

Multiflavor QCD* on $R_3 \times S_1$: Studying Transition From Abelian to Non-Abelian Confinement

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Abstract

The center-stabilized multiflavor QCD* theories formulated on $R_3 \times S_1$ exhibit both Abelian and non-Abelian confinement as a function of the S_1 radius, similar to the Seiberg–Witten theory as a function of the mass deformation parameter. For sufficiently small number of flavors and small $r(S_1)$, we show occurrence of a mass gap in gauge fluctuations, and linear confinement. This is a regime of confinement without continuous chiral symmetry breaking (χ SB). Unlike one-flavor theories where there is no phase transition in $r(S_1)$, the multiflavor theories possess a single phase transition associated with breaking of the continuous χ S. We conjecture that the scale of the χ SB is parametrically tied up with the scale of Abelian to non-Abelian confinement transition.

1 Introduction

In supersymmetric $\mathcal{N} = 2$ Yang–Mills theories slightly deformed to $\mathcal{N} = 1$ by a mass term $\mu \text{Tr} \Phi^2$ for chiral superfield Φ linear confinement is a result of the dual Meissner effect [1]. In the limit of small μ amenable to analytic studies [1] confinement is Abelian (for a definition of Abelian vs. non-Abelian confinement see e.g. [2]). Many theorists believe that in passing to large μ , (i.e. $\mu \gtrsim \Lambda$), a smooth transition to non-Abelian confinement – pertinent to pure $\mathcal{N} = 1$ supersymmetric Yang–Mills – takes place in the Seiberg–Witten model. This is sometimes referred to as the same universality class hypothesis. In non-supersymmetric theories a construction serving the same purpose – studying the transition from Abelian to non-Abelian confinement by tuning an adjustable parameter – was engineered in [3] (see also [4]). There we considered $SU(N)$ Yang–Mills theories on $R_3 \times S_1$ treating the radius of the compact dimension $r(S_1)$ as a free parameter. At small $r(S_1)$ (i.e. $r(S_1) \ll \Lambda^{-1}$) we introduced a double-trace deformation stabilizing the vacuum of the theory at a center-symmetric point. With this stabilization, the Polyakov mechanism [5] guarantees linear Abelian confinement both, in pure Yang–Mills theories and in those with one massless quark in various representations of the $SU(N)$ gauge group [3]. A discrete chiral symmetry (χ S) inherent to two-index representations is spontaneously broken. Both effects, linear confinement and χ SB were caused by topological excitations in the vacuum (monopole-instantons, bions, etc.) which are under complete control at small $r(S_1)$. No obvious phase transitions in passing from the weak coupling Abelian regime at small- $r(S_1)$ to the strong coupling decompactification/non-Abelian confinement regime at $r(S_1) \gg \Lambda^{-1}$ was detected. The trace of the Polyakov line $\mathcal{U} = \text{Pexp} \left\{ i \int_0^L A_z dz \right\}$ remains vanishing in both regimes. Thus, the same universality class hypothesis is not necessarily tied up with supersymmetry.

In this paper we turn to $SU(N)$ gauge theories with several flavors, within the same theoretical framework.¹ A crucial distinction with the previously considered cases is the presence of *continuous* chiral symmetries. At small $r(S_1)$, at weak coupling, the continuous chiral symmetries remain unbroken, while Abelian confinement of the Polyakov type sets in, much in the same way as in pure Yang–Mills or single-flavor theories. Linear confinement coexists with the unbroken chiral symmetry in the quark sector.²

In the strong coupling decompactification limit one expects non-Abelian confinement and spontaneous χ SB. We study the dynamical origin and other details of the χ SB phenomenon within our theoretical framework. We observe a chiral phase transition in passing from small to large $r(S_1)$ in the multiflavor case. We con-

¹Neither the number of colors N nor the number of flavors N_f are assumed to be large.

²There is no contradiction with the Casher argument [6] since the latter does not apply in 2+1 dimensions.

ture that the scale of the χ SB is tied with the passage from Abelian to non-Abelian confinement, and is of the order $L_{\chi SB} \sim \Lambda^{-1}/N$. This surprising suppressed scale would be a natural scale of χ SB were the center symmetry stable all the way down to arbitrarily small $r(S_1)$.

2 Theoretical framework

The general design is as follows. We consider $SU(N)$ Yang–Mills theories with N_f flavors where $N_f > 1$. Each flavor is described by the Dirac fermion field in the complex representation

$$\mathcal{R} = \{\text{F, S, AS, BF}\} \quad (1)$$

where F stands for fundamental, AS/S/BF stand for two index antisymmetric, symmetric and bifundamental representations. We assume N_f to be sufficiently small so that asymptotic freedom is preserved and the theory at hand is below the lower boundary of the conformal window. For simplicity we will focus on $N_f = 2$. The action for multiflavor QCD-like theories on $R_3 \times S_1$ takes the form

$$S = \int_{R_3 \times S_1} \frac{1}{g^2} \left[\frac{1}{2} \text{Tr} F_{MN}^2 + i \bar{\Psi}^a \not{D} \Psi_a \right] \quad (2)$$

where a is the flavor index and $\not{D} = \gamma_M (\partial_M + iA_M)$ is the covariant derivative acting in the representation \mathcal{R} . For QCD(BF) the gauge group is $SU(N) \times SU(N)$, and the gauge part of the action (2) must be replaced by

$$F_{MN}^2 \rightarrow F_{1,MN}^2 + F_{2,MN}^2. \quad (3)$$

On a small cylinder $r(S_1) \ll \Lambda^{-1}$, one can deform the original theory by adding a double-trace operator $P[U(\mathbf{x})]$ where

$$P[U(\mathbf{x})] = \frac{2}{\pi^2 L^4} \sum_{n=1}^{\lfloor \frac{N}{2} \rfloor} d_n |\text{Tr} U^n(\mathbf{x})|^2, \quad (4)$$

d_n are numerical parameters of order one, and $\lfloor \dots \rfloor$ denotes the integer part of the argument in the brackets. The deformed action is

$$S^* = S + \int_{R_3 \times S_1} P[U(\mathbf{x})]. \quad (5)$$

For judiciously chosen d_n , the center symmetry remains unbroken in the vacuum while – due to weak gauge coupling *and* center symmetric holonomy – the gauge symmetry $SU(N)$ spontaneously breaks,

$$SU(N) \rightarrow U(1)^{N-1}. \quad (6)$$

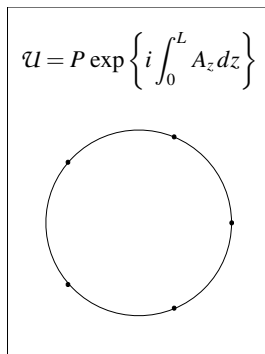


Figure 1: The center-symmetric configuration of the eigenvalues of the Polyakov line for SU(5).

The eigenvalues of the Polyakov line \mathcal{U} in the vacuum have a regular pattern

$$u_k = e^{\frac{2\pi i k}{N}}, \quad k = 0, 1, \dots, N-1 \quad (7)$$

depicted in Fig. 1. $N-1$ diagonal gauge bosons – photons – remain perturbatively massless, while off-diagonal gauge bosons acquire masses $\sim 1/L$. For what follows it will be convenient to introduce

$$\langle A_z \rangle = \frac{1}{L} \text{diag} \left\{ -\frac{2\pi[N/2]}{N}, -\frac{2\pi([N/2]-1)}{N}, \dots, \frac{2\pi[N/2]}{N} \right\}, \quad (8)$$

a matrix whose main diagonal is proportional to $\ln u_k$. If N is odd, one of the eigenvalues is zero, and there is a fermionic mode which is massless.

On fermions we will impose a boundary condition with a U(1)-twist

$$\Psi^a(\mathbf{x}, x_4 + L) = e^{2\pi i \omega} \Psi^a(\mathbf{x}, x_4), \quad g_{4D}^2 \ll \omega \ll 1. \quad (9)$$

which is equivalent to turning on an overall U(1) Wilson line for the background holonomy. The U(1)-shifted holonomy $\langle A_z \rangle - \omega \frac{2\pi}{L}$ generates three-dimensional real mass terms for the fermion fields which does not break any of the chiral symmetries (10) inherent to the multiflavor theories on R_4 . This is unlike the complex four-dimensional mass which would explicitly violate χ S. The U(1)-twist $\omega = 0^+$ plays the role of an infrared regulator in loops with (otherwise massless) fermions.

The chiral symmetry group of the action (2) is

$$\frac{\text{SU}(N_f)_L \times \text{SU}(N_f)_R \times \text{U}(1)_V \times Z_{2hN_f}}{Z_{N_f} \times Z_{N_f} \times Z_2}. \quad (10)$$

The factors in the denominator eliminate double-counting of the symmetries. $2N_f h$ is the number of fermion zero modes in the background of the Belavin–Polyakov–Schwarz–Tyupkin (BPST) instanton [7] on R_4 . For the fermionic representations of interest

$$2h = \{2, 2N + 4, 2N - 4, 2N\} \quad \text{for } \mathcal{R} = \{\text{F, S, AS, BF}\}. \quad (11)$$

The action of (10) on the Weyl fermions is

$$\begin{aligned} \text{SU}(N_f)_L : \quad & \lambda^a \rightarrow (U\lambda)^a, & \bar{\psi}^a & \rightarrow \bar{\psi}^a, \\ \text{SU}(N_f)_R : \quad & \lambda^a \rightarrow \lambda^a, & \bar{\psi}^a & \rightarrow (V\bar{\psi})^a, \\ \text{U}(1)_V : \quad & \lambda^a \rightarrow e^{i\delta}\lambda^a, & \bar{\psi}^a & \rightarrow e^{i\delta}\bar{\psi}^a, \\ Z_{2hN_f} : \quad & \lambda^a \rightarrow e^{\frac{2\pi ik}{2hN_f}}\lambda^a, & \bar{\psi}^a & \rightarrow e^{-\frac{2\pi ik}{2hN_f}}\bar{\psi}^a. \end{aligned} \quad (12)$$

Note that the four-dimensional Weyl fermion reduces to the Dirac fermion in the long-distance three-dimensional gauge theory. Thus, the relation between the four-dimensional Dirac fermion and the three-dimensional Dirac fermion obtained upon reduction is

$$\Psi = \begin{pmatrix} \lambda \\ \bar{\psi} \end{pmatrix}. \quad (13)$$

At small $r(S_1)$ the eigenvalues of the Polyakov line weakly fluctuate near their vacuum values depicted in Fig. 1. It is only the sum of the eigenvalues that vanishes in the vacuum. At large $r(S_1)$ each eigenvalue is expected to wildly fluctuate and average to zero. The same universality class hypothesis (say, for pure Yang–Mills) is that the passage from one regime to another is smooth. In principle, the smoothness, as opposed to a phase transition on the way from small to large values of $r(S_1)$, can be tested on lattices.

3 QCD* with two flavors

For definiteness, let us start from the case of fundamental fermions. Relevant introductory material and notation can be found in [3]. There are $N - 1$ distinct $\text{U}(1)$'s in this model, corresponding to $N - 1$ distinct electric charges. Each component Ψ_i^a ($i = 1, \dots, N$ and $a = 1, 2$) will be characterized by a set of $N - 1$ charges, which we will denote by \mathbf{q}_{Ψ_i} ,

$$\mathbf{q}_{\Psi_i} = g \mathbf{H}_{ii} \equiv g \left([H^1]_{ii}, [H^2]_{ii}, \dots, [H^{N-1}]_{ii} \right), \quad i = 1, \dots, N, \quad (14)$$

where \mathbf{H} is the set of $N - 1$ Cartan generators. If N is odd, $2N_f = 4$ fermion components remain massless (two ψ^a 's and two λ^a 's). More exactly, these modes

are nearly massless, with mass ω/L . Other components become massive and can be integrated out. If N is even, to keep fermions in the low-energy limit, we will have to tune $2\pi\omega = \pi/N + 0^+$. The fermions that survive in the low-energy limit are charged and therefore appear in loops.

The infrared dynamics can be described as compact QED₃ with light fermionic matter. Due to gauge symmetry breaking (6) via a *compact scalar* (the holonomy), there are N types of elementary instanton-monopoles. These topological excitations are uniquely labeled by their magnetic charges valued in the affine root system $\Delta_{\text{aff}}^0 = \{\boldsymbol{\alpha}_1, \boldsymbol{\alpha}_2, \dots, \boldsymbol{\alpha}_N\}$. The operators corresponding to the topological excitations are expressed in terms of the dual variables for the photons $d\boldsymbol{\sigma} = *d\mathbf{F}$ by using Abelian duality. Summing over the instanton-monopole contributions, the non-perturbative low-energy effective Lagrangian takes the form

$$S^{\text{QCD(F)}^*} = \int_{R_3} \left[\frac{1}{4g_3^2} \mathbf{F}^2 + \frac{1}{g_3^2} i\bar{\Psi}^a \left[\gamma^\mu (\partial_\mu + i\mathbf{q}_\Psi \mathbf{A}_\mu) + \gamma^4 \frac{2\pi i\omega}{L} \right] \Psi_a \right. \\ \left. + e^{-S_0} \left(\tilde{\mu} e^{i\boldsymbol{\alpha}_1 \boldsymbol{\sigma}} \det_{a,b=1,2} \{\lambda^a \psi^b\} + \mu \sum_{\boldsymbol{\alpha}_j \in (\Delta_{\text{aff}}^0 - \boldsymbol{\alpha}_1)} e^{i\boldsymbol{\alpha}_j \boldsymbol{\sigma}} + \text{H.c.} \right) + \dots \right], \quad (15)$$

where μ and $\tilde{\mu}$ are dimensionful coefficients of the monopole operators and the ellipses stand for higher order terms in the topological e^{-S_0} expansion and ignored massive modes. The $N - 1$ linearly independent instanton-monopole operators render all $N - 1$ dual photons $\boldsymbol{\sigma}$ massive, with masses proportional to $e^{-S_0/2}$. This switches on the Polyakov linear confinement.³

Let us now discuss the fermion sector of the low-energy theory (15). To establish the vacuum structure we note that at distances larger than $e^{S_0/2}$, the photons are gapped and are pinned at the bottom of the instanton-monopole induced potential. Consequently, to find the symmetry of the vacuum we explore the long-distance Lagrangian

$$S^{\text{NJL}} = \int_{R_3} \left\{ \frac{1}{g_3^2} i\bar{\Psi}^a \left[\gamma^\mu \partial_\mu + \gamma^4 \frac{2\pi i\omega}{L} \right] \Psi_a + e^{-S_0} \left(\tilde{\mu} \det_{a,b=1,2} \{\lambda^a \psi^b\} + \text{H.c.} \right) \right\}. \quad (16)$$

In essence, it describes a three-dimensional Nambu–Jona-Lasinio (NJL) model with $SU(N_f)_L \times SU(N_f)_R \times U(1)_V$ chiral symmetry at the Lagrangian level. It is known that at arbitrarily weak coupling (in the units of the cut-off scale), the chiral symmetry of the NJL model remains unbroken. This is the phase of confinement without χ SB, with massless (or light) fermions in the spectrum whose masslessness is protected by unbroken χ S.

As the coupling increases and approaches unity in the domain $r(S_1)\Lambda \sim 1$, the chiral symmetry (10) is expected to spontaneously break down to the diagonal vector

³There are $N - 1$ distinct strings. In principle, they can break due to the fermion pair creation, but the breaking is exponentially suppressed.

subgroup, $SU(N_f)_D$. This breaking must result in massless $N_f^2 - 1$ Nambu–Goldstone (NG) bosons. The χ SB phase transition occurs at the boundary of the region of validity of the low-energy theory (15).

Now let us extend the above discussion to fermions in the two-index representations (still keeping $N_f = 2$).

QCD(AS/S/BF)*: In QCD* theories with two-index fermions in the representations $\mathcal{R} = \{\text{AS/S/BF}\}$ we also observe two different phases as is the case with QCD(F)*. These are

$$\begin{aligned} L < L_{\chi SB}, & \quad \text{with massless (or light) fermions,} \\ L \geq L_{\chi SB}, & \quad \text{with massless NG-bosons.} \end{aligned} \quad (17)$$

In the latter phase we have confinement with (continuous) χ SB while in the former confinement without χ SB. At any radius, the discrete chiral symmetry pattern is $Z_{2h} \rightarrow Z_2$ [3], probed by a determinantal, continuous χ S-singlet order parameter, $\langle \det \lambda^a \psi^b \rangle$. Thus, at small S_1 , these theories have h isolated vacua and at large $r(S_1)$, h isolated coset spaces. Below, we highlight the differences in the analyses of two- and one-index representation fermions. We take QCD(BF)* as our main example due to its simplicity. QCD(AS/S) analysis is analogous, up to minor differences that can be filled in by using Ref. [3].

In two-flavor QCD(BF)*, the $4N$ zero modes of the BPST instanton (which can be viewed as N instanton-monopoles) split into N groups of four zero modes each [8]. (This is the reason why the instanton-monopoles must play a more prominent role on $S_1 \times R_3$ than the four-dimensional BPST instanton.) Thus, unlike QCD(F)*, the instanton-monopoles appearing at the order e^{-S_0} do not cause confinement, but may induce χ SB. The magnetic bions which appear at the order e^{-2S_0} lead to confinement, and mass gap for the gauge field fluctuations. The leading monopole and bion induced nonperturbative effects are

$$\begin{aligned} L_{\text{nonpert}}^{\text{QCD(BF)*}} &= \sum_{\alpha_i \in \Delta_{\text{aff}}^0} \left[\tilde{\mu} e^{-S_0} (e^{+i\alpha_i \sigma_1} + e^{+i\alpha_i \sigma_2}) (\det_{a,b} \lambda_i^a \psi_i^b + \det_{a,b} \lambda_{i+1}^a \psi_{i+1}^b) \right. \\ &+ \mu e^{-2S_0} \left[c_1 (e^{i(\alpha_i - \alpha_{i-1}) \sigma_1} + e^{i(\alpha_i - \alpha_{i-1}) \sigma_2}) \right. \\ &+ \left. \left. c_2 (2e^{i(\alpha_i \sigma_1 - \alpha_i \sigma_2)} + e^{i(\alpha_i \sigma_1 - \alpha_{i-1} \sigma_2)} + e^{i(\alpha_i \sigma_1 - \alpha_{i+1} \sigma_2)}) \right] \right] + \text{H.c.} \end{aligned} \quad (18)$$

Consequently, the gauge structure of the theory undergoes a two-stage breaking:⁴

$$SU(N) \times SU(N) \xrightarrow{\text{Higgsing}} [U(1)]^{N-1} \times [U(1)]^{N-1} \xrightarrow{\text{nonperturbative}} Z_N. \quad (19)$$

⁴In the second stage of the gauge structure reduction, the gauge group is not Higgsed, but, rather, the dual photons acquire gauge invariant masses.

The discrete gauge group (DGG) appearing at the final stage is another important difference between various $\text{QCD}(\mathcal{R})^*$ theories. In general, the gauge symmetry breaking pattern pertinent to $\text{QCD}(\mathcal{R})^*$ theories can be described as

$$G \xrightarrow{\text{Higgsing}} \mathbf{Ab}(G) \xrightarrow{\text{nonperturbative}} \text{DGG}(\mathcal{R}) \quad (20)$$

where $\mathbf{Ab}(G)$ is the Abelian gauge structure, as in the Seiberg–Witten theory, and DGG is the discrete gauge group which survives in the infrared. DGG is equal to Z_κ where κ is determined by the representation of massless fermions. For even N

$$\kappa = \{N, N, 2, 2, 1\}, \quad \text{for } \mathcal{R} = \{\text{Adj, BF, AS, S, F}\}, \quad (21)$$

while for odd N , $\kappa = \{N, N, 1, 1, 1\}$, respectively. Note that DGG is a subgroup of the center group and can be obtained as the quotient of the center group by equivalences imposed by massless matter. In $\text{QCD}(\mathcal{R})^*$, the charges which are non-neutral under $\text{DGG}=Z_\kappa$ are confined. This leads to the second difference between $\text{QCD}(\text{F})^*$ and $\text{QCD}(\mathcal{R})^*$ where $\mathcal{R} = \{\text{BF, AS, S, Adj}\}$. In the first problem, strictly speaking, the strings can break. In the latter case, the area law behavior of large Wilson loops (typically due to magnetic bions) is exact. This guarantees that charges nonvanishing under DGG are confined. Nonetheless, the gauge fluctuations are gapped in both cases.

As stated above (see (17)), the $\text{QCD}(\mathcal{R})^*$ theories possess two phases of confinement, with and without χSB . Returning to $\text{QCD}(\text{BF})^*$, at distances larger than $\sim e^{S_0}$, the photons are gapped, and the vacuum structure is determined by the fermion action

$$S = \int_{R^3} \sum_{i=1}^N \left[i\bar{\Psi}_i^a \left[\gamma^\mu \partial_\mu + \gamma^4 \frac{2\pi i\omega}{L} \right] \Psi_{a,i} + 2\tilde{\mu}e^{-S_0} \left(\det_{a,b} \lambda_i^a \psi_i^b + \text{h.c.} \right) \right]. \quad (22)$$

This is again, an NJL-type model, with the same consequences as those discussed around (17). For $L < L_{\chi\text{SB}}$, we have massless or light fermions in the spectrum – no complex Dirac mass is generated. For $L > L_{\chi\text{SB}}$, the chiral symmetry is broken via the bilinear $\langle \lambda^a \psi^b \rangle \sim \delta^{ab}$, inducing a *complex* four-dimensional Dirac mass for the fermion. This phase possesses massless NG-bosons due to χSB .

4 Abelian to non-Abelian confinement

In the small- $r(S_1)$ regime, the mechanism of confinement is Abelian, by virtue of $\mathbf{Ab}(\mathbf{G})$ in the gauge structure chain (20).⁵ At $e^{-S_0} \sim r(S_1)\Lambda \sim 1$, one loses the

⁵In fact, this is true for all the analytically controlled mechanisms of confinement known so far, including the Seiberg–Witten theory [1] or Polyakov’s mechanism [5]. The reason for the applicability of the semiclassical analysis is the appearance of an $\mathbf{Ab}(G)$ structure at some length scale.

separation of scales between the lightest W -bosons and the heaviest nonperturbatively gapped photons, so that the long distance theory based on $\mathbf{Ab}(\mathbf{G})$ loses its validity. This is also the scale at which one expects the long-distance NJL-model to induce the χ SB. We believe that, the scale of the passage from the Abelian to non-Abelian confinement and that of the chiral phase transition are parametrically tied up, and the bilinear chiral order parameter probes both.

This suggests that, in multiflavor QCD-like gauge theories, confinement without χ SB is a property of the Abelian confinement, whereas, continuous χ SB is associated with non-Abelian confinement.

One other surprising aspect of this chiral transition is its scale. In this work we dealt with small N , small N_f theories. However, if we let N to be arbitrarily large, we would observe that the scale of the chiral transition is a *sliding* (or suppressed) scale as a function of N . We found, by either an order of magnitude estimate based on the NJL Lagrangian, or by employing more powerful large- N volume independence theorem of the center symmetric theories, that $L_{\chi SB} \sim \Lambda^{-1}/N$. This also means that in the $N = \infty$ limit, the region of Abelian confinement shrinks to zero, in compliance with the volume independence. (See section 5 of Ref. [4] and references therein.) The emergence of such N -suppressed physical scales in QCD-like theories is rather surprising by itself, and is outside the reach of perturbation theory and non-perturbative holographic (supergravity) constructions. It is testable by numerical lattice simulations.

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