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Real next-to-next-to-leading-order QCD corrections to J/ψ and Υ hadroproduction in association with a photon

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Abstract

We update the study of the QCD corrections to direct J/ψ and Υ hadroproduction in association with a photon in the QCD-based approach of the Colour-Singlet (CS) Model. After comparison with the recent full next-to-leading-order (NLO) computation for this process, we provide an independent confirmation to the inclusive case that NLO QCD corrections to quarkonium-production processes whose LO exhibits a non-leading P_T behaviour can be reliably computed at mid and large P_T by considering only the real emission contributions accompanied with a kinematical cut. In turn, we evaluate the leading part of the $\alpha_S^4\alpha$ contributions, namely those coming from $(J/\psi, \Upsilon) + \gamma$ associated with two light partons. We find that they are dominant at mid and large P_T. This confirms our expectations from the leading P_T scaling of the new topologies appearing at NNLO. We obtain that the yield from the CS becomes one order of magnitude larger than the upper value of the potential Colour-Octet yield. The polarisation of the $^3S_1$ quarkonia produced in association with a photon is confirmed to be longitudinal at mid and large P_T .

Key words: Quarkonium production, QCD corrections
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1. Introduction

For a long time, the many difficulties to correctly predict quarkonium-production rates at hadron colliders have been attributed to non-perturbative effects [1, 2, 3]. It appears now that quantum-number conservation (J, P, C and colour) has been responsible for this longstanding confusion, by preventing leading P_T scaling at LO and NLO. An additional source of confusion was brought in by two unexpected features of fragmentation approximations: heavy-quark fragmentation only dominates at very high P_T and gluon fragmentation rates are below t-channel gluon exchange ones, in contrast with the standard perturbative QCD (pQCD) calculations for jet or single hadron production, for instance. Yet, when leading kinematical contributions are correctly accounted for, the observables related to quarkonium production seem to be explained by pQCD.

This became clear thanks to the several recent computations of QCD corrections to quarkonium hadroproduction processes. The Next-to-Leading Order (NLO) (α_S^4) corrections to the inclusive yield of J/ψ and Υ were computed [4, 5] in the QCD-based approach of the Colour-Singlet (CS) Model [2] [6]. Its polarisation was in turn computed [8] at NLO. These computations were recently complemented [9] by the addition of to the real Next-to-Next-to-Leading Order (NNLO) corrections –thereafter referred to as NNLO*–. It was then shown that there is no need to incorporate Colour-Octet transition (higher-v corrections of NRQCD) to describe the hadroproduction of Υ at the Tevatron [10, 11, 12, 13]. In the case of the J/ψ and ψ' [14, 15], the CS contributions are significantly enhanced and brought very near the experimental data of CDF although the large P_T direct yield seems not to be fully accounted for in the case of the ψ' for instance. As regards the Colour-Octet (CO) channels for J/ψ production, their NLO QCD corrections were recently computed in [16]. It was seen that they minimally affect both the P_T dependence and the normalisation of the partonic matrix elements, thus also the values of the CO Long-Distance Matrix Elements (LDME) fit to the data. Similarly, the polarisation prediction are not modified and remains in disagreement with the latest CDF measurements [17]. For recent reviews, the reader is guided to [1, 2, 3] along with some perspectives for the LHC [18] and a recent discussion on those aforementioned QCD correction computations [15].

Beside the studies of inclusive production, efforts are being made to obtain improved theoretical predictions for complementary observables to the inclusive yield, such as hadroproduction of J/ψ and Υ (thereafter commonly named $\mathcal{Q}$) in association with a photon [19, 20, 21, 22, 23, 24, 25]. Recently, the NLO corrections to the hadroproduction of $\mathcal{Q} + \gamma$ in the CS channel were computed by Li and Wang in [26]. As in the inclusive case, NLO corrections are significant in the large P_T region since new topologies appear with slower P_T falloff (see Fig. 1 (c)), in comparison to LO topologies Fig. 1 (a) and other NLO topologies as the loop

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[2] The CSM can be also regarded as the leading order contributions in the heavy-quark-velocity (v) expansion of the effective theory, Non-Relativistic Quantum Chromodynamics (NRQCD) [7].

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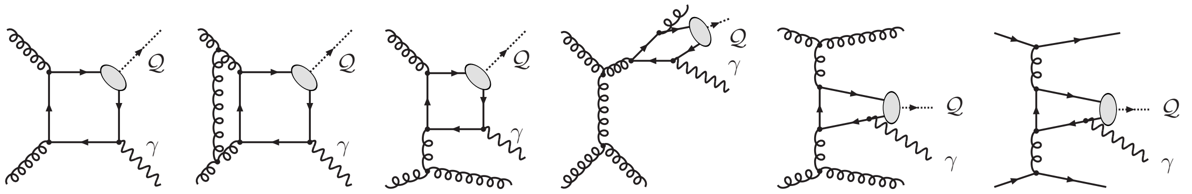


(a) (b) (c) (d) (e) (f)

Figure 1: Representative diagrams contributing to the hadroproduction of a J/ψ in association with a photon at orders $\alpha_S^2\alpha$ (a), $\alpha_S^3\alpha$ (b,c), $\alpha_S^4\alpha$ (d,e,f). See discussions in the text.

corrections Fig. 1 (b).

In [9], it was shown that some α_S^5 contributions to the inclusive yield coming from three-jet configurations, i.e. $\mathcal{Q} + jjj$, such as those arising from gluon-fragmentation and "high-energy enhanced" (or t-channel gluon exchange) channels are important and clearly dominate over the other contributions at mid and large P_T. In the case which interests us here, $\mathcal{Q} + \gamma$, those two channels appear at order $\alpha_S^4\alpha$ – see for instance Fig. 1 (d) and (e,f)– and are expected to dominate over the $\mathcal{Q} + \gamma$ yield at mid and large P_T.

In this work, we therefore apply the same procedure as in [9] to evaluate their contributions. First, we will check that the NLO computation of Li and Wang [26] can indeed be reproduced by an evaluation of the real $\alpha_S^3\alpha$ contributions ($\mathcal{Q} + \gamma$ + one light partons) complemented by a cut-off on low invariant masses for any pairs of external light partons in the process. The procedure is explained in the next section. We will also check that the sensitivity of our computation on this cut-off dies away when P_T grows and that the theoretical uncertainty attached to its choice is typically smaller than the ones attached to the choices of the renormalisation scale μ_r for α_S and the heavy-quark mass $m_{\mathcal{Q}}$.

Having performed those checks, we will apply the same procedure for the evaluation of the contribution for $\mathcal{Q} + \gamma$ + two light partons –namely the real NNLO contributions to $\mathcal{Q} + \gamma$ production– arguing that they provide with a first reliable estimate of the complete NNLO contributions to $\mathcal{Q} + \gamma$ production at large enough P_T. As regards the polarisation, the quarkonia directly produced in association with a photon via those channels are mainly longitudinally polarised, as in the inclusive case.

2. Cross section at NLO

2.1. Inclusive case

As we argued in [9], the NLO contributions to the inclusive yield can be approximated at large enough P_T in a relatively simple and reliable manner by computing the α_S^4 contributions consisting in the production of a $\mathcal{Q}$ with 2 light partons (denoted j thereafter) on which we apply a cut-off on low invariant masses for any light parton pairs in the process. Computations of such cross sections can be done reliably using the automated generator of matrix elements MadOnia [27].

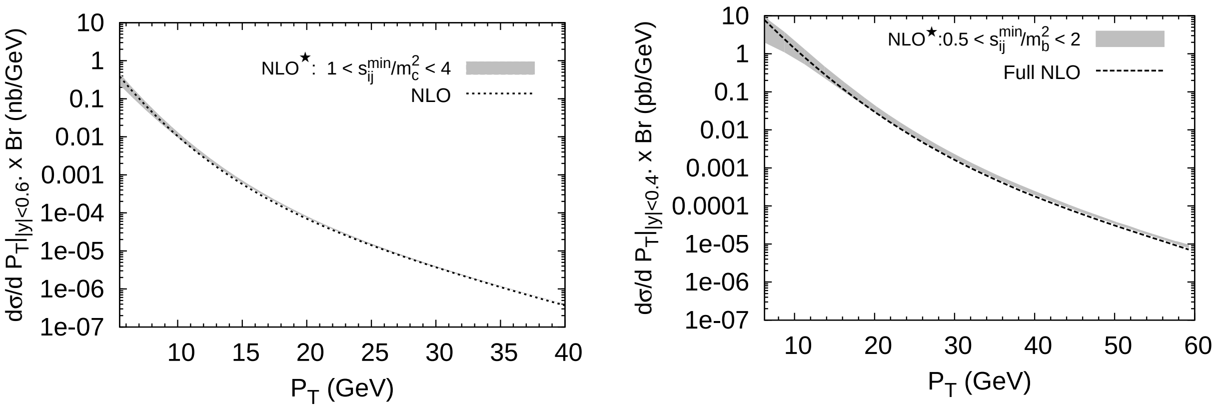
The underlying idea supporting this was twofold:

- First, at large enough P_T, topologies with the leading P_T behaviour will dominate and those are wholly included in this subset of α_S^4 contributions (the production of a $\mathcal{Q}$ with 2 light partons);
- Second, this subset accounts for a physical process at LO. Its contribution is therefore finite except for soft and collinear divergences. The purpose of the cut-off is to avoid such divergences by imposing a lower bound on the invariant-mass of any light partons (s_{ij}). For the new channels (with leading P_T scaling) opening up at α_S^4, the dependence on this cut gets smaller for large P_T since no collinear or soft divergences can appear there. For other channels, whose LO contribution is at α_S^3, the cut would produce logarithms of $s_{ij}/s_{ij}^{\min}$. Those can be large. Nevertheless, they can be factorised over their corresponding LO contribution, which scales at most as P_T^{-8}. The sensitivity on $s_{ij}^{\min}$ is thus expected to vanish at large P_T.

This insensitivity to the cut and the good agreement between the NLO* ($\mathcal{Q} + jj$ with a s_{ij} cut) and the full NLO result is recalled in Fig. 2 for the case of the inclusive J/ψ and $\Upsilon(1S)$ production. The gray band illustrates the sensitivity to the invariant-mass cut $s_{ij}^{\min}$ between any pairs of light partons when it is varied from m_c^2 to $4m_c^2$ and $0.5m_b^2$ to $2m_b^2$. In both NLO and NLO* computations, the value of all parameters were set to the same values. For the J/ψ, we have $m_c = 1.5$ GeV, $|R(0)|^2 = 0.810$ GeV^3, $\mu_f = \mu_r = \mu_0 = \sqrt{4m_c^2 + P_T^2}$ and $\text{Br}(J/\psi \to \mu^+\mu^-) = 0.0588$ and, for the $\Upsilon(1S)$, $m_b = 4.75$ GeV, $|R(0)|^2 = 6.48$ GeV^3, $\mu_f = \mu_r = \mu_0 = \sqrt{4m_b^2 + P_T^2}$ and $\text{Br}(J/\psi \to \mu^+\mu^-) = 0.0218$. The gluon distribution set used was CTEQ6_M [28]. The yield becomes insensitive to the value of $s_{ij}^{\min}$ as P_T increases, and it reproduces very accurately the differential cross section at NLO accuracy, both for the J/ψ and $\Upsilon(1S)$ case.

2.2. Production in association with a photon

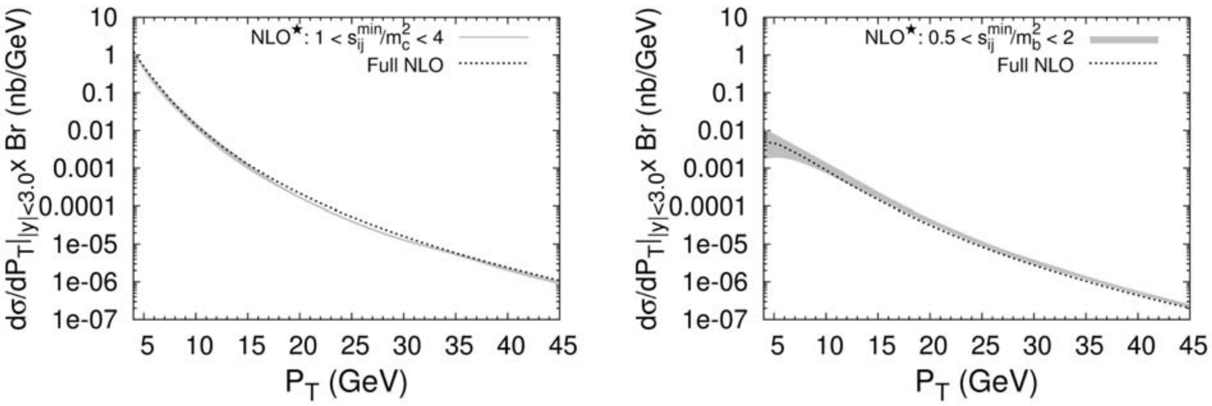
In the more exclusive case $\mathcal{Q} + \gamma$, similar topologies are present with the same P_T scaling and we also expect to reproduce accurately the yield at NLO accuracy ($\alpha_S^3\alpha$) computed in [26] by computing the yield from the production



(a) $J/\psi + X$

(b) $\Upsilon + X$

Figure 2: (a) full computation at NLO for $J/\psi + X$ (dashed line) [4] vs. NLO* (J/ψ + 2 light partons with a cut on s_{ij}) (gray band) at $\sqrt{s}$ = 1.96 TeV; (b) full computation at NLO for $\Upsilon(1S) + X$ (dashed line) [4] vs. NLO* ($\Upsilon(1S)$ + 2 light partons with a cut on s_{ij}) (gray band) at $\sqrt{s}$ = 1.96 TeV. See text for details.



(a) $J/\psi + \gamma + X$

(b) $\Upsilon + \gamma + X$

Figure 3: full computation at NLO for $J/\psi + \gamma + X$ (dashed line) [26] vs. $J/\psi + X$ + 1 light parton with a cut on s_{ij} (gray band) at $\sqrt{s}$ = 14 TeV; (b) full computation at NLO for $\Upsilon(1S) + \gamma + X$ (dashed line) [26] vs. $\Upsilon(1S) + \gamma$ + 1 light parton with a cut on s_{ij} (gray band) at $\sqrt{s}$ = 14 TeV. The absolute value of the rapidity of both the Q and the γ is limited to 3.

of $Q + \gamma$ with one light parton with the invariant-mass cut $s_{ij}^{\rm min}$ between any pairs of light partons, also referred to as NLO*.

This is indeed the case. For instance, the differential cross section for $Q+\gamma$ at NLO accuracy from Li and Wang [26] is displayed in Fig. 3 and is very well reproduced by the NLO* computed for different values of $s_{ij}^{\rm min}$. When P_T grows, the latter becomes completely insensitive to the value chosen for $s_{ij}^{\rm min}$. The same parameter values were used for Fig. 3 as for Fig. 2 and α was set to 1/137.

This result is a clear and completely independent confirmation of the validity of the reasoning initially given in [9] that NLO QCD corrections to quarkonium-production processes whose LO shows a non-leading P_T behaviour can be reliably computed at mid and large P_T by considering only the real emission contributions accompanied with a kinematical cut. In turn, this reinforces our confidence that the impact of NNLO contributions can evaluated likewise by computing the NNLO* contributions, as done in the following section.

This also gives us confidence that much better Monte Carlo simulations of inclusive production at mid and large P_T could be achieved using NLO* and NNLO* partonic matrix elements, which possess the advantage of being interfaced [29] easily with event-generators such as PYTHIA [30]. In any case, they would give results much more reliable than simulations based on matrix elements for CS channels at LO only.

3. Cross section and Polarisation at NNLO*

3.1. Cross section

Among the contributions appearing at $\alpha_S^4\alpha$, we find the topologies of Fig. 1 (d) (gluon fragmentation) and Fig. 1 (e,f) ("high-energy enhanced" or t-channel gluon exchange), those exhibit the last kinematical enhancements appearing in higher-order QCD corrections. In other words, those provide us with new mechanisms to produce a high-P_T $\mathcal{Q}$ with a γ with a lower kinematic suppression, still via CS transitions. They are therefore expected to dominate the differential cross section at NNLO accuracy in the region of large transverse momentum.

Those are also entirely contained in the contributions to $pp \to \mathcal{Q} + \gamma + jj$, namely the real $\alpha_S^4\alpha$ corrections, and we can follow the procedure validated in the previous section, by "simply" adding one light partons in the final state.

The computation of $pp \to \mathcal{Q}+\gamma+jj$ at tree level is in principle systematic, but technically quite challenging: a dozen parton-level subprocesses contribute, most involving a few hundred Feynman diagrams. As done in [9], we follow the approach described in Ref. [27], which allows the automatic generation of both the subprocesses and the corresponding scattering amplitudes.

The differential cross-sections for $J/\psi + \gamma$ and $\Upsilon(1S) + \gamma$ are shown in Fig. 4. The gray band (referred to as NLO*) corresponds to the sum of the LO and the real $\alpha_S^3\alpha$ contributions. The red (or dark) band (referred to as NNLO*) corresponds to the sum of the LO, the real $\alpha_S^3\alpha$ and the real $\alpha_S^4\alpha$ contributions. The $\alpha_S^4\alpha$ contributions in both case dominate over the yield at large P_T. The uncertainty bands are obtained from the combined variations $0.5\mu_0 \le \mu_r \le 2\mu_0$ with for the J/ψ, $m_c = 1.5 \pm 0.1$ GeV and $1 \le s_{ij}^{\min}/(1.5\text{ GeV})^2 \le 2$ and, for the $\Upsilon(1S)$, $m_b = 4.75 \pm 0.25$ GeV and $0.5 \le s_{ij}^{\min}/(4.5\text{ GeV})^2 \le 2$.

At the leading order in the heavy-quark velocity (v), the results for the radially excited states $\psi(2S)$, $\Upsilon(2S)$ and $\Upsilon(3S)$ are readily obtained by changing $|R_\mathcal{Q}(0)|^2$ and the branching ratio into dileptons.

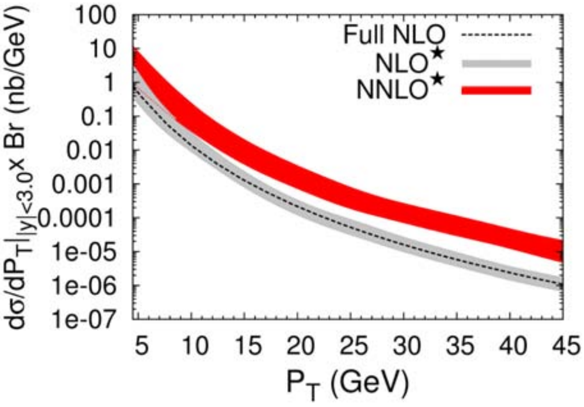
3.2. Polarisation

As regards the polarisation parameter λ, it is computed by analysing the angular distribution (θ) between the ℓ^+ direction in the quarkonium rest frame and the quarkonium direction in the laboratory frame. The normalised angular distribution $I(\cos\theta)$ then reads

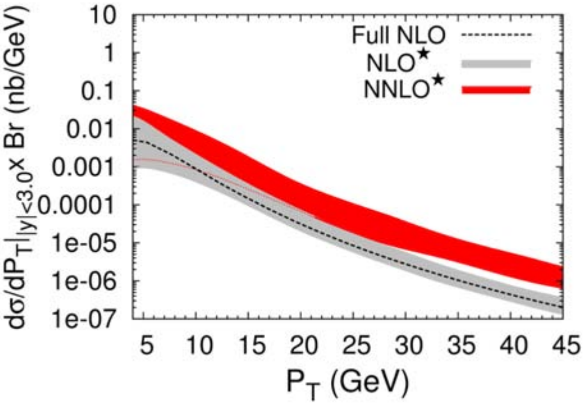
$$I(\cos\theta) = \frac{3}{2(\lambda+3)}(1 + \lambda\cos^2\theta)\,, \qquad (1)$$

from which we can extract λ bin by bin in P_T.

Our results for the J/ψ are shown in Fig. 5 along with the curves for the NLO*. Our predictions for the polarisation parameter λ for the NLO* are in qualitative agreement with those of [26], i.e. the J/ψ's produced in association with a photon are dominantly longitudinal. We did not go further in the comparison since, for both numerical computations, 5% precision in λ could only be reached at a high cost of computing time and since the NNLO* is anyhow larger. As regards the latter, it confirms the trends of the NLO* (and thus NLO) results. This is not a surprise knowing the NNLO* results for the inclusive yield, differing essentially[3] in the replacement of the photon by a gluon. This replacement is indeed not expected to change results concerning the polarisations of the particles produced.



(a) $J/\psi + \gamma$



(b) $\Upsilon(1S) + \gamma$

Figure 4: Results for (a) $J/\psi+\gamma$ and (b) $\Upsilon(1S)+\gamma$ from full NLO, NLO* and NNLO* contributions at $\sqrt{s}$ = 14 TeV. The theoretical-error bands for NLO* and NNLO* come from combining the uncertainties resulting from the choice of μ_f, μ_r, m_q and $s_{ij}^{\min}$.The absolute value of the rapidity of both the $\mathcal{Q}$ and the γ is limited to 3.

4. Discussion of the results at NNLO*

First, let us stress that although the uncertainty associated with the choice of the cut $s_{ij}^{\min}$ is somewhat larger than at NLO*, it is nevertheless smaller than the one attached to the mass and the renormalisation scale. The latter dependence is expected: on the one hand, we miss the virtual part at low

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[3] at least as far as the dominant channels are concerned.

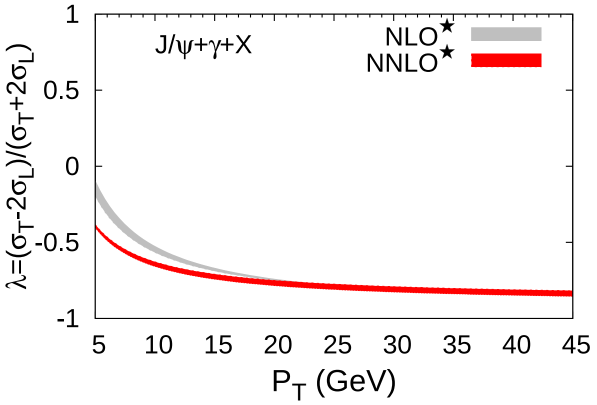

Figure 5: Polarisation of the J/ψ produced in association with a photon at $\sqrt{s} = 14$ TeV up to the order $\alpha_S^4\alpha$ (NNLO*). Most of the uncertainties on λ from the choice of m_c and μ_r cancel. The uncertainty band of the NLO* and NNLO* result comes from the variation of the cutoff $s_{ij}^{\rm min}$.

P_T where it is sizable and where it is expected to reduce the renormalisation-scale dependence; on the other hand, the contributions dominating at large P_T are directly sensitive to the fourth power of α_S. Yet, the dependence is smaller than in the inclusive case [9] where five powers of α_S are involved.

Second, we find that the subprocess $gg \to \mathcal{Q} + \gamma + gg$ dominates, providing with about 70% of the whole yield in the J/ψ case. In addition, we have checked that this fraction is slightly increasing with P_T and only weakly dependent on the value of the invariant mass cut-off of light partons $s_{ij}^{\rm min}$, removing the collinear and infrared divergences of $gg \to \mathcal{Q} + \gamma + gg$. This indicates that those divergences are not –after being cut– artificially responsible for a large part of the NNLO* yield. Another complementary observation is that the size of the yield from $gg \to \mathcal{Q} + \gamma + gg$ does not vary much when $s_{ij}^{\rm min}$ is changed from 2.25 to 9 GeV^2 in the J/ψ case for instance.

Third, it is likely that the largest part of this contribution is not from gluon fragmentation topologies, but rather from t-channel gluon exchange ones[4], keeping in mind that such a decomposition in terms of the corresponding Feynman graphs is not gauge invariant. There are a couple of indications supporting this:

- In the very similar inclusive case, the evaluation of the contribution of the corresponding process $gg \to \mathcal{Q} + ggg$ using the fragmentation approximation seems to significantly underestimate its actual value [9].
- Such fragmentation contributions are expected to provide transverse quarkonia up to corrections due to the off-shellness of the fragmenting gluons, whereas $\mathcal{Q}$ produced via $gg \to \mathcal{Q} + \gamma + gg$ appear to be longitudinally polarised.
- Processes such as $qq' \to \mathcal{Q} + \gamma + qq'$, which proceed uniquely via t-channel gluon exchange, have the same P_T dependence as the process $gg \to \mathcal{Q} + \gamma + gg$ and the difference in normalisation is naturally accounted for by colour factors and the smaller value of the quark PDF compared to the gluon one at low x. Another similarity with $gg \to \mathcal{Q} + \gamma + gg$ is that the polarisation of the yield from $qq' \to \mathcal{Q} + \gamma + qq'$ is strongly longitudinal, for P_T larger than 5 GeV, as observed for the full NNLO* yield dominated by $gg \to \mathcal{Q} + \gamma + gg$.

A careful kinematical analysis of the yield from $gg \to \mathcal{Q} + \gamma + gg$ would certainly be helpful. Yet, it would be highly computer-time demanding, especially to obtain a distribution[5] of the relative momentum between the J/ψ and its closest gluon precise enough to unequivocally attribute the most part of the yield to the t-channel gluon exchange channels rather than to the fragmentation ones. Indeed, it concerns the most complicated process with a couple of hundreds of diagrams. This work is left for a future analysis and would certainly be expediently done along with the one for the inclusive case.

5. Phenomenology

5.1. Photon detectability

In order to detect the photon, we evidently have to impose that it possesses a finite transverse momentum for it not to go in the beam-pipe. At LO, this is trivially required for a $\mathcal{Q}$ with a finite $P_T^{\mathcal{Q}}$ since P_T^{γ} is balancing $P_T^{\mathcal{Q}}$. As discussed by Li and Wang [26], this is not automatically the case at NLO and a minimum P_T^{γ} cut has to be applied and it affects the yield vs $P_T^{\mathcal{Q}}$ up to roughly three times the value of this cut. Typically a cut $P_{T,min}^{\gamma} = 2$ GeV will affect the yield up to $P_T^{\mathcal{Q}} = 6 - 8$ GeV.

In our case such an analysis is less relevant for two reasons: first, our computation cannot be reliably extended to low P_T and, second, the main contributions –the t-channel gluon exchange and gluon-fragmentation channels– create a γ with a similar momentum as that of the $\mathcal{Q}$ (see the graphs of Fig. 1 (d) & (e)). Overall, such a kinematical cut insuring its detectability does not affect the NNLO* yield for P_T larger than 10 GeV.

Beside the problem of the photon detectability, we have to make sure that other processes will not contribute to the yield of $\mathcal{Q} + \gamma$. In the J/ψ case, we expect the non-prompt background to be properly subtracted (by vertex-displacement method for instance). However, in this case, the ratio "non-prompt over prompt" at very large P_T is expected to be lower than in the inclusive case since the photon emission is required for both processes, whereas in the

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[4] In the inclusive case, those have been previously discussed [31] in the k_t factorisation formalism. See [32] for a recent application to Υ hadroproduction.

[5] The evolution of the distribution for different P_T would be even better.

inclusive case the B feed-down can proceed at a lower order in α_S. Similarly, the χ_Q feed-down will not be significant (except in the region where the invariant mass of the pair $Q+\gamma$ is close to the mass of the χ_Q) since suppressed by v^2 and by the branching.

5.2. *Colour-octet yield*

Now, let us discuss the possible colour octet contributions, ignoring here the modifications –most probably reductions– of the Colour-Octet matrix elements fit to the Tevatron data induced by the NLO corrections for the CO [16] and by the NNLO* for the NLO CS [9]. First, as discussed in [26], a quick comparison with the results obtained in [25] shows that the CS and the CO yields are of the same order at the LHC. For instance, the sum of the CO yield (times the branching in dileptons) at $P_T = 10$ GeV is one tenth of the CS at NLO (about 2 pb compared to 20 pb, see Fig. 4 (a)) but, at $P_T = 20$ GeV, is already one fourth of the CS (about 50 fb compared to 200 fb). At larger P_T, the CO really competes with the CS at NLO. Li and Wang therefore suggested that the polarisation measurement (transverse for the CO and longitudinal for the CS) would discriminate between them.

In the light of the NNLO* results, this is no longer the case. Indeed, when the real $\alpha_S^4\alpha$ contributions are taken into account, the CS contributions are about one order of magnitude larger than the most conservative (upper) evaluations of the CO. This is expected since the photon has to be emitted by quarks. If the hard scattering part is $gg \to gg$, the photon can only be emitted by the heavy quarks with topologies similar to CS channels. At $\alpha_S^2\alpha$, the gluon-fusion CO channels do not scale like P_T^{-4} (only one initiated by quark, and thus sub-dominant at low x, $q\bar{q} \to \gamma + (Q\bar{Q})^{[8]}$, with a quark in the t-channel shows a P_T^{-4} scaling). One has to go to $\alpha_S^3\alpha$ to have the first gluon fragmentation channels initiated by gluon fusion. In this case, the gluon fragments into a CO C=+1 and a photon[6].

Finally, while in the low P_T region contributions from s-channel cut to J/ψ production could appear as in the inclusive case [33] (the final state gluon of the $gg \to J/\psi g$ being simply replaced by a photon), in the large P_T region, the color-transfer-enhancement mechanism discussed in [34] is not expected to matter for the present process. Indeed, we have checked that the contribution to $J/\psi + \gamma$ from the process $gg \to J/\psi + c\bar{c} + \gamma$ is sub-dominant and found that it is one order of magnitude smaller than the process from $gg \to J/\psi + \gamma + gg$ with the same P_T dependence between 20 and 50 GeV.

6. Conclusion

In conclusion, we have computed the real next-to-next-to-leading order QCD contributions to the hadroproduction of a $J/\psi + \gamma$ and $\Upsilon + \gamma$ via color singlet transitions along the same lines as [9] for the inclusive case and argued that it provided a first reliable evaluation of the corresponding yield at NNLO accuracy.

Indeed, prior to that, we have shown that the full NLO evaluation of Li and Wang [26] is very accurately reproduced for $P_T > 5$ GeV by the sole evaluation of the real emission contributions up to order $\alpha_S^3\alpha$, namely the NLO*. In the inclusive case [9], a similar observation was made and motivated the study of the NNLO* contributions to evaluate the yield at NNLO accuracy. We have thus reach a completely independent, but similar, conclusion for the production of a quarkonium in association with a photon.

A priori, integrating the amplitudes for such real emission contributions leads to divergences in some phase-space regions. Those can be avoided by imposing a minimal invariant mass between the external light partons, while in a complete calculation those divergences are canceled by the virtual corrections. Yet, at NLO, the latter scale as P_T^{-8} and, at NNLO, as P_T^{-6}, hence are suppressed compared to respectively the real NLO part scaling as P_T^{-6} and the real NNLO part scaling as P_T^{-4}. This explains why we can neglect the virtual corrections at mid and large P_T and use a cut-off on which the results become insensitive when P_T grows, and also why the sole NLO* reproduces the full NLO accurately.

We have also shown that the differential cross section in P_T at NNLO* is one order of magnitude larger than at NLO for large P_T –as expected from their P_T scaling–. We have also identified the process responsible for the most part of it, i.e. $gg \to Q + \gamma + gg$. Although, the distinction between the gluon-fragmentation and the t-channel gluon exchange graphs cannot be carried out in gauge invariant way, we provided some hints that the second type of topology is dominating, similar to the inclusive case [9]. One of this hint is the polarisation of the quarkonia produced in association with a photon.

Indeed, we have computed the polarisation parameter λ of the NNLO* yield which is negative, indicating a longitudinally polarised yield. This confirms the trend observed at NLO and hints at the dominance of t-channel gluon exchange contributions.

When the NNLO* contributions are incorporated in the CS yield, it becomes one order of magnitude larger than the potential CO yield, which would mainly produce transversally polarised Q. The measurements of the cross section for the production of $Q + \gamma$ would directly measure the size of the CS without being sensitive to the non-perturbative CO parameters. This is a complementary case to the study of $J/\psi + c\bar{c}$ and $\Upsilon + b\bar{b}$ as discussed in [5, 35, 15].

Similarly to the analysis of the inclusive case [9], this analysis cannot be extended to too low P_T, where the approximations on which it is based no longer hold. One way

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[6] If one required that P_T^γ exactly balance P_T^Q in order to select the LO CSM, aiming at the extraction of the gluon distributions for instance, one would have to refine the analysis to take into account NNLO* corrections in the CS channels and the CO yield [24] which would then be dominated by $gg \to g^\star \to (Q\bar{Q})^{[8]} + \gamma$ via ${}^1S_0^{[8]}$.

to improve the predictions could be achieved by merging the matrix elements with parton showers using one of the approaches available in the literature [36], or by performing an analytic re-summation [37].

Finally, the results presented here strongly support the procedure used for the very first evaluation of the inclusive yield at NNLO accuracy [9] and which showed an agreement with the Tevatron measurements. This in turn confirms that much better Monte Carlo simulations than the ones based on matrix elements for CS channels at LO only are now possible at mid and large P_T. This could be achieved using NLO* and NNLO* partonic matrix elements generated by MadOnia [27, 29], for the process studied here $pp \to \mathcal{Q} + \gamma + X$, but also for the inclusive measurements to be performed at the LHC.

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