# INCLUSIVE AND EXCLUSIVE PROCESSES AT LARGE TRANSVERSE MOMENTUM: A COMPENDIUM\*

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The primary goal of this talk will be to summarize in a unified way the predictions of the constituent interchange theory<sup>1</sup>,<sup>2</sup> (CIM) of high transverse momenta phenomena. A unified approach is possible due to the nature of the theory to be described in which predictions for all such processes are determined by a few simple assumptions. Success in the exclusive realm of hadron physics <u>must</u> go hand in hand with success in the inclusive realm. Most derivations and experimental comparisons will be omitted except where difficulties may be present. The present approach was developed by myself, S. J. Brodsky, R. Blankenbecler, G. Farrar and R. Savit in varying degrees of collaboration.

We begin with the naive assumption that mesons are composed of a quark and antiquark +  $q\bar{q}$  pairs while baryons contain  $3q + q\bar{q}$ pairs. Next we assume<sup>3</sup> that the quark-quark interaction is characterized by

a) dimensionless coupling constant(s)

p+q)/2

(p+q)/2

Pion form factor

at large

<sub>q</sub><sup>2</sup>.

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b) absence of anomalous dimensions both for quark quark scattering and for the hadronic quark wave functions.

(Assumption b) has not been rigorously demonstrated in any field theory.) These assumptions, combined with assumed dominance of the quark interchange type of process<sup>1</sup> in hadronic reactions, determine completely the behavior of a very large number of hadronic and electromagnetic processes.

#### I. Form Factors:

p/2

Fig. 1.

a

The form factor of an N quark state is predicted to behave asymptotically as

$$F(t) \sim \frac{1}{t^{N-1}}$$
 (1)

This is illustrated for the qq meson state in a vector gluon-fermion quark theory in Fig. 1.

The probability for a particle H to emit a secondary particle, H',

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(quark, quark pair, hadron ...) with fraction x of its linear momentum (x ~  $p_{\rm H}$  / $p_{\rm H}$ ) is

$$f_{H'/H} \sim (1-x)^{2n_{\bar{H}}H^{-1}}$$
(2)

where  $n_{\bar{H}'H}$  is the number of quarks not common to H' and H, i.e. the number in an  $\bar{H}'$ -H state. Of particular importance is the special case H' = q. Equation (2) yields

$$f_{q/H} \sim (1-x)^{2N-3}$$
(3)

for extracting a quark from an N quark state of a hadron. (These distributions are, of course, directly measured in deep inelastic scattering,  $F_2(x \approx 1/\omega) \sim \Sigma_a e_a^2 xf_a(x)$ .) The relation (3) corresponds to the well known Drell-Yan-West (DYW) relation which can be proven (as indeed can all the results (2)) in theories of the present type (provided infinite momentum frame Z graphs, which also give rise to hadronic J = 0 fixed poles, are eliminated by a not yet understood bound state mechanism).

For a proton (3) yields  $f(x) \sim (1-x)^3$  for a valence state quark and  $f(x) \sim (1-x)^7$  for the lst "sea" or qq pair proton state. The consistency of this "sea" threshold suppression with the deep inelastic scattering data and with the expected valence quark forms may be directly tested<sup>4</sup> with very good results. The only complication is that  $f_{\mathcal{N}/p} \sim (1-x)^4$  (instead of  $(1-x)^3$ ) simply because  $F_2^{en}/F_2^{ep} \approx 1 1/4$ . This may be evidence for some kind of  $\mathcal{N}-\mathcal{P}$ quark pairing within the proton.<sup>4,7</sup> Additional tests should be possible in the  $pp \rightarrow \mu^+ \mu^-$  massive  $\mu$  pair production process.<sup>5,8</sup> The hadronic bremmstrahlung probabilities (H' = hadron in (2)) can also be seen to be consistent with the quark extraction results

(3) in a slightly more intuitive way. For instance, visualize obtaining a  $\bar{q}$  from a proton either a) from the  $3q + q\bar{q}$  sea state or





b) from the valence state of a meson which in turn arose from the proton (Fig. 2).9

The demand that these two visualizations be equivalent requires (generally)

$$f_{q/H} (x)$$

$$\sim \sum_{H' x} \int_{q/H}^{1} f_{H'/H}(z) dz$$

(4)

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- 3 -

This equation is satisfied as  $x \rightarrow l$  by the threshold powers (2) and (3). Important examples of  $f_{q/H}$  and  $f_{H'/H}$  are listed below

$$f_{q/p} \sim (1-x)^3 \qquad \text{valence state} \\ \sim (1-x)^7 \qquad \text{sea state} \\ f_{q/\pi} \sim (1-x) \qquad \text{valence state} \\ \sim (1-x)^5 \qquad \text{sea state} \end{cases}$$

$$f_{M/B} \sim f_{B/M} \sim (1-x)^5 n = 3 \text{ e.g. } \pi^{\dagger}/p, \ \bar{p}/\pi^{\dagger}$$

$$f_{M/B} \sim f_{B/M} \sim (1-x)^9 n_{\overline{MB}} = 5 \text{ e.g. } \text{K}^{-}/\text{p}$$

These results will be crucial in understanding high  $\mathbf{p}_{\mathrm{T}}$  inclusive reactions.

III. Large angle exclusive processes:

If A and B scatter to produce C and D via the interchange graph (illustrated for meson baryon scattering in Fig.3), we obtain at fixed angles.<sup>3</sup>,10



Fig. 3. u - t and s - tinterchange graphs (MB  $\rightarrow$  MB).

$$s^{2} \frac{d\sigma}{dt} \sim \mathscr{A}^{2}$$
with  $\mathscr{A} \sim \frac{1}{s^{n}ABCD}$  (6)

$$n_{ABCD} = (n_A + n_B + n_C + n_D)/2 - 2$$

n<sub>H</sub> is the minimal number of quarks in hadron H required in order that an interchange graph be drawable. (Usually the valence number suffices). Photons, as well as quarks, are taken to be elementary. Simple examples are

$$\frac{\mathrm{d}\sigma}{\mathrm{d}t} \begin{bmatrix} \pi^{\pm} \mathbf{p} \to \pi^{\pm} \mathbf{p} \\ \mathbf{p} \mathbf{p} \to \mathbf{p} \mathbf{p} \\ \gamma \mathbf{p} \to \pi \mathbf{p} \\ \gamma \mathbf{p} \to \pi \mathbf{p} \\ \gamma \mathbf{p} \to \gamma \mathbf{p} \end{bmatrix} \sim \frac{1/s^{10}}{\sim 1/s^{7}}$$
(6)

(5)

These results are all quite successful. In particular the  $\gamma p \rightarrow \pi p$  prediction is borne out in contradiction to vector meson dominance expectations.

Complications abound for many choices of particles, however. For instance, for  $K^0 p \rightarrow K^0 p$  the  $\mathscr{N}$  quark of the proton must be employed. The deep inelastic data extraction (suggesting quark pairing) referred to earlier implies that this process might behave as  $d\sigma/dt(K^0 p \rightarrow K^0 p) \sim 1/s^{9-7}$  at fixed angle. Particularly curious cases are those of the type  $\pi \bar{p} \rightarrow \Delta \bar{\pi}^+$  for which the only quark graph which one can draw is of the form illustrated in Fig. 4. Such graphs should also,







if present at all, contribute to  $\pi^{\pm}p$ elastic scattering, but would appear to destroy the good angular dependence predictions we are about to discuss.<sup>11</sup> Thus observation of processes for which only this graph can contribute, at a valence state level, are important. Interchange graphs can, of course, be drawn by going to nonvalence state components for some of the particles involved. These, of course, fall more rapidly in s, at fixed angle; for instance the minimal interchange topology for  $\pi^- p \rightarrow \Delta^- \pi^+$  gives  $d\sigma/dt (\pi^- p \rightarrow \Delta^- \pi^+) \sim 1/s^{12}$ .

Finally there are processes such as  $\bar{K}^0 p \to \bar{K}^0 p$  for which nonvalence components (of the proton as it turns out) <u>must</u> be employed. Thus  $d\sigma/dt(\bar{K}^0 p \to \bar{K}^0 p) \sim 1/s^{12}$ .

Similar complications are present in baryon baryon scattering.

The angular dependence about 90° (we continue to restrict ourselves to two body processes) is also completely determined. Ignoring for the moment the complications due to spin we obtain this most easily by employing the "effective trajectory"<sup>12</sup> approach. Consider the u - t topology of Fig. 3. At fixed, but substantial, t, s  $\rightarrow \infty$ 

$$\alpha_{\rm eff}^{\rm ut} \sim \frac{1}{\sqrt{s}^{n_{\rm A}+n_{\rm C}+n_{\rm t}-4}} \sim s^{\alpha_{\rm eff}(t)}$$
(8)  
$$\alpha_{\rm eff}(t) \sim +2 - \frac{(n_{\rm A}+n_{\rm C}+n_{\rm t})}{2}$$

where  $n_t$  is the number of quarks (2 in the figure) exchanged in the t channel. (This result is immediate from (6) since only the upper half of the diagram provides damping in s at fixed t; note: we assume particle B to have the most strongly damped form factor!) At fixed u

$$\alpha_{eff}(u) \sim 2 - \frac{(n_A + n_D + n_u)}{2}$$

(9)

 $n_u$  = number of quarks exchanged in the u channel (3 for Fig. 3). We thus find

 $\mathcal{A}^{ut} \sim s^{\alpha} eff^{(u)}$ 

$$\mathcal{A}^{ut} \sim \frac{1}{\binom{(n_{A}-n_{B})/2 + (n_{u}+n_{t})/2 - 2}{s}} \frac{1}{\binom{(n_{B}+n_{D}-n_{t})/2}{t}} \frac{1}{\binom{(n_{B}+n_{C}-n_{u})/2}{u}} = \frac{1}{\binom{(-n_{ABCD}-\alpha_{eff}(u)-\alpha_{eff}(t))}{s}} \frac{1}{\binom{n_{ABCD}+\alpha_{eff}(t)}{t}} \frac{1}{\binom{n_{ABCD}+\alpha_{eff}(u)}{u}}$$
(10)

with  $\mathscr{A}^{st}$  obtained by  $s \leftrightarrow u, C \leftrightarrow A$ . For example,  $\mathscr{A}^{ut}(\pi^+ p \rightarrow \pi^+ p) \sim 1/ut^2$ .

Spin modifies these results. For instance in MB  $\rightarrow$  MB and  $\gamma B \rightarrow MB$  reactions the overall s channel spin, in general, cannot be conserved in at least one (forward or backward) direction. Spin flip forces introduction of extra kinematic zeroes appearing multiplicatively with the kinematic singularity free invariant amplitudes. If the spin is forced to flip by 1 unit in the forward direction  $\alpha_{eff}(t)$  is increased by 1/2 unit from that given in Eq. (8), with a similar backward direction result. To understand this result consider  $\pi p \rightarrow \pi p$ .  $\gamma_5$  counting in vector gluon theories tells us that the nucleon helicity is conserved. Thus in the forward direction no spin flip is present while in the backward direction the spin flips by one unit. Thus we must have an extra kinematic factor, √s √u at the amplitude level. However, we must keep the fixed angle s dependence the same which implies that the invariant amplitude multiplying this kinematic singularity must fall more rapidly than in the spinless case. For the ut topology (for which the upper  $q\pi \rightarrow q\pi$  amplitude, Fig. 3, controlling the spin considerations is a function of u only) we must have a  $1/u^2$  (vs. 1/u) behavior for the invariant amplitude. Thus (adding in the st topology at the amplitude level)

$$s^{2} \frac{d\sigma}{dt} (MB \to MB) \sim (\sqrt{s} \sqrt{u})^{2} \left( \frac{a^{ut}}{\frac{2}{u}} + \frac{a^{st}}{s^{2}} \right)^{2} \frac{1}{\frac{1}{4}}$$

$$\alpha_{eff}(t) = -1 \qquad \alpha_{eff}(u) = -3/2 \qquad (11)$$

Note that if only the st graph is present,  $\alpha_{eff}(u) = -7/2$ . The relative weighting of ut and st topologies, is determined by quark

counting. In some cases, e.g.,  $K^{\pm}p \rightarrow K^{\pm}p$ , one or the other is absent with often striking and experimentally observed implications for the angular dependence about 90°.1,7

For  $\gamma p \rightarrow \pi p$  the angular dependence is more difficult to obtain, because of the necessity of including gauge invariant pole terms,<sup>13</sup> Fig. 5. The  $\alpha_{\text{eff}}$  rules modified for spin flip remain valid, how-

ever. Thus if we consider  $\gamma n \rightarrow$ 

 $\pi^{T}p$  in a model in which only one  $\pi$  quark constituent is charged we obtain





Fig. 5.  $\gamma$  + quark  $\rightarrow \pi$  + quark assuming only one quark charged. One should be cautious of the J = 0"fixed pole" result for  $\alpha_{off}(t)$ 

as the full meson bound state complexity may lead (and accord-

ing to most prejudice) should lead to its removal and the persistence of the naive  $\alpha_{eff}(t) \sim -1/2$ . For pp elastic scattering one naively obtains (only ut graphs contribute, and these must be symmetrized among the protons in the final state)

$$s^{2} \frac{d\sigma}{dt} \sim \left(\frac{1}{st^{2}u} + \frac{1}{stu^{2}}\right)^{2}$$
$$\frac{d\sigma}{dt} \sim \frac{1}{s^{2}} \frac{1}{t^{\frac{1}{4}}u}$$
$$\alpha_{eff} \sim -2$$
(13)

Spin flip is not required (nucleon <u>helicity</u> is conserved in one of the two ways: a)  $A \leftrightarrow D$  and  $B \leftrightarrow C$ ; or b)  $A \leftrightarrow C$  and  $B \leftrightarrow D$ , corresponding to the two diagrams in which the final state protons exchange roles) suggesting no modification of the naive result (13), a form which is not in good agreement with the observed angular dependence (which requires  $1/s^{10} (1-z^2)^{-7}$  <sup>14</sup> at the least).  $\mathcal{P} \mathcal{N}$ quark pairing<sup>7</sup> effects within the proton are capable of improving the situation but lead to violation, for this case, of the naive scaling laws given by quark counting. The "good" scaling laws of Eq. (8) are not affected, only pp  $\rightarrow$  pp.

i.e.

It is clear from the effective trajectory approach that as, t, for instance, becomes nonsubstantial, i.e. the true Regge region is approached the effective trajectory rises<sup>9</sup> from its ultimate asymptotic value (as predicted by interchange theory). This rise could easily be due to t channel iteration of interchange diagrams,<sup>9</sup> Fig. 6.





The effects of signature are easily included. In some cases the u or t channel is exotic in which case the iteration may not lead to an increase in the effective trajectory level (though perhaps the residue would be changed from its lowest order value). Such cases include pp backward scattering for which  $\alpha_{eff}(u)$ = - 4 and

$$\frac{\mathrm{d}\sigma}{\mathrm{d}t}\Big)_{\mathrm{u}}^{\mathrm{pp}} \xrightarrow{\mathrm{pp}} \sim \frac{1}{\mathrm{s}^{10}}$$

(the same as the fixed angle

scaling law) and  $K^{-}p \rightarrow K^{-}p$  backward

$$\alpha_{\text{eff}}(u \sim 0) = -\frac{7}{2}; \quad \frac{d\sigma}{dt} \bigg|_{u \sim 0} \sim \frac{1}{s^9}.$$

Even more exotic are cases which violate quark selection rules at the valence level. For instance  $\pi p \rightarrow \phi + n$  can proceed (if  $\phi$  is pure  $\lambda \bar{\lambda}$ ) via interchange graphs only by adding a  $\lambda \bar{\lambda}$  pair to the  $\pi$  valence state and a  $\overline{\mathscr{P}}\mathscr{P}$  or  $\mathcal{N}\mathscr{N}$  pair to the  $\phi$  valence state. At fixed angle do/dt is expected to fall as  $1/s^{12}$ . Even in the forward direction if the interchange value,  $\alpha_{\text{eff}} \approx -3$  is appropriate (due to absence of normal Regge trajectories)

$$\frac{\mathrm{d}\sigma}{\mathrm{d}t}\Big)_{t=0}^{(\pi^{-}p \to \phi_{n})} \sim \frac{1}{s^{8}}$$

 $\gamma p \rightarrow \oint p$  is also interesting for similar reasons. A  $\lambda \overline{\lambda}$  pair must be added to both the initial and final proton states yielding

$$\frac{d\sigma}{dt} (\gamma p \to \phi p) \sim \frac{1}{11}$$

In general processes characterized by exotic trajectories and/or valence quark forbiddeness become particularly interesting.

Finally it should be stressed that 2 body  $\rightarrow 3$  body (etc.) processes should also obey exactly analogous scaling laws when all relative angles are held fixed as the energy is increased. For  $A + B \rightarrow C + D + E$ 

$$\mathcal{A} \sim \frac{1}{(\sqrt{s})^{n} A^{+n} B^{+n} C^{+n} D^{+n} E^{-4}}$$
(14)

## IV. Inclusive Processes:

We turn now to inclusive reactions at large transverse momentum.<sup>1,6,15</sup> We shall see, however, that the interchange graphs which provide a beautiful description of existing experimental data may even yield important effects at low  $p_m$ .

The general form of the diagram is given in Fig. 7. A particle,







$$E \frac{d\sigma}{d^{3}p} \sim \int dx \ x \ f_{b/B}(x) \ dz \ z \ f_{a/A}(z) \ \frac{d\sigma}{dt} \Big)_{xzs, zt, xu}^{a+b \to C+d}$$

 $\times s\delta(xzs + xu + zt)/\pi$  (15)

In order to explore the predictions of this form it is best to make a symmetric variable change

$$x = \frac{x_2}{x_2 + \epsilon\beta} \qquad z = \frac{x_1}{x_1 + \epsilon(1-\beta)}$$
(16a)  
$$x_1 = \frac{|u = (p_B - p_C)^2|}{s}, \quad x_2 = \frac{|t = (p_A - p_C)^2|}{s}$$
  
$$\epsilon = 1 - x_1 - x_2 \approx [(\text{missing mass})^2/s = \mathcal{M}^2/s]$$

where

$$\bar{xu} = \frac{p_{\perp}^{2}}{x_{2}^{+\epsilon\beta}}, \quad zt = \frac{p_{\perp}^{2}}{x_{1}^{+\epsilon(1-\beta)}}, \quad xzs = \frac{p_{\perp}^{2}}{(x_{2}^{+\epsilon\beta})(x_{1}^{+\epsilon(1-\beta)})}$$

Other convenient variables are

$$x_{L} = x_{1} - x_{2}$$
 the Feynman longitudinal fraction and  
 $x_{L} = \sqrt{4x_{1}x_{2}} = \frac{2p_{L}}{\sqrt{s}}$  the transverse momentum fraction

In terms of these

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 $\beta$  varies between 0 and 1 independently of the kinematic configuration. Let us also employ a slightly simplified form for  $f_{a/A}$  etc.

$$f_{a/A} \sim \frac{(1-x)^{2n_{\bar{a}A}-1}}{\frac{\alpha_{a/A}}{x^{a/A}}}$$
 (17)

 $\alpha_{a/A}$  is the trajectory controlling a-A forward scattering (usually taken = 1 for the leading Pomeron term). For do/dt we use the scaling laws and forms of Eqs. (6) and (10). (We, for simplicity, treat the cross section as though it has only one term which as we have already seen is not generally the case. Care must be taken, in applying the results to be given, to incorporate the exact cross section expression, though predictions are only slightly altered for most cases.) The result is

$$E \frac{d\sigma}{d^{2}p} = \epsilon \int_{0}^{1} d\beta \left(\frac{x_{2}}{x_{2}^{+\epsilon\beta}}\right)^{1-\alpha} b/B} \left(\frac{\epsilon\beta}{x_{2}^{+\epsilon\beta}}\right)^{2n} \overline{b}B^{-1}$$

$$\left(\frac{x_{1}}{x_{1}^{+\epsilon}(1-\beta)}\right)^{1-\alpha} a/A \left(\frac{\epsilon(1-\beta)}{x_{1}^{+\epsilon}(1-\beta)}\right)^{2n} \overline{a}A^{-1}$$

$$(18)$$

$$\times \frac{1}{(p_{\perp}^{2})^{n} a^{+n} b^{+n} c^{+n} d^{-2}} (x_{1}^{+\epsilon}(1-\beta))^{1-2\alpha} eff^{(xu)} (x_{2}^{+\epsilon\beta})^{1-2\alpha} eff^{(zt)}$$

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where the  $\alpha_{\text{eff}}$ 's are those appropriate to  $ab \rightarrow Cd$ . Provided one is supplied with the correct trajectory forms (say from small t or u 2-body data) (18) should have considerable validity even in the small t or u region. In general one must symmetrize (18) with respect to the role of the incoming hadrons A and B and sum over all subprocesses  $ab \rightarrow Cd$ . One should also bear in mind that there are, socalled, "direct" processes in which either A or B is directly involved in the fixed angle subprocesses. For direct scattering involving A these involve the absence of bremmstrahlung a/A, i.e. z is set = 1 or  $\beta = 1$  ( $\delta(z-1) = (1/x_1 \in )\delta(1-\beta)$ ).

A number of special limits are of particular physical interest.

A) Exclusive Limit: First is the exclusive limit  $\epsilon \rightarrow 0$  in which the missing mass gets small. From (18) we have

$$E \frac{d\sigma}{d^{3}p} = \frac{2(n_{\bar{a}A}^{+n}h_{\bar{b}B}^{-1}) - 1}{(n_{\bar{a}A}^{+n}h_{\bar{b}}^{+n}h_{\bar{c}}^{+n}h_{\bar{c}}^{-2}} (x_{1})^{2(1-\alpha_{eff}(u)-n_{\bar{a}A}^{-1})} (x_{2})^{2(1-\alpha_{eff}(t)-n_{\bar{b}B}^{-1})} (19)$$

i.e. the cross section vanishes in  $\epsilon$  as  $\epsilon$  where n is the number of passive spectators not actively participating in the large momentum transfer subprocess a + b  $\rightarrow$  C + d. The more spectators, the more the suppression. In general the number of spectators required increases as the valence quarks of C have less in common with A and B.<sup>6,15</sup> In general one must also include spectators to the decay c  $\rightarrow$  C, in which case,  $E(d\sigma/d^3p)$  vanishes as

$$E \frac{d\sigma}{d^{3}p} \stackrel{\epsilon \to 0}{\sim} \stackrel{2(n_{\bar{a}A} + n_{\bar{b}B} + n_{\bar{c}C}) - 1}{\epsilon}$$
(20)

A tabulation of particular cases will be given shortly.

B) Second we obtain the <u>Feynman</u> scaling regime in which  $x_2$  and  $x_1$  are small compared to  $\epsilon\beta$  and  $\epsilon(1-\beta)$ , respectively. (Roughly if  $\beta$  is approximately 1/2 on the average this means  $\epsilon \geq .75$  or so.)

$$E \frac{d\sigma}{d^{3}p} \sim \frac{\frac{3-2\alpha_{eff}(xu=p_{\perp}^{2}/\epsilon\bar{\beta})-2\alpha_{eff}(zt=p_{\perp}^{2}/\epsilon(1-\bar{\beta}))}{\epsilon}}{(p_{\perp}^{2})^{n_{a}+n_{b}+n_{c}+n_{d}-2} \alpha_{b}/B^{-1} \alpha_{a}/A^{-1}}$$
(21)

For large  $p_1^2$  we employ the results (8) and (9) for  $a + b \rightarrow C + d$ .

$$\alpha_{eff}(u) \rightarrow 2 - (n_{a} + n_{d} + n_{u})/2 + n_{SF}^{u}$$

$$\alpha_{eff}(t) \rightarrow 2 - (n_{a} + n_{C} + n_{t})/2 + n_{SF}^{t}$$
(22)

 $(n_u, n_t \text{ are the numbers of quarks exchanged in the u and t channels of ab <math>\rightarrow$  cd and  $n_{SF}^u$  and  $n_{SF}^t$  are the spin flip corrections to the  $\alpha_{eff}$ 's discussed earlier.) Thus, if one specializes to 90°,  $x_1 = x_2 = p_1/\sqrt{s}$ ,

$$\frac{\frac{d\sigma}{d^{3}p}}{\sim} \frac{s^{(\alpha_{a}/A^{+\alpha_{b}}/B^{-2})/2} (1-x_{i})^{1+(n_{a}+n_{d}+n_{u})-3+(n_{a}+n_{c}+n_{t})-3+(\alpha_{a}/A^{+\alpha_{b}/B^{-2}})}{(1-x_{i})^{n_{a}+n_{b}+n_{c}+n_{d}+(\alpha_{b}/B^{+\alpha_{a}/A^{-2}})/2 - 2}}}{(p_{i}^{2})^{n_{a}+n_{b}+n_{c}+n_{d}+(\alpha_{b}/B^{+\alpha_{a}/A^{-2}})/2 - 2}}}$$
(23)

which exhibits Feynman scaling for the standard choice  $\alpha_{a/A} = \alpha_{b/B} = 1$ (Pomeron behavior) as  $x_{\perp} \to 0$ .

It should be noted that both regions will be treated to a good approximation by using a mean value evaluation of the general integral (18) which approximately requires

$$\bar{\beta} = \frac{2n_{\bar{b}B} - 1}{2(n_{\bar{b}B} + n_{\bar{a}A}) - 2}$$

C) The triple Regge (TR) region: The term given in (18) is appropriate to discussing the small t,  $x_L > 0$ , triple Regge region. (The  $x_L < 0$  region requires the A  $\leftrightarrow$  B symmetric version of (18).) The TR region is  $s \gg \mathcal{M} \gg t$ , i.e.  $1 \sim x_1 \gg \epsilon \gg x_2$ . From (18)

$$\mathbb{E}_{d^{3}p}^{\frac{d\sigma}{3}p} \sim \frac{\epsilon^{2n} \tilde{a} A^{-2\alpha} eff^{(t)+\alpha} b/B s^{\alpha} b/B^{-1}}{t^{n} a^{+n} b^{+n} c^{+n} d^{+\alpha} b/B^{-2}}$$
(24)

where  $\epsilon = \mathcal{M}/s$  the missing mass squared fraction. This clearly takes the form of the standard triple Regge expression ( $\alpha_{\rm B}(0)$  and  $\alpha_{\rm b/B}$  are identical)

$$E \frac{d\sigma}{d^{3}p} \sim \left(\frac{M^{2}}{s}\right)^{\alpha_{B}(0)-2\alpha_{AC}(t)} s^{\alpha_{B}(0)-1} \beta_{AC}(t)$$
(25)

with the AC trajectory and residue given by

$$\alpha_{AC}(t) = \alpha_{eff}^{aC}(t) - n_{\overline{a}A}$$

$$\beta_{AC}(t) \stackrel{t \text{ large}}{\simeq} \frac{1}{n_{a}^{n+n}b^{+n}c^{+n}d^{+\alpha}B^{(0)-1}}$$
(26)

(The superscript on  $\alpha_{\text{eff}}$  is merely to remind you that it refers to the aC channel of  $a + b \rightarrow C + d$ .) "Direct processes," in which A participates in the large momentum transfer subprocesses of Fig. 7 directly, give equivalent results with  $n_{\overline{a}A} = 0$  and  $n_{\overline{a}} \equiv n_{\overline{A}}$ . It



"Bremstrahlung" Triple Regge Term



"Direct" Triple Regge Term

Fig. 8. "Bremmstrahlung" and "Direct" Triple Regge diagrams.

should be noted however that even for small t, bremmstrahlung processes such as the one just considered starting from (18) will be present and reasonably substantial. Thus one should not be surprised to see, phenomenologically, effective triple Regge trajectories substantially below those expected on the basis of naive Mueller analysis. Indeed this is the case except for photon initiated processes in which the present bremmstrahlung processes will occur only for the vector meson dominated part of the photon. The point-like portion of the photon cannot, by definition, bremmstrahlung. For large t this vector meson contribution is especially suppressed relative to the point like portion of the photon and only the "direct" triple Regge contribution will contribute.<sup>16</sup> See Fig. 8.

It is also true that for  $A \equiv C$ , for which  $\alpha_{AC} = 1$  is possible, the "direct" process should tend to dominate. The mystery which reappears in other guises not considered here, is that it doesn't, at least for  $p \rightarrow p$ .

D) Exclusive Inclusive Connection and High  $p_T$  Phenomenology: To discuss this topic<sup>17</sup> we employ the  $\epsilon \to 0$ , small  $\mathcal{M}^2$ , result (19). If we integrate  $d\sigma/dt d\mathcal{M}^2 = (1/s) E(d\sigma/d^3p)$  over a small  $\mathcal{M}^2$ range we obtain (including final state decay  $c \to C$ )

$$\left\langle \frac{d\sigma}{dt} \right\rangle \sim \frac{1}{\sum_{s=1}^{n_{s}+n_{s}+n_{s}+n_{s}-2+2n_{s}}} \sim \frac{1}{\sum_{s=1}^{p_{exclusive}} \sum_{s=1}^{p_{E}}}$$
 (27)

 $n_s = n_{\overline{a}A} + n_{\overline{b}B} + n_{\overline{c}C}$  is the number of passive spectators. We have, of course, rewritten  $p_{\perp}^2$  in terms of s and the angular variables,  $x_1$  and  $x_2$ .

Clearly for any observed particle C there is a minimum possible value for p found by isolating the exclusive final state which contains C and the minimal number of additional particles consistent with quark assignments.

<u>Meson production</u>: For instance for pp collisions producing a  $\pi^+$  (BB  $\rightarrow$  M) the minimal exclusive channel is pp  $\rightarrow$  p + n +  $\pi^+$ (B<sup>\*</sup>B<sup>\*</sup>M generally), which by dimensional counting should have  $d\sigma/dt \sim 1/s^{12}$ . This behavior can be achieved inclusively as 17 (remember  $\langle \epsilon \rangle \sim 1/s$ ):

Table A. a) BB 
$$\rightarrow B^* B^* M; p_E = 12$$

ai)	$\frac{1}{24} \frac{1}{s} \frac{1}{\epsilon} ; n_s = 0;$ P1	corresponding to the exclusive large angle process itself interpreted in- clusively, Fig. 9a.
aii)	$\frac{1}{20} \frac{1}{s} \in ; n_s = 1:$ $p_{\perp}$	corresponding to a fixed angle subprocess $qq + B \rightarrow M + B^* + qq$ , Fig. 9b.
aiii)	$\frac{1}{16} \frac{1}{s} \epsilon^3$ ; $n_s = 2$ :	corresponding to the subprocess $q + B \rightarrow B^* + M + q$ , Fig. 9c.
aiv)	$\frac{1}{12} \frac{1}{s} \epsilon^5; n_s = 3$ , Pi	corresponding subprocesses $q + qq \rightarrow M + B^*$ , Fig. 9d, and $q + B \rightarrow q + B^*$ , Fig. 9e.
av)	$\frac{1}{8} \frac{1}{s} \epsilon^7; n_s = 4$ $p_{\perp}$	corresponding subprocess $q + 2q \rightarrow 2q + q$ , Fig. 9f h + q







Within interchange theory  $p_{\perp}^{8}$  is the minimal  $p_{\perp}$  dependence possible. The next largest  $p_{exclusive}$  allowed is  $p_{exclusive} = 14$  from (we list only phenomenologically useful subprocesses from here on):

Table B. b) 
$$BB \rightarrow M + M^* + B^* + B^*$$
;  $p_E = 14$   
bi)  $\frac{1}{12} \frac{1}{5} \epsilon^7$ ;  $n_s = 3$ :  $q + q \rightarrow M + B^* + \bar{q}$  Fig. 10a  
bii)  $\frac{1}{8} \frac{1}{5} \epsilon^9$ ;  $n_s = 5$ :  $M + q \rightarrow M + q$  Fig. 10b and  
 $q + q \rightarrow \bar{q} + B^*$  Fig. 10c  
 $q + q \rightarrow \bar{q} + B^*$  Fig. 10c  
 $h_M + \bar{q}$ 

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Fig. 10. Inclusive graphs for  $pp \rightarrow \pi^+ + X$  based upon  $p + p \rightarrow \pi^+ + M^* + B + B^*$ 

A 
$$p_{exclusive} = 16$$
 channel is  
Table C: c)  $B + B \rightarrow M + M^* + M + B^* + B^*$ ;  $p_E = 16$   
ci)  $\frac{1}{8} \frac{1}{s} \epsilon^{11}$ ;  $n_s = 6$ :  $q + \bar{q} \rightarrow M + M^*$  Fig. lla  
 $q + M \rightarrow q + M^*$  Fig. llb  
 $\downarrow M + q$ 

In the vicinity of 90° these forms with  $\epsilon = (1-x_1)$  are good approximations to use in fitting data for  $(1/s) E(d\sigma/d^{2}p)$ . For instance for  $pp \rightarrow \pi^{+-0}$  a combination of aiv), bi), bii) and ci) is easily able to describe all existing data. However av) the minimally  $\epsilon$  damped term  $\frac{\text{does not seem to be present}}{(\text{the observed } \epsilon \text{ dependence is})}$ steeper) indicating possible complications in this approach, i.e. perhaps one should not allow 2q pairs on the same footing as mesons, M, thus eliminating av) in favor of bii). In general, one should perhaps require that there be enough



Fig. 11. Inclusive graphs for  $pp \rightarrow \pi^+ + X$  based  $upon p + p \rightarrow M + M^* + M^* + B^* + B^*$ . hadrons present for the subprocess to proceed even in a bag model, where the interaction of the quarks with the walls provides the only force. The absence of av) is a worrisome point in the phenomenology.

For other final states the  $p_E = 12$  exclusive channel may not be allowed. For K<sup>+</sup> production there is no problem whereas for K production the minimal exclusive channel is  $pp \rightarrow K^+ + K^- + p + p$ i.e. a  $p_E = 14$  channel. Thus for any given  $p_\perp$  power the  $\epsilon$ dependence of  $K^-/[\pi^{+-0} K^+]$  will be  $\epsilon^2$  if only most favorable terms are kept.

<u>Baryon production</u>: For baryon production from initial BB states the minimal exclusive channel is  $BB \rightarrow B + B^*$  for which  $p_{\rm E} = 10$ .

Table D: d)  $BB \rightarrow B + B^*$ ;  $p_E = 10$ di)  $\frac{1}{20} \frac{1}{5} \epsilon^{-1}$ ;  $n_S = 0$ :  $B + B \rightarrow B + B$ 

dii) 
$$\frac{1}{16} \frac{1}{s} \in ;$$
  $n_s = 1$ :  $B + 2q \rightarrow B + 2q$ 

1111) 
$$\frac{1}{12} \frac{1}{s} \epsilon^3$$
;  $n_s = 2$ :  $B + q \rightarrow B + q$ 

div)  $\frac{1}{8} \frac{1}{s} \epsilon^5$ ;  $n_s = 3$ : no allowed process

Phenomenologically<sup>15</sup> the NAL data may be telling us that none of these terms are present, a rather surprising result particularly when one attempts to compute their normalization and finds that it should be quite large. For  $p_E = 12$  processes and higher we need merely refer to Tables A) through C). The important term in fitting the data appears to be aiv), the same term as is important in describing the  $1/p_{\perp}^{12}$  terms in pion production.  $1/p_{\perp}^{8}$  for p production should be dominated by analogues of av);  $q + 2q \rightarrow q + 2q$ and  $q + q \rightarrow B + \bar{q}$ ; both  $=> (1/p_{\perp}^{8}) \epsilon^{7}$ . In fact the NAL  $\rightarrow B + \bar{q}$ data suggests (and I emphasize this word) a much stronger  $\epsilon$  damping associated with the  $1/p_{\perp}^{8}$  terms. Again 2q pairs in the av) configuration do not seem desirable. Proton target data is needed before any such conclusion can be made with certainty, however. Antibaryon production: The minimal exclusive channel is

Table E: e) 
$$B + B \rightarrow B + B^* + B^* + B^*; p_E = 16$$
  
ei)  $\frac{1}{8} \frac{1}{5} \epsilon^{11}; n_s = 6: q + q \rightarrow B^* + \bar{q}$   
 $p_\perp = 16$   
eii)  $\frac{1}{p_\perp^{12}} \frac{1}{5} \epsilon^9; n_s = 5: q + 2q \rightarrow \bar{q} + (B^*q)$   
 $p_\perp = 5: q + 2q \rightarrow \bar{q} + (B^*q)$ 

Thus, for instance, for any given  $p_{\perp}$  power we should, if minimal terms dominate, see  $\bar{p}/p \approx \epsilon^6$ , or if no direct  $p_E = 10$  processes are present for some reason,  $\bar{p}/p \sim \epsilon^4$  ( $p_E = 12$  does contribute to B production). We should also find  $\bar{p}/\pi^+ \approx \epsilon^4$ ,  $\bar{p}/K^- \approx \epsilon^2$ .

<u>General Rule</u>: Assume that for particle A production the minimal exclusive channel type contributing has  $p_E = p_E(A)$ , while for particle B we have  $p_E = p_E(B)$ . Then for any given common  $p_1$  power

$$\frac{E(d\sigma/d^{3}p) \rightarrow A + X}{E(d\sigma/d^{3}p) \rightarrow B + X} \sim \epsilon^{p_{E}(A) - p_{E}(B)}$$
(28)

and the  $1/p_{\perp}^{8}$ ,  $1/p_{\perp}^{12}$ , and  $1/p_{\perp}^{16}$  cross sections will have the form

$$E \frac{d\sigma}{d^{3}p} \xrightarrow{\rightarrow A + X} \sim \begin{bmatrix} \frac{1}{8} e^{p_{E}(A) - 5} \\ p_{L} \\ \frac{1}{12} e^{p_{E}(A) - 7} \\ \frac{1}{16} e^{p_{E}(A) - 9} \\ \frac{1}{16} e^{p_{E}(A) - 9} \end{bmatrix}$$
(29)

A table of minimal and phenomenologically preferred  $p_E$  values for proton proton collisions appears below. The phenomenlogical preferences depend upon the  $p_1$  power.

C	$p_{E}(C)$ minimal	$p_E(C)$ phenomenolo $1/p_{\perp}^8$	ogical preference l/p_
·			
π	12	14 + 16	12
$\pi^{-}$	12	14 + 16	12
к <sup>+</sup>	12	14 + 16	12
к-	14	16 + 18	14 or 16
P	10 (12 for $1/p_1^8$	.) ~ ~ 22	12
Ē	16	õ 22	16 or 18

In short the  $1/p_{\perp}^{12}$  forms correspond for the most part, to the minimal  $p_E$  forms. In contrast none of the minimal  $p_E 1/p_{\perp}^8$  forms appear to be present. Why this is so and why the "direct"  $p_E = 10$  processes for p production are absent is a mystery.

Finally we remark on Meson + Baryon processes yielding mesons. Clearly the minimal exclusive channel is often (e.g. for  $\pi^+ p \rightarrow \pi^+ + X$  but not  $\pi^+ p \rightarrow \pi^- + X$ ).

Table E. f)  $M + B \rightarrow M + B^*$ ;  $p_E = 8$ 

fi)  $\frac{1}{p_{\perp}^{16}} \frac{\epsilon^{-1}}{s}$ ;  $n_s = 0$ :  $M + B \rightarrow M + B$ fii)  $\frac{1}{\frac{12}{p_{\perp}}} \frac{\epsilon}{s}$ ;  $n_s = 1$ :  $\bar{q} + B \rightarrow M + qq$ fiii)  $\frac{1}{\frac{1}{p_{\perp}}} \frac{\epsilon^{3}}{s}$ ;  $n_s = 2$ :  $q + M \rightarrow q + M$ 

fiii) should dominate the  $p_{\perp}^{8}$  behavior (over any of the higher  $p_{E}$  possibilities) but our experience with the "direct" processes (based on the exclusive channel  $p + p \rightarrow p + B^{*}$ ) for p production should make us cautious.

The next exclusive channel (and the first one capable of contributing to  $\pi^+ p \rightarrow \pi^- + X$ ) for  $M + B \rightarrow M + X$  is

## Table I. $p + p \rightarrow C + X$

••••

Table G. g) 
$$M + B \rightarrow M + M^* + B^*$$
;  $p_E = 10$   
.gi)  $\frac{1}{p_L} \frac{\epsilon^3}{s}$ ;  $n_s = 2$ :  $2q + M \rightarrow B + \bar{q}$   
 $p_L \longrightarrow M + \bar{q}$   
gii)  $\frac{1}{p_L} \frac{\epsilon^5}{8}$ ;  $n_s = 3$ :  $q + \bar{q} \rightarrow M + M^*$ 

Thus terms of given  $p_{\perp}$  power should be in the ratio  $\epsilon^2$  for  $\pi^+ p \rightarrow \pi^- / \pi^+ p \rightarrow \pi^+$ .

Before ending it is perhaps worth noting that there is a variable in high  $p_{\perp}$  scattering which like  $\omega' = \omega + M^2/q^2$  in deep inelastic scattering is able to phenomenologically simulate the corrections to  $1/p_{\perp}^{0}$  scaling when higher inverse  $p_{\perp}$  powers become important. First note that even in deep inelastic scattering calculations, for proton targets, in addition to the standard  $F_2 \sim (\omega-1)^3$  scaling contribution there are diagrams with nonscaling behavior. Thus in general we have  $(\omega-1 = 2m\nu/q^2-1)$  the usual Bjorken variable)

Table H (for  $W_{\gamma}$ ): h)  $\gamma + p \rightarrow \gamma + p$ 

hi) 
$$\frac{1}{Q^2} (\omega - 1)^3$$
;  $n_s = 2$ :  $\gamma + q \rightarrow \gamma + q$   
hii)  $\frac{1}{(Q^2)^3} (\omega - 1)$ ;  $n_s = 1$ :  $\gamma + 2q \rightarrow \gamma + 2q$   
hiii)  $\frac{1}{(Q^2)^5} \frac{1}{(\omega - 1)}$ ;  $n_s = 0$ :  $\gamma + p \rightarrow \gamma + p$   
elastic form factor

contribution.

Each contribution is presumably positive definite. If we write hi) in terms of  $\omega^{i}$  we have

$$\frac{1}{q^{2}} (\omega'-1)^{3} \sim \frac{1}{q^{2}} \left( \omega - 1 + \frac{M^{2}}{q^{2}} \right)^{3}$$

$$\sim \frac{1}{q^{2}} (\omega-1)^{3} + \frac{3}{q^{4}} (\omega-1)^{2} M^{2} + \frac{3}{(q^{2})^{3}} (\omega-1) M^{4} \quad (30)$$

$$+ \frac{1}{(q^{2})^{4}} M^{6}$$

of which the 3rd term simulates hii) while the 2nd and 4th are missing from Table H. It is difficult to say with any certainty which approach should be considered more fundamental. Nonetheless attempts to fit high  $p_1$  data with forms of the type

$$p_{\perp}^{8} = \frac{E \, d\sigma^{\rightarrow A} + X}{d^{3}p} \sim (\epsilon')^{p_{E}^{(A)-5}}$$
(31)

with

$$\epsilon' = \epsilon + \frac{\widetilde{M}^2(A)}{p_{\perp}^2}$$
(32)

should prove interesting. Such a form "sums up" the  $1/p_{\perp}^{12}$ ,  $1/p_{\perp}^{16}$  etc. corrections to the  $1/p_{\perp}^{8}$  term in any given order,  $p_{E}(A)$ . The various allowed possibilities or a sum of possibilities should be considered for  $p_{E}(A)$ . It is clear from the data that one  $p_{E}(A)$  for each produced particle type is not enough. Inclusive exclusive normalization tests suggest that values for  $\tilde{M}^{2}(A)$  may be larger than those encountered in the Bloom-Gilman case. It should be noted that the addition of  $\tilde{M}(A)^{2}/p_{\perp}^{2}$  to  $\epsilon$  appears somewhat ad hoc compared to the Bloom-Gilman case. There  $\omega' - 1 = \mathcal{M}^{2}/Q^{2}$  where  $\mathcal{M}^{2}$  is the exact missing mass. The corrections suggested by the formalism above to  $\epsilon = \mathcal{M}^{2}/s$  are not of this form.

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