

# Measurement of $|V_{us}|$ using hadronic tau decays from *BABAR* & Belle

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We report on measurements of branching fractions for several hadronic tau decays to final states with kaons, which can be used to determine the strange quark mass and the element  $|V_{us}|$  of the Cabibbo-Kobayashi-Maskawa quark-mixing matrix. The results are obtained from data collected with the *BABAR* and Belle detectors at the PEP-II and KEKB asymmetric-energy  $e^+e^-$  colliders at SLAC and KEK, respectively, both operating at center-of-mass energies near the  $Y(4S)$  resonance.

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## 1. Introduction

The weak interaction universality between quarks was asserted by Cabibbo with the introduction of a mixing angle between the first and second quark generations [1]. Three-generation quark mixing between the mass eigenstates and the flavor eigenstates is described by the unitary Cabibbo-Kobayashi-Maskawa (CKM) matrix [2].

The magnitude of the largest off-diagonal element of this CKM matrix,  $|V_{us}|$ , has historically been measured from three-body kaon ( $K\ell 3$ ) and hyperon decays [3,4].  $|V_{us}|$  has also been extracted from a comparison of the radiative inclusive rates of two-body kaon ( $K\ell 2$ ) and pion decays, along with lattice-QCD results of the meson decay constants [5]. Recently, the analysis of flavor-breaking sum rules has shown that inclusive measurements of the strange spectral function, obtained from hadronic  $\tau$  decays having net strangeness of unity in the final state, can provide a direct determination of  $|V_{us}|$  and the strange quark mass,  $m_s$  [6–12].

Present generation  $B$  factories,  $BABAR$  and Belle, also serve as  $\tau$  factories thanks to the large  $\tau$  pair production cross-section  $\sigma_{e^+e^- \rightarrow \tau^+\tau^-} = 0.919 \pm 0.003 \text{ nb}$  [13], as determined using the  $\text{KK}2\text{f}$  Monte Carlo (MC) generator [14] at a center-of-mass (CM) energy of  $\sqrt{s} = 10.58 \text{ GeV}$ . Using the world's largest sample of hadronic  $\tau$  decays collected with the  $BABAR$  and Belle detectors at the PEP-II and KEKB asymmetric-energy  $e^+e^-$  colliders, measurement of  $m_s$  and  $|V_{us}|$  can be performed with unprecedented precision [16,17].

Study of the strange spectral function from these  $\tau$  data is still in progress. However, using the updated knowledge of  $m_s(2\text{GeV}) = 94 \pm 6 \text{ MeV}/c^2$  from lattice calculations [18],  $|V_{us}|$  can be extracted with relatively small theoretical uncertainties [15,16,19], using available measurements of branching fractions of all  $\tau$  decays into final states containing an odd number of kaons.

## 2. Hadronic $\tau$ decays

Here we report on the recent  $BABAR$  [20,21] and Belle [22] measurements:

$$\begin{aligned} \mathcal{B}(\tau^- \rightarrow K^- \pi^0 \nu_\tau) &= (0.416 \pm 0.003 \pm 0.018)\% \text{ [20]}, \\ \mathcal{B}(\tau^- \rightarrow \pi^- \pi^- \pi^+ \nu_\tau) &= (8.83 \pm 0.01 \pm 0.13)\% \text{ [21]}, \\ \mathcal{B}(\tau^- \rightarrow K^- \pi^- \pi^+ \nu_\tau) &= (0.273 \pm 0.002 \pm 0.009)\% \text{ [21]}, \\ \mathcal{B}(\tau^- \rightarrow K^- \pi^- K^+ \nu_\tau) &= (0.1346 \pm 0.0010 \pm 0.0036)\% \text{ [21]}, \\ \mathcal{B}(\tau^- \rightarrow K^- K^- K^+ \nu_\tau) &= (1.58 \pm 0.13 \pm 0.12) \times 10^{-5} \text{ [21] and} \\ \mathcal{B}(\tau^- \rightarrow K_s^0 \pi^- \nu_\tau) &= (0.404 \pm 0.002 \pm 0.013)\% \text{ [22]}, \end{aligned}$$

where the uncertainties are statistical and systematic, respectively, and the charge-conjugate modes are implied. These results are more precise than the previously published measurements [23].

The vector  $K^*(892)^-$  resonance is expected to nearly saturate the  $(K\pi)^-$  final state. The Belle analysis of the  $K_s^0 \pi^-$  invariant mass spectrum reveals contributions from the vector  $K^*(892)^-$  resonance as well as other states [22]. For the first time, the  $K^*(892)^-$  mass and width have been measured in  $\tau$  decays:  $m(K^*(892)) = (895.47 \pm 0.20(\text{stat}) \pm 0.44(\text{syst}) \pm 0.59(\text{mod})) \text{ MeV}/c^2$ ,  $\Gamma(K^*(892)) = (46.2 \pm 0.6(\text{stat}) \pm 1.0(\text{syst}) \pm 0.7(\text{mod})) \text{ MeV}$ . The  $K^*(892)^-$  mass is significantly different from the current world-average value of  $891.66 \pm 0.26 \text{ MeV}/c^2$  [23].

We also report on the first measurement of  $\mathcal{B}(\tau^- \rightarrow \phi K^- \nu_\tau) = (4.05 \pm 0.25 \pm 0.26) \times 10^{-5}$  by the Belle experiment [24], which is consistent with the new *BABAR* measurement of  $\mathcal{B}(\tau^- \rightarrow \phi K^- \nu_\tau) = (3.39 \pm 0.20 \pm 0.28) \times 10^{-5}$ , and show that the  $\tau^- \rightarrow \phi K^- \nu_\tau$  decay saturates the  $K^- K^- K^+$  final state [21]. The first measurement of  $\mathcal{B}(\tau^- \rightarrow \phi \pi^- \nu_\tau) = (3.42 \pm 0.55 \pm 0.25) \times 10^{-5}$  has been performed by the *BABAR* experiment [21], which provides an interesting laboratory to study OZI suppression [25], because the  $\tau^- \rightarrow \phi K^- \nu_\tau$  decay, with a comparable rate, is not OZI suppressed.

### 3. $|V_{us}|$ from $\tau$ decays

The hadronic width of the  $\tau$  is normalized as:  $R_\tau = \Gamma[\tau^- \rightarrow \text{hadrons}^- \nu_\tau(\gamma)] / \Gamma[\tau^- \rightarrow e^- \bar{\nu}_e \nu_\tau(\gamma)]$ . The spectral moments are defined as:  $R_\tau^{kl} = \int_0^1 dz (1-z)^k z^l \frac{dR_\tau}{dz}$ , where  $z = \frac{q^2}{m_\tau^2}$  is the square of the scaled invariant mass of the hadronic system. The flavor-breaking term  $\delta R_\tau^{kl} = \frac{R_{\tau,\text{non-strange}}^{kl}}{|V_{ud}|^2} - \frac{R_{\tau,\text{strange}}^{kl}}{|V_{us}|^2}$  is sensitive to  $m_s$ . This term for the  $kl = 00$  moment has the smallest theoretical uncertainty (given in parentheses):  $\delta R_{\tau,th}^{00} = 0.1544(37) + 9.3(3.4)m_s^2 + 0.0034(28) = 0.240(32)$  [16,18].

We can then measure  $|V_{us}| = \sqrt{R_{\tau,\text{strange}}^{00} / \left[ \frac{R_{\tau,\text{non-strange}}^{00}}{|V_{ud}|^2} - \delta R_{\tau,th}^{00} \right]}$  from the measured  $\tau$  branching fraction into strange final states, where we use  $|V_{ud}| = 0.97377 \pm 0.00027$  [23]. The modest 13% error on  $\delta R_{\tau,th}^{00}$  gives a relatively small contribution to the error on  $|V_{us}|$  [19].

Here, we use the values of  $\tau$  branching fractions as estimated in Reference [15]. The electronic branching fraction  $\mathcal{B}_e^{\text{uni}} = (17.818 \pm 0.032)\%$  is obtained by averaging the direct measurements of the electronic and muonic branching fractions and the lifetime of the  $\tau$  lepton. Using lepton universality, the total hadronic  $\tau$  branching fraction is  $\mathcal{B}_{\text{had}} = 1 - 1.97257 \mathcal{B}_e^{\text{uni}} = (64.853 \pm 0.063)\%$ , and the total  $\tau$  hadronic width is  $R_\tau = 3.640 \pm 0.010$ . The non-strange width is  $R_{\text{non-strange}} = R_\tau - R_{\text{strange}}$ , where the branching fractions into strange final states are listed in Table 1.

**Table 1:**  $\tau$  branching fractions into strange final states from [15], updated with results from [20–22].

Mode	$\mathcal{B}(10^{-3})$ [15]	Updated $\mathcal{B}(10^{-3})$ with results from [20–22]
$K^-$	$6.81 \pm 0.23$	[Replace with $7.15 \pm 0.03$ ]
$K^- \pi^0$	$4.54 \pm 0.30$	Average with $4.16 \pm 0.18 \Rightarrow 4.26 \pm 0.16$ ( $S = 1.0$ )
$\bar{K}^0 \pi^-$	$8.78 \pm 0.38$	Average with $8.08 \pm 0.26 \Rightarrow 8.31 \pm 0.28$ ( $S = 1.3$ )
$K^- \pi^0 \pi^0$	$0.58 \pm 0.24$	
$\bar{K}^0 \pi^- \pi^0$	$3.60 \pm 0.40$	
$K^- \pi^+ \pi^-$	$3.30 \pm 0.28$	Average with $2.73 \pm 0.09 \Rightarrow 2.80 \pm 0.16$ ( $S = 1.9$ )
$K^- \eta$	$0.27 \pm 0.06$	
$(\bar{K}^0 3\pi)^-$ (estimated)	$0.74 \pm 0.30$	
$K_1(1270)^- \rightarrow K^- \omega$	$0.67 \pm 0.21$	
$(\bar{K}^0 4\pi)^-$ (estimated) and $K^{*-} \eta$	$0.40 \pm 0.12$	
Sum	$29.69 \pm 0.86$	Updated Estimate: $28.44 \pm 0.74$ [ $28.78 \pm 0.71$ ]

The results for the  $K^- \pi^0$ ,  $K^- \pi^+ \pi^-$  and  $\bar{K}^0 \pi^-$  branching fractions in [15] have been averaged with the results presented here [20–22], where the errors include a scale factor  $S$  following the PDG

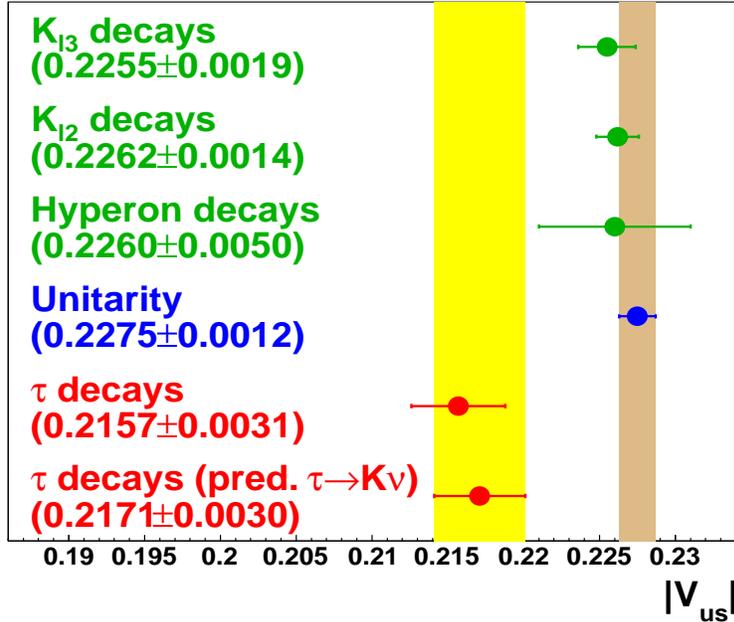
prescription [23]. The total branching fraction into strange final states has also been calculated by replacing  $\mathcal{B}(\tau^- \rightarrow K^- \nu_\tau)$  with  $(7.15 \pm 0.03) \times 10^{-3}$ , obtained from theoretical predictions using the much better known  $K^- \rightarrow \mu^- \nu_\mu(\gamma)$  decay rate and assuming  $\tau$ - $\mu$  universality.

The updated value is  $|V_{us}| = 0.2157 \pm 0.0031$  using measured  $\tau$  branching fractions alone, and  $|V_{us}| = 0.2171 \pm 0.0030$  using the predicted  $\tau^- \rightarrow K^- \nu_\tau$  branching fraction as well. The uncertainties are dominated by the  $\sim 2\%$  experimental errors on the measurements of the  $\tau$  branching fractions. Some of the uncertainties have been reduced almost by a factor of two, because of the new *BABAR* and *Belle* results. In the near future, we expect to reduce the uncertainty on the strange branching fractions by an additional factor of two, corresponding to a precision of 0.7% on  $|V_{us}|$ .

#### 4. Summary

In Figure 1, updated estimates of  $|V_{us}|$  from  $\tau$  decays are compared with those extracted from  $K\ell 3$  [26],  $K\ell 2$  [27] and hyperon decays [28]. The values obtained from  $\tau$  decays with and without the predicted  $\tau^- \rightarrow K^- \nu_\tau$  branching fraction are lower than the estimate of  $|V_{us}| = 0.2275 \pm 0.0012$  obtained by using the unitarity constraint from the value of  $|V_{ud}|$ , mentioned above, by  $3.2\sigma$  and  $3.5\sigma$ , respectively. Possible implications for new physics due to this departure from the unitarity constraint are discussed in Reference [29].

**Figure 1:** Comparison of different estimates of  $|V_{us}|$ .



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