

# On Condensates in Strongly Coupled Gauge Theories

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We analyze quark and gluon condensates in quantum chromodynamics. We suggest that these are localized inside hadrons, because the particles whose interactions are responsible for them are confined within these hadrons. This can explain the results of recent studies of gluon condensate contributions to vacuum correlators. We also give a general discussion of condensates in asymptotically free vectorial and chiral gauge theories.

PACS numbers: 11.15.-q, 11.30.Rd, 12.38.-t

Hadronic condensates play an important role in quantum chromodynamics (QCD). Two important examples are  $\langle \bar{q}q \rangle \equiv \langle \sum_{a=1}^{N_c} \bar{q}_a q^a \rangle$  and  $\langle G_{\mu\nu} G^{\mu\nu} \rangle \equiv \langle \sum_{a=1}^{N_c^2-1} G_{\mu\nu}^a G^{a\mu\nu} \rangle$ , where  $q$  is a light quark (i.e., a quark with current-quark mass small compared with the confinement scale),  $G_{\mu\nu}^a = \partial_\mu A_\nu^a - \partial_\nu A_\mu^a + g_s c_{abc} A_\mu^b A_\nu^c$ ,  $a, b, c$  denote the color indices, and  $N_c = 3$ . For QCD with  $N_f$  light quarks, the  $\langle \bar{q}q \rangle = \langle \bar{q}_L q_R + \bar{q}_R q_L \rangle$  condensate spontaneously breaks the global chiral symmetry  $SU(N_f)_L \times SU(N_f)_R$  down to the diagonal, vectorial subgroup  $SU(N_f)_{diag}$ , where  $N_f = 2$  (or  $N_f = 3$  if one includes the  $s$  quark). (Pre-QCD studies of spontaneous chiral symmetry breaking,  $S\chi SB$ , include [1].) In an otherwise massless theory, the  $\langle G_{\mu\nu} G^{\mu\nu} \rangle$  condensate breaks dilatation invariance. Conventionally, these condensates are considered to be properties of the QCD vacuum and hence to be constant throughout spacetime [2].

Some insight into spontaneous chiral symmetry breaking in QCD was obtained via an approximate solution of the Schwinger-Dyson equation for a massless quark propagator; if the running coupling  $\alpha_s = g_s^2/(4\pi)$  exceeds a value of order 1, this yields a nonzero dynamical (constituent) quark mass  $\Sigma$  [3]. Since in the path integral,  $\Sigma$  is formally a source for the operator  $\bar{q}q$ , one associates  $\Sigma \neq 0$  with a nonzero quark condensate (related studies of  $S\chi SB$  include [4]-[10]). However, this Schwinger-Dyson equation, by itself, does not determine where this condensate has spatial support and does not imply that it is a spacetime constant.

Here we analyze the condensates  $\langle \bar{q}q \rangle$  and  $\langle G_{\mu\nu} G^{\mu\nu} \rangle$  and, in particular, the question of where they have spatial and temporal support. We suggest that their spatial support is restricted to the interiors of hadrons, since these condensates are due to quark and gluon interactions, and these particles are confined within hadrons. Higher-order condensates such as  $\langle (\bar{q}q)^2 \rangle$ ,  $\langle (\bar{q}q) G_{\mu\nu} G^{\mu\nu} \rangle$ , etc. are also present, and our discussion implicitly also applies to these [11]. We first emphasize the subtlety in characterizing the formal quantity  $\langle 0|\mathcal{O}|0 \rangle$ , where  $\mathcal{O}$  is a product of quantum field operators, by recalling that one can render this automatically zero by normal-ordering  $\mathcal{O}$ . This subtlety is especially delicate in a confining theory, since the vacuum state in such a theory is not defined relative to the fields in the Lagrangian, quarks and gluons,

but to the actual physical, color-singlet, states.

A formulation using a Euclidean path-integral (vacuum-to-vacuum amplitude),  $Z$ , provides a precise meaning for  $\langle \mathcal{O} \rangle$  as  $\langle \mathcal{O} \rangle = \lim_{J \rightarrow 0} (\delta \ln Z / \delta J)$ , where  $J$  is a source for  $\mathcal{O}$ . The path integral for QCD, integrated over quark fields and gauge links using the gauge-invariant lattice discretization exhibits a formal analogy with the partition function for a statistical mechanical system. In this correspondence, a condensate such as  $\langle \bar{q}q \rangle$  or  $\langle G_{\mu\nu} G^{\mu\nu} \rangle$  is analogous to an ensemble average in statistical mechanics [12]. To develop a physical picture of the QCD condensates, we pursue this analogy. In a superconductor, the electron-phonon interaction produces a pairing of two electrons with opposite spins and 3-momenta at the Fermi surface, and, for  $T < T_c$ , an associated nonzero Cooper pair condensate  $\langle ee \rangle_T$  [16], (here  $\langle \dots \rangle_T$  means thermal average). Since this condensate has a phase, the phenomenological Ginzburg-Landau free energy function  $F = |\nabla\Phi|^2 + c_2(\Phi^*\Phi) + c_4(\Phi^*\Phi)^2$  uses a complex scalar field  $\Phi$  to represent it. The formal treatment of a phase transition such as that in a superconductor begins with a partition function calculated for a finite  $d$ -dimensional lattice, and then takes the thermodynamic (infinite-volume) limit. The non-analytic behavior associated with the superconducting phase transition only occurs in this infinite-volume limit; for  $T < T_c$ , the (infinite-volume) system develops a nonzero value of the order parameter, namely  $\langle \Phi \rangle_T$ , in the phenomenological Ginzburg-Landau model, or  $\langle ee \rangle_T$ , in the microscopic Bardeen-Cooper-Schrieffer theory. In the formal statistical mechanics context, the minimization of the  $|\nabla\Phi|^2$  term implies that the order parameter is a constant throughout the infinite spatial volume.

However, the infinite-volume limit is an idealization; in reality, superconductivity is experimentally observed to occur in finite samples of material, such as Sn, Nb, etc., and the condensate clearly has spatial support only in the volume of these samples. This is evident from either of two basic properties of a superconducting substance, namely, (i) zero-resistance flow of electric current, and (ii) the Meissner effect, that  $|\mathbf{B}(z)| \sim |\mathbf{B}(0)|e^{-z/\lambda_L}$  for a magnetic field  $\mathbf{B}(z)$  a distance  $z$  inside the superconducting sample, where the London penetration depth  $\lambda_L$  is given by  $\lambda_L^2 = m_e c^2 / (4\pi n e^2)$  ( $n$  = electron con-

*Submitted to Physical Review Letters*

Work supported in part by US Department of Energy contract DE-AC02-76SF00515

centration); both of these properties clearly hold only within the sample. The same statement applies to other phase transitions such as liquid-gas or ferromagnetic; again, in the formal statistical mechanics framework, the phase transition and associated symmetry breaking by a nonzero order parameter at low  $T$  occur only in the thermodynamic limit, but experimentally, one observes the phase transition to occur effectively in a finite volume of matter, and the order parameter (e.g., magnetization  $M$ ) has support only in this finite volume, rather than the infinite volume considered in the formal treatment. Similarly, the Goldstone modes that result from the spontaneous breaking of a continuous symmetry (e.g., spin waves in a Heisenberg ferromagnet) are experimentally observed in finite-volume samples. There is, of course, no conflict between the experimental measurements and the abstract theorems; the key point is that these samples are large enough for the infinite-volume limit to be a useful idealization.

This condensed-matter physics helps to motivate our suggestion for QCD. In our picture, the spatial support for QCD condensates is where the particles are whose interactions give rise to them, just as the spatial support of a magnetization  $M$ , say, is inside, not outside, of a piece of iron. The physical origin of the  $\langle \bar{q}q \rangle$  condensate in QCD can be argued to be due to the reversal of helicity (chirality) of a massless quark as it moves outward and reverses its three-momentum at the boundary of a hadron due to confinement [7]. This argument implies that the condensate has support only within the spatial extent where the quark is confined; i.e., the physical size of a hadron. Another way to infer this is to note that in the light-front Fock state picture of hadron wavefunctions [17], a valence quark can flip its chirality when it interacts or interchanges with the sea quarks of multi-quark Fock states, thus providing a dynamical origin for the quark running mass. In this description, the  $\langle \bar{q}q \rangle$  and  $\langle G_{\mu\nu}G^{\mu\nu} \rangle$  condensates are effective quantities which originate from  $q\bar{q}$  and gluon contributions to the higher Fock state light-front wavefunctions of the hadron and hence are localized within the hadron. Similarly, approaches to  $S\chi SB$  based on the use of an effective instanton operator [6] lead one to infer that the  $S\chi SB$  occurs in the region where these instantons affect quark propagation, i.e., the interior of hadrons, where these quarks are confined.

The Anti-De Sitter/conformal field theory (AdS/CFT) correspondence between string theory in AdS space and CFT's in physical spacetime has been used to obtain an analytic, semi-classical model for strongly-coupled QCD which has scale invariance and dimensional counting at short distances and color confinement at large distances [18]. Color confinement can be imposed by introducing hard-wall boundary conditions at  $z = 1/\Lambda_{QCD}$  ( $z = \text{AdS}$  fifth dimension) or by modification of the AdS metric. This AdS/QCD model gives a good representation of the mass spectrum of light-quark mesons and baryons as well as the hadronic wavefunctions [19]. One can also study the propagation of a scalar field  $X(z)$  as a model for the

dynamical running quark mass [19]. The AdS solution has the form [20]  $X(z) = a_1 z + a_2 z^3$ , where  $a_1$  is proportional to the current-quark mass. The coefficient  $a_2$  scales as  $\Lambda_{QCD}^3$  and is the analog of  $\langle \bar{q}q \rangle$ ; however, since the quark is a color nonsinglet, the propagation of  $X(z)$ , and thus the domain of the quark condensate, is limited to the region of color confinement.

The AdS/QCD picture of effective confined condensates is in general agreement with results from chiral bag models [21], which modify the original MIT bag by coupling a pion field to the surface of the bag in a chirally invariant manner. Since explicit breaking of  $SU(2)_L \times SU(2)_R$  chiral symmetry is small, and hence  $m_\pi$  is small relative to typical hadronic scales like  $m_\rho$  or  $m_N$ , these condensates can be treated as approximately constant throughout much of the volume of a hadron. In each of these pictures, the QCD condensates actually have spatial support extending out a distance of order  $1/m_\pi$  around hadrons.

Several studies have reported values of  $\langle (\alpha_s/\pi) G_{\mu\nu}G^{\mu\nu} \rangle$  from the vacuum-to-vacuum current correlators relevant to  $e^+e^- \rightarrow$  charmonium and hadronic  $\tau$  decays [13]-[15]. In the pioneering work on QCD sum rules [13] the authors obtained an estimate  $\simeq 0.01 \text{ GeV}^4$ . Some recent values (in  $\text{GeV}^4$ ) include  $0.006 \pm 0.012$  [15](a),  $0.009 \pm 0.007$  [15](b), and  $-0.015 \pm 0.008$  [15](c). These values show significant scatter and even differences in sign. Our explanation would be that the gluon condensate is confined within hadrons, rather than extending throughout all of space, as would be true of a vacuum condensate.

In our picture, the QCD condensates should be considered as contributing to the masses of the hadrons where they are located. This is clear, since, e.g., a proton subjected to a constant electric field will accelerate and, since the condensates move with it, they comprise part of its mass. Similarly, when a hadron decays to a non-hadronic final state, such as  $\pi^0 \rightarrow \gamma\gamma$ , the condensates in this hadron contribute their energy to the final-state photons. Thus, over long times, the dominant regions of support for these condensates would be within nucleons, since the proton is effectively stable (with lifetime  $\tau_p \gg \tau_{univ} \simeq 1.4 \times 10^{10} \text{ yr.}$ ), and the neutron can be stable when bound in a nucleus. In a process like  $e^+e^- \rightarrow$  hadrons, the formation of the condensates occurs on the same time scale as hadronization. In accord with the Heisenberg uncertainty principle, these QCD condensates also affect virtual processes occurring over times  $t \lesssim 1/\Lambda_{QCD}$ .

Our suggestion implies that condensates  $\langle \bar{q}q \rangle$  in different hadrons may be chirally rotated with respect to each other, somewhat analogous to disoriented chiral condensates in heavy-ion collisions [22]. Our suggestion can, in principle, be verified by careful lattice gauge theory measurements. Note that the lattice measurements that have inferred nonzero values of  $\langle \bar{q}q \rangle$  were performed in finite volumes [9], although these were usually considered as approximations to the infinite-volume limit.

Having discussed QCD, we next consider, as an exercise, how our observation would apply to several hypothetical asymptotically free gauge theories. We begin with a vectorial gauge theory with the gauge group  $SU(N_c)$ , allowing  $N_c$  to be generalized to values  $N_c \geq 3$ . First, consider a theory of this type with no fermions, so that only  $\langle G_{\mu\nu}G^{\mu\nu} \rangle$  need be considered. This condensate would then have support within the interior of the glueballs. Second, consider a theory with  $N_f = 1$  massless or light fermion transforming according to some nonsinglet representation  $R$  of  $SU(N_c)$ . The  $\langle \bar{q}q \rangle$  and  $\langle G_{\mu\nu}G^{\mu\nu} \rangle$  condensates in this theory would have support in the interior of the mesons, baryons, and glueballs (or mass eigenstates formed from glueballs and mesons). Here, the condensate  $\langle \bar{q}q \rangle$  does not break any non-anomalous global chiral symmetry, so there would not be any Nambu-Goldstone boson (NGB). In both of these theories, the sizes of the mesons, baryons, and glueballs are  $\simeq 1/\Lambda$ , where  $\Lambda$  is the confinement scale.

We next consider asymptotically free chiral gauge theories (which are free of gauge and global anomalies) with massless fermions transforming as representations  $\{R_i\}$  of the gauge group. The properties of strongly coupled theories of this type are not as well understood as those of vectorial gauge theories [23]-[25]. One possibility is that, as the energy scale decreases from large values and the associated running coupling  $g$  increases, it eventually becomes large enough to produce a (bilinear) fermion condensate, which thus breaks the initial gauge symmetry [25]. This is expected to form in the most attractive channel (MAC)  $R_1 \times R_2 \rightarrow R_{cond.}$ , which maximizes the quantity  $\Delta C_2 = C_2(R_1) + C_2(R_2) - C_2(R_{cond.})$ , where  $C_2(R)$  is the quadratic Casimir invariant. Depending on the theory, several stages of self-breaking may occur [25, 26]. Let us consider an explicit model of this type, with gauge group  $SU(5)$  and massless left-handed fermion content consisting of an antisymmetric rank-2 tensor representation,  $\psi_L^{ij}$ , and a conjugate fundamental representation,  $\chi_{i,L}$ . This theory is asymptotically free and has a formal  $U(1)_\psi \times U(1)_\chi$  global chiral symmetry; both  $U(1)$ 's are broken by  $SU(5)$  instantons, but the linear combination  $U(1)'$  generated by  $Q = Q_\psi - 3Q_\chi$  is preserved. The MAC for condensation is  $10 \times 10 \rightarrow \bar{5}$ , with  $\Delta C_2 = 24/5$ , and the associated condensate is  $\langle \epsilon_{ijkln} \psi_L^{jk} {}^T C \psi_L^{\ell n} \rangle$ , which breaks  $SU(5)$  to  $SU(4)$ . Thus, as the energy scale decreases and the running  $\alpha = g^2/(4\pi)$  grows, at a scale  $\Lambda$  at which  $\alpha \Delta C_2 \sim O(1)$ , this condensate is expected to form. Without loss of generality, we take  $i = 1$ , and note

$$\begin{aligned} \langle \epsilon_{1jkl n} \psi_L^{jk} {}^T C \psi_L^{\ell n} \rangle &\propto \langle \psi_L^{23} {}^T C \psi_L^{45} - \psi_L^{24} {}^T C \psi_L^{35} \\ &+ \psi_L^{25} {}^T C \psi_L^{34} \rangle \end{aligned} \quad (1)$$

The nine gauge bosons in the coset  $SU(5)/SU(4)$  gain masses of order  $\Lambda$ . The six components of  $\psi_L^{ij}$  involved in the condensate (1) also gain dynamical masses of order  $\Lambda$ . These components bind to form an  $SU(4)$ -singlet meson whose wavefunction is given by the operator in (1).

This binding involves the exchange of the various (perturbatively massless) gauge bosons of  $SU(4)$ . The condensate (1) breaks the global  $U(1)'$ , but the would-be resultant NGB is absorbed by the gauge boson corresponding to the diagonal generator in  $SU(5)/SU(4)$ . We infer that this condensate (1) has spatial support in the meson with the same wavefunction. Aside from the  $SU(4)$ -singlet  $\chi_{1,L}$ , the remaining massless fermion content of the  $SU(4)$  theory is vectorial, consisting of a 4,  $\psi_L^{1j}$ , and a  $\bar{4}$ ,  $\chi_{j,L}$ ,  $j = 2 \dots 4$ . The formal global flavor symmetry of this effective  $SU(4)$  theory at energy scales below  $\Lambda$  is  $U(1)_L \times U(1)_R = U(1)_V \times U(1)_A$ , and the  $U(1)_A$  is broken by  $SU(4)$  instantons. This low-energy effective field theory is asymptotically free, so that at lower energy scales, the coupling  $\alpha$  that it inherits from the  $SU(5)$  theory continues to increase, and the theory confines and produces the condensate  $\langle \psi_L^{1j} {}^T C \chi_{j,L} \rangle$ , which preserves the gauged  $SU(4)$  and global  $U(1)_V$ . We infer that  $\langle \psi_L^{1j} {}^T C \chi_{j,L} \rangle$  and the  $SU(4)$  gluon condensate  $\langle G_{\mu\nu}G^{\mu\nu} \rangle$  have spatial support in the  $SU(4)$ -singlet baryon, meson, and glueball states of this theory.

Although our suggestion associates condensates in a confining gauge theory  $G$  with  $G$ -singlet hadrons, these condensates can affect properties of  $G$ -singlet particles if they both couple to a common set of fields. For example, the  $\langle \bar{F}F \rangle$  condensate and the corresponding dynamical mass  $\Sigma_F$  of technifermions in a technicolor (TC) theory give rise to the masses of the (TC-singlet) quarks and leptons via diagrams involving exchanges of virtual extended technicolor gauge bosons. Our analysis could also be extended to supersymmetric gauge theories, but we shall not pursue this here.

Our argument is only intended to apply to asymptotically free gauge theories. However, we offer some remarks on the situation for a particular infrared-free theory here, namely a  $U(1)$  gauge theory with gauge coupling  $e$  and some set of fermions  $\psi_i$  with charges  $q_i$ . Here there are several important differences with respect to an asymptotically free non-abelian gauge theory. First, while the chiral limit of QCD, i.e., quarks with zero current-quark masses, is well-defined because of quark confinement, a  $U(1)$  theory with massless charged particles is unstable, owing to the well-known fact that these would give rise to a divergent Bethe-Heitler pair production cross section. It is therefore necessary to break the chiral symmetry explicitly with bare fermion mass terms  $m_i$ . If the running coupling  $\alpha_1 = e^2/(4\pi)$  at a given energy scale  $\mu$  were sufficiently large,  $\alpha_1(\mu) \gtrsim O(1)$ , an approximate solution to the Schwinger-Dyson equation for the propagator of a fermion  $\psi_i$  with  $m_i \ll \mu$  would suggest that this fermion gains a nonzero dynamical mass  $\Sigma_i$  [3] and hence, presumably, there would be an associated condensate  $\langle \psi_i \psi_i \rangle$  (no sum on  $i$ ). However, in analyzing  $S_\chi$ SB, it is important to minimize the effects of explicit chiral symmetry breaking due to the bare masses  $m_i$ . The infrared-free nature of this theory means that for any given value of  $\alpha_1$  at a scale  $\mu$ , as one decreases  $m_i/\mu$  to reduce explicit breaking of chiral symmetry,  $\alpha_1(m_i)$  also decreases, ap-

proaching zero as  $m_i/\mu \rightarrow 0$ . Since  $\alpha_1(m_i)$  should be the relevant coupling to use in the Schwinger-Dyson equation, it may in fact be impossible to realize a situation in this theory in which one has small explicit breaking of chiral symmetry and a large enough value of  $\alpha_1(m_i)$  to induce spontaneous chiral symmetry breaking. A full analysis would require knowledge of the bound state spectrum of the hypothetical strongly coupled U(1) theory, but this spectrum is not reliably known.

So far, we have discussed QCD and other theories at zero temperature. For QCD in thermal equilibrium at a finite temperature  $T$ , as  $T$  increases above the deconfinement temperature  $T_{dec}$ , both the hadrons and the associated condensates eventually disappear, although experiments at CERN and BNL-RHIC show that the situation for  $T \gtrsim T_{dec}$  is more complicated than a weakly coupled quark-gluon plasma. Our picture of the QCD con-

densates here is especially close to experiment, since, although finite-temperature QCD makes use of the formal thermodynamic, infinite-volume limit, actual heavy ion experiments and resultant transitions from confined to deconfined quarks and gluons take place in the finite volume and time interval provided by colliding heavy ions.

In conclusion, we have suggested a picture in which the quark and gluon condensates in QCD are localized to the interiors of hadrons, the reason being that the particles whose interactions give rise to the condensates are confined within these hadrons. Our work has important implications for cosmology [27].

This research was partially supported by grants DE-AC02-76SF00515 (SJB) and NSF-PHY-06-53342 (RS). SJB thanks Guy de Teramond for useful discussions. Preprint SLAC-PUB-13154, YITP-SB-08-07.

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