

EXAMINATION OF FLAVOR SU(3) IN B , $B_s \rightarrow K\pi$ DECAYS ¹

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We update a discussion of the relation between the weak phase γ and the rates and CP asymmetries of several $K\pi$ decays of B^+ , B^0 , and B_s . We emphasize the impact of measurements of $B_s \rightarrow K\pi$. Current data indicate large SU(3) breaking in the strong phases or failure of factorization (including its application to penguin amplitudes) in $K\pi$ modes of B^0 and B_s . SU(3) and factorization only remain approximately valid if the branching ratio for $B_s \rightarrow K^-\pi^+$ exceeds its current value of $(5.00 \pm 1.25) \times 10^{-6}$ by at least 50%, or if a parameter ξ describing ratios of form factors and decay constants is shifted from its nominal value by more than twice its estimated error.

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Several methods have been proposed to measure the Cabibbo-Kobayashi-Maskawa (CKM) phase γ from B meson decays into DK final states [1, 2, 3] and in charmless strange final states using flavor SU(3) symmetry [4, 5, 6, 7, 8]. Ref. [9] proposed using $B^+ \rightarrow K^0\pi^+$, $B^0 \rightarrow K^+\pi^-$, and $B_s \rightarrow K^-\pi^+$, the last two related by the U-spin symmetry $d \leftrightarrow s$, to obtain γ . (A recent analysis employing this method is described in Ref. [10].) Ignoring $\mathcal{O}(\lambda^2)$ terms in the $B^\pm \rightarrow K^0\pi^\pm$ decay amplitude,³ where $\lambda = 0.2257$ [12, 13], γ is obtained from the ratios of decay widths.

The decay width of the $B_s \rightarrow K^-\pi^+$ mode has recently been measured [14]. In this paper, we include the newly observed decay width along with direct CP asymmetries in $B^0 \rightarrow K^+\pi^-$ and $B_s \rightarrow K^-\pi^+$ to solve for γ , the strong phases, and the ratio

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³Ref. [11] illustrates the effect of a $\mathcal{O}(\lambda^2)$ term from the penguin amplitude, but a color-suppressed penguin amplitude of the same order is not included.

between tree and penguin amplitudes. We find in general a two-fold ambiguity in the solutions for weak and strong phases. Moreover, we find a large SU(3)-breaking effect either between the strong phases or between the magnitudes of strangeness-conserving and strangeness-changing amplitudes, given the present experimental situation [14].

We review the method proposed in Ref. [9]. Employing U-spin symmetry, the decay amplitudes of the relevant modes are

$$A(B^+ \rightarrow K^0 \pi^+) = P, \quad (1)$$

$$A(B^0 \rightarrow K^+ \pi^-) = T e^{i(\delta_d + \gamma)} + P, \quad (2)$$

$$\xi A(B_s \rightarrow K^- \pi^+) = \frac{1}{\tilde{\lambda}} T e^{i(\delta_s + \gamma)} - \tilde{\lambda} P, \quad (3)$$

where the explicit t -quark dependence is removed using CKM unitarity. Here T and P denote “tree” and “penguin” amplitudes, proportional to the CKM factors $V_{us}^* V_{ub}$ and $V_{cs}^* V_{cb}$, respectively. The parameter $\tilde{\lambda} \equiv |V_{us}/V_{ud}| \simeq 0.2317$ using $\lambda = 0.2257$ [13] and $V_{ud} = \sqrt{1 - \lambda^2}$. We also include an overall SU(3)-breaking factor

$$\xi \equiv \frac{f_K F_{B^0 \pi}(m_K^2) m_{B^0}^2 - m_\pi^2}{f_\pi F_{B_s K}(m_\pi^2) m_{B_s}^2 - m_K^2} \quad (4)$$

according to the factorization assumption for the amplitudes.⁴ Its value is $0.97_{-0.11}^{+0.09}$ [13, 15], corresponding to almost exact SU(3).⁵ This should be compared with global fits done within flavor SU(3) [16, 17], which associated the breaking factor $f_K/f_\pi \simeq 1.2$ with tree-type amplitudes only. In that case, the predicted branching ratios of the $B_s \rightarrow K^- \pi^+$ and $K^+ K^-$ modes [17] agreed with the later experimental measurements. The relative strong phases between T and P are denoted by δ_d and δ_s for $B^0 \rightarrow K^+ \pi^-$ and $B_s \rightarrow K^- \pi^+$, respectively. For consistency, terms of $\mathcal{O}(\tilde{\lambda}^2)$ have been ignored in these amplitudes. Since interactions directly involving the spectator quark are expected to be dynamically suppressed, we also ignore their contributions.

Consider the charge-averaged ratios [9]

$$R_d \equiv \frac{\Gamma(B^0 \rightarrow K^+ \pi^-) + \Gamma(\bar{B}^0 \rightarrow K^- \pi^+)}{\Gamma(B^+ \rightarrow K^0 \pi^+) + \Gamma(B^- \rightarrow \bar{K}^0 \pi^-)}, \quad (5)$$

$$R_s \equiv \frac{\Gamma(B_s \rightarrow K^- \pi^+) + \Gamma(\bar{B}_s \rightarrow K^+ \pi^-)}{\Gamma(B^+ \rightarrow K^0 \pi^+) + \Gamma(B^- \rightarrow \bar{K}^0 \pi^-)}, \quad (6)$$

and the CP-violating rate pseudo-asymmetries:

$$A_d \equiv \frac{\Gamma(\bar{B}^0 \rightarrow K^- \pi^+) - \Gamma(B^0 \rightarrow K^+ \pi^-)}{\Gamma(B^- \rightarrow \bar{K}^0 \pi^-) + \Gamma(B^+ \rightarrow K^0 \pi^+)} = R_d A_{CP}(B^0 \rightarrow K^+ \pi^-), \quad (7)$$

$$A_s \equiv \frac{\Gamma(\bar{B}_s \rightarrow K^+ \pi^-) - \Gamma(B_s \rightarrow K^- \pi^+)}{\Gamma(B^- \rightarrow \bar{K}^0 \pi^-) + \Gamma(B^+ \rightarrow K^0 \pi^+)} = R_s A_{CP}(B_s \rightarrow K^- \pi^+). \quad (8)$$

⁴This includes the assumption that the penguin and tree amplitudes scale in the same way. The consequence of relaxing this assumption will be explored.

⁵We have assumed a vector dominance pole model to extrapolate the form factors from the $q^2 = 0$ point computed in Ref. [15].

Observable	Exp. Value	Ref.
$\mathcal{B}(B^+ \rightarrow K^0 \pi^+)$	23.1 ± 1.0	[18]
$\mathcal{B}(B^0 \rightarrow K^+ \pi^-)$	19.4 ± 0.6	[18]
$A_{CP}(B^0 \rightarrow K^+ \pi^-)$	-0.097 ± 0.012	[18]
$\mathcal{B}(B_s \rightarrow K^- \pi^+)$	5.00 ± 1.25	[14]
$A_{CP}(B_s \rightarrow K^- \pi^+)$	0.39 ± 0.17	[14]

Table I: Experimental values of observables used in this analysis. Branching ratios are charge-averaged and in units of 10^{-6} . To convert their ratios to those of rates we use the lifetime ratios [18] $\tau(B^+)/\tau(B^0) = 1.071 \pm 0.009$ and $\tau(B_s)/\tau(B^0) = 0.939 \pm 0.021$.

Defining the ratio $r \equiv T/P$, we derive

$$R_d = 1 + r^2 + 2r \cos \gamma \cos \delta_d, \quad (9)$$

$$\xi^2 R_s = \tilde{\lambda}^2 + \left(\frac{r}{\tilde{\lambda}}\right)^2 - 2r \cos \gamma \cos \delta_s, \quad (10)$$

$$A_d = 2r \sin \gamma \sin \delta_d, \quad (11)$$

$$\xi^2 A_s = -2r \sin \gamma \sin \delta_s. \quad (12)$$

Here we have ignored very small phase space differences. Eqs. (11) and (12) imply a simple relation between the strong phases:

$$\frac{\sin \delta_d}{\sin \delta_s} = -\frac{A_d}{\xi^2 A_s} = -\frac{R_d A_{CP}(B^0 \rightarrow K^+ \pi^-)}{\xi^2 R_s A_{CP}(B_s \rightarrow K^- \pi^+)}. \quad (13)$$

Numerically, this ratio is 0.96 ± 0.54 according to the data in Table I.

First, we consider the SU(3) limit where $\delta_d = \delta_s \equiv \delta$. In this case, γ and δ always appear in the combinations $\cos \gamma \cos \delta$ and $\sin \gamma \sin \delta$ in Eqs. (9), (10), (11) and (12). This set of equations has the discrete symmetries (i) $\gamma \leftrightarrow \delta$ and r invariant; (ii) $\gamma \rightarrow \gamma + \pi$, $\delta \rightarrow \delta + \pi$, and r invariant; (iii) $\gamma \rightarrow \gamma + \pi$, $r \rightarrow -r$, and δ invariant; and (iv) $\delta \rightarrow \pi - \delta$, $\gamma \rightarrow \pi - \gamma$, and r invariant. The amplitude ratio r is negative according to the factorization assumption. In the following analysis, we therefore consider only solutions with $0 \leq \gamma \leq 90^\circ$ and $r < 0$. This still leaves the two-fold ambiguity (i) mentioned above.

Eqs. (9) and (10) give the absolute value of the ratio between the redefined tree and penguin amplitudes

$$|r| = \tilde{\lambda} \sqrt{\frac{R_d + \xi^2 R_s}{1 + \tilde{\lambda}^2} - 1}. \quad (14)$$

Using the experimental inputs listed in Table I, we have $R_d = 0.899 \pm 0.048$, $R_s = 0.247 \pm 0.063$, $A_d = 0.087 \pm 0.012$, and $A_s = -0.096 \pm 0.049$. Eq. (14) implies $|r| \simeq 0.063 \pm 0.036$ with the SU(3) breaking factor ξ included. If ξ is set to (1, 1.2), $|r|$ increases to $(0.069 \pm 0.029, 0.101 \pm 0.026)$. The condition $R_d < 1$ demands $r \cos \gamma \cos \delta < 0$ according to Eq. (9).

The $B^0 \rightarrow K^+\pi^-$ and $B_s \rightarrow K^-\pi^+$ rate asymmetries satisfy the relation

$$\Gamma(B_s \rightarrow K^-\pi^+) - \Gamma(\overline{B}_s \rightarrow K^+\pi^-) = -\frac{1}{\xi^2} [\Gamma(B^0 \rightarrow K^+\pi^-) - \Gamma(\overline{B}_d \rightarrow K^-\pi^+)] \quad (15)$$

by U-spin symmetry. We can thus use $A_{CP}(B^0 \rightarrow K^+\pi^-)$ to predict $A_{CP}(B_s \rightarrow K^-\pi^+) \simeq 0.37 \pm 0.13$. This is consistent with the measured value in Table I.

As $\mathcal{B}(B^+ \rightarrow K^0\pi^+)$ and $\mathcal{B}(B^0 \rightarrow K^+\pi^-)$ have been determined to about 5%, their current central values are not likely to vary much in the future. In contrast, \mathcal{B} and A_{CP} of $B_s \rightarrow K^-\pi^+$ have only been measured by the CDF Collaboration for the first time. The quoted value of $\mathcal{B}(B_s \rightarrow K^-\pi^+)$ [14] depends on a value of the fragmentation function $f_s/(f_u + f_d) = 0.160 \pm 0.005_{-0.010}^{+0.011+0.057}$ [19], slightly higher than the Particle Data Group average [20] $f_s/(f_u + f_d) = 0.135 \pm 0.015$ (see also Ref. [21].) In the following, we discuss the dependence of solutions on the central value of $\mathcal{B}(B_s \rightarrow K^-\pi^+)$. As δ_s has been fixed to be the same as δ_d , we omit $A_{CP}(B_s \rightarrow K^-\pi^+)$ from the fit and predict its value from the fit parameters. Errors and other measurements are kept at their current values.

Fig. 1 shows the dependence of r on $\mathcal{B}(B_s \rightarrow K^-\pi^+)$; the χ^2 for the fit to R_d , R_s , and $A_{CP}(B^0 \rightarrow K^+\pi^-)$; and the predicted $A_{CP}(B_s \rightarrow K^-\pi^+)$. For $\mathcal{B}(B_s \rightarrow K^-\pi^+) \gtrsim 7.5 \times 10^{-6}$, a solution with strong phases satisfying exact SU(3) can be obtained, as indicated by the vanishing χ_{\min}^2 value. This can be attributed to an overall SU(3) breaking factor of at least $\xi \simeq 1.2$, 2.5σ from the central value 0.97 given by factorization and echoing the observation in Ref. [17] mentioned above. For $\mathcal{B}(B_s \rightarrow K^-\pi^+) < 7.5 \times 10^{-6}$, one cannot obtain a satisfactory solution if $\delta_s = \delta_d$. In that case, the value $|r| = 0.063 \pm 0.036$ from Eq. (14) is too small to account for R_d and R_s . The value of $|r|$ is increased to 0.100 (or larger) if we increase R_s by a factor 1.5 (or larger). With such values of r one may obtain the central value of R_d and a suitable value of R_s using Eqs. (9) and (10). This is the essence of the need for either a larger $\mathcal{B}(B_s \rightarrow K^-\pi^+)$ or a larger ξ . Indeed, Fleischer [10] obtained a solution with $\delta_s - \delta_d \simeq 10^\circ$ by increasing R_d and R_s by 1σ .

Current data thus call for SU(3) breaking in amplitudes at the level of 20% or very different strong phases. As shown in Fig. 1, both r and $A_{CP}(B_s \rightarrow K^-\pi^+)$ decrease with increasing $\mathcal{B}(B_s \rightarrow K^-\pi^+)$. These conclusions are qualitatively unchanged if we allow δ_d and δ_s to differ by $\lesssim 10^\circ$ for small SU(3) breaking.

We show the dependence of γ and δ on $\mathcal{B}(B_s \rightarrow K^-\pi^+)$ in Fig. 2. Their values coincide with each other for small values of $\mathcal{B}(B_s \rightarrow K^-\pi^+)$, and start to split into three curves when it is greater than 7.5×10^{-6} . This occurs when χ_{\min}^2 becomes zero for the upper (solid) and lower (dashed) branches. For the dash-dotted branch in the middle, γ and δ still coincide with each other and continue to decrease with $\mathcal{B}(B_s \rightarrow K^-\pi^+)$. The χ_{\min}^2 values along this branch are small but non-vanishing, corresponding to a ‘‘saddle’’ region in parameter space. The upper and lower branches can represent either γ or δ due to the $\gamma \leftrightarrow \delta$ symmetry. However, the weak phase given by the solid curve is more consistent with other analyses. In that case, the corresponding strong phase is given by the dashed curve. As shown in the plot, γ (δ) grows (decreases) monotonically with $\mathcal{B}(B_s \rightarrow K^-\pi^+)$ above the fork point.

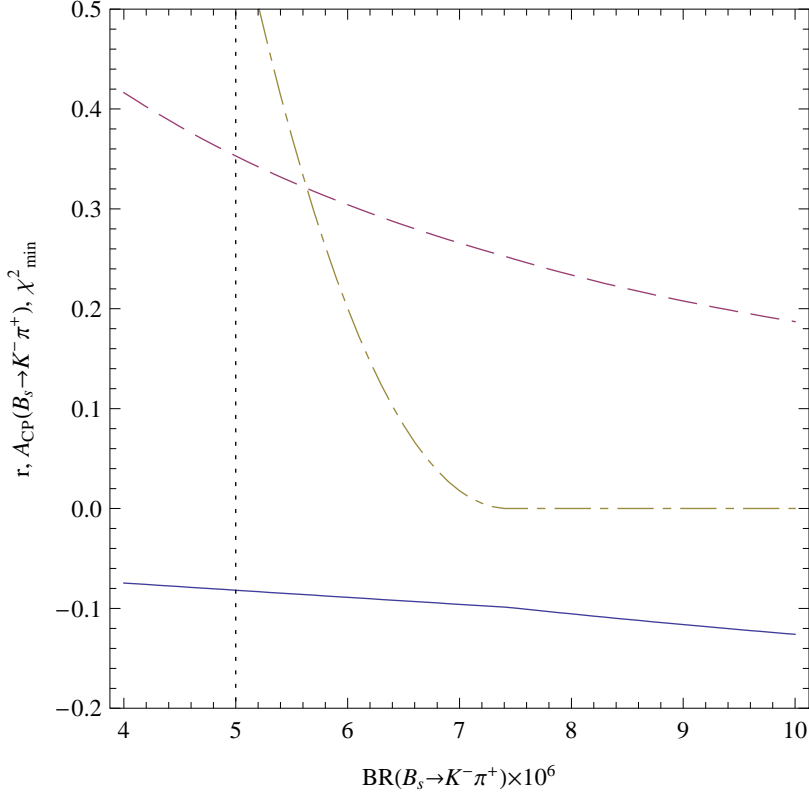


Figure 1: Behavior of solutions as a function of $\mathcal{B}(B_s \rightarrow K^- \pi^+)$, assuming $r < 0$ and $\delta_d = \delta_s \equiv \delta$. The solid, dashed, and dot-dashed curves represent r , preferred $A_{CP}(B_s \rightarrow K^- \pi^+)$, and χ_{\min}^2 , respectively. The vertical dotted line indicates the current central value of $\mathcal{B}(B_s \rightarrow K^- \pi^+)$.

We now let $\delta_d \neq \delta_s$, permitting a test of the SU(3) symmetry assumption. With four observables R_d , R_s , $A_{CP}(B^0 \rightarrow K^+ \pi^-)$, and $A_{CP}(B_s \rightarrow K^- \pi^+)$, one can solve for all four parameters r , γ , δ_d and δ_s in the decay amplitudes.

As shown in Fig. 3, there are two sets of possible solutions (left and right) as a function of $\mathcal{B}(B_s \rightarrow K^- \pi^+)$. For the solution on the left, even though γ falls within the expected range, δ_d and δ_s differ significantly from each other. For the solution on the right, the strong phases are also quite different and γ is too small when $\mathcal{B}(B_s \rightarrow K^- \pi^+) \lesssim 6.5 \times 10^{-6}$. However, when $\mathcal{B}(B_s \rightarrow K^- \pi^+) \geq 7.5 \times 10^{-6}$, γ becomes reasonable, δ_d is between 20° and 30° , and δ_s approaches 50° . As the current measurement of the CP asymmetry of $B_s \rightarrow K^- \pi^+$ has an error over 40%, we expect it to have a weaker constraint on the parameters, δ_s . For the current data, two solutions are found, corresponding to the parameters:

$$\begin{aligned}
 (r, \gamma, \delta_d, \delta_s) &= (-0.126, 60^\circ, 24^\circ, 155^\circ), \\
 (r, \gamma, \delta_d, \delta_s) &= (-0.123, 25^\circ, 59^\circ, 117^\circ).
 \end{aligned}
 \tag{16}$$

In the former, γ is more consistent with results using other methods (for example, adding information based on $B^0 \rightarrow \pi^+ \pi^-$ [22]), and a small strong phase δ_d as

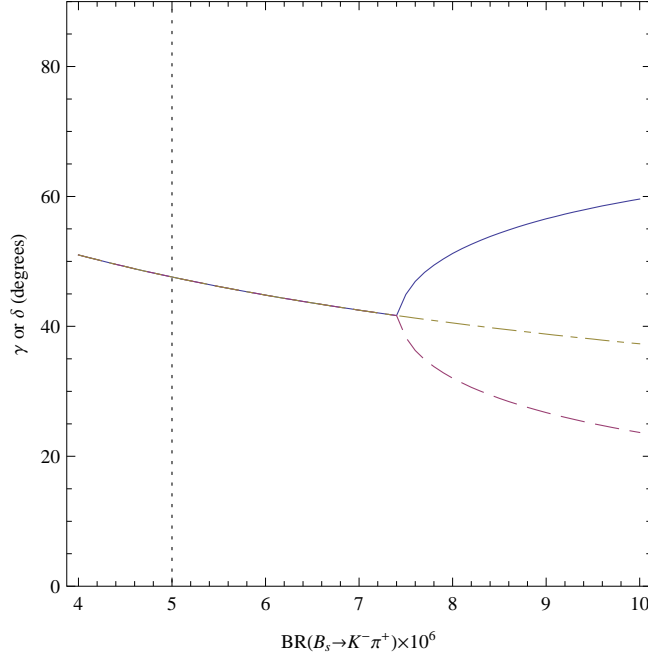


Figure 2: Solutions as function of $\mathcal{B}(B_s \rightarrow K^- \pi^+)$, for $r < 0$ and $\delta_d = \delta_s \equiv \delta$. The fork point corresponds to $\mathcal{B}(B_s \rightarrow K^- \pi^+) \simeq 7.5 \times 10^{-6}$. The solid and dashed curves represent γ and δ , respectively, as preferred by other analyses. A saddle point solution with $\delta_s = \delta_d$ and small nonzero χ^2 is indicated by the dash-dotted curve. The vertical dotted line indicates the current central value of $\mathcal{B}(B_s \rightarrow K^- \pi^+)$.

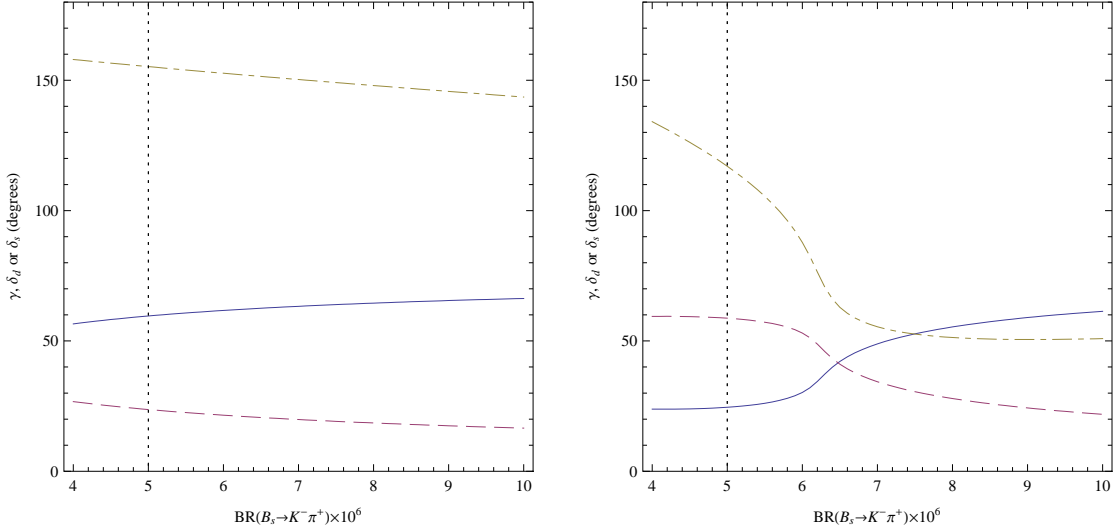


Figure 3: Behavior of solutions as a function of $\mathcal{B}(B_s \rightarrow K^- \pi^+)$, assuming $r < 0$. There are two sets of solutions (left and right) when δ_d and δ_s are treated as independent parameters. The solid, dashed and dash-dotted curves represent γ , δ_d and δ_s , respectively. The vertical dotted line indicates the current central value of $\mathcal{B}(B_s \rightarrow K^- \pi^+)$.

expected in perturbative QCD [23, 24]. However, the strong phase δ_s in both solutions is unexpectedly large. The 1σ ranges around the former are

$$-0.141 \lesssim r \lesssim -0.108, \quad 47^\circ \lesssim \gamma \lesssim 72^\circ. \quad (17)$$

The result for $|r|$ here is larger than that from Eq. (14) with $\delta_d = \delta_s$.

Even though we no longer have the symmetries between the weak and strong phases mentioned before because of the introduction of an additional strong phase δ_s , we still obtain two possible solutions roughly corresponding to $\gamma \leftrightarrow \delta_d$. Within this set of observables, it is impossible to resolve the two-fold ambiguity without resorting to some other methods or observables.

For the solutions in Eq. (16), δ_s is very different from δ_d , contrary to the SU(3) symmetry assumption. More likely possibilities are a B_s branching ratio larger than the current value or a value of ξ larger than the factorization estimate given above. These alternatives are impossible to distinguish from one another as the parameters ξ and R_s always appear in the combination $\xi^2 R_s$ [even in $\xi^2 A_s = \xi^2 R_s A_{CP}(B_s \rightarrow K^- \pi^+)$]. A larger left-hand side of Eq. (10) would entail $\cos \delta_s > 0$ rather than the current situation, permitting δ_s to be closer to δ_d . With $\xi = 1.2$, one would obtain a solution $r = -0.11$, $\gamma = 53^\circ$, $\delta_d = 30^\circ$, and $\delta_s = 52^\circ$. The reason that $\delta_s - \delta_d$ is still as large as 22° is because of the pull from $A_{CP}(B_s \rightarrow K^- \pi^+)$. As shown in Fig. 1, a smaller asymmetry is preferred if one hopes to have $\delta_s \simeq \delta_d$.

Even though one often assumes the same SU(3) breaking factor for the tree and penguin amplitudes, they can *a priori* scale differently. Denote the scaling factors associated with T and P by ξ_T and ξ_P , respectively. By fixing $\xi_T = \xi$ and allowing ξ_P to vary around 1, we find that for $\xi_P > 1.2$ the strong phase δ_s can lie in the first quadrant, but is still too large ($> 75^\circ$). The weak phase γ also falls below 50° in this case. However, if we fix $\xi_P = \xi$ instead and vary ξ_T , the solution improves with increasing ξ_T . Taking $\xi_T = 1.5$ as an example, we find $r = -0.123$, $\gamma = 59^\circ$, $\delta_d = 25^\circ$, and $\delta_s = 42^\circ$. This shows that the scaling behavior of T plays a more dominant role.

Next, we allow both ξ_T and ξ_P to vary by including $\gamma = (67.6 \pm 4.5)^\circ$ [25] obtained from other methods as another observable constraint. We find that if $\delta_s - \delta_d \gtrsim 20^\circ$, it is possible to obtain a perfect fit to the data. In these cases, the preferred values of r , γ , and δ_d become fixed at -0.182 , 67.6° , and 15° .

The preferred values of ξ_T and ξ_P as a function of $\delta_s - \delta_d$ are shown in Fig. 4. When $\delta_s - \delta_d \lesssim 20^\circ$, no perfect solution exists. But the most favored ξ_T increases linearly with the strong phase difference, while ξ_P stays almost as a constant. If the equality between the two strong phases is imposed, we find $\chi_{\min}^2 = 0.82$ with $r = -0.114$, $\gamma = 67^\circ$, $\delta_d = \delta_s = 26^\circ$, $\xi_T = 1.63$, and $\xi_P = 0.77$. When $\delta_s - \delta_d \gtrsim 20^\circ$, ξ_T drops while ξ_P increases.

In Table II we compare pairs of solutions for $\mathcal{B}(B_s \rightarrow K^- \pi^+) = 7.5 \times 10^{-6}$ and 10^{-5} with those for the current value $\mathcal{B}(B_s \rightarrow K^- \pi^+) = (5.00 \pm 1.25) \times 10^{-4}$, keeping the same 25% error. As $\mathcal{B}(B_s \rightarrow K^- \pi^+)$ increases, the values of γ and those of δ_d in the two solutions become closer to each other. However, the values of δ_s remain significantly different from δ_d .

To summarize, the U-spin relation between $B^0 \rightarrow K^+ \pi^-$ and $B_s \rightarrow K^- \pi^+$ [9] has been utilized to obtain a range of values of the CKM phase γ , thanks to new data on

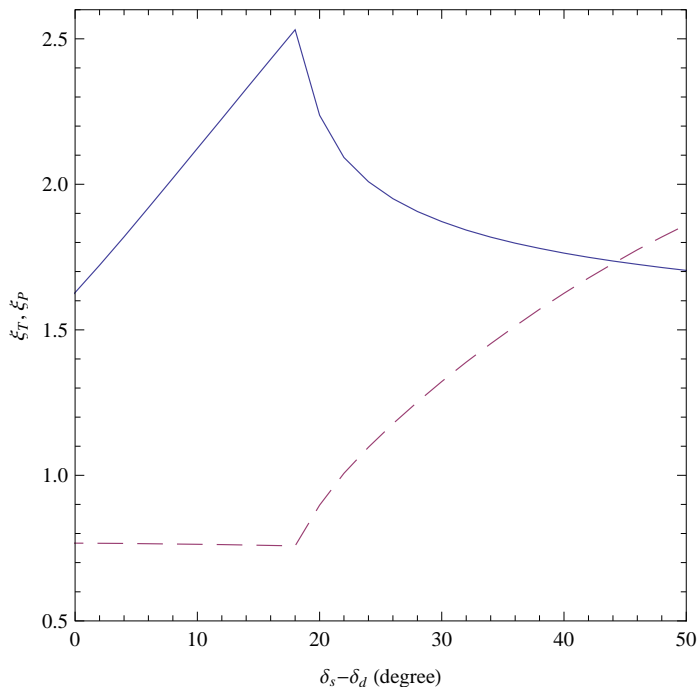


Figure 4: Dependence of preferred values of ξ_T (solid) and ξ_P (dashed) on the strong phase difference $\delta_s - \delta_d$.

Table II: Comparison of solutions for various values of $\mathcal{B}_s \equiv \mathcal{B}(B_s \rightarrow K^- \pi^+)$

\mathcal{B}_s (10^{-6})	Solution 1				Solution 2			
	r	γ	δ_d	δ_s	r	γ	δ_d	δ_s
5.0	-0.126	60°	24°	155°	-0.123	25°	59°	117°
7.5	-0.148	64°	19°	149°	-0.108	53°	31°	53°
10.0	-0.167	66°	17°	144°	-0.133	61°	22°	51°

the decay $B_s \rightarrow K^- \pi^+$ obtained by the CDF Collaboration [14]. Values of γ consistent with other determinations and strong phases δ_d and δ_s not differing substantially from one another may be obtained only if the branching ratio $\mathcal{B}(B_s \rightarrow K^- \pi^+)$ is at least 50% larger than its currently quoted value of $(5.00 \pm 1.25) \times 10^{-6}$, or if the parameter ξ [Eq. (4)] describing the ratio of decay constants and form factors is more than about 1.2 (vs. its nominal value of $0.97_{-0.11}^{+0.09}$).

For the nominal values of $\mathcal{B}(B_s \rightarrow K^- \pi^+)$ and ξ , one obtains a solution with a two-fold ambiguity, whose value of γ in the solution closer to other determinations (using such processes as $B^0 \rightarrow \pi^+ \pi^-$ [22]) is $\simeq 60^\circ$. In this solution, however, the strong phases are $\delta_d \simeq 23^\circ$ and $\delta_s \simeq 155^\circ$. The latter is inconsistent with perturbative QCD calculations and its large difference from δ_d would signal significant SU(3) breaking or failure of factorization. Solutions with smaller SU(3) breaking, such as those which would result if $\mathcal{B}(B_s \rightarrow K^- \pi^+)$ were at least 50% larger than its nominal

value, would be suggested if recent measurements of b quark fragmentation [19] had overestimated the fraction of b quarks which end up as B_s . Alternatively, the SU(3) breaking factor ξ could be larger than estimated, or could differ in tree and penguin amplitudes. Further studies of the $B_s \rightarrow K^- \pi^+$ decay and b fragmentation at the Fermilab Tevatron and at LHCb may help to illuminate this question.

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