SLAC-PUB-13550
Revised -- March 2009

Strong-interaction theories based on gauge/gravity duality[*]

Stanley J. Brodsky

SLAC National Accelerator Laboratory,
Stanford University, Stanford, California 94309, USA

Guy F. de Téramond

Universidad de Costa Rica, San José, Costa Rica

Abstract

Recent developments in the theory of strong interactions are discussed in the framework of the AdS/CFT duality between string theories of gravity in a higher dimension Anti-de Sitter space and conformal quantum field theories in physical space-time. This novel theoretical approach, combined with "light-front holography", leads to new insights into the quark and gluon structure of hadrons and a viable first approximation to quantum chromodynamics, the fundamental theory of the strong and nuclear interactions.

Invited contribution to the McGraw-Hill Yearbook of Science & Technology 2010

---

[*] This research was supported by the Department of Energy contract DE-AC02-76SF00515.

Strong-interaction theories based on gauge/gravity duality[*]

One of the triumphs of theoretical physics of the twentieth century was the development of Quantum Electrodynamics (QED), the fundamental theory of electrons and photons. QED not only describes all of electrodynamics, atomic physics, and chemistry with extraordinary precision, but also the basic properties of the electron itself. For example, the electron *g* factor measured from the ratio of its spin precession to its Larmor rotation in a magnetic field is correctly predicted by QED to 10 significant figures.

Quantum Chromodynamics. The corresponding problem in particle and nuclear physics is to be able to describe the structure and properties of hadrons, such as protons and neutrons, in terms of their fundamental constituents, quarks and gluons. High-energy experiments, such as the deep inelastic electron-proton scattering pioneered at SLAC, which revealed the quark structure of the proton, have shown that the basic elementary interactions of quarks and gluons are well described by a remarkable generalization of QED called Quantum Chromodynamics (QCD). In QCD quarks and gluons interact with each other via a new type of charge called “color”. However, unlike the photons of QED, gluons also interact with each other, and these “non-Abelian” gauge couplings lead to color confinement -i.e., the impossibility of free colored quarks and gluons.

The strong couplings and confinement of quarks and gluons into hadrons in QCD makes the calculation of hadronic properties such as hadron masses and their interactions a much more difficult problem than solving QED. The most successful theoretical approach thus far has been to employ very large numerical simulations on advanced computers using an approximate representation of QCD called lattice gauge theory.

AdS/CFT correspondence. The AdS/CFT correspondence introduced by Juan Maldacena (also called gauge-gravity duality) between string theories of gravity in Anti-de Sitter (AdS) space and conformal (scale-invariant) gauge field theories (CFT) in physical space-time has brought a completely new set of tools for studying the dynamics of strongly coupled quantum field theories such as QCD. In effect, the strong interactions of quarks and gluons are represented by a simpler semi-classical (without quantum effects such as particle creation and annihilation) gravity theory in higher dimensions. Although a perfect string theory dual of QCD is not yet known, the AdS/CFT correspondence has already provided many new and remarkable insights into QCD, including color confinement, quantitative predictions for the meson and baryon spectra, mass generation from chiral symmetry breaking, and the wavefunctions which describe the structure of hadrons at a fundamental level. AdS/CFT predicts the power-law fall-off of the rate for hard exclusive reactions (such as proton-proton elastic scattering and electron-proton scattering) as the momentum transfer increases, in striking agreement with many experiments. The AdS/CFT correspondence has also been applied to the

---

[*] This research was supported by the Department of Energy contract DE-AC02-76SF00515.

behavior of quark and gluon matter at extreme conditions, such as high temperatures and pressure, thus providing insight into phenomena now being experimentally studied in relativistic heavy ion collisions at the Brookhaven National Laboratory (BNL). For example, AdS/CFT predicts a very low ratio of the viscosity to entropy density for the quark-gluon plasma phenomena seen at BNL, in apparent agreement with experiment.

Geometry of AdS space. The constant radial distance r between any point on a sphere and its center is given by Euclid (300 BC) formula $r^2 = x^2 + y^2 + z^2$. In the special theory of relativity an invariant space-time distance interval σ also involves the time dimension as measured by the speed of light c: $\sigma^2 = x^2 + y^2 + z^2 - c^2 t^2$. Anti-de Sitter space has one extra mathematical space dimension labeled u. Unlike ordinary space, AdS space is intrinsically curved, consequently the distance between neighboring points ds also depends on a "warp" factor R/u, where R is the radius of curvature of AdS space. This is represented mathematically by a "metric" formula

$$(ds)^2 = \frac{R^2}{u^2}\left[(dx)^2 + (dy)^2 + (dz)^2 - c^2(dt)^2 - (du)^2\right]$$

In AdS space the size of an object as seen by an observer in four dimensions depends on its location inside the higher dimensional space (**Fig. 1**). Technically AdS is a space of negative curvature with a four-dimensional space-time boundary. Due to the warp factor R/u in the metric, the object shrinks near the boundary of AdS space at $u = 0$ represented by the outer sphere in Fig 1. The picture is reminiscent of a hologram in the two-dimensional surface of a sphere, depicting objects located in the three-dimensional interior volume of the sphere. Thus the name "holographic theories" used in gauge-gravity dual theories. The holographic description requires that the quantum theory defined in the four-dimensional boundary "surface" is equivalent to the gravity theory defined in the "volume" at the "surface" interior (**Fig. 1**).

Changes of scale. It is easy to see that the interval ds remains invariant if one simultaneously changes the length and time scale of ordinary space-time: $x \to \lambda x,\ y \to \lambda y,\ z \to \lambda z,\ t \to \lambda t$ by an arbitrary factor λ, with a corresponding change of length scale in the fifth dimension $u \to \lambda u$. Thus changes in the length scale in the physical four-dimensional space-time world are equivalent to a change of scale in the fifth dimension.

Towards a dual theory of QCD. The original application of the AdS/CFT correspondence by Maldacena connected a gravity theory in AdS space to a particular type of supersymmetric theory of colored quarks and gluons in physical space-time. In this example, one can compute physical observables in a strongly coupled gauge theory in terms of a classical gravity theory. However the supersymmetric theory studied by Maldacena is conformal; i.e., it has neither an inherent length dimension nor color confinement. Moreover, in a supersymmetric theory, every particle has a partner of the same mass, but with different intrinsic spin. Such a theory is thus far different than QCD, the confining theory of quarks and gluons which underlies the real world of hadrons and nuclei.

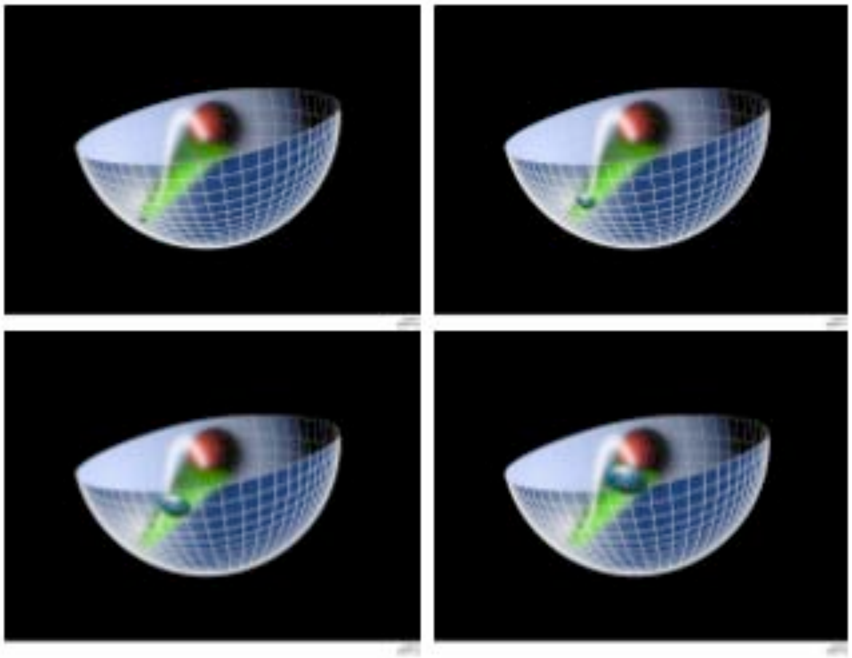

Fig. 1. Geometry of AdS space and the size of a proton. Different values of the fifth dimensional AdS coordinate u (the radius of the sphere) correspond to different scales at which the proton is examined. The outer sphere corresponds to events at very short distances in the usual four-dimensional space; i.e., the size of the proton (blob with three quarks inside), as seen by a four-dimensional observer, shrinks to zero near the $u = 0$ boundary of AdS space. Large distance confinement physics corresponds to the inner sphere at $u = u_0$. To help visualize the five dimensions in the figure we may think in two tangent spatial dimensions x and y, with origin of coordinates in the small proton at the outer sphere, and a fifth dimension coordinate u pointing inwards to the center of the sphere. The string wave amplitude Φ is illustrated by the white stream. The warp-shrinking factor is represented by the green stream.

QCD is not a conformal scale-invariant theory, as evidenced simply from the fact that its bound states, such as the proton, have a specific mass. Furthermore, the strength of the interaction between quarks and gluons falls off logarithmically at large momentum transfer Q a property known as “asymptotic freedom”. Although QCD depends on a mass scale Λ_{QCD}, which sets the spectrum and the confining dimensions, it nevertheless shows some manifestations of a conformal field theory. For example, recent experiments at small momentum transfer carried out at the Thomas Jefferson National Accelerator Facility indicate that the effective coupling strength $\alpha_s(Q)$ between quarks and gluons becomes independent of Q, a property called an “infrared fixed point”. This property which mimics conformal invariance in a limited domain, can be understood as a consequence of the fact that quarks and gluons have a maximum wavelength due to color confinement within hadrons. Consequently, there is a window of momentum transfer where the coupling is large but constant, a “conformal window”, where AdS/CFT can be applied. Thus, in effect, one can use a theory of gravity in AdS space to characterize a strongly interacting theory of quarks and gluons in physical space-time.

AdS/QCD. In fact, it is not difficult to modify the AdS/CFT theory to incorporate quark and gluon confinement. The simplest method, developed by Joseph Polchinski and Mathew Strassler, is to simply introduce a boundary at a point $u = u_0$ in the fifth dimension of AdS space, so that the quarks and gluons are restricted to propagate inside the domain $0 < u < u_0$. This is effectively a "bag" model in the AdS fifth dimension, similar to the well known MIT bag model where quarks are permanently confined inside a "bag" of given radius, but with the important feature that Lorentz invariance is preserved. Since the change in the length scale in physical space is equivalent to a change of scale in the fifth dimension, this also restricts the separation between quarks and gluons in physical space-time to a finite domain. Moreover, the value of u_0 introduces a unit of mass $\Lambda_{QCD} = \hbar c / u_0$ which characterizes the mass spectrum of the quark and gluon bound states such as the proton ($\hbar$ is Plank's constant). Inside the bag the quarks and gluons propagate freely, but their configuration depends on the different energy scales at which the proton is examined. For example, in a very high energy collision experiment the three quarks in the proton are close together near the boundary of AdS space at small u; conversely a large-size proton, with far-separated quarks as perceived in a low energy experiment, has been pulled by the gravitational field in AdS space up to the largest size allowed by confinement, as limited by the inner sphere in (**Fig. 1**).

In the "bottom-up" approach, known as AdS/QCD, an effective gravitational theory is constructed which encodes salient properties of the QCD dual theory, such as color confinement and chiral symmetry breaking, as shown by Joshua Erlich, Emanuel Katz, Dam T. Son and Mikhail A. Stephanov and by Leandro Da Rold and Alex Pomarol. In a recent collaboration of Andreas Karch with Katz, Son and Stephanov it has been observed that one can modify the metric of AdS space with a harmonic oscillator confining potential $U = \kappa^4 u^2$ to reproduce the observed linear behavior in the hadronic spectrum, the well known "Regge trajectories". In this "soft-wall" model, the value of κ breaks conformal invariance and sets the mass scale for the hadronic spectrum.

Form Factors. One can easily use the AdS/QCD models to predict the fall-off of hadronic form factors $F(Q)$, the quantity which measures the charge distribution of quarks within a hadron and whose square characterizes its probability to stay intact when hit by an electron. As first shown by Polchinski and Strassler, the AdS/CFT duality, modified to incorporate a mass scale, provides a derivation of constituent counting rules for the leading power-law fall-off of form factors and other high energy (hard) scattering rates. One finds that the form factor falls off as a simple power of the momentum transfer Q: $F(Q) \propto 1/Q^{2(N-1)}$ at large Q, where the value of the power is set by the number N of quark and gluons inside the hadron. Thus the form factor measures the degree of compositeness of the hadron. The constituent counting rule is consistent with experiment. For example, the proton form factor is measured to decrease as $1/Q^4$ at large momentum transfer, consistent with AdS/QCD predictions for a bound state of three quarks. The form factor of the deuteron (the simplest two-nucleon nucleus) falls off as $1/Q^{10}$ consistent with six fundamental constituents.

Dirac's amazing idea. One of the most useful theoretical tools for describing bound states is “light-front quantization”, inspired by an insightful paper by Paul A. M. Dirac in 1949. Dirac showed that there are extraordinary advantages for relativistic theories if one replaces ordinary time t with the time marked by the front of a light wave τ. For example, when one takes a flash photograph, one obtains an image at the specific values of τ, not t. Light-front quantization is the ideal framework to describe the structure of hadrons in terms of their quark and gluon degrees of freedom since wavefunctions defined at fixed “light-front” time τ are independent of the total momentum of the state. In contrast, wavefunctions defined at fixed “instant” time t depend in a very complicated way on the total momentum. Even worse, the computation of even the simplest processes, such as form factors, requires one to take into account processes in which currents arise spontaneously from the vacuum. In contrast, the simple structure of the light-front vacuum allows an unambiguous definition of the quark and gluon content of a hadron and their wavefunctions. The light-front wavefunctions of relativistic bound states thus can provide description of the structure and internal dynamics of hadronic states in terms of their fundamental constituents.

Light-front holography. In fact, as we have shown recently, there is a remarkable and direct connection between AdS space and the light-front formalism, called “light-front holography”. This procedure allows information of the wave amplitude $\varphi(u)$ which propagates in AdS space to be precisely mapped to the light-front wave functions of hadrons in physical space-time in terms of a specific light-front variable ζ which measures the separation of the quark and gluonic constituents within the hadron, independent of its total momentum. The correspondence between the variables u and ζ can be identified by comparing the formulae for hadron form factors derived in AdS space by Polchinski and Strassler, with the light-front formula for hadron form factors derived in ordinary space-time by Sidney Drell, Tung-Mow Yan and Geoffrey West.

A Schrödinger equation for hadrons. We can also use “light-front holography” to transform the bound state equations for the wavefunction in AdS space to a corresponding bound-state equation in physical space at fixed light-front time τ. The resulting light-front equation is similar to the celebrated Schrödinger radial wave equation at fixed t which describes the quantum-mechanical structure of atomic systems. Internal orbital angular momentum L and its effect on quark kinetic energy play an explicit role. Thus by using the AdS/CFT correspondence one obtains a relativistic wave equation applicable to hadron physics, where the light-front coordinate ζ has the role of the radial variable r of the nonrelativistic theory. The solutions of the wave equation determine the mass spectrum M and the wavefunctions $\phi(\zeta)$ of hadrons. The Light-front Schrödinger wave equation for a pion in holographic QCD is

$$\left[\frac{d^2}{d\zeta^2} - \frac{1-4L^2}{4\zeta^2} + U(\zeta)\right]\phi(\zeta) = M^2\phi(\zeta),$$

where the vast complexity of the QCD interactions among constituents is summed up in the addition of the effective potential $U(\zeta)$, which is then modeled to enforce confinement. For example, in the soft wall model the potential is $U = \kappa^4 \zeta^2 + 2\kappa^2 (J - 1)$ where J is the total angular momentum of the hadron. The corresponding wavefunctions of a pion describe the probability distribution of its constituents for the different orbital and radial states. The separation of the constituent quark and anti-quark in AdS space get larger as the orbital angular momentum increases. Radial excitations are also located deeper inside AdS space (**Fig. 2**).

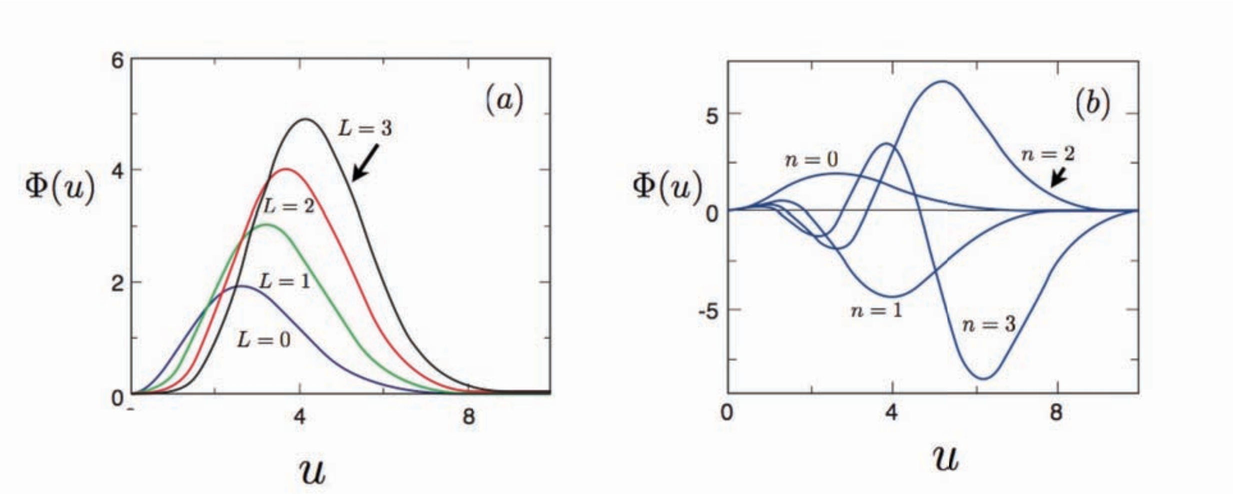

Fig. 2. Meson wavefunctions is AdS space in the soft-wall holographic model of confinement: (a) orbital modes (n=0) and (b) radial modes (L=0). Constituent quark and antiquark fly away from each other as the orbital and radial quantum number increases.

Hadronic spectrum. Thus AdS/CFT and light-front holography provide a quantum mechanical wave equation formalism for hadron physics. The soft-wall model, in particular, appears to provide a very useful first approximation to QCD. The solutions of the light-front equation determine the masses of the hadrons, given the total internal spin S, the orbital angular momenta L of the constituents, and the index n, the number of nodes of the wavefunction in ζ. For example, if the total quark spin S is zero, the meson bound state spectrum follows the quadratic form $M^2 = 4\kappa^2 (n + L)$. The pion with $n = 0$ and $L = 0$ is massless for zero quark mass, in agreement with general arguments based on chiral symmetry. If the total spin of the constituents is $S = 1$, the corresponding mass formula for the orbital and radial spectrum of the ρ and ω vector mesons is $M^2 = 4\kappa^2 (n + L + 1/2)$. The states are aligned along linear Regge trajectories (**Fig.3**).

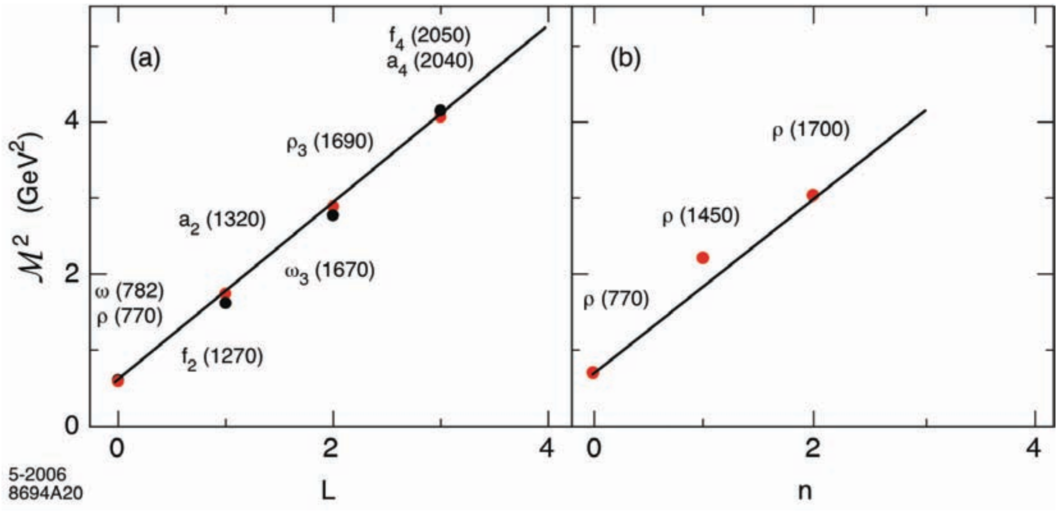

Fig. 3. **Vector meson states in the soft-wall holographic model: (a) orbital states (n=0) and (b) radial states (L= 0). The value of the mass scale is κ= 0.54 GeV. Experimental values are marked by dots.**

The light-front wavefunctions describe the quark and gluon composition of the hadrons in close analogy to the way positronium is described as an electron-positron bound state by the Schrödinger equation. The hadron spectra and their wavefunctions are obtained for general spin and orbital angular momentum. The predicted form factors for the pion and nucleons agree well with experiment. The nucleon has both S- and P wave components, allowing one to compute both Dirac (spin-conserving) and Pauli (spin-flip) form factors. Even the spectrum of glueballs - the bound states of two or three gluons - is predicted within this framework.

A significant theoretical advance in recent years has been the application of the AdS/CFT correspondence to study the dynamics of strongly coupled quantum field theories in terms of classical gravity in a higher dimensional space. Extensions of the AdS/CFT method that have been recently developed by theorists have led to new analytical insights into the confining dynamics of QCD, a concept difficult to realize using other methods. The AdS/QCD model, together with light-front holography, provides a semi-classical first approximation to strongly coupled QCD. The model does not account for particle creation and absorption, and thus it is expected to break down at very short distances where such relativistic quantum effects become important. However, the model can be systematically improved, for example, by using AdS/QCD solutions as the basis functions to compute higher order corrections to the full theory at fixed light-front time.

For background information *see* HOLOGRAPHY, ADS/CFT CORRESPONDENCE; QUARK-GLUON PLASMA; QUANTUM MECHANICS, QUANTUM FIELD THEORY; ELEMENTARY PARTICLES, NUCLEI, QUARKS, QCD; GAUGE INVARIANCE; SPECIAL RELATIVITY; GENERAL RELATIVITY; SUPERSYMMETRY, STRING THEORY in the McGraw-Hill Encyclopedia of Science & Technology.

Stanley J. Brodsky; Guy F. de Téramond

Bibliography. J. Maldacena, The illusion of gravity, *Sci. Amer*., 293(5):56-63, 2005; N. Evans, The gravity of hadrons, *Phys. World*, 18:26-27, 2005; J. Polchinski and M. J. Strassler, Hard scattering and gauge/string duality, *Phys. Rev. Lett*., 88:031601 (2002); G. F. de Téramond and S. J. Brodsky, The hadronic spectrum of a holographic dual of QCD, *Phys. Rev. Lett.*, 94:201601 (2005); J. Erlich, E. Katz, D. T. Son and M. A. Stephanov, QCD and a holographic model of hadrons, *Phys. Rev. Lett*., 95:261602 (2005); L. Da Rold and A. Pomarol, Chiral symmetry breaking from five dimensional spaces, *Nucl. Phys. B*, 721:79 (2005); A. Karch, E. Katz, D. T. Son and M. A. Stephanov, Linear confinement and AdS/QCD, *Phys. Rev. D*, 74:015005 (2006); S. J. Brodsky and G. R. Farrar, Scaling laws at large transverse momentum, *Phys. Rev. Lett*., 31: 1153 (1973); S.D. Drell and Tung-Mow Yan, Connection of elastic electromagnetic nucleon form-factors at large Q^2 and deep inelastic structure functions near threshold, *Phys. Rev. Lett*., 24:181-185, (1970); Geoffrey B. West, Phenomenological model for the electromagnetic structure of the proton, *Phys. Rev. Lett*., 24:1206-1209, (1970); S. J. Brodsky and G. F. de Téramond, Hadronic spectra and light-front wavefunctions in holographic QCD, *Phys. Rev. Lett*., 96:201601 (2006); G. F. de Téramond and S. J. Brodsky, Light-front holography: A first approximation to QCD, *Phys. Rev. Lett*., 102:081601 (2009).