetilde{C}_{i-k} \equiv \widetilde{C}_i - \widetilde{C}_k$. In general, one cannot predict the rare decay rate by knowing just the mixing rate, even if both $x_D$ and $\mathcal{B}_{D^0\to\ell^+\ell^-}$ are dominated by a given NP contribution. We shall see, however, that this is possible for a restricted subset of NP models.

## III. NP MODELS WITH TREE-LEVEL AMPLITUDES

This is the most obvious situation for producing a correlation between mixing and decay because there is a factorization between the initial and final interaction vertices. In the following, it will be convenient to consider separately the propagation of a spin-1 boson V and of a spin-0 boson S as the intermediate particle in the tree-level amplitudes. The bosons V and S can be of either parity.

*Spin-1 Boson V*: Assuming that the spin-1 particle $V$ has flavor-changing couplings and keeping all the operators in the effective Lagrangian up to dimension 5, the most general Lagrangian can be written as

$$\mathcal{H}_V = \mathcal{H}_V^{\rm FCNC} + \mathcal{H}_V^L \quad , \tag{10}$$

where the quark part $\mathcal{H}_V^{\rm FCNC}$ is

$$\mathcal{H}_V^{\rm FCNC} = g_{V1}\overline{u}_L\gamma_\mu c_L V^\mu + g_{V2}\overline{u}_R\gamma_\mu c_R V^\mu + g_{V3}\overline{u}_L\sigma_{\mu\nu}c_R V^{\mu\nu} + g_{V4}\overline{u}_R\sigma_{\mu\nu}c_L V^{\mu\nu} \tag{11}$$

and the part that describes interactions of $V$ with leptons $\mathcal{H}_V^L$ is

$$\mathcal{H}_V^L = g'_{V1}\overline{\ell}_L\gamma_\mu\ell_L V^\mu + g'_{V2}\overline{\ell}_R\gamma_\mu\ell_R V^\mu + g'_{V3}\overline{\ell}_L\sigma_{\mu\nu}\ell_R V^{\mu\nu} + g'_{V4}\overline{\ell}_R\sigma_{\mu\nu}\ell_L V^{\mu\nu}. \tag{12}$$

Here $V_\mu$ is the vector field and $V_{\mu\nu} = \partial_\mu V_\nu - \partial_\nu V_\mu + ...$ is the field-strength tensor for $V_\mu$. For this study it is not important whether the field $V$ corresponds to an abelian or non-abelian gauge symmetry group.

In order to see the leading contribution to $D$ mixing from Eq. (11), let us consider a correlator,

$$\Sigma_D(q^2) = (-i)\int d^4x\ e^{i(q-p)\cdot x}\langle\overline{D}^0(p)|T\left\{\mathcal{H}^{\Delta C=1}(x)\mathcal{H}^{\Delta C=1}(0)\right\}|D^0(p)\rangle. \tag{13}$$

This correlator is related to the mass and lifetime differences of a $D$-meson as [8],

$$\Sigma_D(M_D^2) = 2M_D\left(\Delta M_D - \frac{i}{2}\Delta\Gamma_D\right). \tag{14}$$

Inserting Eq. (11) for $\mathcal{H}^{\Delta C=1}$, we obtain

$$\begin{aligned}\Sigma_D(q^2) = (-i)\ \beta\int d^4x\ e^{i(q-p)\cdot x}&\langle 0|T\left\{V^\mu(x)V^\nu(0)\right\}|0\rangle\langle\overline{D}^0(p)|\ g_{V1}^2\ \overline{u}_L\gamma_\mu c_L(x)\ \overline{u}_L\gamma_\nu c_L(0)\\ &+ g_{V1}g_{V2}\ \overline{u}_L\gamma_\mu c_L(x)\ \overline{u}_R\gamma_\nu c_R(0) + g_{V1}g_{V2}\ \overline{u}_R\gamma_\mu c_R(x)\ \overline{u}_L\gamma_\nu c_L(0)\\ &+ g_{V2}^2\ \overline{u}_R\gamma_\mu c_R(x)\ \overline{u}_R\gamma_\nu c_R(0)\ +\ \mathcal{O}(1/M_V)\ |D^0(p)\rangle\ ,\end{aligned} \tag{15}$$

where $\mathcal{O}(1/M_V)$ denotes terms additionally suppressed by powers of $1/M_V$ and $\beta \sim \mathcal{O}(1)$ denotes all relevant traces over group indices associated with $V_\mu$, with $\beta = 1$ for an Abelian symmetry group. The leading-order $D = 6$ contribution is found by expanding the vector boson propagator in the large $M_V$-limit and then performing the resulting elementary integral,

$$\Sigma_D(M_D^2) = \frac{\beta}{M_V^2} \langle \overline{D}^0(p)|\ g_{V1}^2 Q_1 + 2 g_{V1} g_{V2} Q_2 + g_{V2}^2 Q_6\ |D^0(p)\rangle\ . \tag{16}$$

Taking into account RG-running between the heavy scale $M_V$ and the light scale $\mu = m_c$ at which the matrix elements are computed, we obtain a subcase of the general Eq. (6),

$$x_{\rm D}^{\rm (V)} = \frac{\beta f_D^2 M_D B_D}{2 M_V^2 \Gamma_D} \left[ \frac{2}{3} \left( C_1(m_c) + C_6(m_c) \right) - \left[ \frac{1}{2} + \frac{\eta}{3} \right] C_2(m_c) + \left[ \frac{1}{12} + \frac{\eta}{2} \right] C_3(m_c) \right] \tag{17}$$

where the superscript on $x_{\rm D}^{\rm (V)}$ denotes propagation of a vector boson in the tree amplitude. The Wilson coefficients evaluated at scale $\mu = m_c$ are

$$\begin{aligned} &{\rm C}_1(m_c) = r(m_c, M_V)\ g_{V1}^2\ , && {\rm C}_3(m_c) = \tfrac{4}{3} \left[ r(m_c, M_V)^{1/2} - r(m_c, M_V)^{-4} \right] g_{V1} g_{V2}\ , \\ &{\rm C}_2(m_c) = 2\ r(m_c, M_V)^{1/2} g_{V1} g_{V2}\ , && {\rm C}_6(m_c) = r(m_c, M_V)\ g_{V2}^2\ . \end{aligned} \tag{18}$$

Similar calculations can be performed for the $D^0 \to \ell^+ \ell^-$ decay. The effective Hamiltonian in this case is

$$\mathcal{H}_{c \to u \ell^+ \ell^-}^{\rm (V)} = \frac{1}{M_V^2} \left[ g_{V1} g_{V1}' \widetilde{Q}_1 + g_{V1} g_{V2}' \widetilde{Q}_7 + g_{V1}' g_{V2} \widetilde{Q}_2 + g_{V2} g_{V2}' \widetilde{Q}_6 \right]\ , \tag{19}$$

which leads to the branching fraction,

$$\mathcal{B}_{D^0 \to \ell^+ \ell^-}^{\rm (V)} = \frac{f_D^2 m_\ell^2 M_D}{32 \pi M_V^4 \Gamma_D} \sqrt{1 - \frac{4 m_\ell^2}{M_D^2}} \left( g_{V1} - g_{V2} \right)^2 \left( g_{V1}' - g_{V2}' \right)^2\ . \tag{20}$$

Clearly, Eqs. (17) and (20) can be related to each other only for a specific set of NP models. We shall consider those shortly.

*Spin-0 Boson S*: Analogous procedures can be followed if now the FCNC is generated by quarks interacting with spin-0 particles. Again, assuming that the spin-0 particle $S$ has flavor-changing couplings and keeping all the operators in the effective Hamiltonian up to dimension five, we can write the most general Hamiltonian as

$$\mathcal{H}_S = \mathcal{H}_S^{\rm FCNC} + \mathcal{H}_S^L\ , \tag{21}$$

where the quark FCNC part is given by

$$\mathcal{H}_S^{\rm FCNC} = g_{S1} \overline{u}_L c_R S + g_{S2} \overline{u}_R c_L S + g_{S3} \overline{u}_L \gamma_\mu c_L \partial^\mu S + g_{S4} \overline{u}_R \gamma_\mu c_R \partial^\mu S \tag{22}$$

and the part that is responsible for the interactions of $S$ with leptons is

$$\mathcal{H}_S^L = g'_{S1}\overline{\ell}_L \ell_R S + g'_{S2}\overline{\ell}_R \ell_L S + g'_{S3}\overline{\ell}_L \gamma_\mu \ell_L \partial^\mu S + g'_{S4}\overline{\ell}_R \gamma_\mu \ell_R \partial^\mu S \quad . \tag{23}$$

Inserting this Hamiltonian into the correlator Eq. (13) and performing steps similar to the spin-one case leads to

$$\Sigma_D(M_D^2) = -\frac{1}{M_S^2}\langle \overline{D}^0(p)|\ g_{S1}^2 Q_7 + 2g_{S1}g_{S2}Q_3 + g_{S2}^2 Q_4\ |D^0(p)\rangle \quad . \tag{24}$$

Evaluation at scale $\mu = m_c$ gives

$$x_D^{(\mathrm{S})} = -\frac{f_D^2 M_D B_D}{2\Gamma_D M_S^2}\left[\left[\frac{1}{12}+\frac{\eta}{2}\right] C_3(m_c) - \frac{5\eta}{12}(C_4(m_c)+C_7(m_c)) + \eta(C_5(m_c)+C_8(m_c))\right] \tag{25}$$

with the Wilson coefficients defined as

$$\begin{aligned}
C_3(m_c) &= -2r(m_c, M_S)^{-4}\ g_{S1}g_{S2} \\
C_4(m_c) &= -\left[\left(\frac{1}{2}-\frac{8}{\sqrt{241}}\right) r_+(m_c, M_S) + \left(\frac{1}{2}+\frac{8}{\sqrt{241}}\right) r_-(m_c, M_S)\right] g_{S2}^2 \\
C_5(m_c) &= -\frac{1}{8\sqrt{241}}\left[r_+(m_c, M_S) - r_-(m_c, M_S)\right] g_{S2}^2 \\
C_7(m_c) &= -\left[\left(\frac{1}{2}-\frac{8}{\sqrt{241}}\right) r_+(m_c, M_S) + \left(\frac{1}{2}+\frac{8}{\sqrt{241}}\right) r_-(m_c, M_S)\right] g_{S1}^2 \\
C_8(m_c) &= -\frac{1}{8\sqrt{241}}\left[r_+(m_c, M_S) - r_-(m_c, M_S)\right] g_{S1}^2 \quad ,
\end{aligned} \tag{26}$$

where for notational simplicity we have defined $r_\pm \equiv r^{(1\pm\sqrt{241})/6}$ (*cf* Eq. (A1)).

The effective Hamiltonian for the $D^0 \to \ell^+\ell^-$ decay is

$$\mathcal{H}^{(\mathrm{S})}_{c\to u\ell^+\ell^-} = -\frac{1}{M_S^2}\left[g_{S1}g'_{S1}\widetilde{Q}_9 + g_{S1}g'_{S2}\widetilde{Q}_8 + g'_{S1}g_{S2}\widetilde{Q}_3 + g_{S2}g'_{S2}\widetilde{Q}_4\right] \ , \tag{27}$$

and from this, it follows that the branching fraction is

$$\mathcal{B}^{(\mathrm{S})}_{D^0\to\ell^+\ell^-} = \frac{f_D^2 M_D^5}{128\pi m_c^2 M_S^4 \Gamma_D}\sqrt{1-\frac{4m_\ell^2}{M_D^2}}\,(g_{S1}-g_{S2})^2\left[(g'_{S1}+g'_{S2})^2\left(1-\frac{4m_\ell^2}{M_D^2}\right)+(g'_{S1}-g'_{S2})^2\right] . \tag{28}$$

Note that if the spin-0 particle $S$ only has *scalar* FCNC couplings, *i.e.* $g_{S1} = g_{S2}$, no contribution to $D^0 \to \ell^+\ell^-$ branching ratio is generated at the tree level; the non-zero contribution to rare decays is produced at one-loop level. This follows from the *pseudoscalar* nature of the $D$-meson.

If there are not one but several particles mediating those processes (assuming that they all couple to quarks and leptons), the above generic Lagrangians would need to be modified. For example, in the spin-0 case, one would have to replace $g_i\ S$ with $\sum_k g_{ik} S_k F(k)$, where $F(k)$ is a numerical factor, as $S_k$ are the mediating fields. For example, in models with extra dimensions the factor $F(k)$ would be related to Kaluza-Klein decompositions of bosons living in the bulk. Similar corrections have to be performed in the case of a bulk spin-1 boson.

Below, we consider generic models where the correlations between the $D^0 - \overline{D}^0$ mixing rates and $D^0 \to \ell^+\ell^-$ rare decays can be found.

### A. Heavy Vector-like Quarks: Q = +2/3 Singlet Quark

Scenarios with heavy quarks beyond the three generations are severely constrained experimentally, if those quarks have chiral couplings. We thus examine the case where the heavy quarks are $SU(2)_L$ singlets (so-called *vector-like* quarks) [22]. Here, we consider the charge assignment $Q = +2/3$ for the heavy quark and then $Q = -1/3$ in the next Section. Weak isosinglets with $Q = +2/3$ occur in Little Higgs theories [23, 24] in which the Standard Model Higgs boson is a pseudo-Goldstone boson, and the heavy iso-singlet $T$ quark cancels the quadratic divergences generated by the top quark when performing quantum corrections to the mass of the Higgs boson. Weak isosinglets with $Q = -1/3$ appear in $E_6$ GUTs [25, 26], with one for each of the three generations ($D$, $S$, and $B$).

The presence of such quarks violates the Glashow-Weinberg-Paschos naturalness conditions for neutral currents [27]. Since their electroweak quantum number assignments are different than those for the SM fermions, flavor changing neutral current interactions are generated in the left-handed up-quark sector. Thus, in addition to the charged current interaction

$$\mathcal{L}_{\rm int}^{(ch)} = \frac{g}{\sqrt{2}}\ V_{\alpha i} \bar{u}_{\alpha,L} \gamma_\mu d_{i,L} W^\mu \quad , \tag{29}$$

there are also FCNC couplings with the $Z^0$ boson [22],

$$\mathcal{L}_{\rm int}^{(ntl)} = \frac{g}{2\sqrt{2}\cos\theta_w}\ \lambda_{ij} \bar{u}_{i,L} \gamma_\mu u_{j,L} Z^{0\mu} \quad . \tag{30}$$

Here, $g$ is the SM $SU(2)$ gauge coupling and $V_{\alpha i}$ is a $4 \times 3$ mixing matrix with $\alpha$ running over $1 \to 4$, $i = 1 \to 3$, with the CKM matrix comprising the first $3 \times 3$ block.

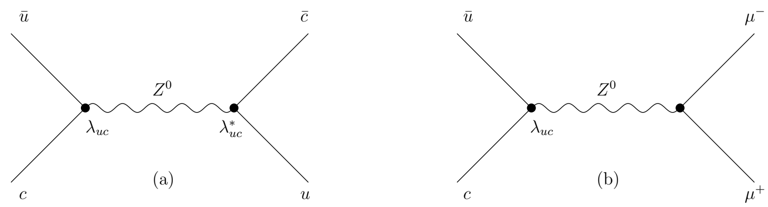


FIG. 1: (a) $D^0$-$\bar{D}^0$ Mixing, (b) $D^0 \to \mu^+\mu^-$.

*$D^0$-$\bar{D}^0$ Mixing*: In this case, a tree-level contribution to $\Delta M_D$ is generated from $Z^0$-exchange as shown in Fig. 1. This is represented by an effective hamiltonian at the scale $M_Z$ as

$$\mathcal{H}_{2/3} = \frac{g^2}{8\cos^2\theta_w M_Z^2}\lambda_{uc}^2\, Q_1 \;=\; \frac{G_F \lambda_{uc}^2}{\sqrt{2}} Q_1 \quad , \tag{31}$$

where from unitarity,

$$\lambda_{uc} \equiv -\left(V_{ud}^* V_{cd} + V_{us}^* V_{cs} + V_{ub}^* V_{cb}\right) \quad . \tag{32}$$

Thus, we find

$$x_{\rm D}^{(+2/3)} = \frac{2 G_F \lambda_{uc}^2 f_D^2 M_D B_D r(m_c, M_Z)}{3\sqrt{2}\Gamma_D} \tag{33}$$

Using $r(m_c, M_Z) = 0.778$ and demanding that the NP contribution is responsible for the observed mixing value yields $\lambda_{uc} = 2.39 \times 10^{-4}$.

*$D^0 \to \mu^+\mu^-$ Decay*: For this case, the leptons have SM couplings to the $Z^0$. We then have

$$A_{D^0\to\ell^+\ell^-} = 0 \qquad B_{D^0\to\ell^+\ell^-} = \lambda_{uc}\frac{G_F f_{\rm D} m_\mu}{2} \quad . \tag{34}$$

Restricting our attention to the $\mu^+\mu^-$ final state because the decay amplitude is proportional to lepton mass, we find

$$\lambda_{uc}^2 \le 8\pi \mathcal{B}_{D^0\to\mu^+\mu^-}\frac{\Gamma_{\rm D}}{M_{\rm D}}\left(\frac{2}{G_F f_{\rm D} m_\mu}\right)^2\left[1 - \frac{4m_\mu^2}{M_{\rm D}}\right]^{-1/2} \quad . \tag{35}$$

From the branching fraction bound of Eq. (2), we obtain $\lambda_{uc} \le 4.17 \times 10^{-2}$, which is much less restrictive than the value from $D^0$ mixing.

*Combining the Mixing and Decay Relations*: A correlation will exist in this case because the coupling between $Z^0$ and the lepton pair is known from the Standard Model. Thus, if we assume that all the $D$ meson mixing comes from the $Q = +2/3$ heavy quark (*i.e.* $x_{\rm D}^{(+2/3)} = x_{\rm D}$), then we can remove the dependence on the NP parameter $\lambda_{uc}$ and predict

$\mathcal{B}_{D^0\to\mu^+\mu^-}$ in terms of $x_{\rm D}$,

$$\begin{aligned}\mathcal{B}_{D^0\to\mu^+\mu^-} &= \frac{3\sqrt{2}}{64\pi}\ \frac{G_F m_\mu^2 x_{\rm D}}{B_{\rm D} r(m_c, M_Z)}\left[1-\frac{4m_\mu^2}{M_{\rm D}}\right]^{1/2} \\ &\simeq\ 4.3\times 10^{-9} x_{\rm D}\ \le\ 4.3\times 10^{-11}\quad .\end{aligned} \tag{36}$$

### B. New Gauge Boson $Z'$

New heavy neutral gauge bosons can exist in a variety of NP models [28]. In these scenarios, there are in general five parameters that describe the two processes under consideration here, namely $g_{Z'1}$, $g_{Z'2}$, $g'_{Z'1}$, $g'_{Z'2}$, and $M_{Z'}$, where the coupling constants are defined as in Eqs. (11, 12) by substiuting $V\to Z'$. There are, of course, many ways to reduce this number. In the following, let us assume that $Z'$ couples only to left-handed quarks and has SM-like diagonal couplings to leptons,

$$g_{Z'2}=0,\quad g'_{Z'1}=\frac{g}{\cos\theta_W}\left(-\frac{1}{2}+\sin^2\theta_W\right),\quad g'_{Z'2}=\frac{g\sin^2\theta_W}{\cos\theta_W}\quad , \tag{37}$$

where $g$ is again the SM $SU(2)$ gauge coupling. This procedure reduces the number of unknowns to two, $g_{Z'1}$ and $M_{Z'}$. Note that for purely vector couplings of a $Z'$ to leptons, *i.e.* $g'_{Z'1}=g'_{Z'2}$ no contributions are generated for $D^0\to\mu^+\mu^-$ due to conservation of vector current.

$D^0$-$\bar{D}^0$ *Mixing*: The contribution of the $Z'$ model to mixing is given by Eq. (17),

$$x_{\rm D}^{({\rm Z}')}=\frac{f_D^2 M_D B_D r(m_c, M_{Z'})}{3\Gamma_D}\ \frac{g_{Z'1}^2}{M_{Z'}^2}\quad . \tag{38}$$

For the very slowly varying RG factor, we have taken $r(m_c, M_{Z'})=0.71$, which is typical of values for a $Z'$ mass in the TeV range. From Eq. (38), we obtain the bound $M_{Z'}/g_{Z'1}\ge 1.7\times 10^6$ GeV.

$D^0\to\mu^+\mu^-$ *Decay*: In this model, the contribution to the rare decay branching fraction can be written in the form,

$$\mathcal{B}^{({\rm Z}')}_{D^0\to\mu^+\mu^-}=\frac{G_F f_D^2 m_\mu^2 M_D}{16\sqrt{2}\pi\Gamma_D}\sqrt{1-\frac{4m_\mu^2}{M_D^2}}\ \frac{g_{Z'1}^2}{M_{Z'}^2}\cdot\frac{M_Z^2}{M_{Z'}^2}\quad . \tag{39}$$

Besides the $g_{Z'1}^2/M_{Z'}^2$ dependence which appears in the above $D$ mixing relation of Eq. (38), there is now an additional factor of $M_Z^2/M_{Z'}^2$. The bound obtained from Eq. (2) implies the restriction $M_{Z'}/g^{1/2}_{Z'1} \geq 8.7\ 10^2$ GeV, which is weaker than the constraint from $D^0$-$\bar{D}^0$ mixing.

*Combining the Mixing and Decay Relations*: Assuming that $Z'$ saturates the observed experimental value for $x_D$, the bound obtained from the $D^0 \to \mu^+\mu^-$ branching fraction as a function of $M_{Z'}$ is

$$\begin{aligned}
\mathcal{B}_{D^0\to\mu^+\mu^-} &= \frac{3G_F m_\mu^2 M_Z^2 x_D}{16\sqrt{2}\pi B_D r(m_c, M_{Z'})}\sqrt{1-\frac{4m_\ell^2}{M_D^2}}\frac{1}{M_{Z'}^2} \\
&\simeq\ 2.4\times 10^{-10}\ \left(x_{\rm D}/M_{Z'}^2({\rm TeV})\right)\ \leq\ 2.4\times 10^{-12}/M_{Z'}^2({\rm TeV})\quad . \qquad (40)
\end{aligned}$$

### C. Family (Horizontal) Symmetries

The gauge sector in the Standard Model has a large global symmetry which is broken by the Higgs interaction [29]. By enlarging the Higgs sector, some subgroup of this symmetry can be imposed on the full SM lagrangian and break the symmetry spontaneously. This family symmetry can be global as well as gauged [30]. If the new gauge couplings are very weak or the gauge boson masses are large, the difference between a gauged or global symmetry is rather difficult to distinguish in practice. In general there would be FCNC effects from both the gauge and scalar sectors. Here we consider the gauge contributions.

Consider the group $SU(2)_G$ acting only on the first two left-handed families (it may be regarded as a subgroup of an $SU(3)_G$, which is broken). Spontaneous breaking of $SU(2)_G$ makes the gauge bosons $G_i$ massive. For simplicity we assume that after symmetry breaking the gauge boson mass matrix is diagonal to a good approximation in which case $G_{i\mu}$ are physical eigenstates and any mixing between them is neglected. Leaving further discussion to Refs. [14, 20], we write down the couplings in the fermion mass basis as

$$\begin{aligned}
\mathcal{H}_{hs} = -f\Big[&G_{1\mu}\Big\{\sin 2\theta_d\left(\overline{d}_L\gamma_\mu d_L - \overline{s}_L\gamma_\mu s_L\right) + \sin 2\theta_u\left(\overline{u}_L\gamma_\mu u_L - \overline{c}_L\gamma_\mu c_L\right) \\
&+ \sin 2\theta_l\left(\overline{e}_L\gamma_\mu e_L - \overline{\mu}_L\gamma_\mu \mu_L\right) + \cos 2\theta_d\left(\overline{d}_L\gamma_\mu s_L + \overline{s}_L\gamma_\mu d_L\right) \\
&+ \cos 2\theta_u\left(\overline{u}_L\gamma_\mu c_L + \overline{c}_L\gamma_\mu u_L\right) + \cos 2\theta_l\left(\overline{e}_L\gamma_\mu \mu_L + \overline{\mu}_L\gamma_\mu e_L\right)\Big\} \\
&+ iG_{2\mu}\Big\{\left(\overline{s}_L\gamma_\mu d_L - \overline{d}_L\gamma_\mu s_L\right) + \left(\overline{c}_L\gamma_\mu u_L - \overline{u}_L\gamma_\mu c_L\right) + \left(\overline{\mu}_L\gamma_\mu e_L - \overline{e}_L\gamma_\mu \mu_L\right)\Big\} \\
&+ G_{3\mu}\Big\{\cos 2\theta_d\left(\overline{d}_L\gamma_\mu d_L - \overline{s}_L\gamma_\mu s_L\right) + \cos 2\theta_u\left(\overline{u}_L\gamma_\mu u_L - \overline{c}_L\gamma_\mu c_L\right) \\
&+ \cos 2\theta_l\left(\overline{e}_L\gamma_\mu e_L - \overline{\mu}_L\gamma_\mu \mu_L\right) - \sin 2\theta_d\left(\overline{d}_L\gamma_\mu s_L + \overline{s}_L\gamma_\mu d_L\right) \\
&- \sin 2\theta_u\left(\overline{u}_L\gamma_\mu c_L + \overline{c}_L\gamma_\mu u_L\right) - \sin 2\theta_l\left(\overline{e}_L\gamma_\mu \mu_L + \overline{\mu}_L\gamma_\mu e_L\right)\Big\}\Big]\quad . \qquad (41)
\end{aligned}$$

Applications of this general interaction yield expressions for $D^0$-$\bar{D}^0$ mixing,

$$x_{\rm D}^{\rm (FS)} = \frac{2f_D^2 M_D B_D r(m_c,M)}{3\Gamma_D}\ f^2\left(\frac{\cos^2 2\theta_u}{m_1^2}+\frac{\sin^2 2\theta_u}{m_3^2}-\frac{1}{m_2^2}\right)\quad , \tag{42}$$

for $D^0 \to \mu^+\mu^-$ decay,

$$\mathcal{B}^{\rm (FS)}_{D^0\to\mu^+\mu^-} = \frac{M_D f_D^2 m_\mu^2}{64\pi\Gamma_D} f^4\left(\frac{\sin 2\theta_u \cos 2\theta_\ell}{m_3^2}-\frac{\cos 2\theta_u \sin 2\theta_\ell}{m_1^2}\right)^2\quad , \tag{43}$$

and for $D^0 \to \mu^+ e^-$ decay,

$$\mathcal{B}^{\rm (FS)}_{D^0\to\mu^+e^-} = \frac{M_D f_D^2 m_\mu^2}{64\pi\Gamma_D} f^4\left(\frac{\cos 2\theta_u \cos 2\theta_\ell}{m_1^2}+\frac{1}{m_2^2}+\frac{\sin 2\theta_u \sin 2\theta_\ell}{m_3^2}\right)^2\quad . \tag{44}$$

In Eq. (42) the very-slowly varying RG factor $r(m_c,M)$ is set to the scale $M \sim 1$ TeV.

Precise predictions for the above three processes are not immediate due to the large number of NP parameters. Different patterns can be obtained depending on the region of parameter space:

*Case A* [$m_1 = m_3 \ll m_2$ and $\theta_u - \theta_\ell = \pi/4$]:

Here, $\mathcal{B}^{\rm (FS)}_{D^0\to\mu^+e^-}$ is suppressed and a parameter-free prediction for $\mathcal{B}_{D^0\to\mu^+\mu^-}$ in terms of $x_D$ occurs,

$$\mathcal{B}_{D^0\to\mu^+\mu^-} = \frac{9\Gamma_D m_\mu^2 x_D^2}{256\pi M_D f_D^2 B_D^2 r(m_c,m_1)^2} \simeq 0.7\times 10^{-14}\ x_D^2 \le 0.7\times 10^{-18}\quad . \tag{45}$$

Note that here we related the $\mathcal{B}_{D^0\to\mu^+\mu^-}$ to the *square* of $x_D$.

*Case B* [$m_1 = m_2 = m_3$ and $\theta_u - \theta_\ell = \pi/2$]:

In this case, the amplitudes for all three processes vanish.

*Case C* [$m_1 = m_2 \ll m_3$ and $\theta_u - \theta_\ell = \pi/2$]:

Now, the mixing contribution vanishes but the branching fractions for $D^0 \to \mu^+\mu^-$ and $D^0 \to \mu^+e^-$ are equal, although undetermined due to NP parameter dependence.

*Case D* [$m_1 = m_3 \gg m_2$]:

In this limit, $D^0 \to \mu^+\mu^-$ is negligible and there is a parameter-free prediction for $\mathcal{B}_{D^0\to\mu^+\mu^-}$ in terms of $x_D$, but $x_D$ has the wrong sign.

## IV. NP MODELS WITH LOOP AMPLITUDES

Although tree amplitudes represent the most obvious situation for producing a correlation between mixing and decay, it turns out that loop amplitudes can have the same effect. As is well known [31], low energy effective lagrangians continue to provide the most useful description. In the following, we consider three examples of NP models with loop amplitudes.

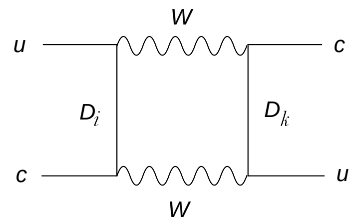


FIG. 2: Box contribution from heavy weak-isosinglet quarks.

### A. Heavy Vector-like Quarks: Q = -1/3 Singlet Quark

We first consider models with a heavy vector-like $Q = -1/3$ singlet quark. Note that essentially identical results hold for a SM fourth quark generation as well, since in each case, the fermions will interact with a SM $W^{\pm}$ gauge boson and thus the charged leptons have SM interactions [32]. It is this which allows for correlations between $D^0$-$\bar{D}^0$ mixing and $D^0 \to \ell^+\ell^-$ decay.

For the class of models with $Q = -1/3$ down-type singlet quarks, the down quark mass matrix is a $4 \times 4$ array if there is just one heavy singlet (or $6 \times 6$ for three heavy singlets as in $E_6$ models). As a consequence, the standard $3 \times 3$ CKM matrix is no longer unitary. Moreover, the weak charged current will now contain terms that couple up-quarks to the heavy singlet quarks. For three heavy singlets, we have

$$\mathcal{L}_{\rm int}^{(ch)} = \frac{g}{\sqrt{2}}\, V_{i\alpha} W^{\mu} \bar{u}_{i,L}\gamma_{\mu} D_{\alpha} \quad , \tag{46}$$

where $u_{i,L} \equiv (u, c, t)_L$ and $D_{\alpha} \equiv (D, S, B)$ refer to the standard up quark and heavy isosinglet down quark sectors. The $\{V_{i\alpha}\}$ are elements of a $3 \times 6$ matrix, which is the product of the $3 \times 3$ and $6 \times 6$ unitary matrices that diagonalize the $Q = +2/3$ and $Q = -1/3$ quark sectors, respectively.

*$D^0$-$\bar{D}^0$ Mixing*: The box diagram contribution to $\Delta M_{\rm D}$ from these new quarks is displayed in Fig. 2. Assuming that the contribution of one of the heavy quarks (say the $S$ quark, of mass $m_S$) dominates, one can write an expression for $x_{\rm D}$ [32],

$$|x_{\rm D}^{(-1/3)}| \simeq \frac{G_F^2 M_W^2 f_D^2 M_D}{6\pi^2 \Gamma_D} B_D \left(V_{cS}^* V_{uS}\right)^2 r(m_c, M_W) |\overline{E}(x_S)| \quad , \tag{47}$$

where $x_S \equiv (m_S/M_W)^2$. The Inami-Lin [31] function $\overline{E}(x_S)$ is defined as

$$\overline{E}(x_S) \equiv x_S \left[\frac{1}{4} - \frac{9}{4(x_S - 1)} - \frac{3}{2(x_S - 1)^2} + \frac{3x_S^3}{2(x_S - 1)^3} \ln x_S\right] \quad . \tag{48}$$

For our numerical work, we assume a default value of $m_S = 500$ GeV, but express our result for variable $m_S$ by noting that the functions $\overline{E}(x_S)$ and $\bar{C}(x_S)$ (*cf* Eq. (53) below) are proportional to $x_S$ within ten per cent over the mass region $400 \leq m_S(\mathrm{GeV}) \leq 700$. The light-heavy mixing angles $|V_{cS}^* V_{uS}|^2$ should go as $1/m_S$ for large $m_S$ to keep the contribution under control. The current bound on $|V_{cS}^* V_{uS}|^2$ from unitarity of the CKM matrix is not very stringent, $|V_{cS}^* V_{uS}|^2 < 4 \times 10^{-4}$ [16].

In the $E_6$-based model proposed by Bjorken *et al* [33], the $6 \times 6$ mass matrix has an especially simple form. The resulting $6 \times 6$ mass matrix has a pseudo-orthogonality property which implies that the $3 \times 3$ CKM matrix, although not unitary, satisfies

$$\sum_{i=1}^{3} \left(V_{\mathrm{CKM}}\right)_{bi}^* \left(V_{\mathrm{CKM}}\right)_{is} = 0 \quad . \tag{49}$$

The analog of this condition in the up quark sector does not hold, and as a result, there are no new FCNC effects in the down quark sector. For the CKM elements participating in $D^0$-$\bar{D}^0$ mixing, the prediction is now (recall capital lettering is used to denote the heavy quark)

$$|V_{cS}^* V_{uS}|^2 = s_2^2 \, |V_{cs}^* V_{us}|^2 \simeq s_2^2 \lambda^2 \quad , \tag{50}$$

where $|V_{cs}^* V_{us}| \simeq \lambda \simeq 0.22$ and $s_2$ is the (small) mixing parameter describing the mixing between the light $s$ quark and the heavy $S$ quark. Thus, we rewrite $|x_{\mathrm{D}}^{(-1/3)}|$ in the modified form,

$$|x_{\mathrm{D}}^{(-1/3)}| \simeq \frac{G_F^2 M_W^2 f_D^2 M_D}{6\pi^2 \Gamma_D} B_D s_2^2 \lambda^2 r(m_c, M_W) |\overline{E}(x_S)| \quad . \tag{51}$$

*$D^0 \to \mu^+\mu^-$ Decay*: For $D^0 \to \mu^+\mu^-$, the effective Lagrangian is given in Eq. (2.2) of Ref. [31],

$$\mathcal{L}_{\mathrm{eff}} = \frac{G_F^2 M_W^2}{\pi^2} \bar{C}(x_S) \lambda s_2 \, \widetilde{Q}_1 \quad , \tag{52}$$

where

$$\bar{C}(x_S) \equiv \frac{x_S}{4} - \frac{3x_S}{4(x_S - 1)} - \frac{3}{4}\left(\frac{x_s}{x_S - 1}\right)^2 \ln x_S \quad . \tag{53}$$

In this model, the $D^0 \to \mu^+\mu^-$ branching fraction becomes

$$\mathcal{B}_{D^0 \to \mu^+\mu^-} = \frac{M_D \sqrt{1 - 4\frac{m_\mu^2}{M_D^2}} \; (G_F M_w)^4 \cdot \left(s_2 \lambda f_D m_\mu \bar{C}(x_S)\right)^2}{32\pi^5 \Gamma_D} \quad . \tag{54}$$

*Combining the Mixing and Decay Relations*: If we eliminate $s_2^2$ from the mixing and decay relations, we obtain

$$\mathcal{B}_{D^0\to\mu^+\mu^-} = \frac{6}{32\pi^3}\cdot\frac{x_{\rm D}\sqrt{1-4m_\mu^2/M_D^2}\,\left(m_\mu G_F M_W \bar{C}(x_S)\right)^2}{B_D r(m_c,M_W)|\bar{E}(x_S)|}$$
$$\simeq 1.0\times 10^{-9}\, x_{\rm D}\left(\frac{m_S}{500\ \text{GeV}}\right)^2 \le\ 1.0\times 10^{-11}\left(\frac{m_S}{500\ \text{GeV}}\right)^2 \quad . \tag{55}$$

### B. Minimal Supersymmetric Standard Model

We next consider the Minimal Supersymmetric Standard Model (MSSM) with unbroken R-parity. Conservation of R-parity implies that only pairs of sparticles can be produced or exchanged in loops. We will assume that neither squarks nor gluinos are decoupled (direct collider searches for squark and gluino pair production place the bound $m_{\tilde{q},g} \gtrsim 330$ GeV [16] in the MSSM with minimal gravity mediated Supersymmetry breaking), so the MSSM can in principle give a dominant contribution to the processes under consideration here.

We will not assume any particular SUSY breaking mechanism, and hence parameterize all possible soft SUSY-breaking terms. We work in the so-called super-CKM basis, where flavor violation is driven by non-diagonal squark mass insertions (see [14] for a discussion of this mechanism in $D^0-\overline{D}^0$ mixing). In this case, the squark-quark-gluino couplings are flavor conserving, while the squark propagators are expanded to include the non-diagonal mass terms. The $6\times 6$ mass matrix for the $Q=+2/3$ squarks can be divided into $3\times 3$ sub-matrices,

$$\widetilde{M}^2 = \begin{pmatrix} \widetilde{M}^2_{LL} & \widetilde{M}^2_{LR} \\ \widetilde{M}^{2\ T}_{LR} & \widetilde{M}^2_{RR} \end{pmatrix} \quad , \tag{56}$$

and the mass insertions can be parameterized in a model independent fashion as

$$\left(\delta_{ij}\right)_{MN} = \frac{\left(V_M \widetilde{M}^2 V_N^\dagger\right)_{ij}}{m_{\tilde{q}}^2} \quad . \tag{57}$$

Here, $i,j$ are flavor indices, $M,N$ refers to the helicity choices $LL$, $LR$, $RR$, and $m_{\tilde{q}}$ represents the average squark mass. The squark-gluino loops with mass insertions are by far the largest supersymmetric contribution to $D^0-\overline{D}^0$-mixing and can dominate the transition. The effective hamiltonian relevant for this contribution to $D^0-\overline{D}^0$-mixing is given by

$$\mathcal{H}^{mix}_{MSSM} = \frac{\alpha_s^2}{2m_{\tilde{q}}^2}\sum_{i=1}^{8} C_i(m_{\tilde{q}})Q_i \quad , \tag{58}$$

where all eight operators contribute in the MSSM. Evaluating the Wilson coefficients at the SUSY scale gives,

$$
\begin{aligned}
C_1(m_{\tilde{q}}^2) &= \frac{1}{18}\,(\delta_{12}^u)_{LL}^2\,[4xf_1(x)+11f_2(x)]\ , \\
C_2(m_{\tilde{q}}^2) &= \frac{1}{18}\,\{(\delta_{12}^u)_{LR}\,(\delta_{12}^u)_{RL}\ 15f_2(x)-(\delta_{12}^u)_{LL}\,(\delta_{12}^u)_{RR}\,[2xf_1(x)+10f_2(x)]\}\ , \\
C_3(m_{\tilde{q}}^2) &= \frac{1}{9}\,\{(\delta_{12}^u)_{LL}\,(\delta_{12}^u)_{RR}\,[42xf_1(x)-6f_2(x)]-(\delta_{12}^u)_{LR}\,(\delta_{12}^u)_{RL}\ 11f_2(x)\}\ , \\
C_4(m_{\tilde{q}}^2) &= \frac{1}{18}\,(\delta_{12}^u)_{RL}^2\,37xf_1(x)\ , \\
C_5(m_{\tilde{q}}^2) &= \frac{1}{24}\,(\delta_{12}^u)_{RL}^2\ xf_1(x)\ , \\
C_6(m_{\tilde{q}}^2) &= \frac{1}{18}\,(\delta_{12}^u)_{RR}^2\,[4xf_1(x)+11f_2(x)]\ , \\
C_7(m_{\tilde{q}}^2) &= \frac{1}{18}\,(\delta_{12}^u)_{LR}^2\,37xf_1(x)\ , \\
C_8(m_{\tilde{q}}^2) &= \frac{1}{24}\,(\delta_{12}^u)_{LR}^2\ xf_1(x)\ ,
\end{aligned}
\tag{59}
$$

where $x \equiv m_{\tilde{g}}^2/m_{\tilde{q}}^2$, with $m_{\tilde{g}}$ being the mass of the gluino. The equations above are symmetric under the interchange $L \leftrightarrow R$. These contributions to $x_D$ are found to be large [14], and the observation of $D$ mixing constrains the mass insertions to be at the percent level, or less, for Tev-scale sparticles.

We now examine the squark-gluino contribution to rare decays, which proceeds through $Z$ penguin diagrams for on-shell leptons. The relevant $c \to u\ell^+\ell^-$ Lagrangian is given, for example, in Ref. [20]. Electromagnetic current conservation forbids the contribution of the photonic penguin diagram for on-shell leptons in the final state. In addition, the vector leptonic operator $\bar{\ell}\gamma_\mu\ell$ also does not contribute for on-shell leptons as $p_D^\mu(\bar{\ell}\gamma_\mu\ell) = (p_{\ell^+}^\mu + p_{\ell^-}^\mu)(\bar{\ell}\gamma_\mu\ell) = 0$. The effective hamiltonian is then given by

$$
\mathcal{H}_{MSSM}^{rare} = -\frac{4G_F}{\sqrt{2}}\frac{e^2}{16\pi^2}\left[c_{10}\ (\bar{\ell}\gamma_\mu\gamma_5\ell)\ (\overline{u}_L\gamma^\mu c_L) + c_{10}'\ (\bar{\ell}\gamma_\mu\gamma_5\ell)\ (\overline{u}_R\gamma^\mu c_R)\right], \tag{60}
$$

where $c_{10}$ and $c_{10}'$ are given by [34]

$$
c_{10} = -\frac{1}{9}\frac{\alpha_s}{\alpha}(\delta_{22}^u)_{LR}(\delta_{12}^u)_{RL}P_{032}\,, \qquad c_{10}' = -\frac{1}{9}\frac{\alpha_s}{\alpha}(\delta_{22}^u)_{RL}(\delta_{12}^u)_{LR}P_{122}\,. \tag{61}
$$

$P_{032,122}$ are kinematic loop functions and are defined in the above reference. The double mass insertion is required to induce a helicity flip in the squark propagator. Due to this double mass insertion, this contribution to $D \to \ell^+\ell^-$ is completely negligible. We note that the chargino contribution to the $Z$ penguin for $D \to \ell^+\ell^-$ also contains a double mass

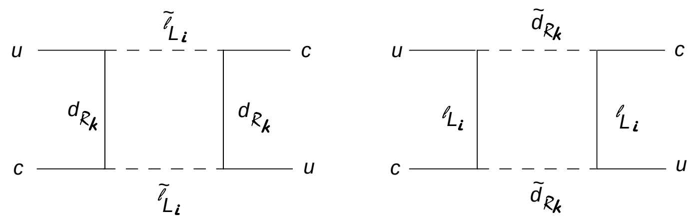


FIG. 3: Contributions to $D^0$-$\overline{D}^0$ mixing from the $\lambda'$ superpotential terms in supersymmetric models with R-parity violation.

insertion. The leading MSSM contribution to this rare decay is thus most likely mediated by a box diagram with squark-chargino-sneutrino exchange. This precludes a relation to $D^0 - \overline{D}^0$ mixing.

Lastly, we note that in contrast to the $B_s$ system [35], $D \to \ell^+\ell^-$ does not receive a sizable contribution from Higgs boson exchange with large $\tan\beta$. This is because in this case, the loop-induced term to the Yukawa couplings is proportional to $v_d$ (*i.e.,* the vev of the Higgs doublet that generates masses for the down-type quarks) which is the smaller of the two vevs and hence does not compensate for the small loop factor.

### C. R Parity Violating Supersymmetry

Finally, we consider Supersymmetry with R-Parity violation (RPV). We refer the reader to Refs. [14, 20] for discussions and earlier references of RPV-SUSY relevant to this paper. Suffice it to say that the lepton number violating RPV-SUSY interactions can be expressed as

$$W_{\lambda'} = \tilde{\lambda}'_{ijk} \left\{ V_{jl} \left[ \tilde{\nu}^i_L \bar{d}^k_R d^l_L + \tilde{d}^l_L \bar{d}^k_R \nu^i_L + (\tilde{d}^k_R)^* (\bar{\nu}^i_L)^c d^l_L \right] - \tilde{e}^i_L \bar{d}^k_R u^j_L - \tilde{u}^j_L \bar{d}^k_R u^j_L - (\tilde{d}^k_R)^* (\bar{e}^i_L)^c u^j_L \right\}, \tag{62}$$

in terms of the coupling parameters $\{\tilde{\lambda}'_{ijk}\}$. The generation indices denote the correspondences $i \Leftrightarrow$ leptons or sleptons, $j \Leftrightarrow$ up-type quarks and $k \Leftrightarrow$ down-type quarks or squarks.

$D^0$-$\bar{D}^0$ *Mixing*: As described at the high mass scale by the effective hamiltonian

$$\mathcal{H}_{R_p} = \frac{1}{128\pi^2} (\tilde{\lambda}'_{i2k} \tilde{\lambda}'_{i1k})^2 \left[ \frac{1}{m^2_{\tilde{\ell}_{L,i}}} + \frac{1}{m^2_{\tilde{d}_{R,k}}} \right] Q_1 \quad , \tag{63}$$

this implies constraints on the product of couplings $\tilde{\lambda}'_{i2k}\tilde{\lambda}'_{i1k}$. Here, we have assumed that only one set of the R-parity violating couplings $\tilde{\lambda}'_{i2k}\tilde{\lambda}'_{i1k}$ is large and dominant. This is equivalent to saying that, *e.g.*, both the sleptons and both the down-type quarks being exchanged in the first contribution to the box diagram of Fig. 3 are from the same generation. In general, this need not be the case and the coupling factor would then be the product $\tilde{\lambda}'_{i2k}\tilde{\lambda}'_{m1k}\tilde{\lambda}'_{m2n}\tilde{\lambda}'_{i1n}$, with, *e.g.*, the set of $\tilde{\ell}_{L,i}, d_{R,k}, \tilde{\ell}_{L,m}, d_{R,n}$ being exchanged. Computing the evolution to the charm-quark scale yields

$$\mathcal{H}_{R_p} = \frac{1}{2m^2_{\tilde{d}_{R,k}}} C_1(m_c) Q_1 \ , \tag{64}$$

with $C_1(m_c) = r(m_c, m_{\tilde{q}})C_1(m_{\tilde{q}})$. The mixing contribution from the R-parity violating $\tilde{\lambda}'$ terms then implies

$$(\tilde{\lambda}'_{i2k}\tilde{\lambda}'_{i1k})^2 = \frac{192\pi^2 \Gamma_D m^2_{\tilde{d}_{R,k}}}{(1+\epsilon) f_D^2 M_D B_D r(m_c, m_{\tilde{q}})} x_{\rm D} \quad , \tag{65}$$

where $\epsilon \equiv m^2_{\tilde{d}_{R,k}}/m^2_{\tilde{\ell}_{L,i}}$. For definiteness, we shall scale the results to the value $m_{\tilde{d}_{R,k}} =$ 300 GeV, so that

$$\tilde{\lambda}'_{i2k}\tilde{\lambda}'_{i1k} = 0.0053 \cdot \frac{m_{\tilde{d}_{R,k}}}{300 \text{ GeV}} \cdot \sqrt{\frac{2}{1+\epsilon}} \cdot \sqrt{\frac{x_{\rm D}}{0.01}} \quad . \tag{66}$$

$D^0 \to \mu^+\mu^-$ *Decay*: In RPV-SUSY, the underlying transition for $D^0 \to \mu^+\mu^-$ is $c + \bar{u} \to \mu^+ + \mu^-$ via tree-level d-squark exchange. The coupling constant dependence for $D^0 \to \ell^+\ell^-$ would therefore generally involve $\tilde{\lambda}'_{i2k}\tilde{\lambda}'_{i1k}$. For the specific mode $D^0 \to \mu^+\mu^-$, we take $i = 2$ to get $\tilde{\lambda}'_{22k}\tilde{\lambda}'_{21k}$. The effective hamiltonian of Eq. (74) in Ref. [20], but with $\ell \to \mu$, reads

$$\delta\mathcal{H}_{\rm eff} = -\frac{\tilde{\lambda}'_{22k}\tilde{\lambda}'_{21k}}{2m^2_{\tilde{d}^k_R}} \widetilde{Q}_1 \quad . \tag{67}$$

This leads to the branching fraction

$$\mathcal{B}^{R\!\!\!/_p}_{D^0\to\mu^+\mu^-} = \frac{f_D^2\, m_\mu^2\, M_D}{\Gamma_D} \left[1 - \frac{4m_\mu^2}{M_D^2}\right]^{1/2} \frac{\left(\tilde{\lambda}'_{22k}\tilde{\lambda}'_{21k}\right)^2}{128\pi\, m^4_{\tilde{d}_k}} \quad , \tag{68}$$

and so the constraint

$$\left(\tilde{\lambda}'_{22k}\tilde{\lambda}'_{21k}\right)^2 \le \mathcal{B}_{D^0\to\mu^+\mu^-} \frac{128\pi\, m^4_{\tilde{d}_k}}{f_D^2 m_\mu^2} \frac{\Gamma_{\rm D}}{M_{\rm D}} \left[1 - \frac{4m_\mu^2}{M_{\rm D}}\right]^{-1/2} \quad , \tag{69}$$

which reads numerically

$$\tilde{\lambda}'_{22k}\tilde{\lambda}'_{21k} \le 0.088 \left(\frac{m_{\tilde{d}_k}}{300 \text{ GeV}}\right)^2 \quad . \tag{70}$$

| Model | $\mathcal{B}_{D^0\to\mu^+\mu^-}$ |
|---|---|
| Experiment | $\leq 1.3\times 10^{-6}$ |
| Standard Model (SD) | $\sim 10^{-18}$ |
| Standard Model (LD) | $\sim$ several $\times 10^{-13}$ |
| $Q=+2/3$ Vectorlike Singlet | $4.3\times 10^{-11}$ |
| $Q=-1/3$ Vectorlike Singlet | $1\times 10^{-11}\ (m_S/500\ \text{GeV})^2$ |
| $Q=-1/3$ Fourth Family | $1\times 10^{-11}\ (m_S/500\ \text{GeV})^2$ |
| $Z'$ Standard Model (LD) | $2.4\times 10^{-12}/(M_{Z'}(\text{TeV}))^2$ |
| Family Symmetry | $0.7\times 10^{-18}$ (Case A) |
| RPV-SUSY | $4.8\times 10^{-9}\ (300\ \text{GeV}/m_{\tilde{d}_k})^2$ |

TABLE I: Predictions for $D^0 \to \mu^+\mu^-$ branching fraction for $x_D \sim 1\%$. Experimental upper bound is a compilation from [16].

*Combining the Mixing and Decay Relations*: Note the mixing constraint involves $\tilde{\lambda}'_{i2k}\tilde{\lambda}'_{i1k}$, whereas the decay constraint has $\tilde{\lambda}'_{22k}\tilde{\lambda}'_{21k}$. If the $i=2$ case dominates, then we arrive at the prediction (here we set $\epsilon = 1$)

$$\begin{aligned}
\mathcal{B}^{R\!\!\!/_p}_{D^0\to\mu^+\mu^-} &= \frac{3\pi m_\mu^2\left[1-4m_\mu^2/M_{\rm D}\right]^{1/2} x_D}{4m_{\tilde{d}_k}^2 B_D r(m_c, m_{\tilde{q}})} \\
&\simeq\ 4.8\times 10^{-7}\ x_{\rm D}\left(\frac{300\ \text{GeV}}{m_{\tilde{d}_k}}\right)^2 \ \leq\ 4.8\times 10^{-9}\left(\frac{300\ \text{GeV}}{m_{\tilde{d}_k}}\right)^2 \quad . \qquad (71)
\end{aligned}$$

## V. CONCLUSIONS

The search for New Physics will in general involve many experiments, including the measurement of rare decay branching fractions and observation of particle-antiparticle mixing. Such experiments are essentially competitors, each seeking to be the first to indirectly detect physics beyond the Standard Model. At any given point, which measurements are more sensitive to New Physics must be determined on a case by case basis. Our earlier work of Ref. [14] already pointed out that the observed $D^0$-$\bar{D}^0$ signal imposes severe limits for a large number of New Physics models. If the $D^0$-$\bar{D}^0$ mixing is dominated by one of those New Physics contributions, what does it imply for rare decays such as $D^0 \to \mu^+\mu^-$? Not only have we been able to answer this question in several specific scenarios, but we find a striking correlation in some of the models, wherein the branching fraction for the decay mode $D^0 \to \mu^+\mu^-$ is completely fixed in terms of the mixing parameter $x_{\rm D}$.

For convenience we have gathered our results in Table I. For all but one case (Family Symmetry), we find the NP branching fraction exceeds the SM branching fraction. All the NP branching fractions are, however, well below the current experimental bounds of Eq. (2). Anticipating future improvements in sensitivity, the first NP model to be constrained will be R-parity Violating supersymmetry. This will require lowering of the current bound by a factor of a few hundred.

**Acknowledgments**

The work of E.G. was supported in part by the U.S. National Science Foundation under Grant PHY–0555304, J.H. was supported by the U.S. Department of Energy under Contract DE-AC02-76SF00515, S.P. was supported by the U.S. Department of Energy under Contract DE-FG02-04ER41291 and A.A.P. was supported in part by the U.S. National Science Foundation under CAREER Award PHY–0547794, and by the U.S. Department of Energy under Contract DE-FG02-96ER41005.

## APPENDIX A: RG RUNNING AND MIXING MATRIX ELEMENTS

NP contributions are affected by RG-running. The relevant $8 \times 8$ anomalous dimension matrix, derived to NLO in Ref. [21], is applied to LO here (see also Ref. [14]) to yield the Wilson coefficients,

$$
\begin{aligned}
C_1(\mu) &= r(\mu, M)\ C_1(M) \\
C_2(\mu) &= r(\mu, M)^{1/2}\ C_2(M) \\
C_3(\mu) &= r(\mu, M)^{1/2}\ \frac{2C_2(M)}{3} + r(\mu, M)^{-4}\ \left[ C_3(M) - \frac{2C_2(M)}{3} \right] \\
C_4(\mu) &= r(\mu, M)^{(1+\sqrt{241})/6}\ \left[ \left( \frac{1}{2} - \frac{8}{\sqrt{241}} \right) C_4(M) - \frac{30C_5(M)}{\sqrt{241}} \right]
\end{aligned}
$$

$$
\begin{aligned}
&+r(\mu,M)^{(1-\sqrt{241})/6} \ \left[\left(\frac{1}{2}+\frac{8}{\sqrt{241}}\right)C_4(M)+\frac{30C_5(M)}{\sqrt{241}}\right] \\
C_5(\mu) = \ & r(\mu,M)^{(1+\sqrt{241})/6} \ \left[\left(\frac{1}{2}+\frac{8}{\sqrt{241}}\right)C_5(M)+\frac{C_4(M)}{8\sqrt{241}}\right] \\
&+r(\mu,M)^{(1-\sqrt{241})/6} \ \left[\left(\frac{1}{2}-\frac{8}{\sqrt{241}}\right)C_5(M)-\frac{C_4(M)}{8\sqrt{241}}\right]
\end{aligned}
\tag{A1}
$$

where (presuming that $M > m_t$),

$$
r(\mu,M) = \left(\frac{\alpha_s(M)}{\alpha_s(m_t)}\right)^{2/7}\left(\frac{\alpha_s(m_t)}{\alpha_s(m_b)}\right)^{6/23}\left(\frac{\alpha_s(m_b)}{\alpha_s(\mu)}\right)^{6/25} \quad . \tag{A2}
$$

Regarding the remaining Wilson coefficients, $C_6$ runs analogous to $C_1$ and $C_{7,8}$ run analogous to $C_{4,5}$. The presence of operator mixing in Eq. (A1) is a consequence of the nondiagonal structure of the anomalous dimension matrix.

We also need to evaluate the $D^0$-to-$\bar{D}^0$ matrix elements of the eight dimension-six basis operators. In general, this implies eight non-perturbative parameters that would have to be evaluated by means of QCD sum rules or on the lattice. We choose those parameters (denoted by $\{B_i\}$) as follows,

$$
\begin{aligned}
\langle Q_1\rangle &= \tfrac{2}{3}f_{\rm D}^2M_{\rm D}^2B_1 \ , & \langle Q_5\rangle &= f_{\rm D}^2M_{\rm D}^2B_5 \ , \\
\langle Q_2\rangle &= -\tfrac{5}{6}f_{\rm D}^2M_{\rm D}^2B_2 \ , & \langle Q_6\rangle &= \tfrac{2}{3}f_{\rm D}^2M_{\rm D}^2B_6 \ , \\
\langle Q_3\rangle &= \tfrac{7}{12}f_{\rm D}^2M_{\rm D}^2B_3 \ , & \langle Q_7\rangle &= -\tfrac{5}{12}f_{\rm D}^2M_{\rm D}^2B_7 \ , \\
\langle Q_4\rangle &= -\tfrac{5}{12}f_{\rm D}^2M_{\rm D}^2B_4 \ , & \langle Q_8\rangle &= f_{\rm D}^2M_{\rm D}^2B_8 \ \ ,
\end{aligned}
\tag{A3}
$$

where $\langle Q_i\rangle \equiv \langle \bar{D}^0|Q_i|D^0\rangle$, and $f_D$ represents the $D$ meson decay constant. By and large, the compensatory $B$-factors $\{B_i\}$ are unknown, except in vacuum saturation and in the heavy quark limit; there, one has $B_i \to 1$.

Since most of the matrix elements in Eq. (A3) are not known, we will need something more manageable in order to obtain numerical results. The usual approach to computing matrix elements is to employ the vacuum saturation approximation. However, because some of the $B$-parameters are known, we introduce a 'modified vacuum saturation' (MVS), where all matrix elements in Eq. (A3) are written in terms of (known) matrix elements of

$(V-A)\times(V-A)$ and $(S-P)\times(S+P)$ matrix elements $B_{\rm D}$ and $B_{\rm D}^{\rm (S)}$,

$$\begin{aligned}
\langle Q_1\rangle &= \frac{2}{3} f_{\rm D}^2 M_{\rm D}^2 B_D \ , & \langle Q_5\rangle &= \frac{3}{N_c} f_{\rm D}^2 M_{\rm D}^2 B_D\ \eta \ , \\
\langle Q_2\rangle &= f_{\rm D}^2 M_{\rm D}^2 B_D \left[-\frac{1}{2} - \frac{\eta}{N_c}\right] \ , & \langle Q_6\rangle &= \langle Q_1\rangle \ , \\
\langle Q_3\rangle &= f_{\rm D}^2 M_{\rm D}^2 B_D \left[\frac{1}{4N_c} + \frac{\eta}{2}\right] \ , & \langle Q_7\rangle &= \langle Q_4\rangle \ , \\
\langle Q_4\rangle &= -\frac{2N_c-1}{4N_c} f_{\rm D}^2 M_{\rm D}^2 B_D\ \eta \ , & \langle Q_8\rangle &= \langle Q_5\rangle \ ,
\end{aligned} \tag{A4}$$

where we take $N_c = 3$ as the number of colors and define

$$\eta \equiv \frac{B_{\rm D}^{\rm (S)}}{B_D} \cdot \frac{M_{\rm D}^2}{m_c^2} \ . \tag{A5}$$

In our numerical work, we take

1. $B_{\rm D} = 0.82$, which is the most recent result from the quenched lattice calculation.

2. For $\eta$, we use $B_{\rm D}^{\rm (S)} \simeq B_{\rm D}$ [36] so that $\eta \simeq M_{\rm D}^2/m_c^2 \simeq 2$.

3. Regarding the decay constant $f_D$, there is now good agreement [37] between determinations from QCD-lattice simulations $f_D^{\rm (latt.)} = 0.207(4)$ GeV and various experiments $f_D^{\rm (expt.)} = 0.206(9)$ GeV. For definiteness, we adopt the value $f_D = 0.207$ GeV.

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