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NUCLEON STRUCTURE FUNCTIONS AT LARGE MOMENTUM TRANSFER*

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ABSTRACT

Several field theoretical models predict a squeeze of the nucleon structure functions towards small values of the Bjorken variable x for very large momentum transfer. We present an intuitive argument for this behavior in the framework of a simple quark-gluon parton model.

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The description of all deep inelastic lepton-hadron scattering processes with light, structureless quark partons has been very successful until now.[1] The main result, which follows from such a model, is that the structure functions of the nucleon depend solely on the dimensionless Bjorken variable x provided the momentum transfer Q^2 exceeds a certain minimum value.[2] But it would not be surprising if this situation did not continue indefinitely: most nontrivial field theories predict deviations from the SLAC structure functions at (possibly much) higher momentum transfers.[3] So far, scale breaking effects in nucleon structure functions have been discussed in the context of two different approaches:

(i) Wilson expansions of current-current commutators with anomalous dimensions, leading to scale breaking effects proportional to powers of Q^2; or asymptotically free theories which break scale invariance logarithmically.[4]

(ii) Quark-gluon models à la Chanowitz and Drell,[5] endowing quarks with a small finite size and allowing for gluon radiation.

As a consequence of attributing form factors to quarks, the nucleon structure functions $F_{1,2}$, in general dependent on x and Q^2, decrease uniformly in x with increasing momentum transfer Q^2 below the production threshold for gluons. This decrease is partly balanced by gluon production above threshold.[5] The first approach leads directly to a drop of the structure functions for large x (defined for the interval $0 \leq x \leq 1$) whereas for small x the structure functions increase.[6]

In this note we want to illustrate how the squeeze of the structure functions to small x values in field theoretical models originates in the interplay of a form factor effect and a phase space effect. For this purpose, a slightly generalized version of the Chanowitz-Drell model will be adopted, incorporating light quarks and massive vector gluons; but the gluon mass will not be assumed to be much larger than all momentum transfers involved. Choosing the simplest Abelian internal symmetry group does not constitute a severe restriction as long as we deal only with kinematical questions. Thus, quarks and gluons should be coupled by $f \sum_i \bar{q}_i \not{G} q_i$ as in massive QED, and we will discuss a case in which the gluon mass is roughly of the same order as the energies involved: $m_G \sim$ GeV. This coupling induces the following effects in electron scattering processes:

1) Diagrams of type (a) in Fig. 1 endow quarks with form factors[5] (likewise, anomalous magnetic moments[7] are induced, but they can be disregarded in the present context).

2) Gluons can be produced by bremsstrahlung[5] (diagram (b) in Fig. 1). That does not necessarily mean that they exist as free particles like ordinary hadrons, but their existence within hadrons can be as slavish as the existence of light quarks. Multigluon production is suppressed if the coupling constant f is small and/or the effective gluon mass is not much smaller than the energies involved. Both criteria apply to our example as explained below in more detail.

3) Gluons split into charged quark-antiquark pairs on which electrons can scatter (diagram (c) in Fig. 1).

Qualitative consequences, which follow from these subprocesses for the nucleon structure functions, can easily be drawn in a parton-type picture of the nucleon. We disregard the confinement problem (dealing with particles with effective masses and coupling constants) and apply the impulse approximation to the scattering processes. In doing so, we go beyond what is rigorously proved in the field theoretical framework by Chanowitz and Drell.[5] However, our discussions should remain true as long as we are only interested in qualitative kinematical effects. We will restrict ourselves to discussing the proton structure function $F_2(x, Q^2)$; there is no essential difference in the behavior of F_1. The SLAC structure function of the proton, as measured at moderate momentum transfers $1 \lesssim Q^2 \lesssim 10\ \mathrm{GeV}^2$, will be denoted by $\mathscr{F}_2(x)$. In the point-like quark picture, $\mathscr{F}_2(x)$ is given in terms of the quark-parton distributions $f_i(x)$ as

$$\mathscr{F}_2(x) = \sum_{\text{quarks } i} e_i^2\, x\, f_i(x) \tag{1}$$

where, in the infinite momentum frame, the Bjorken variable x can be interpreted as the fraction of the longitudinal momentum of the quark type i relative to the proton momentum. Taking into account gluon effects at still larger Q^2, $\mathscr{F}_2(x)$ is to be replaced by

$$\mathscr{F}_2(x) \rightarrow F_2(x, Q^2) = \mathscr{F}_2(x) + \Delta F_2^a(x, Q^2) + \Delta F_2^b(x, Q^2) + \Delta F_2^c(x, Q^2) + \ldots\ ; \tag{2}$$

the upper indices refer to the corresponding diagrams in Fig. 1. The corrections to this representation of F_2 comprise quark binding effects and multi-gluon contributions (which of course are suppressed by phase space effects and the small coupling constant).

a) Without radiation effects ΔF_2^a would change the structure function in the following way[5]:

$$\Delta F_2^a(x, Q^2) = g(Q^2)\, \mathscr{F}_2(x) \quad . \tag{3}$$

Most important, $g(Q^2)$ turns out to be negative definite. This feature can be believed to be quite model independent, for it reflects the physical intuition that form factors decrease in the deep space-like region. Because ΔF_2^a factorizes in x and Q^2, this contribution leads to a decrease of the structure function which is uniform in x. This is shown in Fig. 2, where the dashed lines represent the sum $\mathscr{F}_2 + \Delta F_2^a$ for momentum transfers $Q^2 = 20$ and 50 GeV^2. The SLAC structure function $\mathscr{F}_2(x)$ is parametrized according to Ref. 8. Mass parameters and coupling constants are chosen as

effective quark mass $m_q = 0.3$ GeV

effective gluon mass $m_G = 2$ GeV

effective quark-gluon coupling constant $f^2/4\pi = 1/5$

(these values, leading to results compatible with present experimental data, should not be taken too literally; compare with discussions in Ref. 9). At $Q^2 = 50$ GeV^2, the original scaling curve is lowered by about 20% (this effect is not terribly different from monopole form factors with a pole mass $\Lambda \sim 10$ GeV).

b) By contrast, gluon radiation processes as in diagram (1b) give positive contributions to the proton structure function. Because at fixed Q^2 the phase space for particle production is maximum at small values of x, we expect $\Delta F_2^b(x, Q^2)$ (and $\Delta F_2^c(x, Q^2)$ as well) to be largest near x=0 and to fall off rapidly towards x=1. This of course can be verified in our gluon model. We attribute a radiation function r to diagram (1b), formally defined as

$$\frac{1}{8\pi} \sum_{\text{spins}} \sum\!\!\!\!\!\!\int_{k,q'} (2\pi)^4 \delta_4(\Sigma P) <q|j_\mu|q',k> <q',k|j_\nu|q>$$

$$= \nu_q \left\{ \frac{q_\mu}{\nu_q} + \frac{Q_\mu}{Q^2} \right\} \left\{ \frac{q_\nu}{\nu_q} + \frac{Q_\nu}{Q^2} \right\} r(\nu_q, Q^2) + \dots \tag{4}$$

with $\nu_q = q \cdot Q$ and $Q^2 \equiv -Q^\mu Q_\mu \geq 0$. Then it can easily be shown in the infinite momentum frame that

$$\Delta F_2^b(x, Q^2) = \sum_{\text{quarks } i} e_i^2 \int_{\eta_{\min}}^{1} d\eta \cdot \eta \; f_i(\eta) \left[\frac{1}{\eta} \, r\left(\frac{\eta}{x} \frac{Q^2}{2}, Q^2 \right) \right]$$

$$= \int_{\eta_{\min}}^{1} \frac{d\eta}{\eta} \, \mathscr{F}_2(\eta) \, r\left(\frac{\eta}{x} \frac{Q^2}{2}, Q^2 \right) \tag{5}$$

where η denotes the fraction of the proton momentum carried by the struck quark $\left(\nu_q = \eta\nu = \frac{\eta}{x}\frac{Q^2}{2}\right)$.

Because the invariant mass of the final quark-gluon system is variable, η and x are no longer identical as in elastic electron-quark scattering, where their identity is a consequence of all in- and out-going quarks being on-shell.

Most important is the dependence of the lower integration limit η_{min} on x and Q^2 (reflecting the dependence of the phase space boundary on these variables); from $(q+Q)^2 \geq m_G^2$ we derive:

$$\eta_{min}(x, Q^2) = x\left(1 + \frac{m_G^2}{Q^2}\right) \leq \eta \leq 1 \quad . \tag{6}$$

One sees immediately that the available phase space increases with Q^2 and even more strongly with x^{-1}. The phase space goes rapidly to zero as x approaches 1. Thus, the gluon contribution to the structure function is relevant only for x smaller than some x_0 which increases with increasing Q^2. The actual behavior of $\Delta F_2^b(x, Q^2)$ is shown by dotted lines in Fig. 3, again for $Q^2 = 20\ \mathrm{GeV}^2$ and $50\ \mathrm{GeV}^2$, and for the same coupling constant and mass parameters as before. The curves agree with what has been expected on simple physical grounds in this picture.

c) We complete our discussion with some comments on the gluon splitting into quark-antiquark pairs. If the gluon carries a fraction η of the longitudinal momentum of the proton in the infinite momentum frame, then the decay products, quarks and antiquarks, carry fractions of η itself. Current estimates of parton distribution functions suggest that for $x > 0.2$ there are only a few antiquarks present in the nucleon, which is true, a forteriori, for antiquarks due to the splitting of gluons into quark-antiquark pairs at moderate values of the momentum transfer; it can be concluded that such a process is important only for small x where it just enhances the effect due to bremsstrahlung of gluons, as discussed before. Assuming that the gluon distributions have a shape similar to the quark distributions,[10] one finds Q^2 dependent effects in $\Delta F_2^c(x, Q^2)$ of the same size as in case (b). Due to the vagueness of the estimate and because, on general grounds, this effect can only reinforce the bremsstrahlung effect, we do not go into greater detail in the present paper.

Two main features emerge from the discussion of the diagrams (a) to (c):

1) The form factor diagram (a) reduces the structure functions uniformly in x.

2) The contribution due to gluon bremsstrahlung and gluon splitting into quark-antiquark pairs enhances the structure functions for small x.

The overall effect is that the structure functions show a deficit with respect to the SLAC curve for large x (growing with Q^2) and a surplus for small x (also growing with Q^2). As a result, the structure functions are squeezed towards small values with (almost) constant area below the curve. We have sketched this behavior in Fig. 3, assuming roughly equal contributions to the structure functions from gluon bremsstrahlung and gluon splitting. It is well known that such features are present also in asymptotically free gauge theories[6] (on a less rapidly changing logarithmic scale, of course; this behavior could easily be achieved in our context by incorporating more complicated group structures). The purpose of these notes was merely to investigate a model which yields similar results to those obtained in the renormalization group approaches, yet which is simple enough to allow some intuitive physical insight into the results. In particular, it points out the important role played by simple kinematical effects.

We conclude with a few remarks:

(i) The implications of gluon radiation in quark models for the e^+e^- puzzle are not large as long as effective masses of the order of 2 GeV are employed; these masses keep the 3-particle phase space at presently available energies small. In terms of the familiar variable z (= gluon energy/beam energy E) this can be seen from

$$\frac{m_G}{E} \leq z \leq 1 + \frac{m_G^2}{4E^2} \quad . \tag{7}$$

For $m_G = 2$ GeV and $E = 2.5$ GeV, z can vary only from 20/25 to 29/25. More detailed calculations show that radiation contributions do not exceed a few percent of the corresponding cross section for pointlike quark-pair production. Thus nothing essential has changed compared with what has been found by Chanowitz and Drell[5] and West.[7]

(ii) Denoting the differential cross section (with respect to Q^2) for inelastic electron-proton scattering at fixed electron energy E by $d\sigma/dQ^2$, we expect for large Q^2 a drop of $d\sigma/dQ^2$ compared with the extrapolated SLAC value because in this case only large x values contribute (where the form factor effect is dominant). By contrast, small x values, too, are important for moderate Q^2 values (but large E), and we expect to get back the SLAC result for the cross section.

(iii) Cross sections for neutrino and antineutrino scattering are expected to follow closely their scaling pattern because the area below the structure functions stays (even in our crude example) almost constant in Q^2. This is quite different from the result one gets by attaching a monopole form factor to the quarks without allowing any particles to be radiated off the quark lines.[11]

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FIGURE CAPTIONS

1. (a) Form factor diagram in electron-quark scattering;

   (b) gluon bremsstrahlung;

   (c) gluon splitting into a quark-antiquark pair.

2. Structure function of the proton; at moderate momentum transfer: solid line; quark form factor included: dashed lines; bremsstrahlung contribution: dotted lines.

3. Qualitative change of the structure function from moderate to large momentum transfer.

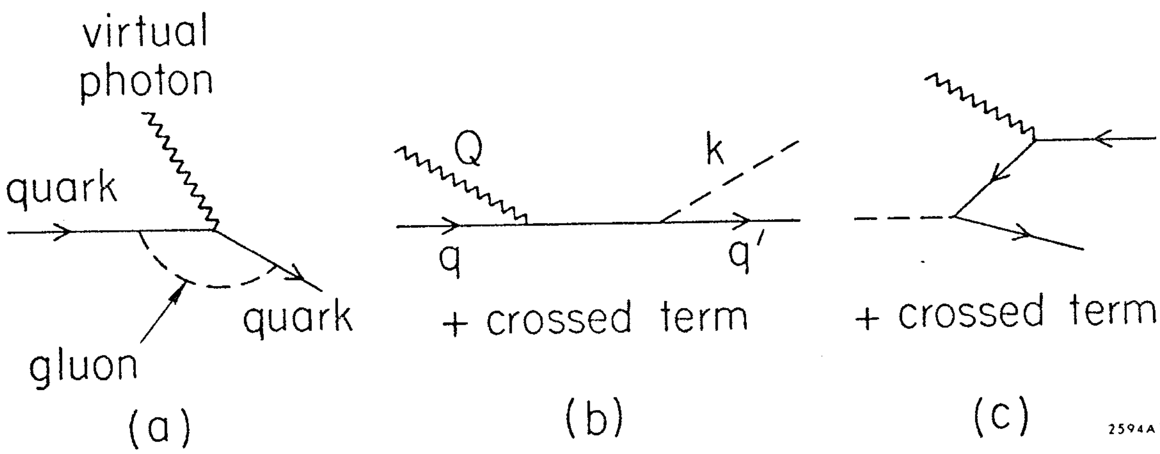
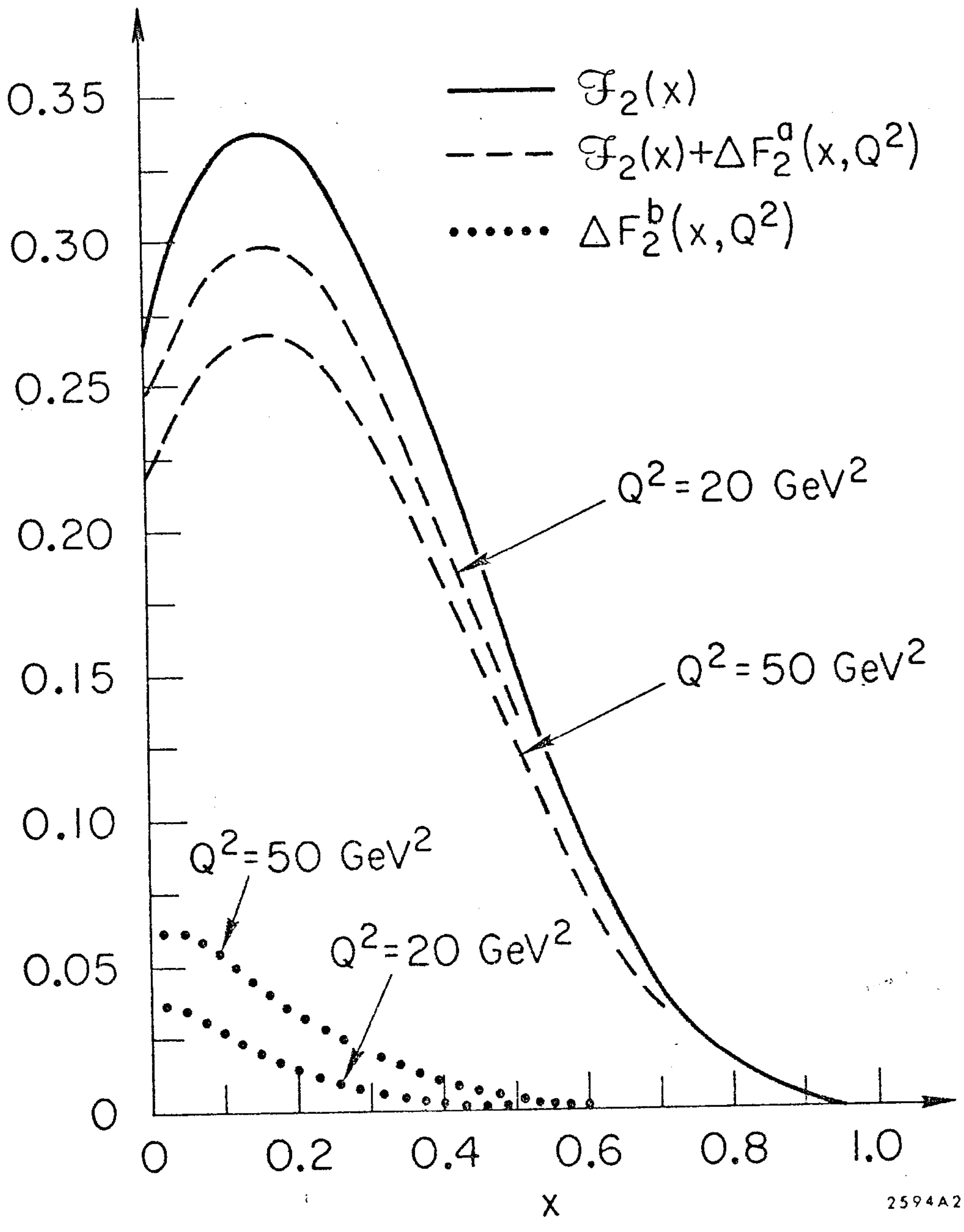
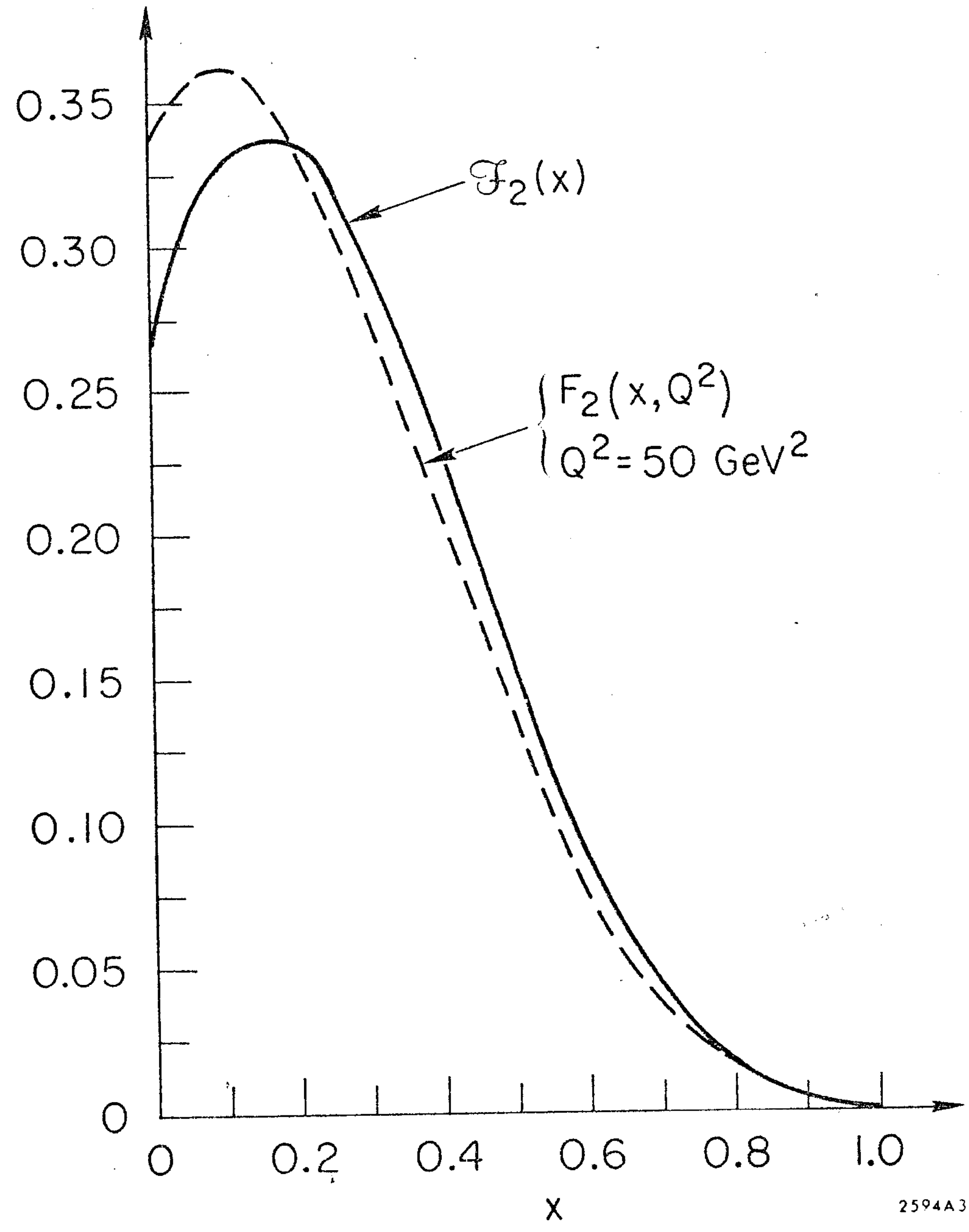

virtual
photon
quark
quark
gluon
(a)
Q
k
q
q′
+ crossed term
(b)
+ crossed term
(c)
2594A1

Fig. 1



0.35
0.30
0.25
0.20
0.15
0.10
0.05
0
$\mathcal{F}_2(x)$
$\mathcal{F}_2(x)+\Delta F_2^a(x,Q^2)$
$\Delta F_2^b(x,Q^2)$
$Q^2=20$ GeV^2
$Q^2=50$ GeV^2
$Q^2=50$ GeV^2
$Q^2=20$ GeV^2
0
0.2
0.4
0.6
0.8
1.0
x
2594A2

Fig. 2



0.35
0.30
0.25
0.20
0.15
0.10
0.05
0
$\mathcal{F}_2(x)$
$F_2(x, Q^2)$
$Q^2 = 50\ \mathrm{GeV}^2$
0
0.2
0.4
0.6
0.8
1.0
X
2594A3

Fig. 3