SLAC-PUB-13042
arXiv:0707.1340v1[astro-ph]

Plasma Suppression of Large Scale Structure Formation in the Universe

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(Dated: July 10, 2007)

We point out that during the reionization epoch of the cosmic history, the plasma collective effect among the ordinary matter would suppress the large scale structure formation. The imperfect Debye shielding at finite temperature would induce a residual long-range electrostatic potential which, working together with the baryon thermal pressure, would counter the gravitational collapse. As a result the effective Jean's length, $\tilde{\lambda}_J$, is increased by a factor, $\tilde{\lambda}_J/\lambda_J = \sqrt{8/5}$, relative to the conventional one. For scales smaller than the effective Jean's scale the plasma would oscillate at the ion-acoustic frequency. The modes that would be influenced by this effect depend on the starting time and the initial temperature of reionization, but roughly lie in the range $0.5h\mathrm{Mpc}^{-1} < k$, which corresponds to the region of the Lyman-α forest from the inter-galactic medium. We predict that in the linear regime of density-contrast growth, the plasma suppression of the matter power spectrum would approach $1-(\Omega_{dm}/\Omega_m)^2 \sim 1-(5/6)^2 \sim 30\%$.

PACS numbers: 98.65.-r, 98.80.Bp, 98.80.Es

The matter power spectrum of the large scale structure, alongside with the cosmic microwave background (CMB) fluctuations, provides important information on the composition and the evolution history of the universe. Unlike the CMB fluctuations, which was frozen since the recombination time, the matter power spectrum continued to evolve until now, during which the universe has been essentially in a reionization, or plasma, state. Plasmas are known to support numerous electron and ion collective oscillation modes. Although in the pre-recombination era the universe was also in a plasma state, the extreme dominance of radiation pressure over that of matter (by a factor 10^{10}) before decoupling renders the contribution of (ep) plasma oscillations totally negligible. In the post-decoupling era, the CMB pressure became negligible. Thus during the reionization epoch[1] one might expect collective plasma effects to exhibit influence on the structure formation.

A perfectly uniform plasma is charge-neutral and has no long-range electrostatic potential. The inevitable thermal fluctuations of its electron density at finite temperature would, however, induce a non-vanishing residual electrostatic potential among the ions that defies the Debye shielding. Such a residual electric force that exerts an additional pressure on the ions is non-collisional in nature. When combined with the collision-induced thermal pressure among them, these pressures can support ion-acoustic oscillations, a phenomenon well-known in plasma physics.[2, 3]

In this Letter we point out that the inclusion of such plasma collective effects would result in an increase of the sound speed and therefore an increase of the Jean's scale by a factor $\tilde{\lambda}_J/\lambda_J = \sqrt{8/5}$. The modes that would be influenced by this effect depend on the starting time of reioniation and its initial temperature, but should roughly lie in the range $0.5h\mathrm{Mpc}^{-1} < k$, which corresponds to the region of the Lyman-α forest. This plasma suppression mechanism is effective to both linear and nonlinear regimes of density perturbations. We predict that, the maximum suppression of the power spectrum in the linear regime should approach $\sim 1-(\Omega_{dm}/\Omega_m)^2 \sim 1-(5/6)^2 \sim 30\%$.

When the electrostatic potential is included, the standard Einstein-Boltzmann equation in the Newtonian limit is now extended into a Maxwell-Einstein-Boltzmann (MEB) equation. Let us designate the baryon perturbation as $\Delta \equiv \delta\rho_b/\rho_b$. In the electrostatic and the Newtonian limits and with the density perturbation $\Delta \ll 1$, the MEB equation reads,

$$\ddot{\Delta} + 2H\dot{\Delta} - \nabla^2\Big[\frac{\delta P}{\rho_b} + \frac{e\phi_{em}}{m_b}\Big] = 4\pi G\rho\Delta\,. \qquad (1)$$

where the electrostatic potential satisfies the Poisson equation: $\nabla^2\phi_{em} = 4\pi e\rho_e$. The source of gravity contains both dark matter and baryons, i.e., $\rho = \rho_{dm} + \rho_b$. We assume that the reionization epoch began at a certain time a^*. The adiabatic condition dictates that the initial perturbations are identical between the dark and the baryonic matters, i.e., $\delta\rho_{dm}(a^*)/\rho_{dm}(a^*) = \delta\rho_b(a^*)/\rho_b(a^*) = \delta\rho(a^*)/\rho(a^*) \equiv \Delta^*$. After that, dark matter and baryonic matter would still evolve hand-in-hand and grow linearly for scales larger than the Jean's scale, but would evolve separately for scales smaller than that. It is this latter situation which we are focusing for our plasma effect. Thermodynamics governs that

$$\frac{\delta P}{\rho_b} = \frac{\delta P}{\delta\rho_b}\frac{\delta\rho_b}{\rho_b} = \frac{\gamma_b k_B T_b}{m_b}\Delta\,. \qquad (2)$$

On the other hand, the electrostatic potential at a given

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Submitted to Physical Review Letters

Work supported in part by US Department of Energy contract DE-AC02-76SF00515

temperature is given by

$$e\phi_{\rm em} = \frac{4\pi e^2 \delta\rho_e}{k^2 + \lambda_D^{-2}} \approx \gamma_e k_B T_e \Delta\,. \tag{3}$$

The approximation results from the fact that the typical cosmic scale of interest is much larger than the Debye length, i.e., $\lambda \sim 1/k \gg \lambda_D \equiv \sqrt{\gamma_e k_B T_e/4\pi e^2 \rho_e}$. We have further assumed that $\delta\rho_e/\rho_e \simeq \delta\rho_b/\rho_b = \Delta$. This is because globally the fast-moving electrons tend to rapidly readjust themselves to follow the baryon density perturbations. At first sight, it may seem counter-intuitive that the charge neutrality condition would allow for a non-vanishing electrostatic potential. In reality, due to the inevitable thermal fluctuations of the much lighter electrons the plasma is never perfectly neutral. Balance of forces on electrons requires that the electron number density satisfies the Boltzmann's relation: $n_e = \rho_e/m_e = n_{e0}\exp(-e\phi_{em}/k_B T_e)$. Thus at finite temperature the Debye shielding is never perfect, and there is always a residual electrostatic potential at the level of $e\phi_{em} \sim k_B T_e \delta\rho_e/\rho_e$ in the leading order of Taylor expansion. This, that one assumes $\delta\rho_e/\rho_e = \delta\rho_b/\rho_b$ and $\nabla \cdot E = \nabla^2\phi_{em} \neq 0$ at the same time, is the so-called quasineutrality, or *plasma approximation* in plasma physics[2].

Fourier transforming Eq.(1) leads to

$$\ddot{\Delta}_k + 2H\dot{\Delta}_k + \Big(\frac{k}{a}\Big)^2 \tilde{c}_s^2 \Delta_k = 4\pi G\rho\Delta_k\,, \tag{4}$$

where

$$\tilde{c}_s \equiv \Big(\frac{\gamma_e k_B T_e + \gamma_b k_B T_b}{m_b}\Big)^{1/2} = \frac{\omega}{k} \tag{5}$$

is the 'sound speed' of the ion-acoustic waves. We see that the main change from the ordinary acoustic wave to the ion acoustic wave is that there is an extra contribution to the sound speed from the electron temperature. The fast-moving electrons are isothermal. So $\gamma_e = 1$. But for protons, $\gamma_b = 5/3$. In the case where electrons and ions are in thermal equilibrium, i.e., $T_e = T_b$, the net change is an increase of the sound speed from the ordinary acoustic wave, $\sqrt{5/3}$, to that of the ion-acoustic wave, $\sqrt{1+5/3}$. Both waves, however, are constant-velocity pressure waves that satisfy linear dispersion relations in the long wavelength limit.

So far the temperature and the on-set of the reionization are still unresolved. In the literature (e.g. [4, 5, 6]), the reionization temperature ranges from 10^3K to 10^7K. As for the on-set of reionization, the observation of the Gunn-Peterson trough in the spectra of high redshift quasars indicates that the reionization process was completed by $z^* \approx 6$ [7, 8], while WMAP determines that $z^* = 17 \pm 5$ [9, 10]. For simplicity and without compromising qualitative features of our effect, we model the reionization as an abrupt transition at a given instant $a^* = 1/(1+z^*)$ where the plasma temperature raised instantly to T_b^* homogeneously throughout the universe.

To solve Eq.(4) for a given k-mode, it is essential to recognize its relationship with the initial and the final Jean's scales at a^* and a, respectively. Since the universe expands adiabatically during the matter-dominant epoch, the large-scale baryon/plasma temperature satisfies the equation of state, $T_b V^{\gamma_b - 1} = \text{const.}$, where $\gamma_b = 5/3$ and $V \propto 1/\rho \propto a^3$. Therefore $T_b \propto \tilde{c}_s^2 \propto a^{-2}$. Thus the post-reionization ($a > a^*$) Jean's scale evolves as

$$\tilde{k}_J(a) = \frac{\sqrt{4\pi G\rho(a)}}{\tilde{c}_s(a)/a} = \Big(\frac{3\pi}{2}\frac{m_b G\rho^*}{T_b^*}\Big)^{1/2}\sqrt{a^* a} \geq \tilde{k}_J^*\,. \tag{6}$$

There are three regimes of k-modes that concern us. For $k < \tilde{k}_J^*$, the thermal and the plasma pressures are negligible from the outset. So the baryon perturbation grows linearly in the same way as that of the dark matter: $\delta_b \propto \delta_{dm} \propto a$. The decay mode, $\delta_{b(dm)} \propto a^{-3/2}$, fades away in time and can be ignored.

For $k > \tilde{k}_J^*$, care must be taken regarding k's further relationship with the Jean's scale at the time of interest. If $k > \tilde{k}_J(a) > \tilde{k}_J^*$, then the combined pressure dominates over the gravity throughout the period (a^*, a). Thus the right-hand-side of Eq.(4) is negligible. Changing variable from t to a, with $a \propto t^{2/3}$ in the matter-dominant era, we find

$$\frac{d^2\Delta_k}{da^2} + \frac{3}{2a}\frac{d\Delta_k}{da} + \Big(\frac{k^2\tilde{c}_s^2}{aH_0^2}\Big)\Delta_k = 0\,. \tag{7}$$

Let us introduce a parameter ω_r which satisfies the relation, $k^2\tilde{c}_s^2/H_0^2 \equiv \omega_r^2 a^{-2}$, such that ω_r is independent of a. Since $\tilde{k}_J \propto a^{1/2}$, we can reexpress it as

$$\omega_r^2 = \frac{\tilde{c}_s^2 k^2 a^2}{H_0^2} \simeq \frac{3}{2}\frac{\Omega_m}{\Omega_0}\Big(\frac{k}{\tilde{k}_J^*}\Big)^2 a^*\,. \tag{8}$$

It can be shown, through one more change of variable to $x = a^{-2}$, that the solution to Eq.(7) is oscillatory: $\Delta_k \propto \exp(i2\omega_r/\sqrt{a})$. Physically $\omega_r/\sqrt{a}$ represents the phase of the ripples of the density contrast in k-space due to the ion-acoustic oscillation. Such an oscillation begins at the time of reionization, a^*. Although in reality $\tilde{c}_s k$ is the physical ion-acoustic oscillation frequency, ω_r can be viewed as an effective ion-acoustic frequency since it relates to $\tilde{c}_s k$ straight-forwardly.

Ion-acoustic oscillations are known to suffer Landau damping[2], which was not included in Eq.(7) *a priori*. To include this effect, we invoke a simple empirical formula[2] for the Landau damping dispersion relation that relates the real and the imaginary parts of the ion-acoustic oscillation frequency,

$$\frac{\omega_i}{\omega_r} \simeq 1.1\Big(\frac{T_e}{T_b}\Big)^{7/4} e^{-(T_e/T_b)^2}\,, \quad 1 \leq T_e/T_b \leq 10\,. \tag{9}$$

Since the electron and proton temperatures in the reionization epoch are roughly equal, we find that $\omega_i/\omega_r \sim 0.4$ in our case. This means that within ~ 0.4 of an oscillation, the ion-acoustic wave would be Landau-damped to one e-folding of its initial amplitude. Now we replace the ω_r in the exponent of the solution to Eq.(7) by the complex frequency which satisfies the above dispersion relation. Imposing the initial condition at a^*, we arrive at the perturbation for the k-mode whose scale has been below the Jean's scale all the time from a^* to a, i.e., $k > \tilde{k}_J(a) > \tilde{k}_J^*$,

$$\Delta_k \simeq \Delta_k^* e^{(-0.8+i2)\omega_r \eta}\,, \tag{10}$$

where $\eta = \eta(a^*, a) \equiv \sqrt{1/a^*} - \sqrt{1/a}$.

For the intermediate regime $\tilde{k}_J^* < k < \tilde{k}_J$, the k-mode must have exited the Jean's scale at an intermediate time a_c: $k = \tilde{k}_J(a_c)$, where $a^* < a_c < a$. Such a mode would oscillate for a period $\Delta a = a_c - a^*$. Then it would stop the oscillation at $a = a_c$ and resume its gravitational collapse during the subsequent interval $a - a_c$. Nevertheless, such a resumed growth of perturbation is delayed, because of the ion-acoustic oscillations during the earlier time interval, by the same amount

$$\Delta a \equiv a_c - a^* = \Big[\Big(\frac{k}{\tilde{k}_J^*}\Big)^2 - 1\Big] a^*\,. \tag{11}$$

Thus the evolution of such a mode $\tilde{k}_J^* < k < \tilde{k}_J(a)$ should become

$$\Delta_k = \Delta_k^* \Big(\frac{a - \Delta a}{a^*}\Big) = \Delta_k^* \Big[\frac{a}{a^*} + 1 - \Big(\frac{k}{\tilde{k}_J^*}\Big)^2\Big]\,. \tag{12}$$

Here we have ignored the fact that upon crossing the Jean's scale the k-mode may still carry a non-vanishing oscillation amplitude. We expect that such remaining amplitude should be small due to Landau damping.

Our aim is to identify the imprints of this plasma effect in the large scale structure formation through observations. So we look for its modification to the matter power spectrum[11],

$$P(k,a) = 2\pi^2 \delta_H^2 \frac{k^n}{H_0^{n+3}} T^2(k) D^2(a)\,. \tag{13}$$

Here $\delta_H = 1.9 \times 10^{-5}$ is the amplitude of primordial perturbations, $H_0 = 100h\mathrm{kms}^{-1}\mathrm{Mpc}^{-1}$ the Hubble parameter at present, $T(k)$ the transfer function, and $D(a)$ the growth function. We limit our investigation in the linear regime of power spectrum growth. Due to the onset of the plasma effect, the conventional growth function, $D(a)$, in Eq.(13) has to be modified. Let us denote the modified growth function as $\tilde{D}(k,a)$. Clearly, the plasma suppression would only affect the regime where $k > \tilde{k}_J^*$, which is what we will concentrate below. To track the plasma effect on the baryons for $a > a^*$, we write

$$\begin{aligned}\tilde{D}(k,a) &= D_{dm}(a)\frac{\Omega_{dm}}{\Omega_m} + D_b(a)\frac{\Omega_b}{\Omega_m} \\ &= D_{dm}(a^*)\frac{a}{a^*}\frac{\Omega_{dm}}{\Omega_m} + D_b(a^*)\frac{\tilde{\Delta}_k}{\Delta_k^*}\frac{\Omega_b}{\Omega_m}\,. \end{aligned} \tag{14}$$

Invoking the initial condition, $D_{dm}(a^*) = D_b(a^*) = D(a^*)$, and under the conventional notion of linear growth, $D(a) = D(a^*)(a/a^*)$, we arrive at the relative change of the growth function as

$$\frac{\tilde{D}(k,a)}{D(a)} = 1 - \frac{\Omega_b}{\Omega_m}\Big[1 - \frac{a^*}{a}\frac{\tilde{\Delta}_k}{\Delta_k^*}\Big]\,. \tag{15}$$

Thus for a given k-mode with $k > \tilde{k}_J^*$, the plasma-suppressed matter power spectrum relative to that of the conventional approach reads, for $k > \tilde{k}_J(a) > \tilde{k}_J^*$,

$$\begin{aligned}\Big|\frac{\tilde{D}(k,a)}{D(a)}\Big|^2 \simeq \Big(\frac{\Omega_{dm}}{\Omega_m}\Big)^2 \;\Big\{\; & 1 + 2\Big(\frac{\Omega_b}{\Omega_{dm}}\frac{a^*}{a}\Big)e^{-0.8\omega_r\eta}\cos(2\omega_r\eta) \\ & + \Big(\frac{\Omega_b}{\Omega_{dm}}\frac{a^*}{a}\Big)^2 e^{-1.6\omega_r\eta}\Big\}. \end{aligned} \tag{16}$$

On the other hand we have, for $\tilde{k}_J^* < k < \tilde{k}_J(a)$,

$$\begin{aligned}\Big|\frac{\tilde{D}(k,a)}{D(a)}\Big|^2 \simeq\; & \Big(\frac{\Omega_{dm}}{\Omega_m}\Big)^2\Big\{1 + 2\Big(\frac{\Omega_b}{\Omega_{dm}}\frac{a^*}{a}\Big)\Big[1 + \frac{a}{a^*} - \Big(\frac{k}{\tilde{k}_J^*}\Big)^2\Big] \\ & + \Big(\frac{\Omega_b}{\Omega_{dm}}\frac{a^*}{a}\Big)^2\Big[1 + \frac{a}{a^*} - \Big(\frac{k}{\tilde{k}_J^*}\Big)^2\Big]^2\Big\}. \end{aligned} \tag{17}$$

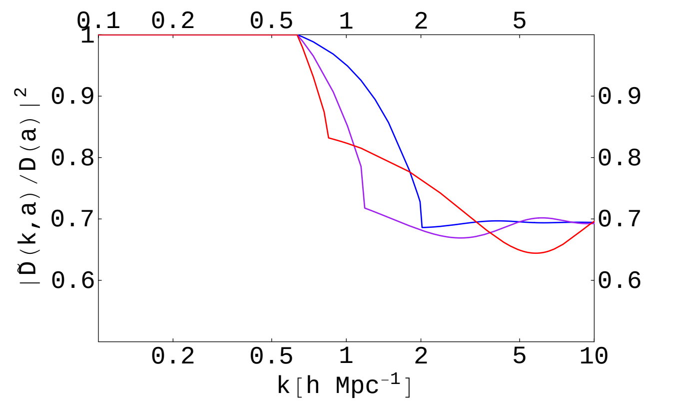

FIG. 1: Suppression of the matter power spectrum. $a^* = 0.1$ and $T_b^* = 5\times10^6$K. The red, purple and blue curves represent the suppressions at $a = 1/6, 1/3$ and 1, respectively. The three regimes of k can be clearly recognized.

To appreciate what this implies in cosmology, we look for the physical scales relevant to the plasma suppression effect. Let us assume that the reionization occurred at $1 + z^* = 10$, or $a^* = 0.1$, with the initial plasma temperature $T_e^* \simeq T_b^* \sim 5 \times 10^6$K. This corresponds to an ion-acoustic wave velocity of $\tilde{c}_s^* \sim 7 \times 10^{-4}c$ and $\tilde{k}_J^* \sim 0.63h\mathrm{Mpc}^{-1}$, and therefore the Jean's length, $\tilde{\lambda}_J^* = 2\pi/\tilde{k}_J^* \sim 1.0h^{-1}\mathrm{Mpc}$. The Jean's scale at present would be $\tilde{k}_J(a = 1) \sim 2.0h\mathrm{Mpc}^{-1}$, or $\tilde{\lambda}_J(a = 1) \sim 3.1h^{-1}\mathrm{Mpc}$. Figure 1 plots the suppression factor, $|\tilde{D}(k,a)/D(a)|^2$, at different times: $a = 1/6, 1/3$ and 1, based on the ΛCDM model with $\Omega_\Lambda = 0.7, \Omega_m = 0.3, \Omega_{dm} = 0.25, \Omega_b = 0.05, a^* = 0.1, a = 1$ and $T_b^* = 5 \times 10^6$K. The kinks of each curve are associated with the transitions at $\tilde{k}_J^*$ and $\tilde{k}_J(a_c)$, respectively. These kinks are the artifact of our approximation, which should be smeared in reality. The three regimes of k, namely, $k < \tilde{k}_J^*$, $\tilde{k}_J(a=1) > k > \tilde{k}_J^*$, and $k > \tilde{k}_J(a=1) > \tilde{k}_J^*$, can be readily recognized in this plot. For a mode at $k = \tilde{k}_J(a=1) = 2.0h\text{Mpc}^{-1}$, for example, it would continue to oscillate until now. Its oscillation 'frequency' is $\omega_r \sim 0.76$. Thus $2\omega_r\eta \sim 3.3 = 0.52(2\pi)$, i.e., it would have oscillated for roughly half a cycle. By then its amplitude would be Landau-damped by more than one e-fold. In the last regime, the suppression reaches an asymptotic value of $\sim 70\%$. Fig.2 compares the conventional and the plasma-suppressed matter power spectra. We have replaced the shape parameter $\Gamma = \Omega_m h$ in the BBKS formula[12] for the transfer function by the Eisenstein-Hu[13] rescaled factor. Thus the baryon effects prior to the reionization epoch has been included in the transfer function, which is common to both cases.

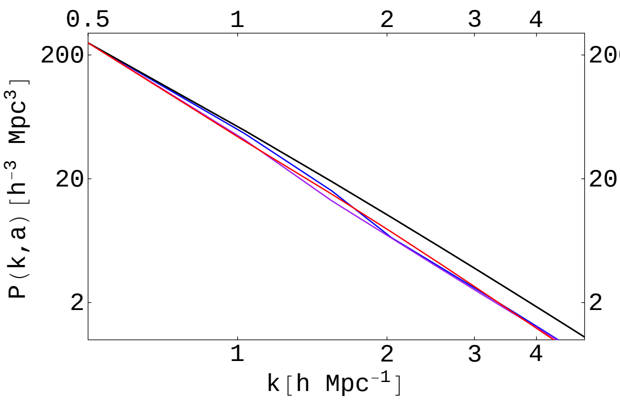

FIG. 2: Matter power spectrum with and without the plasma effect. The conventions are the same as that in Fig.1.

By including the long-range residual electrostatic potential, we have shown that the growth of the baryon density contrast during the reionization epoch should be suppressed by the plasma collective effect in the form of ion-acoustic oscillations. Though subject to Landau damping, these oscillations happen to have both the period and the damping time comparable to the Hubble time. Thus there should exist certain imprints of these oscillations in the large scale structure formation history. In the linear regime of density contrast growth and the asymptotic limit where the oscillations damp away, the amount of suppression of the matter power spectrum due to the plasma effect reaches $\sim 30\%$, which is sizable. In principle the plasma suppression is also effective in the nonlinear regime. It would be interesting to expand our treatment to the nonlinear regime of growth at late times ($a \to 1$) and track its impact.

One major effort in modern cosmology is to understand the cosmic composition through the determination of a set of cosmic parameters. In addition to the CMB fluctuations, large scale structure formation can help further constrain these parameters. In this regard, our predicted plasma suppression of the power spectrum in the Lyman-α forest region may be relevant. We note that the observed power spectrum in the inter-galactic medium (IGM) region indeed appears to be systematically lower than that predicted by the conventional theory[14]. It would be very interesting to confirm this. Aside from this issue, the evolution of the plasma-suppressed power spectrum should provide a unique window to reveal the detail history and dynamics of reionization.

Acknowledgments

We thank T. Abel, R. Blandford, W. Hu, Ue-Li Peng, J. Silk and R. Wechsler for helpful discussions. This work is supported in part by US Department of Energy under Contract No. DE-AC03-76SF00515.

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