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DIFFERENTIAL DISPERSIVE ANALYSIS OF e-PLUS e-MINUS ANNIHILATION INTO HADRONS*

B. F. L. Ward

Stanford Linear Accelerator Center
Stanford University, Stanford, California 94305

ABSTRACT

Using the natural differential dispersive character of the renormalization group, we give a possible explanation of the present high energy data for $e^+e^- \rightarrow$ hadrons.

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One of the most perplexing of the present paradoxes in high energy physics is the apparent difference in scale between $e^+e^- \rightarrow$ hadrons and $e + p \rightarrow e +$ anything. More precisely, while Bjorken scaling[1,2] occurs already at $Q^2 \gtrsim 1_+ (\mathrm{GeV})^2$ in the latter process, there have been reported[3] data which would appear to exhibit pronounced deviations from Bjorken scaling in the former reaction all the way up to $S \sim 25(\mathrm{GeV})^2$! No satisfactory explanation of this paradox has been advanced, although there have been numerous attempts.[4] The present note will be seen to remedy this situation. In particular, using the differential dispersive aspect (this will be defined presently) of the renormalization group equation, as introduced in a previous Letter,[5] we have found what we consider to be a natural resolution of this paradox.

Specifically, as was shown in Ref. 5, the 1 PI Green's functions $\{\Gamma\}$ of quantum electrodynamics satisfy equations of the type

$$\left(-\lambda \frac{\partial}{\partial\lambda} + \beta \frac{\partial}{\partial g} - (1 + \gamma_\theta)\, m_R \frac{\partial}{\partial m_R} - \gamma_\Gamma + D_\Gamma\right) \Gamma = \sum_\alpha \rho_\alpha\, \delta(\lambda^2 (\Sigma P_{j\alpha})^2 - m_\alpha^2) \tag{1}$$

(see Eq. (14) of Ref. 5); here, λ is the scale, m_R is the renormalized mass, β, γ_θ, and γ_Γ are the usual coefficient functions[6] of the renormalized coupling g, and D_Γ is the engineering dimension of Γ. (We shall suppress all spinor and tensor labels where possible.) The amplitudes ρ_α may be computed in perturbation theory, for example. Eq. (1) was previously shown[5] to be an immediate consequence of the renormalization group equation. We view it as a differential dispersion relation, since in general $\{\rho_\alpha\} \neq \phi$.

We have subsequently studied the solutions of (1) more explicitly and found that at the energy at which conventional perturbation theory begins to falter, the fundamental vertices

$$\Gamma_{\mu}, \; D_{\mu\nu}$$

of QED may begin to behave as

$$\Gamma_{\mu}(\lambda q) \xrightarrow[\lambda^2 \to \infty]{} 1/\lambda$$
$$D_{\mu\nu}(\lambda q) \xrightarrow[\lambda^2 \to \infty]{} \text{const} \tag{2}$$

in such a way that, as usual,

$$\Gamma D \Gamma \sim e^2/\lambda^2 q^2 \tag{3}$$

(The vertices behave conventionally for small $\lambda^2 q^2$.) Assuming a gauge theoretic[7] view of leptons, the energy at which this should happen is the well-known[8]

$$q_0^2 < \frac{3\pi}{\alpha} m_{\mu}^2 \simeq 13(\text{GeV})^2 \; . \tag{4}$$

Of course, since $\Gamma D \Gamma$ is unchanged, there are essentially no physical consequences of Eqs. (2) in the QED interactions of fundamental fermions alone. However, for processes involving other types of interactions, the consequences are supremely interesting.

Indeed, for the process

$$e^+e^- \to \text{hadrons}$$

Eqs. (2) predict, to lowest order in hadronic electromagnetism, that at $S \sim 10\ (\text{GeV})^2$, the total cross section should begin to become constant, under the assumption of Bjorken scaling for the hadronic vertex.[9] Hence, taking scaling to set in at $S \gtrsim 1_+ (\text{GeV})^2$, as observed in[2,10]

$$e + p \to e + \text{anything},$$

we have the following picture of $e^+e^- \to$ hadrons:

Region I. For $1_+ \lesssim S \lesssim 10_- (\text{GeV})^2$ the famous ratio $R = \sigma(e^+e^- \to \text{hadrons})/\sigma(e^+e^- \to \mu^+\mu^-)$ should be given by the quark content of J_{μ}^{EM}, namely, $R = 2$ for the fractionally charged three-triplet model, for example.

Region II. For $10_{+}(\mathrm{GeV})^2 < S$, the ratio R should rise linearly with S/q_0^2, with $q_0^2 \simeq 10(\mathrm{GeV})^2$, on account of the constancy of $\sigma(e^+e^- \rightarrow \text{hadrons})$, the slope being given by the value of R for the Region I - namely, the slope should also be 2 in the three-triplet model. Ignoring the possibility of hitherto unseen quantum numbers, this rise should persist until higher order corrections become important. All of these predictions are consistent with the data,[3] as shown in the figure.

The details underlying the discussion here as well as other applications of differential dispersion relations will appear elsewhere.[11]

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4. See, for example, J. D. Bjorken and B. L. Ioffe, "Annihilation of e-plus e-minus into hadrons," SLAC-PUB-1467 (1974) (unpublished).

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7. S. Weinberg, Phys. Rev. D 7, 1068 (1973); Ibid., 2887; H. Georgi and S. L. Glashow, Phys. Rev. D 7, 2457 (1973); J. D. Bjorken, Talk given at

Stanford University, April 1974 (unpublished); P. Vinciarelli, "Non-perturbative calculation of the electron-muon mass ratio in gauge theories," SLAC-PUB-1352 (1973), and references therein.

8. See, for example, J. D. Bjorken and S. D. Drell, Relativistic Quantum Mechanics (McGraw-Hill Book Co., New York, 1964).

9. The validity of this assumption was the subject of the previous Letter.[5]

10. We should remark that our prediction for $e + p \rightarrow e + x$ is, in essence, unchanged from that of convention.[1,11]

11. B. F. L. Ward, to be published.

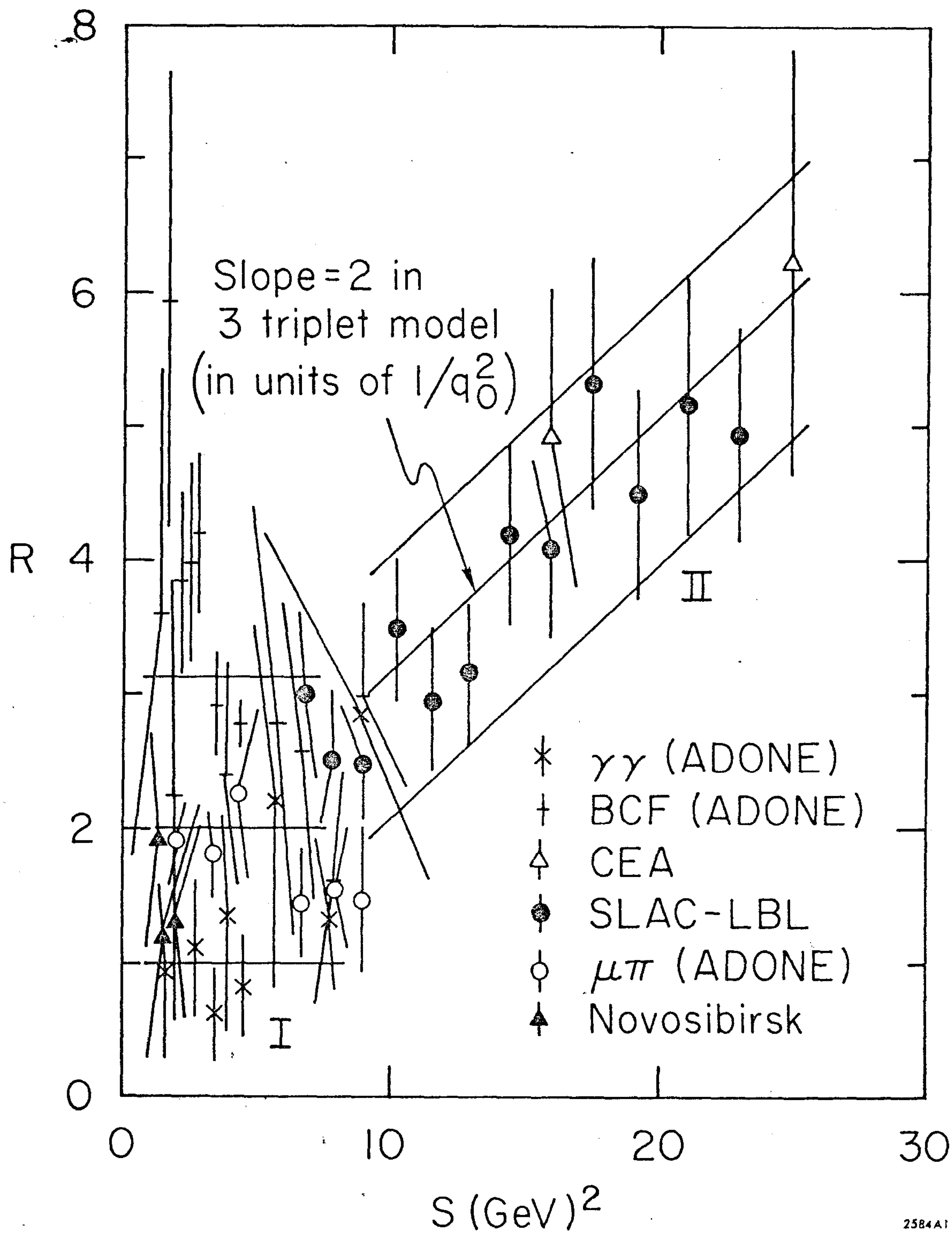

Fig. 1

For $1_+ < S \lesssim 10_-$ (GeV)^2, the data are consistent with the 3 triplet model prediction of $R = 2$. For $10_+ \lesssim S$, the slope of the data is consistent with the 3 triplet prediction of 2 in units of $(1/q_0^2)$, with $q_0^2 \doteq 10$ (GeV)^2.