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FIELD-ANALYSIS OF VACUUM FREE-SPACE LASER ACCELERATION FROM ROUGH-SURFACE AND ABSORBING THIN BOUNDARIES*

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This script presents a field-based analysis of laser acceleration of relativistic electrons in a free space that is bounded by a thin scattering or by a thin absorbing surface. The laser acceleration process is analyzed in terms of the inverse-radiation formalism and compared to the more familiar field path-integral analysis method. When the scattering boundary is modeled as a linear-index medium the predictions for laser-electron interactions from both field methods are found to agree. For the absorbing boundary both interaction pictures are also found to agree provided that the inverse radiation method is generalized to include absorption of energy from the boundary that is modeled as a linear ohmic-loss object.

I. INTRODUCTION

Laser-driven particle acceleration in vacuum is typically understood in terms of the accumulated mechanical work on a relativistic free particle as it samples the external electric field from the laser along its path. In this laser-electron interaction picture the mechanical work of the laser on a free particle of charge q is given by the path integral

$$\Delta U_{P,L} = q\int_{\Gamma} \vec{E}_{L,P}(\vec{r}) \cdot d\vec{r} \tag{1}$$

$\Delta U_{P,L}$ is the mechanical work on the particle P caused by the external laser beam L, $\vec{E}_{L,P}(\vec{r})$ is the electric field of the laser field on the particle at the location $\vec{r}$, and Γ is the particle's trajectory. This first interaction picture described by equation 1 is the simplest and most common model and will be referred to as the field path-integral interaction picture. Here we consider highly relativistic particles with a uniform rectilinear motion. In this limit the integral of equation 1 takes the form

$$\Delta U_{P,L} = \int_{-\infty}^{\infty} qE_z(z(t),t)dz \tag{2}$$

where the position $z(t)$ has the form $z(t) = z_0 + \beta ct$. It is well known that in the discussed limit a free-space electromagnetic field configuration does not exchange energy

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with the free particle in a linear fashion, and in the regime of relatively low laser field intensities considered here the integral in equation 2 becomes vanishingly small. This is the Lawson-Woodward Theorem [1]. The theorem is very specific and does not apply for non-uniform particle motion of the free particle such as in an IFEL. Another important failure mechanism for the Lawson Woodward theorem is the introduction of a mediating object between the particle and the free-space field. In such a scenario the mediating object alters the electromagnetic field components along the particle trajectory in such a way as to produce a nonzero contribution to the mechanical work in equation 2.

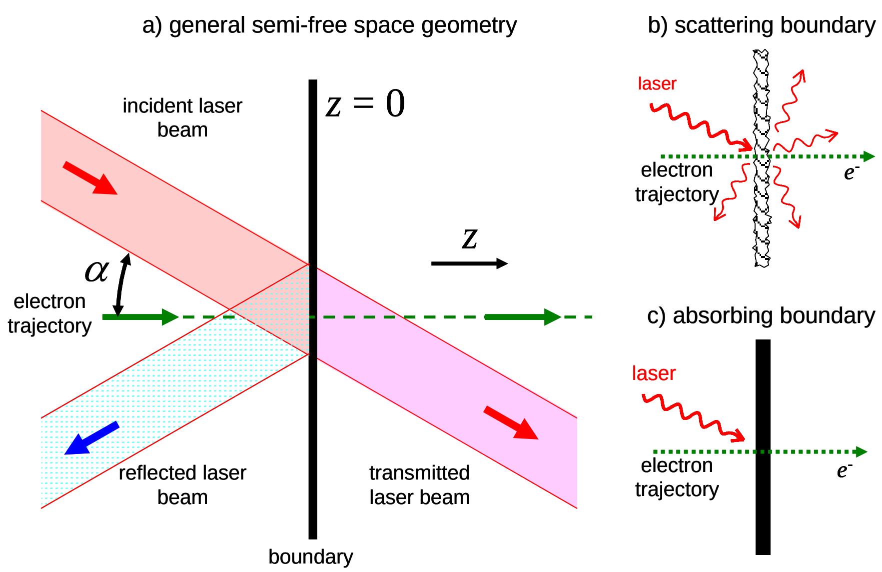

FIG 1. (a) General configuration for thin-boundary terminated vacuum laser-driven particle acceleration. The boundary introduces an alteration to the free-space laser field that breaks the Lawson-Woodward Theorem. (b) A scattering boundary that breaks the incident laser beam into may plane wave components. (c) An absorbing boundary, which heats up as it absorbs energy from the laser beam.

We consider the general geometry where a laser beam is nearly co-propagating with a relativistic electron beam at a very shallow angle α in the upstream space of a boundary. The purpose of the boundary is to alter or to completely stop the laser field in the downstream region. For the thin boundaries as shown in figure 1 the path integral can be decomposed as a sum of an upstream and a downstream interaction

$$\Delta U_{P,L} = \int_{-\infty}^{0} qE_z(z(t),t)dz + \int_{0}^{\infty} qE_z(z(t),t)dz \equiv \Delta U_{upstream} + \Delta U_{downstream} \quad (3)$$

A monochromatic plane wave of amplitude E_0 and wavelength λ co-propagating with the relativistic particle at a shallow angle $|\alpha| \sim 1/\gamma$ can be shown to provide an energy gain in the upstream region of a reflective flat boundary [2]

$$\Delta U_{P,L} = \Delta U_{upstream} = \frac{qE_0\lambda}{\pi}\frac{\alpha}{\alpha^2 + 1/\gamma^2}\cos\rho\sin\varphi \tag{4}$$

where ρ is the polarization angle of the laser beam with respect to the laser-electron beam plane, and φ is the optical phase of the laser beam. The plane wave component that is reflected from the boundary has an angle that is significantly different from $|\alpha| \sim 1/\gamma$ and therefore has a negligible contribution to the field path integral when compared to the contribution from the incident plane wave. For this boundary the downstream region has no laser field ($\Delta U_{downstream} = 0$), and from the field path integral approach one can expect a similar situation for the scattering and the absorbing boundaries, whose downstream region is effectively in the shadow of the incident laser beam. A transparent boundary is the only instance where the downstream region provides a significant interaction to the energy gain. In the limit of no reflection from the surface (such as from a Brewster angle configuration) $\Delta U_{upstream}$ and $\Delta U_{downstream}$ are equal in magnitude and differ only by an optical phase retardation factor φ_{ret} introduced by the boundary, and the particle's energy gain takes the form [3]

$$\Delta U_{\mathrm{P,L}} \sim \frac{qE_0\lambda}{\pi}\frac{\alpha}{\alpha^2 + 1/\gamma^2}\left\{\sin\varphi - \sin(\varphi_{ret} + \varphi)\right\} \tag{5}$$

An alternate interaction model that has been employed to analyze laser-acceleration from the reflective and transparent boundaries is the inverse-radiation interaction picture. It is also field-based and derives from conservation of energy. It is required that the electromagnetic field energy absorbed in the volume of interest, denoted here by $-\Delta U_{\mathrm{rad}}$, be equal to the mechanical work on the free particle plus the mechanical work on any other interacting objects in that volume of interest

$$-\Delta U_{\mathrm{rad}} = \Delta U_P + \Delta U_{\mathrm{M}} \tag{6}$$

For example, ΔU_{M} can represent the change of stored electromagnetic energy in a resonant accelerator cavity, the heating of an ohmic material, or the electronic excitation or chemical transformation of a medium. ΔU_P represents the change of kinetic energy of the free particle. Equation 6 is based on energy balance and describes the inverse-radiation picture of particle acceleration, where commonly the term ΔU_{M} is neglected and all the external electromagnetic energy is assumed to couple to the free particle. Laser-driven particle acceleration in most semi-open vacuum configurations is one such instance where this approximation is valid, and the interaction is

$$-\Delta U_{\text{rad}} = \Delta U_P \tag{7}$$

The minus sign in equation 7 indicates that laser energy has to flow into the volume of interest (be absorbed) when the free particle gains energy. This inverse radiation formulation of particle acceleration can be viewed as a consequence of Poynting's Theorem, and explicit evaluation of the electromagnetic field components reveals that $-\Delta U_{\text{rad}}$ can be expressed as an overlap integral of the particle and driving electromagnetic field plus another overlap integral of the particle's own radiated fields such that the free particle's energy change is described by

$$\Delta U_P = -\frac{1}{\pi Z_0}\int_{-\infty}^{\infty}\oint_S \left(\vec{E}_L(\omega)\cdot\vec{E}_W^{\,*}(\omega)\right) ds\,d\omega - \frac{1}{\pi Z_0}\int_{-\infty}^{\infty}\oint_S \left(\vec{E}_W(\omega)\cdot\vec{E}_W^{\,*}(\omega)\right) ds\,d\omega \tag{8}$$

where $\vec{E}_L$ is the electromagnetic field caused by the laser in the presence of the structure and $\vec{E}_W$ is the wake field from the particle in the presence of the structure. The first overlap integral describes the coupling between the external electromagnetic field and the partice's retarded field, and the second integral describes the energy lost by the particle as a result of interacting with the accelerator structure or medium. In the presence of mediating object such as a boundary equation 8 may become nonzero, allowing for a linear interaction between the external field and the particle. Thus the first overlap integral describes the laser-acceleration process in question and will be denoted by $\Delta U_{P,L}$ in equation 9. The second overlap integral of equation 8 describes the particle wakefield radiation process that already occurs independently of the laser, denoted by $\Delta U_{P,W}$ in equation 9. The total energy change of the particle beam is therefore

$$\Delta U_P = \Delta U_{P,L} + \Delta U_{P,W} \tag{9}$$

For semi-open vacuum laser-driven particle acceleration problems the applied laser field is expected to be large, such that $\left|\Delta U_{P,L}\right| >> \left|\Delta U_{P,W}\right|$. Therefore the wake field energy loss term is usually neglected and $\Delta U_P \sim \Delta U_{P,L}$. With this assumption equation 8 reduces to the common description for particle acceleration as an inverse-radiation process where the interaction strength scales linearly with the overlap integral between the external laser field and the particle's radiation wakefield pattern [4,5]

$$\Delta U_P \sim -\frac{1}{\pi Z_0}\int_{-\infty}^{\infty}\oint_S \left(\vec{E}_L(\omega)\cdot\vec{E}_W^{\,*}(\omega)\right) ds\,d\omega \tag{10}$$

As stated before this picture is based on a simple energy conservation argument. The absorbed electromagnetic energy causes an energy gain on the accelerator medium or structure and on the free particle. For the reflective, transparent and scattering surfaces the boundary does not absorb energy, and therefore all the absorbed electromagnetic energy is channeled into gain of kinetic energy of the free particle. For these boundaries we expect equations (7-10) to hold. For the reflective and the transparent boundaries the transition radiation components have been evaluated analytically and the overlap integrals for these cases have been found to match the respective path-integral energy gain formulas of equations 4 and 5 [2,3]. The following section will analyze the inverse-radiation picture of laser-electron interaction in the presence of a scattering-surface boundary.

II. THE ROUGH-SURFACE SCATTERING BOUNDARY

An explicit evaluation of the individual scattered laser or particle fields is not possible to perform for the scattering rough-surface boundary of Figure 1b because the details of the individual random scatterer boundaries are unknown. However, assuming the slippage distance between the laser and the electron is much longer than the boundary thickness one can still make an explicit evaluation of the overlap integral of equation 10, and the procedure for this is described in this section. Here we model the scattering layer as having a local linear susceptibility matrix $\tilde{\chi}(\vec{r}_\perp)$ that when subjected to an external field produces a local polarization

$$\vec{P}(\vec{r}_\perp) = \tilde{\chi}(\vec{r}_\perp)\vec{E}_{inc}(\vec{r}_\perp) \tag{11}$$

where $\vec{E}_{inc}(\vec{r}_\perp)$ is the local incident field. Hence the boundary can be regarded as a collection of local dipoles that radiate a new field with a distinctive far-field distribution that depends on both the specific properties of the boundary and on the polarization and angular distrubution of the incident field. Figure 2 is an illustration of the model of the boundary.

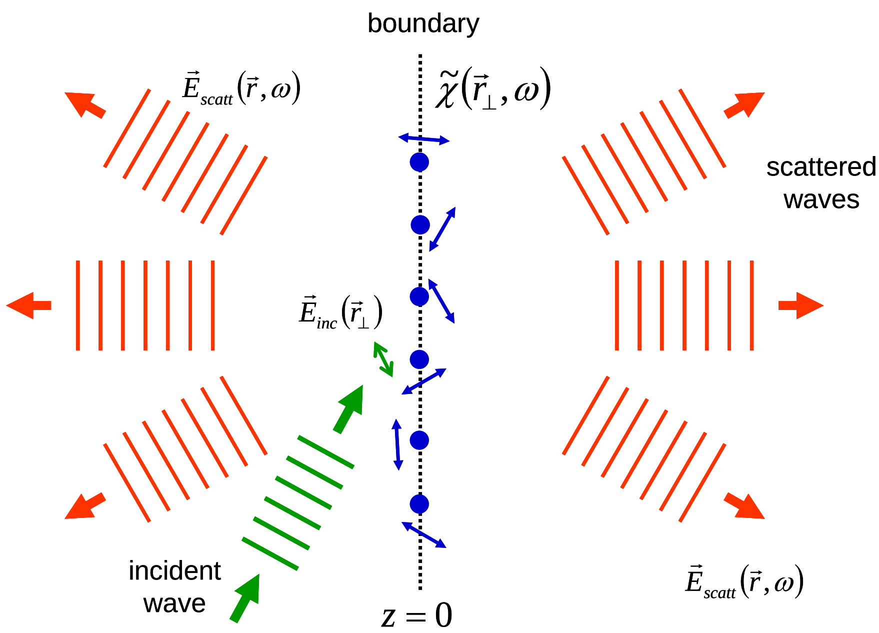

FIG 2. the boundary modeled as a thin scatterer of an incident plane wave into a spectrum of far-field plane waves.

As indicated in equation 11 a linear response is assumed. The incident and the scattered far field distributions can be described in terms of plane wave components with a specific direction of propagation $\vec{k}$ and a polarization $\hat{p}$. Denoting the amplitude of an incident plane wave as $\psi_i\left(\vec{k}_i,\hat{p}_i\right)$ and the amplitude of the scattered plane wave as $\psi_s\left(\vec{k}_s,\hat{p}_s\right)$ these waves can be related by a scattering matrix $\tilde{S}_{\chi(\omega)}\left(\vec{k}_s,\hat{p}_s;\vec{k}_i,\hat{p}_i\right)$, such that

$$\psi_s\left(\vec{k}_s,\hat{p}_s\right)=\sum_i \int_{\vec{k}_{\perp,i}} \tilde{S}_{\chi(\omega)}\left(\vec{k}_s,\hat{p}_s;\vec{k}_i,\hat{p}_i\right)\psi_i\left(\vec{k}_i,\hat{p}_i\right)d\vec{k}_i \tag{12}$$

where the integral extends over all $\vec{k}_i$ and the discrete sum includes the two possible polarizations $i=1,2$. In this notation a particular incident free-space plane wave with direction $\vec{k}_i$ and polarization $\vec{p}_i$ is described by

$$\vec{E}_{inc}\left(\vec{r}=\vec{r}_\perp+z\hat{z}\right)=\psi_i\left(\vec{k}_i,\hat{p}_i\right)e^{i\left(\vec{k}_{\perp,i}\cdot\vec{r}_\perp+k_{z,i}z\right)}\hat{p}_i \tag{13}$$

A similar expression holds for a scattered plane wave. The total scattered electromagnetic field is the sum of all plane wave components of equation 12, namely

$$\begin{aligned}\vec{E}_{scatt}\left(\vec{r}\right)&=\int_{\vec{k}_{\perp,s}}\psi_s\left(\vec{k}_s,\hat{p}_s\right)e^{i\left(\vec{k}_{\perp,s}\cdot\vec{r}_\perp+k_{z,s}z\right)}\hat{p}_s d\vec{k}_{\perp,s}\\&=\int_{\vec{k}_{\perp,s}}\sum_j\int_{\vec{k}_{\perp,i}}\tilde{S}_{\chi(\omega)}\left(\vec{k}_s,\hat{p}_s;\vec{k}_i,\hat{p}_{i,j}\right)\psi_i\left(\vec{k}_s,\hat{p}_s\right)e^{i\left(\vec{k}_{\perp,s}\cdot\vec{r}_\perp+k_{z,s}z\right)}\hat{p}_s d\vec{k}_i d\vec{k}_s\end{aligned} \tag{14}$$

To simplify these expressions it will be more convenient to describe the collection of the incident and scattered wave amplitudes as state vectors, such that

$$\left|\psi_s\left(\vec{k}_s,\hat{p}_s\right)\right\rangle=\tilde{S}_{\chi(\omega)}\left(\vec{k}_s,\hat{p}_s;\vec{k}_i,\hat{p}_i\right)\left|\psi_i\left(\vec{k}_i,\hat{p}_i\right)\right\rangle \tag{15}$$

Thus, for example, the scattering matrix for free space is simply the identity matrix, $\tilde{S}_{\chi(\omega)}=\tilde{I}$, such that the scattered and the incident plane wave spectra are the same; $\left|\psi_s\left(\vec{k}_s,\hat{p}_s\right)\right\rangle=\delta\left(\vec{k}_s,\vec{k}_i;\hat{p}_s,\hat{p}_i\right)\left|\psi_i\left(\vec{k}_i,\hat{p}_i\right)\right\rangle$. Next, assume a dielectric layer. Neglecting the relatively small reflection coefficients and assuming the plane wave spectrum has a small angular spread the scattering matrix of such a layer is a simple retardation factor of the form $\tilde{S}_{\chi(\omega)}=\tilde{I}e^{i\varphi_{ret}}$, where φ_{ret} is an optical phase retardation. For a high-reflector flat boundary the z-component of the plane wave reverses direction; $k_{z,s}=-k_{z,i}$, and there is also a polarization dependent phase change $R(\vec{p}_i)$ for the reflected wave, such that the scattering matrix has a form $\tilde{S}_{\chi(\omega)} = R(\hat{p}_i)\delta(\vec{k}_{\perp,s}, \vec{k}_{\perp,i})\delta(k_{z,s}, -k_{z,i})\delta(\hat{p}_s, \hat{p}_i)$. For more complex-shaped boundaries $\tilde{S}_{\chi(\omega)}$ will not have such a simple structure. However, since we are limiting our discussion to linear-response media time-reversal symmetry is expected to apply and thus $\tilde{S}_{\chi(\omega)}$ should reproduce the original incident field spectrum from the scattered fields if they are propagating backwards into the scattering layer;

$$\left\langle \psi_i\left(\vec{k}_i, \hat{p}_i\right)\right| = \left\langle \psi_s\left(\vec{k}_s, \hat{p}_s\right)\right| \tilde{S}^{\dagger}_{\chi(\omega)} \tag{16}$$

Equations 15 and 16 imply that $\tilde{S}_{\chi(\omega)}$ is unitary, that is, $\tilde{S}^{\dagger}_{\chi(\omega)} \tilde{S}_{\chi(\omega)} = \tilde{I}$. For free-space waves there is a redundancy in specifying all the components of $\vec{k}$ for the scattering matrix since $k_z = \pm\sqrt{(\omega/c)^2 - \vec{k}_{\perp}^2}$. Therefore for a given wavelength the scattering matrix can be described without specifying k_z, such that it takes the form

$$\tilde{S}_{\chi(\omega)} \equiv \tilde{S}_{\chi(\omega)}\left(\vec{k}_{\perp,s}, d_s, \hat{p}_s; \vec{k}_{\perp,i}, d_i, \hat{p}_i\right) \tag{17}$$

The terms d_s and d_i indicate the direction of the plane wave, which can have only two values; either upstream or downstream direction, e.g. $d = \pm 1$. Note that for an absorbing boundary $\tilde{S}_{\chi(\omega)}$ is no longer unitary. Radiation from such a boundary is a broadband blackbody spectrum where the phase and amplitude of the outgoing blackbody radiation waves are not linear functions of the input wave. Hence an absorbing boundary is poorly described by this formalism.

To proceed with the analysis for the laser-electron interaction from a linear boundary the scattering process of the particle's fields from the boundary needs to be described. A key assumption in the analysis presented here is that the local susceptibility $\tilde{\chi}(\vec{r}_{\perp})$ of the scattering plane does not distinguish between the electric field from a free-space wave or from a particle retarded field. Figure 3 illustrates the scattering model for the particle field components. The retarded electron field also produces a scattered plane wave spectrum where the only difference is that the incident plane wave spectrum from the particle field is not free-space, but the scattered wave spectrum still is. Since $\tilde{\chi}(\vec{r}_{\perp})$ is assumed not to differentiate between these fields a plane wave from the electron field with a given $\vec{k}_{\perp,i}$ and $\vec{p}_i$ will experience the same scattering matrix coefficient as the equivalent free-space wave. This incident field differs from the free-space counterpart by a phase-slippage factor $e^{i\Delta k_z z_b}$ because the electron field moves with the electron that creates it at a subluminal velocity (k_z is smaller). Since the boundary is assumed to be much thinner than the slippage distance all the locations of the boundary observe nearly the same phase offset. Thus $e^{i\Delta k_z z_b}$ is only an overall common phase factor for the entire plane wave spectrum of the particle field, and in figure 3 $z_b = 0$. The other difference to free space waves is a nonzero longitudinal component to the particle's field, but as shown later this component becomes negligibly small compared to the transverse component for relativistic particles.

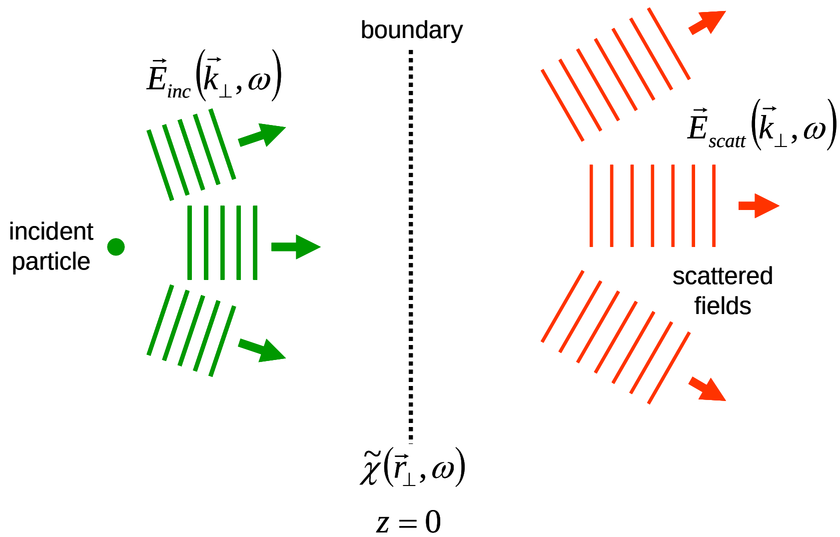

FIG 3: scattering of the "bound" particle fields into a spectrum of free-space plane waves by the dielectric scattering boundary.

With these assumptions in mind the particle's retarded field plane wave spectrum $\left|\psi_i^p(\vec{k}_{\perp,i})\right\rangle$ is scattered by the same boundary matrix $\tilde{S}_{\chi(\omega)}$ into a free-space plane wave spectrum, that is,

$$\left|\psi_s(\vec{k}_{\perp,s})\right\rangle = \tilde{S}_{\chi(\omega)}(\vec{k}_{\perp,s}, d_s, \hat{p}_s; \vec{k}_{\perp,i}, d_i, \hat{p}_i)\left|\psi_i^p(\vec{k}_{\perp,i})\right\rangle e^{i\Delta k_z z_b} \tag{18}$$

The superscript "p" in $\left|\psi_i^p(\vec{k}_{\perp,i})\right\rangle$ indicates that this free-space field spectrum was generated by the electron field. The factor $e^{i\Delta k_z z_b}$ takes into account the phase shift at the boundary due to the slower phase velocity of the particle's waves. Besides this phase factor there is one additional key difference to the scattering of purely free-space waves; since $\tilde{S}_{\chi(\omega)}$ was defined for scattering of free-space waves equation 18 effectively creates an artificial free-space plane wave that co-propagates with the incident particle field and that has to be subtracted out. Therefore the scattering matrix of the boundary on the electron field has the form

$$\left|\psi_s(\vec{k}_{\perp,s})\right\rangle = \left(\tilde{S}_{\chi(\omega)}(\vec{k}_{\perp,s}, d_s, \hat{p}_s; \vec{k}_{\perp,i}, d_i, \hat{p}_i) - \delta(\vec{k}_{\perp,s}, d_s, \hat{p}_s; \vec{k}_{\perp,i}, d_i, \hat{p}_i)\right)\left|\psi_i^p(\vec{k}_{\perp,i})\right\rangle e^{i\Delta k_z z_b} \tag{19}$$

The second term becomes more obvious when we consider the scattering of the electron field from free space. As described before the free-space scattering matrix is the identity matrix $\tilde{S}_{\chi(\omega)} = \tilde{I}$, which should not produce any additional free-space wave. With this scattering matrix equation 19 yields a scattered free-space spectrum of the form $\left|\psi_s\left(\vec{k}_{\perp,s}\right)\right\rangle = \left(\delta\left(\vec{k}_{\perp,s}, d_s, \vec{p}_s; \vec{k}_{\perp,i}, d_i, \vec{p}_i\right) - \delta\left(\vec{k}_{\perp,s}, d_s, \vec{p}_s; \vec{k}_{\perp,i}, d_i, \vec{p}_i\right)\right)\left|\psi_i^p\left(\vec{k}_{\perp,i}\right)\right\rangle e^{i\Delta k_z z_b} = 0$, which is consistent with a free, uniformly moving particle not producing any free-space radiation. To understand the effect of the second term further consider a flat perfect reflector as a scatterer. For a high reflector the matrix $\tilde{S}_{\chi(\omega)}$ in equation 19 reflects the electron fields into the upstream space producing the backward transition radiation pattern that is radiated into the upstream space, while the delta function in equation 19 produces the forward transition free-space radiation pattern that has the same polarization and angular distribution characteristics as the electron plane wave spectrum.

To quantify the electron-laser interaction with this scattering boundary formalism we need to evaluate the electron's retarded field in terms of a plane wave spectrum, which is described by [6]

$$\vec{E}_p\left(\vec{k}, \omega\right) = 2\pi i q Z_0 c \frac{\beta\omega\hat{z}/c - \vec{k}}{k^2 - \omega^2/c^2} \delta\left(\omega - c\beta\hat{z}\cdot\vec{k}\right) \tag{20}$$

As stated before since this field is originated by the particle it also includes longitudinal components, whereas in the model presented here the scattering matrix acts only on transverse field components from free-space waves. For a relativistic particle β is very close to unity, and with this assumption $\vec{E}_p\left(\vec{k}, \omega\right)$ can readily be decomposed into a longitudinal component expressed in terms of the transverse k-vector component $\vec{k}_\perp$. Notice that the delta function in equation 20 establishes that for the particle fields the z-component of the k-vector is $k_z = \omega/\beta c$. The electromagnetic field in equation 20 can be decomposed into a longitudinal and a transverse component. The longitudinal component is given by

$$\vec{E}_{p,\parallel}\left(\vec{k}_\perp, \omega\right) \sim \frac{-2\pi i q Z_0 c}{k_0 \gamma^2} \frac{\hat{n}}{\left(k_\perp/k_0\right)^2 + 1/\gamma^2} \tag{21}$$

and a transverse component is

$$\vec{E}_{p,\perp}\left(\vec{k}_\perp, \omega\right) \sim \frac{-2\pi i q Z_0 c}{k_0} \frac{\left(k_\perp/k_0\right)}{\left(k_\perp/k_0\right)^2 + 1/\gamma^2} \hat{r} \tag{22}$$

where $\hat{n}$ and $\hat{r}$ describe the longitudinal and radial polarizations. These are given by

$$\hat{n}\left(\vec{k}_\perp\right)=\frac{k_\perp\hat{q}+k_z\hat{z}}{\sqrt{\vec{k}_\perp\cdot\vec{k}_\perp+k_z^{\ 2}}}\ ,\ \hat{r}\left(\vec{k}_\perp\right)=\frac{-k_\perp\hat{z}+k_z\hat{q}}{\sqrt{\vec{k}_\perp\cdot\vec{k}_\perp+k_z^{\ 2}}} \tag{23}$$

We can appreciate that for both longitudinal and transverse field components the amplitudes become very small for $\left|k_\perp\right|/k_0 >> 1/\gamma$. This means that for relativistic particles $(\gamma >> 1)$ the bulk of the plane wave spectrum of the particle's field is concentrated at very shallow angles to the particle's direction of propagation. Furthermore notice that at the $1/\gamma$-angle the magnitudes of the components scale as $\left|\vec{E}_{p,\|}\right|=\left|\vec{E}_{p,\perp}\right|/\gamma$ and therefore the longitudinal component $\vec{E}_{p,\|}$ can be neglected. One can conclude that for a relativistic particle its plane-wave spectrum is described fairly accurately by the transverse field components alone and has an amplitude described by

$$\psi_i^p\left(\vec{k}_{\perp,i},\vec{p}_i\right)=\frac{-2\pi iqZ_0\,c/k_0\left(\left|\vec{k}_{\perp,i}\right|/k_0\right)}{\left(\left|\vec{k}_{\perp,i}\right|/k_0\right)^2+1/\gamma^2}\delta\left(\hat{p}_i,\hat{r}\right) \tag{24}$$

Now an explicit evaluation of the overlap integral of inverse-radiation energy gain picture of equation 10 can be performed. In terms of the scattered field amplitudes the field overlap integral of equation 10 takes the form

$$\Delta U_{P,L}\sim-\frac{1}{\pi Z_0}\left\langle\psi_s^{laser}\middle|\psi_s^{particle}\right\rangle \tag{25}$$

The key is to rewrite this overlap integral of scattered fields in terms of the incident fields by use of the scattering matrix. Thus equation 25 can be transformed by rewriting the overlap integral in the following way;

$$\left\langle\psi_s^{laser}\middle|\psi_s^{particle}\right\rangle=\left\langle\psi_i^{laser}\middle|S^{*T}\left(S-I\right)\middle|\psi_i^{particle}\right\rangle=\left\langle\psi_i^{laser}\middle|\psi_i^{particle}\right\rangle-\left\langle\psi_i^{laser}\middle|S^{*T}\middle|\psi_i^{particle}\right\rangle \tag{26}$$

The S^{*T} matrix results from the transformation of the free-space laser field while the matrix $(S-I)$ is a consequence of the transformation of the particle field, which as discussed earlier in not free-space. Notice that the first overlap integral, $\left\langle\psi_i^{laser}\middle|\psi_i^{particle}\right\rangle$, is completely independent of the properties of scattering layer. Only the second overlap integral shows a dependence on the scattering layer. Here we look at the following cases:

First: there is no scattering layer, such that $\tilde{S}=\tilde{I}$. In this case the total interaction described in equation 26 simplifies to

$$\Delta U_{P,L}=\frac{-1}{\pi Z_0}\left(\left\langle\psi_i^{laser}\middle|\psi_i^{particle}\right\rangle-\left\langle\psi_i^{laser}\middle|\tilde{I}\middle|\psi_i^{particle}\right\rangle\right)=0 \tag{27}$$

This could be regarded as a confirmation of the Lawson-Woodward theorem. There is no overlap between the scattered laser and scattered particle fields in equation 25 and hence the interaction has to be zero.

Second: the interaction surface is a strong "random" scatterer that couples into a large spectrum of k-vectors. For the second term in equation 26, $\left\langle \psi_i^{laser} \middle| \tilde{S}^{*T} \middle| \psi_i^{particle} \right\rangle$, this means that $\left\langle \psi_i^{laser} \middle| \tilde{S}^{*T}\right.$ will have a wide angular spectrum compared to $\left| \psi_i^{particle} \right\rangle$, and therefore in the limit of strong random scattering $\left\langle \psi_i^{laser} \middle| \tilde{S}^{*T} \middle| \psi_i^{particle} \right\rangle \sim 0$. Therefore equation 26 simplifies to

$$\Delta U_{P,L} \sim \frac{-1}{\pi Z_0} \left\langle \psi_i^{laser} \middle| \psi_i^{particle} \right\rangle \tag{28}$$

Note that equation 28 is a function of the input field components alone. If the laser beam is a plane wave at angle α with respect to the electron propagation axis and has a polarization vector $\hat{P}$ it is described by $\vec{E} = E_0 \hat{P} \cos\left(\vec{k}_\alpha \vec{r} - \omega_0 t - \varphi\right)$, where $\omega_0 = ck_0$. This corresponds to an angular plane wave spectrum of the form

$$\psi_i^{laser}(\theta, \phi, \hat{p}_i) = E_0 \left(\hat{p}_i \cdot \hat{P}\right) \frac{\delta(\theta - \alpha)\delta(\phi)}{\sin\theta} \frac{1}{2} \left\{\delta(\omega - ck_0) e^{i\varphi} + \delta(\omega + ck_0) e^{-i\varphi}\right\} \tag{29}$$

Because of the delta functions of the far field angle in equation 29 the only particle plane wave $\psi_i^p\left(\vec{k}_{\perp,i}, \vec{p}_i\right)$ from equation 24 that overlaps with the laser satisfies the conditions $k_\perp / k_0 = \sin\alpha$ and $\omega = \omega_0$. Thus the radiation field overlap integral of equation 10 takes the form

$$\begin{aligned} \Delta U_{P,L} = -\frac{1}{\pi Z_0} \frac{E_0 2\pi i q Z_0 c}{2k_0} \oint_\Omega \frac{\theta}{\theta^2 + 1/\gamma^2} \frac{\delta(\theta - \alpha)\delta(\phi)}{\sin\theta} \left(\hat{r}(\theta, \phi) \cdot \hat{P}\right) \sin\theta d\theta d\phi \\ \times \int_{-\infty}^{+\infty} \left\{\delta(\omega - ck_0) e^{i\varphi} + \delta(\omega + ck_0) e^{-i\varphi}\right\} d\omega \end{aligned} \tag{30}$$

The delta functions simplify equation 30 to

$$\Delta U_{P,L} = -\frac{1}{\pi Z_0} \frac{E_0 2\pi i q Z_0 c}{2k_0} \frac{\alpha}{\alpha^2 + 1/\gamma^2} \left(\hat{r}(\alpha, 0) \cdot \hat{P}\right) \left\{c e^{i\varphi} - c e^{-i\varphi}\right\} \tag{31}$$

The product $\left(\hat{r}(\alpha, 0) \cdot \hat{P}\right)$ denotes the polarization overlap of the laser with respect to the particle's field, which is radially polarized. If we introduce the polarization angle $\cos\rho = \hat{r}(\alpha, 0) \cdot \hat{P}$ equation 31 simplifies further to

$$\Delta U_{P,L} = \frac{q\lambda E_0}{\pi} \frac{\alpha}{\alpha^2 + 1/\gamma^2} \cos\rho \sin\varphi \tag{32}$$

With equation 32 having the identical form to equation 10 we confirm that when applied to the scattering boundary the inverse radiation picture yields the same expected interaction as the field path integral approach.

Third: the scattering surface is a simple phase retarder such as a transparent boundary. Suppose that for the given input laser field spectrum the scattered has the transmission matrix $\tilde{S} = \tilde{I}e^{i\phi}$. Then the total interaction becomes

$$\Delta U_{P,L} = \frac{-1}{\pi Z_0}\left(\left\langle \psi_i^{laser} \middle| \psi_i^{particle} \right\rangle - \left\langle \psi_i^{laser} \middle| \tilde{I}e^{i\phi} \middle| \psi_i^{particle} \right\rangle\right) = \frac{-1}{\pi Z_0}\left\langle \psi_i^{laser} \middle| \psi_i^{particle} \right\rangle \left(1 - e^{i\phi}\right) \tag{33}$$

So depending on the amount of optical delay $\Delta U_{\max} = 2\left\langle \psi_i^{laser} \middle| \psi_i^{particle} \right\rangle$ when $\phi = \pi, 3\pi, ...$ and $\Delta U_{\min} = 0$ when $\phi = 0, 2\pi, ...$

Fourth: Finally, for the high reflector $\tilde{S}$ maps $\left| \psi_i^{laser} \right\rangle$ into a reflected spectrum $\left| \psi_r^{laser} \right\rangle$ that travels in another direction, such that $\left\langle \psi_i^{laser} \middle| \psi_r^{laser} \right\rangle \sim 0$. Hence equation 26 can be rewritten as

$$\Delta U_{P,L} = -\frac{1}{\pi Z_0}\left(\left\langle \psi_i^{laser} \middle| \psi_i^{particle} \right\rangle - \left\langle \psi_r^{laser} \middle| \psi_i^{particle} \right\rangle\right) \tag{34}$$

If the laser is interacting with the particle in the upstream region of the reflector boundary $\left| \psi_i^{laser} \right\rangle$ and $\left| \psi_i^{particle} \right\rangle$ are propagating in the same direction and have a large overlap integral $\left\langle \psi_i^{laser} \middle| \psi_i^{particle} \right\rangle$, while $\left\langle \psi_r^{laser} \middle| \psi_i^{particle} \right\rangle$ is zero, and we thus obtain

$$\Delta U_{P,L} = -\frac{1}{\pi Z_0}\left\langle \psi_i^{laser} \middle| \psi_i^{particle} \right\rangle \tag{35}$$

On the other hand consider a situation where the input laser is counter-propagating to the particle beam in the downstream space. If the boundary is a high-reflector the scattered laser field co-propagates with the particle field. Thus in equation 34 $\left\langle \psi_i^{laser} \middle| \psi_i^{particle} \right\rangle$ is zero while the second term is nonzero. In this situation the laser-electron interaction is described by the overlap integral of the reflected laser beam with the particle field. In such a downstream arrangement the laser-electron interaction becomes sensitive to the properties of the boundary.

$$\Delta U_{P,L} = +\frac{1}{\pi Z_0}\left\langle \psi_r^{laser} \middle| \psi_i^{particle} \right\rangle \tag{36}$$

To summarize, all the non-absorbing thin boundary cases can be understood in terms of the scattering matrix formalism presented here. As mentioned earlier, the absorbing boundary requires a different treatment that takes into account the absorption of electromagnetic energy from the boundary itself.

III. THE ABSORBING BOUNDARY

For the absorbing boundary equations 7-10 are no longer applicable, and equation 6 has to be employed to account for the energy absorbed by the medium. Assume the boundary fully absorbs the electromagnetic energy incident on it, such that ΔU_{rad} corresponds to the energy of the laser flowing into the volume of interest. Next, the energy absorbed by the medium, denoted by ΔU_M, has to be expressed in terms of the electromagnetic fields acting on it. Assume a medium with a local linear conductivity $\rho(\vec{r})$, such that the absorbed energy is

$$\Delta U_M = \int_\tau \int_M \vec{E}(\vec{r}) \cdot \vec{J}(\vec{r}) d^3 r dt = \int_\tau \int_M \rho(\vec{r}) \vec{E}(\vec{r})^2 d^3 r dt \tag{37}$$

The total electric field in the medium is the sum of the laser and the particle field components, hence equation 37 becomes

$$\Delta U_M = \int_\tau \int_M \rho(\vec{r}) \left(\vec{E}_L + \vec{E}_W\right)^2 d^3 r dt = \int_\tau \int_M \rho(\vec{r}) \left(\vec{E}_L{}^2 + 2\vec{E}_L \cdot \vec{E}_W + \vec{E}_W{}^2\right) d^3 r dt \tag{38}$$

The absorbed energy gain in the medium has three contributions; heat deposition caused by the laser alone; $\Delta U_{M,L}$, heat deposition caused by the particle wakefields; $\Delta U_{M,W}$, and finally heat deposition in the presence of both the laser and the particle that appears as a field overlap term; $\Delta U_{M,O}$. To identify these contributions in equation 38 consider the situation where only the laser and the boundary are present. In such an instance we can identify ΔU_{rad} in equation 38 with the laser power absorbed in the medium, such that

$$\Delta U_{\mathrm{rad}} = -\Delta U_{M,L} = -\int_\tau \int_M \rho(\vec{r}) \vec{E}_L{}^2 d^3 r dt \tag{39}$$

Next, consider the opposite case where the free particle interacts with the boundary in the absence of the laser. Applying the same reasoning we find that the particle's kinetic energy lost from heat deposited by its own wakefields in the medium corresponds to the third term in the sum in equation 38. Since ΔU_{rad} is zero in this instance the energy balance reads

$$\Delta U_{P,W} = -\Delta U_{M,W} = \int_{\tau}\int_{M} \rho(\vec{r})\vec{E}_W{}^2 d^3 r dt \tag{40}$$

Therefore remaining field overlap term in equation 38 has to correspond to the particle's energy change in the presence of the laser and in the presence of the boundary;

$$\Delta U_{P,L} = -2\int_{\tau}\int_{M} \rho(\vec{r})\vec{E}_L \cdot \vec{E}_W d^3 r dt \tag{41}$$

Substituting the fields by currents with $\vec{J}(\vec{r}) = \rho(\vec{r})\vec{E}(\vec{r})$, and replacing the conductivity by a resistivity coefficient $\sigma(\vec{r}) = 1/\rho(\vec{r})$ equation 38 becomes

$$\Delta U_{P,L} = -2\int_{\tau}\int_{M} \sigma(\vec{r})\vec{J}_L \cdot \vec{J}_W d^3 r dt \tag{42}$$

Thus in the presence of an absorbing boundary the particle energy gain is equivalent to the current overlap integral of the laser induced current and the particle wakefield induced current in the medium multiplied by the local impedance value. Assuming that the external laser and particle wakefields are absorbed within a very thin distance Δz without producing a reflection (impedance matching) the local conductivity has to be $\rho(\vec{r})\Delta z = 1/Z_0$. A derivation for this is given in appendix 1. This simplifies equation 41 to

$$\Delta U_{P,L} = -\frac{2}{Z_0}\int_{\tau}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \vec{E}_L \cdot \vec{E}_W dx dy dt \tag{43}$$

When rewriting the time dependent fields to the frequency domain equation 43 transforms to the now familiar inverse-radiation field overlap expression of equation 10. $\vec{E}_L$ and $\vec{E}_W$ refer to the incident laser and particle wakefields and are therefore completely independent of the properties of the absorbing boundary.

VI. CONCLUSIONS AND OUTLOOK

The models for the scattering and absorbing boundaries employed here show agreement between the inverse-radiation and the field path integral calculation method for laser particle acceleration in a semi-free space geometry. The laser-electron interaction strength for both boundaries is expected to be similar to that from a flat high-reflector boundary because the interaction of the relativistic electron with the laser field is mostly determined by the incident laser beam. This laser beam component is co-propagating with the electron and thus possesses a slippage distance that extends over many wavelengths. Furthermore the incident laser beam does not depend on the properties of the boundary. The reflected and scattered laser beam components on the other hand do strongly depend on the properties of the boundary. However since most of these waves do not co-propagate with the electron beam and thus only possess very short slippage distances. Furthermore for a strong scatterer these wave components add their acceleration contribution incoherently, and hence their collective interaction becomes negligible when compared to the contribution from the incident laser beam.

The absorbing boundary was modeled as a linear ohmic-loss medium, and under these circumstances an extended inverse-radiation picture could be written down to describe the laser-electron interaction in the presence of such a boundary. Notice that this model for an absorbing boundary is specific to an electromagnetic wave at normal incidence to the surface. The boundary conditions that yield an impedance matching for the normal incidence wave described in appendix 1 change with the incidence angle of the laser wave on the medium, and thus the linear ohmic loss model does not present a fully generalized representation for a "perfect" electromagnetic absorber. This immediately motivates the quest for an alternate description of the laser-electron interaction in the presence of an ideal blackbody object, which will be the subject of an upcoming analysis for linear laser-electron interactions in a semi-open geometry.

APPENDIX 1

The impedance from an absorbing boundary for a free-space wave incident on it is derived. The boundary is assumed to cover an infinite area and to absorb all the radiation within a very thin layer of width Δz. Further, the boundary is assumed to have a uniform conductivity ρ. Assume that in the free space next to the boundary there is an electromagnetic field with an incident and a reflected wave component;

$$\begin{aligned} \vec{E}_{ext} &= \vec{E}_{inc} + \vec{E}_{refl} \\ \vec{B}_{ext} &= \vec{B}_{inc} + \vec{B}_{refl} \end{aligned} \qquad \text{A1}$$

Assume these are plane wave components at normal incidence with respect to the boundary. The average current along a stripe of width w along the surface layer Δz has to be

$$\vec{I}_M = (\rho \cdot \Delta z \cdot w)\vec{E}_M \qquad \text{A2}$$

where $\vec{E}_M$ is the driving electric field that has to be equal to the incident field;

$$\vec{E}_M = \vec{E}_{inc} \qquad \text{A3}$$

By Ampere's law the magnetic field in the free-space region that is produced from that current is

$$w\vec{B}_M = \mu_0\left(\hat{n} \times \vec{I}_M\right) = (\mu_0 \cdot \rho \cdot \Delta z \cdot w)\left(\hat{n} \times \vec{E}_M\right) \qquad \text{A4}$$

where as shown in figure 10 it was assumed that if the absorber is perfect inside the medium $\vec{B}_M = 0$. If there is to be no reflection $\vec{E}_M$ and $\vec{B}_M$ have to match the incident fields, and hence $\left|\vec{B}_M\right| / \left|\vec{E}_M\right| = 1/c$. With this condition and with A3 and A4 one finds

$$\frac{\left|\vec{B}_M\right|}{\left|\vec{E}_M\right|} = \mu_0 \rho \cdot \Delta z = \frac{1}{c} \qquad \text{A5}$$

Therefore $\rho \cdot \Delta z = \sqrt{\mu_0 \varepsilon_0} / \mu_0$, which simplifies to $\rho \cdot \Delta z = 1/Z_0$.

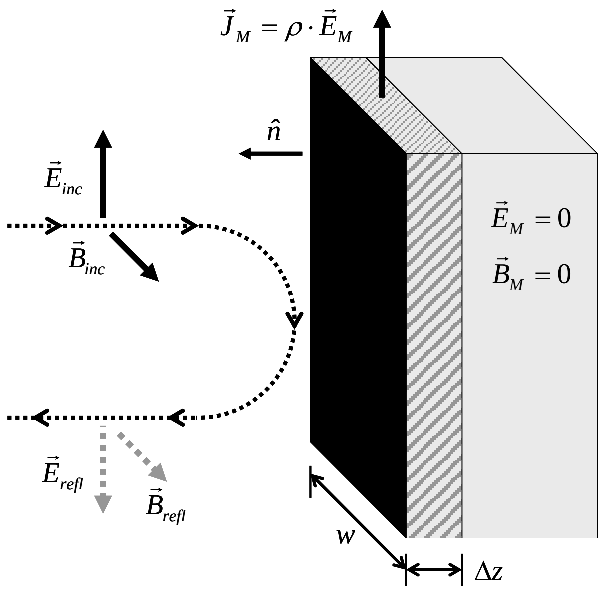

FIG 4. Diagram of a perfect absorber surface

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