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Maximum Wavelength of Confined Quarks and Gluons and Properties of Quantum Chromodynamics

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Because quarks and gluons are confined within hadrons, they have a maximum wavelength of order the confinement scale. Propagators, normally calculated for free quarks and gluons using Dyson-Schwinger equations, are modified by bound-state effects in close analogy to the calculation of the Lamb shift in atomic physics. Because of confinement, the effective quantum chromodynamic coupling stays finite in the infrared. The quark condensate which arises from spontaneous chiral symmetry breaking in the bound state Dyson-Schwinger equation is the expectation value of the operator $\bar{q}q$ evaluated in the background of the fields of the other hadronic constituents, in contrast to a true vacuum expectation value. Thus quark and gluon condensates reside within hadrons. The effects of instantons are also modified. We discuss the implications of the maximum quark and gluon wavelength for phenomena such as deep inelastic scattering and annihilation, the decay of heavy quarkonia, jets, and dimensional counting rules for exclusive reactions. We also discuss implications for the zero-temperature phase structure of a vectorial SU(N) gauge theory with a variable number N_f of massless fermions.

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I. INTRODUCTION

Bethe's remarkable calculation of the Lamb shift in hydrogen in 1947 [1] laid the foundation for the renormalization procedure in quantum field theory and the subsequent development of quantum electrodynamics (QED). The Lamb shift is the change in the bound-state electron energy, in particular, the $2S_{1/2}$ and $2P_{1/2}$ levels of hydrogen, as a result of the effect of fluctuations in the quantized electromagnetic field on the electron. An essential aspect of the Lamb shift calculation in QED is the fact that while the wavefunction renormalization constant of a free electron Z_2 is infrared (IR) divergent, it becomes infrared-finite when the electron is bound in an atom. In the case of a free electron, the IR divergences are cancelled when one properly considers electron propagation together with the emission of soft real photons. In the case of the electron in an atomic bound state, the k integration over photon momenta is cut off in the infrared by the fact that the relevant photon wavelengths have a maximum value set by the size of the atom, i.e., $k \ge k_{min,atom}$,

$$k_{min,atom} \simeq \frac{1}{na_B} \simeq \frac{\alpha_{em} m_e}{n} \ , \tag{1}$$

where a_B is the Bohr radius and n is the radial quantum number characterizing the bound state of the electron in the Coulomb field of the proton. After mass renormalization photon momenta larger than $k \simeq m$ do not appreciably affect the electron motion. Combining these cutoffs then yields the Bethe logarithm, $\ln(1/\alpha_{em})$ in the energy shift [1].

The complete calculation of energy levels of Coulombic bound states in QED, such as hydrogen, positronium, or muonium (μ^+e^-), begins with the Bethe-Salpeter equation for the two bound-state particles. In the case where one of these particles can be taken as very heavy, such as in hydrogen or muonium, the bound-state electron propagator is replaced by the resolvent

$$\frac{1}{\Pi \cdot \gamma - m_e + i\epsilon} \ , \tag{2}$$

where $\Pi^\mu = p^\mu - eA^\mu$ and A^μ is the background electromagnetic field of the heavy particle. An analysis of the bound-state electron self-energy in terms of a gauge-invariant expansion in the electromagnetic field strength of the background field is given in ref. [2].

Quantum chromodynamics (QCD) has provided a remarkably successful theory of hadrons and strong interactions. At short distance; i.e., large Euclidean momentum scales μ, the squared QCD gauge coupling, $\alpha_s(\mu) = g_s^2(\mu)/(4\pi)$ becomes small, as a consequence of asymptotic freedom. As the momentum scale decreases through $\Lambda_{em} \simeq 200$ MeV, the theory exhibits spontaneous chiral symmetry breaking, and quarks and gluons are confined within color-singlet physical states, the hadrons. Thus, because of confinement, quarks and gluons have maximum wavelengths

$$\lambda_{max} \simeq \Lambda_{QCD}^{-1} \simeq 1 \text{ fm} \ . \tag{3}$$

Equivalently, the quantum mechanical wavefunctions for quarks and gluons within hadrons have minimum bound-state momenta,

$$k_{min} = |\mathbf{k}|_{min} \simeq \Lambda_{QCD} \ . \tag{4}$$

For example, in the hard-wall model of AdS/QCD, color confinement of quarks in the AdS fifth dimension gives the frame-independent condition at equal light-front time [3]:

$$\zeta = \sqrt{b_\perp^2 x(1-x)} < \lambda_{max} \tag{5}$$

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where $\vec{b}_\perp$ is the quark-antiquark impact separation and x is the quark light-cone momentum fraction $x = k^+/P^+ = (k^0+k^3)/(P^0+P^3)$. Thus in principle all perturbative and nonperturbative QCD analyses should be performed with the infrared regularization imposed by color confinement.

Here we point out and discuss several important consequences of color confinement and the resultant maximum wavelength of quarks and gluons that do not seem to have received attention in the literature. These include implications for an infrared fixed point of the QCD β function and new insights into spontaneous chiral symmetry breaking which are apparent when one uses bound-state rather than free Dyson-Schwinger equations (DSE's) in close analogy to QED bound state computations. An important consequence is that quark and gluon condensates are confined within hadrons, rather than existing throughout space-time. We also discuss hadron mass calculations using Bethe-Salpeter equations (BSE's), analyze the modifications of instanton physics, and comment on insights that one gains concerning short-distance-dominated processes and dimensional counting for hard exclusive processes.

Parenthetically, we note that if QCD contains $N_f \ge 2$ exactly massless quarks and if one turned off electroweak interactions, then the theory would have a resultant set of $N_f^2 - 1$ exactly massless Nambu-Goldstone bosons (e.g., the pions, for the case $N_f = 2$). Because the size of a nucleon, r_N, is determined by the emission and reabsorption of virtual pions and the resultant pion cloud, and hence is $r_N \sim m_\pi^{-1}$, this nucleon size would be much larger than Λ_{QCD}^{-1}. Since in the real world, m_π is not $<< \Lambda_{QCD}$, we do not pursue the analysis of this *gedanken* world here.

II. IMPLICATIONS OF λ_{max} FOR THE INFRARED BEHAVIOR OF QCD

The fact that quarks and gluons have maximum wavelengths λ_{max} has important consequences for the infrared behavior of QCD. For $\mu^2 >> \Lambda_{QCD}^2$, QCD is weakly coupled, and the evolution of α_s is described by the β function

$$\beta(t) = \frac{d\alpha_s}{dt} = -\frac{\alpha_s^2}{2\pi}\left(b_1 + \frac{b_2\alpha_s}{4\pi} + O(\alpha_s^3)\right) , \qquad (6)$$

where $t = \ln\mu$ and the one- and two-loop coefficients b_ℓ, $\ell = 1,2$ are scheme-independent, while higher-order coefficients are scheme-dependent.

In the standard perturbative calculations of these coefficients, one performs integrations over Euclidean loop momenta ranging from $k = 0$ to $k = \infty$. Although this is a correct procedure for describing the evolution of $\alpha_s(\mu)$ in the ultraviolet region $\mu >> \Lambda_{QCD}$ where the coupling is weak and effects of confinement are unimportant, it does not incorporate the property of confinement at low scales $\mu \lesssim \Lambda_{QCD}$. Confinement implies that both the gluons and quarks have restricted values of momenta $k \ge k_{min}$. Since loop corrections to the gluon propagator vanish as q^2/k_{min}^2 when this ratio is small, it follows that (to the extent that one can continue to use the quark and gluon fields and the associated coupling α_s to describe the physics in this region of momenta), as q^2 decreases in magnitude below Λ_{QCD}^2, the QCD β function measuring the evolution of α at the scale $\mu^2 \simeq q^2$ must also vanish in the infrared. This implies a physical cutoff on the growth of $\alpha_s(\mu)$ at small μ^2; i.e., infrared fixed-point behavior of the QCD coupling. In effect, QCD exhibits a well-defined limiting behavior in the infrared, and the infrared growth of α_s is suppressed, not because the perturbative β function exhibits a zero away from the origin, but because confinement provides an infrared cutoff. In fact, as summarized in reference [14], effective QCD couplings measured in experiments, such as the effective charge $\alpha_{s,g_1}(Q^2)$ defined from the Bjorken sum rule, display a lack of Q^2-dependence in the low Q^2 domain.

III. IMPLICATIONS FOR QCD PHENOMENOLOGY

The Bjorken scaling of the deep inelastic lepton-nucleon scattering (DIS) $\ell N \to \ell' X$ cross section is explained in perturbative QCD by arguing that the emission of gluons from the scattered quark is governed by a coupling α_s which is small because of asymptotic freedom. Thus in the leading-twist DIS regime, the DIS structure functions $F_i(x,Q^2)$ are mainly functions of x, with only small, logarithmic dependence on Q^2. A similar argument underlies the application of perturbative QCD and the parton model to deep inelastic annihilation, $e^+e^- \to$ hadrons with center-of-mass energy squared $s >> \Lambda_{QCD}^2$, away from thresholds, leading to the formula $\sigma(e^+e^- \to \text{hadrons})/\sigma(e^+e^- \to \mu^+\mu^-) = R$ with $R(s) = N_c\sum_j q_j^2$, where the sum is over quarks with $4m_q^2 < s$. Asymptotic freedom in QCD has also been used to explain the narrow widths of $^3S_1$, $J^{PC} = 1^{--}$ $Q\bar{Q}$ states of heavy (i.e., $m_Q >> \Lambda_{QCD}$) quarks with masses below threshold for emission of the associated heavy-flavor mesons. The explanation is, in essence, that hadronic decays proceed by emission of three gluons, and because the corresponding couplings $g_s(\mu)$ are small for $\mu \sim m_Q/3$ due to asymptotic freedom, the resultant decays are suppressed [4]. A fourth property of QCD which makes use of its property of asymptotic freedom is the phenomenon of jets [5].

In each of these examples, the theoretical analyses provide successful descriptions of the physical processes in terms of the asymptotic freedom of QCD. The analyses involve a factorization between the short-distance, perturbatively calculable, part of the process and the long-distance part involving hadronization. However, it is important to ask whether such calculations are stable against multiple gluon emission. For example, if the outgoing struck quark in DIS or the $q\bar{q}$ pair in DIA were to radiate a sufficiently large number of gluons, then, since on average each of these would carry only a small momentum, the associated coupling α_s would not be small, and this could significantly change the prediction for the cross section. A related concern regards the narrow width of orthoquarkonium: if the heavy Q and $\bar{Q}$ annihilate not into three gluons, but into a considerably larger number, ℓ, of gluons, then each would carry a much smaller momentum, $k \sim 2m_Q/\ell$, and thus the associated running coupling $g_s(k)$ would not be small. Similarly, in a hard scattering process that leads to the production of a $q\bar{q}$ pair with invariant mass $\hat{s} = (p_q + p_{\bar{q}})^2 >> \Lambda_{QCD}^2$ and $\hat{s} >> 4m_q^2$, the q and $\bar{q}$ might radiate a large number of gluons, each carrying a small relative momentum; again the resulting running coupling is consequently not small, and this could dilute the jet-like structure of the event. A similar concern applies to jets involving gluons.

Here we provide a simple physical explanation of why gluon emission is stabilized: because of confinement, the gluons have minimum momenta of order Λ_{QCD}, so the apparently dangerous scenario involving emission of a large number of gluons with momenta of order Λ_{QCD} or smaller is kinematically impossible.

An infrared cutoff on the growth of the QCD coupling also helps to explain the remarkable success of dimensional counting rules in describing data on the differential cross sections of exclusive reactions at high-energy and fixed-angle in QCD [6]. Recall that for a reaction $a+b \to c_1 + ... + c_k$ with $s >> \Lambda_{QCD}^2$, $t = Q^2 >> \Lambda_{QCD}^2$, and fixed s/t (i.e., fixed CM scattering angle), dimensional counting gives [6]

$$\frac{d\sigma}{dt} \propto s^{-(n-2)} \ , \tag{7}$$

where the twist n denotes the total number of elementary valence fields entering the hard scattering amplitude. This implies, for example, that, under these conditions, $d\sigma(pp \to pp)/dt \propto s^{-10}$, $d\sigma(\pi p \to \pi p) \propto s^{-8}$, etc. One might worry that since all of the external particles in these reactions are on-shell, eq. (7) might receive large long-distance corrections. An appealing explanation for the absence of such corrections is that they are suppressed by the cutoff in the growth of the running QCD coupling α_s due to the k_{min} of the gluons. This is also consistent with fits to data [7, 8, 9].

IV. IMPLICATIONS FOR SPONTANEOUS CHIRAL SYMMETRY BREAKING

A limit on the maximum of gluon and quark wavelengths also has implications for spontaneous chiral symmetry breaking (SχSB) in QCD. It is clear that because of confinement, analyses based on free quark and gluon propagators, such as the Dyson-Schwinger equation, need to be replaced in principle by analyses which incorporate bound-state dynamics, such as the QCD Bethe-Salpeter equation. Recall that since the u and d current-quark masses are $<< \Lambda_{QCD}$, the QCD Lagrangian theory has a global $SU(N_f)_L \times SU(N_f)_R$ chiral symmetry, broken spontaneously to the diagonal, vector isospin subgroup $SU(N_f)_{diag}$, where $N_f = 2$, by the $\langle \bar{q}q \rangle$ condensates with $q = u, d$. (The analogous statement applies to the corresponding symmetry with $N_f = 3$, with larger explicit breaking via m_s.) Some studies of spontaneous chiral symmetry breaking in QCD are listed in ref. [10].

The inverse quark propagator has the form $S_f(p)^{-1} = A(p^2)\not{p} - B(p^2)$. In the one-gluon exchange approximation, the DSE for S_f^{-1} is

$$S_f(p)^{-1} - \not{p} = -iC_F g^2 \int \frac{d^4k}{(2\pi)^4} D_{\mu\nu}(p-k)\, \gamma^\mu\, S_f(k)\, \gamma^\nu \tag{8}$$

where C_F is the quadratic Casimir invariant and $D_{\mu\nu}(k)$ is the gluon propagator. Chiral symmetry breaking is a gauge-invariant phenomenon, so one may use any gauge in solving this equation. It is convenient to use the Landau gauge since then there is no fermion wavefunction renormalization; i.e., $A(p^2) = 1$. Equation (8) has a nonzero solution for the dynamically generated fermion mass Σ (which can be taken to be $\Sigma(p^2) = B(p^2)$ for Euclidean $p^2 << \Lambda^2$) if $\alpha_s \ge \alpha_{cr}$, where $3\alpha_{cr} C_F/\pi = 1$. Since Σ is formally a source in the path integral for the operator $\bar{q}q$, one associates $\Sigma \neq 0$ with a nonzero quark condensate. Clearly, this only provides a rough estimate of α_{cr}, in view of the strong-coupling nature of the physics and the consequent large higher-order perturbative, and also nonperturbative, contributions.

If one now takes quark and gluon confinement into account, then just as for the Lamb shift, the integral over loop momenta in eq. (8) can extend in the infrared only to $k_{min} \sim \Lambda_{QCD}$. Although the DSE analysis for the free quark propagator may incorporate some of the physics relevant to spontaneous chiral symmetry breaking, it does not incorporate the property of confinement. This is an important omission since a plausible physical explanation for spontaneous chiral symmetry breaking in QCD involves confinement in a crucial manner; this breaking results from the reversal in helicity (chirality) of a massless quark as it heads outward from the center of a hadron and is reflected back at the outer boundary of the hadron [11].

The Dyson-Schwinger equation has been used in conjunction with the Bethe-Salpeter equation for approximate calculations of hadron masses and other quantities in QCD [12]. Our observation implies that here again, the integration over virtual loop momenta in the BSE can only extend down to $k_{min} \sim \Lambda_{QCD}$, not to $k = 0$. This obviates the need for artificial cutoffs on the growth of the QCD coupling occurring in the integrand that have been employed in past studies. Analyses using the DSE and BSE have been used to calculate hadron masses in the confining phase of an abstract asymptotically free, vectorial $SU(N)$ gauge theories with a variable number, N_f, of light fermions [13]. It would be worthwhile to incorporate the effect of k_{min} in these studies, as well as in analyses for actual QCD.

Thus let us consider the propagator of a light quark bound in a light-heavy $q\bar{Q}$ meson, such as $B^+ = (u\bar{b})$ or $B^0_d = (d\bar{b})$. At sufficiently strong coupling α_s, the DSE (in this case, effectively a bound-state Dyson-Schwinger equation) yields a nonzero, dynamically generated mass, Σ for the light quark. One can associate this with a bilinear quark condensate $\langle\bar{q}q\rangle$, as noted above. However, this condensate is the expectation value of the operator $\bar{q}q$ in the background (approximately Coulombic) field of the heavy $\bar{b}$ antiquark, in contrast to a true vacuum expectation value [15]. This is in accord with our argument in [15] that the quark condensate $\langle\bar{q}q\rangle$ and gluon condensate $\langle \mathrm{Tr}(G_{\mu\nu}G^{\mu\nu})\rangle$ have spatial support only in the interior of hadrons, since that is where the quarks and gluons which give rise to it are confined. In [15] we noted that this conclusion is the analogue for quantum field theory of the experimental fact that the spontaneous magnetization below the Curie temperature in a piece of iron has spatial support only within the iron rather than extending to spatial infinity. Our observation in Ref. [15] that QCD condensates have spatial support restricted to the interiors of hadrons has the important consequence that these condensates contribute to the mean baryon mass density in the universe, but not to the cosmological constant or dark energy [16]. As in the case of QED, the bound-state problem of a light quark in a background field can be formulated as a resolvent problem or in terms of an effective theory [17]. Of course, in the hypothetical situation in which the current-quark masses $m_u = m_d = 0$ and one turned off electroweak interactions, so that $m_\pi \to 0$, the size of hadrons, as determined by their meson clouds, would become infinitely large, and the condensates would have infinite spatial extent.

In general, the breaking of a continuous global symmetry gives rise to Nambu-Goldstone modes. It was noted in Ref. [15] that in a sample of a ferromagnetic material below its Curie temperature, these Nambu-Goldstone spin wave modes (magnons) are experimentally measured to reside within the sample. Moreover, via the correspondence between the partition function defining a statistical mechanical system and the path integral defining a quantum field theory, there is an analogy between the spin waves in a Heisenberg ferromagnet and the almost Nambu-Goldstone modes in QCD - the pions. As required for the self-consistency of our analysis of condensates, the wavefunctions for the pions have spatial support where the chiral condensates exist, namely within the pions themselves and, via virtual emission and reabsorption, within other hadrons, in particular nucleons.

We relate these statements to some current algebra results. Consider the vacuum-to-vacuum correlator of the axial-vector current, $\langle 0|J^\mu_5(x_a)J^\mu_5(x_b)|0\rangle$. The Fourier transform of the cut of this propagator can, in principle, be measured in $e^+e^- \to Z^*_0$ axial-vector current events. The pion appears as a pole in this propagator at $q^2 = m^2_\pi$, corresponding to $e^+e^- \to Z^*_0 \to \pi^0$. Here the axial-vector current $J^\mu_5 = \bar{q}\gamma^\mu\gamma^5 q$ creates a quark pair at x_a which propagates to x_b, and as the q and $\bar{q}$ propagate, they interact and bind to create the pion. For example, at fixed light-front time $x^+_a = x^+_b$, the pion pole contribution appears when $-(x_a - x_b)^2 = (x^\perp_a - x^\perp_b)^2 \simeq R^2_\pi$,where R_π is the transverse pion size, of order $1/m_\pi$. The axial current that couples to the pion thus involves $\bar{q}q$ creation within a domain of the size of the pion. Furthermore, any $q\bar{q}$ condensate that appears in the quark or antiquark propagator in this process is created in a finite domain of the pion size, since that is where the quark and antiquark propagate, subject to the confining color interaction. Again, we see that one needs to use a modified form of the DSE for the quark propagator which takes into account the field of the antiquark. And again, the loop momenta have a maximum wavelength corresponding to the finite size of the domain where color can exist.

Since the property that quarks and gluons have a maximum wavelength is a general consequence of confinement, it necessarily appears in specific phenomenological models of confined hadrons, such as bag models [18] (see also [19]) and recent approaches using AdS/CFT methods [9]. It is also evident in lattice gauge simulations of QCD [20]. Sometimes the results of these simulations are phrased as the dynamical generation of an effective "gluon mass"; here we prefer to describe the physics in terms of a maximum gluon wavelength, since this emphasizes the gauge invariance of the phenomenon.

V. IMPLICATIONS FOR INSTANTON EFFECTS IN QCD

The maximum wavelength of gluons also has implications for the role of instantons in QCD. In the semiclassical picture, after a Euclidean rotation, one identifies the gluon field configurations which give the dominant contribution to the path integral as those which minimize the Euclidean action. This requires that the field strength tensor $F_{\mu\nu} \equiv \sum_a T_a F^a_{\mu\nu}$ vanish as $|x| \to \infty$, which implies that $A_\mu = -(i/g_s)(\partial_\mu U)U^{-1}$, where $A_\mu \equiv \sum_a T_a A^a_\mu$ and $U(x) \in \mathrm{SU}(N_c)$. Since the outer boundary of the compactified $\mathbb{R}^4$ in this $|x| \to \infty$ limit is S^3, the gauge fields thus fall into topologically distinct classes, as described by the class of continuous mappings from S^3 to $\mathrm{SU}(N_c)$, i.e., the homotopy group $\pi_3(\mathrm{SU}(N_c)) = \mathbb{Z}$. Instantons appear to play an important role in QCD, explaining, for example, the breaking of the global $\mathrm{U}(1)_A$ symmetry and the resultant fact that the η' meson is not light [21, 22]; however, one should recognize that, because of confinement, the gauge fields actually have no support beyond length scales of order $1/\Lambda_{QCD} \sim 1$ fm. The semiclassical analysis with its $|x| \to \infty$ limit used to derive the result $A_\mu = -(i/g_s)(\partial_\mu U)U^{-1}$ is thus subject to significant corrections due to confinement. This is evident in the BPST instanton solution for $N_c = 2$, namely [23]

$$ig_s A_\mu = \left(\frac{x^2}{x^2+\rho^2}\right)(\partial_\mu U)U^{-1} \ , \qquad (9)$$

where $U(x) = (x^0 + i\tau \cdot \mathbf{x})/|x|$. This form only becomes a pure gauge in the limit $|x| \to \infty$. It has long been recognized that in calculating effects of instantons in QCD, which involve integrations over instanton scale size ρ, uncertainties arise due to the fact that there are big contributions from instantons with large scale sizes, where the semiclassical approximation is not accurate [22]. Our point is different, although related; namely that the accuracy of the semiclassical instanton analysis is also restricted by the property that $\lambda_{max} \sim 1$ fm for the gluon field and hence one cannot really take the $|x| \to \infty$ limit in the manner discussed above.

VI. IMPLICATIONS FOR GENERAL NON-ABELIAN GAUGE THEORIES

Our observations are also relevant to the problem of determining the phase structure (at zero temperature and chemical potential) of a vectorial SU(N) gauge theory with a gauge coupling g and a given content of massless fermions, such as N_f fermions transforming according to the fundamental representation of SU(N). We assume $N_f < (11/2)N$, so that the theory is asymptotically free. Since fermions screen the gauge field, one expects that for sufficiently large N_f, the gauge interaction would be too weak to confine or produce spontaneous chiral breaking (e.g., [24]). An estimate of the critical value, $N_{f,cr}$, beyond which there would be a phase transition from a phase with confinement and SχSB to one without such symmetry breaking (and presumably without confinement) has been obtained combining the perturbative β function and the DSE. We denote this as the βDS method. For sufficiently large N_f, the perturbative β function exhibits a zero away from the origin, at $\alpha_{\rm IR}$, where $\alpha = g^2/(4\pi)$. The value of $\alpha_{\rm IR}$ is a decreasing function of N_f. The value of $N_{f,cr}$ is determined by the condition that α_{IR} decreases below the minimal value α_{cr} for which the approximate solution to the DS equation yields a nonzero solution for the dynamically generated fermion mass. The βDS analysis, to two-loop accuracy, yields the estimate [25]

$$N_{f,cr} = (N_{f,cr})_{2\ell\beta{\rm DS}} = \frac{2N(50N^2-33)}{5(5N^2-3)} \ , \qquad (10)$$

where 2ℓ refers to the two-loop accuracy to which the beta function is calculated. This gives $N_{f,cr} \simeq 8$ for $N = 2$ and $N_{f,cr} \simeq 12$ for $N = 3$. The value of $N_{f,cr}$ is important because if the approximate IR fixed point α_{IR} is larger than, but near to, α_{cr}, the resultant gauge coupling runs slowly over an extended interval of energies. This "walking" behavior is useful for models of dynamical electroweak symmetry breaking [25, 26]. (In such models there are motivations for choosing $N = 2$, including a mechanism to explain light neutrino masses [27].) The effect of the three-loop terms in the β function and the next higher-loop terms in the DSE have been studied in Ref. [28].

Neither the β function nor the DSE used in this βDS method includes the effect of instantons. Studies in QCD have shown that instantons enhance spontaneous chiral symmetry breaking [29]. Analyses of instanton effects on fermion propagation were carried out for general N_f, and these were shown to contribute substantially to SχSB [30]. One would thus expect that if one augmented the βDS approach to include effects of instantons, the resultant improved estimate of $N_{f,cr}$ would be greater than the value obtained from the βDS method without instantons.

In principle, lattice gauge theory can provide a fully nonperturbative approach for calculating $N_{f,cr}$ [31]-[32]. One lattice group has obtained values of $N_{f,cr}$ considerably smaller than the respective βDS values (for $N = 2, 3$) [31]; however, the most recent study of the $N = 3$ case finds evidence that the infrared behavior of the theory is conformal for $N_f \geq 12$ but exhibits confinement and chiral symmetry breaking for $N_f \leq 8$, consistent with the βDS analysis [32]. Since the βDS method does not include instanton effects, there is thus a question why it appears to produce a rather accurate value of $N_{f,cr}$.

Our observation provides a plausible answer to this question. Approaching the chiral boundary from within the phase with confinement and spontaneous chiral symmetry breaking, we note that the confinement-induced k_{min} of the gluons reduces their contribution to the increase of the gauge coupling in the infrared and also to the virtual gluon exchange effects on the fermion propagator. This reduction of gluonic effects acts in the opposite direction relative to the enhancement of chiral symmetry breaking due to instantons, and thus has the potential to explain why the βDS estimate for $N_{f,cr}$, which does not incorporate either confinement or instanton effects, could nevertheless yield a reasonably accurate value for $N_{f,cr}$. Moreover, as noted above, the role of instantons in SχSB is affected by the confinement of gluons and the resultant corrections to the semiclassical approach to QCD.

VII. CONCLUSIONS

Because quarks and gluons are confined within hadrons, there is maximum limit on their wavelengths. Propagators, normally calculated for free quark and gluons using Dyson-Schwinger equations, are modified in the infrared by bound state effects in close analogy to the calculation of the Lamb shift in atomic physics. Thus because of confinement, the effective QCD coupling stays finite and flat at low momenta. The quark condensate which arises from spontaneous chiral symmetry breaking in the bound-state Dyson-Schwinger equation is the expectation value of the operator $\bar{q}q$ evaluated in the background of the fields of the other hadronic constituents, in contrast to a true vacuum expectation value. Thus quark and gluon condensates have support only within hadrons.

We have shown that the limit on the maximum wavelength of gluons and quarks from confinement leads to new insights into a number of phenomena in QCD, including deep inelastic scattering and annihilation, the narrow widths of heavy orthoquarkonium states, jets, instantons, dimensional counting rules for hard exclusive processes, and other phenomena related to the infrared behavior of the theory. We have also given a plausible explanation of how estimates of the value $N_{f,cr}$ in a general asymptotically free, vectorial SU(N) theory based on a method using the perturbative β function and the Dyson-Schwinger equation could be reasonably accurate even though this method does not incorporate the effects of confinement or instantons. Our observations suggest a program of future research devoted to incorporating the effect of the maximum wavelength of quarks and gluons in analytic studies of QCD properties.

VIII. ACKNOWLEDGMENTS

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[1] H. A. Bethe, Phys. Rev. **72** (1947) 339; see also N. M. Kroll and W. E. Lamb, Phys. Rev. **75** (1949) 388; J. B. French and V. F. Weisskopf, Phys. Rev. **75** (1949); H. A. Bethe and E. E. Salpeter, *Quantum Mechanics of One- and Two-Electron Atoms* (Academic Press, New York, 1957).
[2] G. W. Erickson and D. R. Yennie, Ann. Phys. (N.Y.) **35** (1965) 271, 447; S. J. Brodsky and G. W. Erickson, Phys. Rev. **148** (1966) 26.
[3] S. J. Brodsky and G. F. de Teramond, arXiv:0804.3562 [hep-ph].
[4] T. Appelquist and H. D. Politzer, Phys. Rev. Lett. **34** (1975) 43; Phys. Rev. D **12** (1975) 1404.
[5] A review is G. Sterman, in *Proceedings of the Theoretical Advanced Study Institute (TASI 2004)*, arXiv:hep-ph/0412013.
[6] S. J. Brodsky and G. Farrar, Phys. Rev. Lett. **31** (1973) 1153; V. A. Matveev, R. M. Muradian, and A. N. Tavkhelidze, Lett. Nuovo Cim. **7** (1973) 719.
[7] S. J. Brodsky, E. Gardi, G. Grunberg, and J. Rathsman, Phys. Rev. D **63** (2001) 094017.
[8] J. H. Field, Phys. Rev. D **66** (2002) 013013.
[9] S. J. Brodsky and G. F. de Teramond, arXiv:0802.0514.
[10] Early papers include K. Lane, Phys. Rev. D **10** (1974) 2605; H. D. Politzer, Nucl. Phys. B **117** (1976) 397; T. Kugo, Phys. Lett. B **76** (1978) 625; H. Pagels, Phys. Rev. D **19** (1979) 3080; K. Higashijima,, Phys. Rev. D **29** (1984) 1228.
[11] A. Casher, Phys. Lett. B **83** (1979) 395.
[12] K.-I. Aoki, M. Bando, T. Kugo, and M. Mitchard, Prog. Theor. Phys. **85** (1991) 355; K.-I. Aoki, T. Kugo, and M. Mitchard, Phys. Lett. B **266** (1991) 467; V. Miransky, *Dynamical Symmetry Breaking in Quantum Field Theories* (World Scientific, Singapore, 1993); P. Jain and H. Munczek, Phys. Rev. D **48** (1993) 5403; C. J. Burden et al., Phys. Rev. C **55** (1997) 2649; R. Alkofer and L. von Smekal, Phys. Rept. **353** (2001) 281; P. Maris and C. D. Roberts, Int. J. Mod. Phys. E **12** (2003) 297.
[13] M. Harada and Y. Yoshida, Phys. Rev. D **50** (1994) 6902; T. Appelquist and F. Sannino, Phys. Rev. D **59** (1999) 067702; M. Harada, M. Kurachi, and K. Yamawaki, Phys. Rev. D **68** (2003) 076001; Prog. Theor. Phys. **115**, 765 (2006); M. Kurachi and R. Shrock, JHEP **12** (2006) 034; Phys. Rev. D **74** (2006) 056003.
[14] A. Deur, V. Burkert, J. P. Chen and W. Korsch, arXiv:0803.4119 [hep-ph].
[15] S. J. Brodsky and R. Shrock, arXiv:0803.2541.
[16] S. J. Brodsky and R. Shrock, arXiv:0803.2554.
[17] N. Brambilla, A. Pineda, J. Soto and A. Vairo, Rev. Mod. Phys. **77** (2005) 1423.
[18] A. Chodos, R. L. Jaffe, K. Johnson, C. B. Thorn, and V. F. Weisskopf, Phys. Rev. D **9** (1974) 3471; T. Degrand, R. L. Jafffe, K. Johnson, and J. Kiskis, Phys. Rev. D **12** (1975) 2060.
[19] J. M. Cornwall, Phys. Rev. D **26** (1982) 1453; L. von Smekal, A. Hauck, and R. Allkofer, Phys. Rev. Lett. **79** (1997) 3591; A. C. Aguilar, A. Mihara, and A. A. Natale, Phys. Rev. D **65** (2002) 054011; A. C. Aguilar, A. A. Natale, and P. S. da Silva, Phys. Rev. Lett. **90** (2003) 152001.
[20] J. E. Mandula and M. Ogilvie, Phys. Lett. **185** (1987) 127; D. Zwanziger, Phys. Lett. B **257** (1991) 168; C. Bernard, C. Parrinello, and A. Soni, Phys. Rev. D **49** (1994) 1585.
[21] G. 't Hooft, Phys. Rev. Lett. **37** (1976) 8; Phys. Rev. D **14** (1976) 3432.
[22] C. G. Callan, R. Dashen, and D. J. Gross, Phys. Rev. D **16** (1977) 2526.
[23] A. A. Belavin, A. M. Polyakov, A. S. Schwartz, and Yu. S. Tyupkin, Phys. Lett. B **59** (1975) 85.
[24] T. Banks and A. Zaks, Nucl. Phys. B **196** (1982) 189.
[25] T. Appelquist, J. Terning, and L. C. R. Wijewardhana, Phys. Rev. Lett. **77** (1996) 1214; T. Appelquist, A. Ratnaweera, J. Terning, and L. C. R. Wijewardhana, Phys. Rev. D **58** (1998) 105017.
[26] T. Appelquist, D. Karabali, and L. C. R. Wijewardhana, Phys. Rev. Lett. **57** (1986) 957; T. Appelquist and L. C. R. Wijewardhana, Phys. Rev. D **36** (1987) 568; see also B. Holdom, Phys. Lett. **B150** (1985) 301.
[27] T. Appelquist and R. Shrock, Phys. Lett. B **548** (2002) 204; Phys. Rev. Lett. **90** (2003) 201801; T. Appelquist, M. Piai, and R. Shrock, Phys. Rev. D **69** (2004) 015002.
[28] T. Appelquist, K. Lane, and U. Mahanta, Phys. Rev. Lett. **61** (1998) 1553.
[29] D. Caldi, Phys. Rev. Lett. **39** (1997) 121; C. Callan, R. Dashen, and D. Gross, Phys. Rev. D **17** (1978) 2717.
[30] T. Appelquist and S. Selipsky, Phys. Lett. B **400** (1997) 364; M. Velkovsky and E. V. Shuryak, Phys. Lett. B **437**

(1998) 398.
[31] Y. Iwasaki et al., Phys. Rev. Lett. **69** (1992) 21; Phys. Rev. D **69** (2004) 014507.
[32] T. Appelquist, G. Fleming, and E. Neil, Phys. Rev. Lett. **100** (2008) 171607.