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AMPLITUDE STRUCTURE AND GAUGE INVARIANCE CONSTRAINTS FOR DYNAMICAL MODELS OF $\gamma N \to \pi^{\pm}\Delta$

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ABSTRACT

An invariant amplitude formalism is presented which features manifest gauge invariance and provides a convenient separation of the prominent dynamical components for $\gamma N \to \pi^{\pm}\Delta$. Kinematic constraints and low-t theorems are analysed in terms of s- and t-channel helicity amplitudes. Various dynamical models are discussed in terms of the general formalism and constraints.

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I. INTRODUCTION

The $\gamma N \longrightarrow \pi^{\pm}\Delta$ reaction is of interest to both experimentalists and theorists since it provides valuable information about non-diffractive two body reactions with one pion exchange and presents some intriguing mysteries associated with its dynamical behavior in the natural parity exchange component.[1] Data is available over a range of energies[2] for each of the charge states.[3] Results using polarized photons[4,5] allows a clean separation and analysis of the dynamical contributions to natural and unnatural parity exchange components.[6] A variety of dynamical models have been proposed to account for the reaction.[7-14]

The purpose of this paper is to analyse the independent amplitudes describing the process and to investigate the constraints which may be placed on dynamical models from kinematical considerations and by the low-t theorems[15,16] which have been previously derived and shown to play a dominant role in the near forward region. In Sec. II we will briefly review the kinematic structure in terms of invariant amplitudes and the results of the low-t theorems. Sec. III will present an alternate set of independent invariant amplitudes A_j which are manifestly gauge invariant and which allow a convenient separation for exhibiting the contributions of the pion exchange and background terms. In Sec. IV the projection of the amplitudes onto the t- and s-channel helicity amplitudes will be presented and their contribution to $d\sigma^{\parallel}/dt$ and $d\sigma^{\perp}/dt$ exhibited. The final section, Sec. V, is devoted to the brief analysis of dynamical models and their associated expression and constraints in this framework.

II. KINEMATIC REVIEW AND LOW-t THEOREMS

We provide here a brief review of the kinematics and previously derived low-t theorems[15] which will be used in later sections. The reaction matrix for $\gamma N \longrightarrow 0^- + 3/2^+$ is given by

$$f^s_{\lambda_f 0 \lambda_i \lambda_\gamma} = T_\mu \, \epsilon^\mu(k) = \sum_{j=1}^{12} B_j(s,t) \, N^j_{\lambda_f 0 \lambda_i \lambda_\gamma} \tag{1}$$

where

$$N^j_{\lambda_f 0 \lambda_i \lambda_\gamma} = \bar{u}_\nu(p_f,\lambda_f) \, I^{\nu\mu}_j \, u(p_i,\lambda_i) \, \epsilon_\mu(k,\lambda_\gamma) \tag{2}$$

and the $I^{\nu\mu}_j$ are chosen so that the invariant amplitudes $B_j(s,t)$ are free from kinematic singularities. The momenta for the nucleon, delta, pion and photon are given by p_i, p_f, q and k. Similarly the masses and helicities are given by M_i, M_f, μ and λ_i, λ_f and λ_γ.

The kinematic tensors $I^{\nu\mu}_j$ which define the first five B_j, shown previously to be of importance in the forward region, are

$$\begin{aligned} I^{\nu\mu}_1 &= q^\nu q^\mu \qquad & I^{\nu\mu}_3 &= k^\nu q^\mu \qquad & I^{\nu\mu}_5 &= g^{\nu\mu} \\ I^{\nu\mu}_2 &= q^\nu P^\mu \qquad & I^{\nu\mu}_4 &= k^\nu P^\mu & & \end{aligned} \tag{3}$$

where $P = \frac{1}{2}(p_i + p_f)$. The pion contributes only to B_1 and B_3 with the limits

$$\begin{aligned} B_1(s,t) &\xrightarrow[t \to \mu^2]{} \frac{2e_\pi f}{t - \mu^2} \\ B_3(s,t) &\xrightarrow[t \to \mu^2]{} -\frac{2e_\pi f}{t - \mu^2} \, . \end{aligned} \tag{4}$$

Gauge invariance, $T_\mu k^\mu = 0$, gives the constraint relations for the B_j

$$0 = B_1(s,t)k\cdot q + B_2(s,t)k\cdot P$$
$$0 = B_3(s,t)k\cdot q + B_4(s,t)k\cdot P + B_5(s,t) \; . \tag{5}$$

This allows us to determine the exact form of B_2 and $B_4 k\cdot P + B_5$ at $t = \mu^2$ and this defines the low-t theorems for these amplitudes. Experimental results indicate that for $|t| \leq \mu^2$, the contributions from B_4 (and B_{6-12}) are minimal. The low-t theorem which is most valuable to us is given by

$$B_2(s,t) \xrightarrow[t \to \mu^2]{} \frac{2e_\pi f}{s - M_i^2} \; . \tag{6}$$

This relation holds for all s in the limit $t \to \mu^2$ and also appears to provide an excellent approximation to $B_2(s,t)$ (at least in the region $|t| \leq \mu^2$).

III. GAUGE INVARIANT AMPLITUDES

It is useful to define a second set of independent invariant amplitudes A_j which are manifestly gauge invariant and allow a convenient separation of the dynamical features in the near forward direction.

By choosing the appropriate combinations of $I_j^{\nu\mu}$ in Eq. (3) we define a set of gauge invariant tensors as follows

$$
\begin{aligned}
G_1^{\nu\mu} &= (q^\nu - k^\nu)(k\cdot Pq^\mu - k\cdot qP^\mu) & G_5^{\nu\mu} &= (g^{\nu\mu}k\cdot P - k^\nu P^\mu)\not{k} \\
G_2^{\nu\mu} &= g^{\nu\mu}k\cdot q - k^\nu q^\mu & G_6^{\nu\mu} &= q^\nu(P^\mu\not{k} - k\cdot P\gamma^\mu) \\
G_3^{\nu\mu} &= g^{\nu\mu}k\cdot P - k^\nu P^\mu & G_7^{\nu\mu} &= k^\nu\gamma^\mu\not{k} \\
G_4^{\nu\mu} &= g^{\nu\mu}\not{k} - k^\nu\gamma^\mu & G_8^{\nu\mu} &= q^\nu\gamma^\mu\not{k}
\end{aligned}
\tag{7}
$$

and the B_j amplitudes in Eq. (1) are replaced by the independent invariant amplitudes $A_j(s,t)$. The A_j are related to the original kinematic singularity free B_j by the relations

$$
\begin{aligned}
A_1(s,t) &= \frac{2B_2(s,t)}{t-\mu^2} \\
A_2(s,t) &= -(B_1(s,t) + B_3(s,t)) \\
A_3(s,t) &= -(B_2(s,t) + B_4(s,t))
\end{aligned}
\tag{8}
$$

and so on. Since the combinations of right side of Eq. (8) are independent, we are assured that the A_j are generally independent as well.

Now we can investigate the behavior of the A_j in the $t \to \mu^2$ limit. Since B_2 is determined exactly by Eq. (6) as $t \to \mu^2$, A_1 is also given exactly by the low-t theorem with

$$
A_1(s,t) \xrightarrow[t\to\mu^2]{} \frac{4e_\pi f}{(t-\mu^2)(s-M_i^2)} \, . \tag{9}
$$

The appearance of the $1/(t-\mu^2)$ pole in A_1 is expected since $G_1^{\nu\mu}$ was constructed to ensure that the minimal gauge invariant form[15] of the one pion exchange would contribute uniquely to A_1.

Since the pion pole contributions in B_1 and B_3 given in Eq. (4) just cancel as $t \to \mu^2$, A_2 is left with any residual parts of these amplitudes in this limit. Since the next nearest t-singularity is quite distant, this residual combination must exhibit a slow variation with t in the $|t| \leq \mu^2$ region.

A_3 is given by the low-t constrained B_2 amplitudes and the free B_4 term which makes an apparently minimal contribution in the forward direction as discussed above. A_3 is then approximated by the limit

$$A_3(s,t) \underset{t \to \mu^2}{\approx} - \frac{2e_\pi f}{s - M_i^2} \; . \tag{10}$$

This result has no counterpart in $\gamma N \to \pi N$. There the low-t theorem effects only one amplitude, the one to which the pion trajectory contributes.

IV. HELICITY AMPLITUDES: t- AND s-CHANNEL

1. t-Channel Helicity Amplitudes

The contributions of the invariant amplitudes A_{1-3} to the parity conserving t-channel helicity amplitudes $\bar{f}^{tP}_{\lambda_\Delta \lambda_{\bar{N}}, 0\lambda_\gamma}$ may be obtained from the general transformation matrices[17]

$$\bar{f}^{t+}_{\frac{3}{2}\frac{1}{2},01} = - \frac{(t - \mu^2)(t - \epsilon^2)(t - \delta^2)}{4(M_i M_f)^{1/2}} A_3$$

$$\bar{f}^{t-}_{\frac{3}{2}\frac{1}{2},01} = \frac{(t - \mu^2)(t - \epsilon^2)}{2(M_i M_f)^{1/2}} \left[\frac{1}{2}\delta\epsilon A_3 - tA_2\right]$$

$$\bar{f}^{t\pm}_{\frac{3}{2} - \frac{1}{2},01} = \bar{f}^{t+}_{\frac{1}{2}\frac{1}{2},01} = 0$$

$$\bar{f}^{t-}_{\frac{1}{2}\frac{1}{2},01} = \frac{(t-\mu^2)(t-\epsilon^2)}{2\sqrt{3}\,(M_i M_f)^{1/2}\,M_f}\left[\frac{1}{2}(t-\epsilon^2)(t-\delta^2)A_1 + (t+\epsilon\delta)A_2 + \frac{1}{2}(t-\epsilon^2-\epsilon\delta-\delta^2)A_3\right]$$

$$\bar{f}^{t\pm}_{\frac{1}{2}-\frac{1}{2},01} = -\frac{1}{\sqrt{3}}\,\bar{f}^{t\pm}_{\frac{3}{2}\frac{1}{2},01} \qquad (11)$$

where $\epsilon = M_i + M_f$ and $\delta = M_f - M_i$.

We see from these expressions that the A_1 amplitude, which includes the gauge invariant contribution of one-pion exchange, contributes only to the unnatural parity exchange amplitude $\bar{f}^{t-}_{\frac{1}{2}\frac{1}{2},01}$ as expected.

The A_3 amplitudes, however, contributes to both the natural and unnatural parity exchange components.

The first two equations allow us to obtain conspiracy equations relating helicity amplitudes of opposite parity at $t = 0$

$$\bar{f}^{t-}_{\frac{3}{2}\frac{1}{2},01} = \frac{\delta}{\epsilon}\,\bar{f}^{t+}_{\frac{3}{2}\frac{1}{2},01}$$

$$(12)$$

$$\bar{f}^{t-}_{\frac{1}{2}-\frac{1}{2},01} = \frac{\delta}{\epsilon}\,\bar{f}^{t+}_{\frac{1}{2}-\frac{1}{2},01}$$

This is reminiscent of the familiar conspiracy equation obtained[18] for $\gamma N \to \pi N$ at $t = 0$.

2. s-Channel Helicity Amplitudes

In the s-channel it is convenient to consider the helicity combinations which correspond the polarization of the photon beam parallel and perpendicular to the plane of production

$$
\begin{aligned}
f^s_{\lambda_f 0,\lambda_i \parallel} &= \frac{1}{\sqrt{2}} \left(f^s_{\lambda_f 0,\lambda_i +1} - f^s_{\lambda_f 0,\lambda_i -1}\right) \\
f^s_{\lambda_f 0,\lambda_i \perp} &= \frac{i}{\sqrt{2}} \left(f^s_{\lambda_f 0,\lambda_i +1} + f^s_{\lambda_f 0,\lambda_i -1}\right)
\end{aligned}
\tag{13}
$$

General expressions are obtained from the transformation matrices.[19] These complicated expressions are then carefully analyzed in the $t \to t_{min}$ limit, yielding simplified approximate expressions for the relations which are correct to highest order in s and lowest order in t.

$$
\begin{aligned}
f^s_{\frac{3}{2} 0, \frac{1}{2} \parallel} &\xrightarrow[t \to t_{min}]{} \frac{\epsilon}{4\sqrt{2}\,(M_i M_f)^{1/2}} [sA_3 + \mu^2 A_2] \\
f^s_{\frac{3}{2} 0, -\frac{1}{2} \parallel} &\xrightarrow[t \to t_{min}]{} \frac{(-t)^{1/2}}{4\sqrt{2}\,(M_i M_f)^{1/2}} [sA_3 + \mu^2 A_2] \\
f^s_{\frac{1}{2} 0, \frac{1}{2} \parallel} &\xrightarrow[t \to t_{min}]{} \frac{-(-t)^{1/2}}{4\sqrt{6}\,(M_i M_f)^{1/2} M_f} [\{\epsilon^2\delta + (M_i + 2M_f)t\} sA_1 + 2\epsilon sA_2 + (M_i + 2M_f) sA_3] \\
f^s_{\frac{1}{2} 0, -\frac{1}{2} \parallel} &\xrightarrow[t \to t_{min}]{} \frac{\epsilon}{4\sqrt{6}\,(M_i M_f)^{1/2}} [sA_3 + \mu^2 A_2] \\
f^s_{\frac{3}{2} 0, \frac{1}{2} \perp} &\xrightarrow[t \to t_{min}]{} \frac{-i\epsilon}{4\sqrt{2}\,(M_i M_f)^{1/2}} [sA_3 + \mu^2 A_2]
\end{aligned}
\tag{14}
$$

$$f^{s}_{\frac{3}{2}\,0,-\frac{1}{2}\,\perp} \xrightarrow[t \to t_{min}]{} \frac{-i(-t)^{1/2}}{4\sqrt{2}\,(M_i M_f)^{1/2}}\,[sA_3 + \mu^2 A_2]$$

$$f^{s}_{\frac{1}{2}\,0,\frac{1}{2}\,\perp} \xrightarrow[t \to t_{min}]{} \frac{i(-t)^{1/2}}{4\sqrt{6}\,(M_i M_f)^{1/2}}\,[sA_3 + \mu^2 A_2]$$

$$f^{s}_{\frac{1}{2}\,0,-\frac{1}{2}\,\perp} \xrightarrow[t \to t_{min}]{} -\frac{i\epsilon}{4\sqrt{6}\,(M_i M_f)^{1/2}}\,[sA_3 + \mu^2 A_2]$$

In each of the expressions in Eqs. (14) the $(-t)$ is used as shorthand to represent $-(t - t_{min})$, which vanishes in the forward direction for all s and goes to $(-t)$ for high s as $t_{min} \to 0$.

Stichel's theorem,[6] which depends on the spinless naturel of the pion and is valid to $O(t/s)$, allows an identification between the parallel (perpendicular) polarization amplitudes and the unnatural (natural) parity exchange components. Again we see that the A_1 amplitude contributes only to the parallel, unnatural parity exchange, component, where A_2 and A_3 contribute to both.

It is also instructive to note that the A_2 amplitude always contributes to lower order in s than A_3 in the helicity amplitudes which are nonvanishing in the forward direction. This fact can account for the negligiblity of the A_2 contribution for high s.

Equations (14) also allow us to obtain an expression for the differential cross sections in terms of A_{1-3} which will be valid in the near forward direction

$$\left.\frac{d\sigma^{\perp}}{dt}\right|_{\substack{t \to t_{min} \\ s\ \text{large}}} = \frac{(M_i + M_f)^2}{96\pi}\,|A_3(s,t)|^2 \qquad (15)$$

$$\left.\frac{d\sigma^{\parallel}}{dt}\right|_{\substack{t \to t_{min} \\ s \text{ large}}} = \frac{d\sigma^{\perp}}{dt} + \frac{|t|}{384\pi M_f^2} \left|(M_i+M_f)(M_f^2-M_i^2)A_1(s,t) + 2(M_i+M_f)A_2(s,t) + (M_i+2M_f)A_3(s,t)\right|^2 \tag{16}$$

These expressions for the differential cross sections now justify our labeling of the A_3 amplitude as the "background" amplitude since as $t \to t_{min} \to 0$ and s large, A_3 provides the only non-vanishing contribution to both $d\sigma^{\perp}/dt$ and $d\sigma^{\parallel}/dt$.

We can also see why the rapid variation of the pion pole in A_1 accounts for the rapid variation in the parallel polarized and unpolarized photon differential cross sections. Equations 15 and 16 also imply that the asymmetry near $t = 0$ must be negative.

The apparent experimental saturation of the background amplitude A_3 in the near forward direction by the B_2 amplitude and consequent negligibility of the free B_4 amplitude accounts for the success of the low-t theorem amplitudes[15] and the Stichel-Scholz electric Born term amplitudes[7] for $|t| \leq \mu^2$.

It would also be of interest to analyse the A_3 amplitude using finite energy sum rules[19] to form a general pseudomodel similar to that of Jackson and Quigg[21] from $\pi\Delta$ scattering data. It suggests that we would find the $\pi\Delta$ s-channel resonance contributions to B_4 either small or cancelling for small t.

V. DYNAMICAL MODELS

Now we may investigate the components of the various dynamical models in terms of the framework we have presented. It is convenient to consider the dynamical contributions in three classes: 1) pion exchange 2) other t-channel exchanges and 3) the background amplitude.

1. Pion Exchange

Since the importance of this contribution strongly influenced the form of our formalism it is easy to express the pion exchange in a Regge form which is almost universally agreed upon.[11,13] The pion contributes only to A_1 and therefore $\bar{f}^{t-}_{\frac{1}{2}\frac{1}{2},01}$. The known analyticity for A_1 can be used to define the kinematic singularity free parity conserving t-channel helicity amplitude, which is then Reggeized according to the standard prescription and the residue function is defined to provide the proper limit given by the coupling constants at the pion pole as in Eq. (9). The trajectory $\alpha_\pi(t)$ is now determined to be Regge pole like from studies of unnatural parity components in pion exchange reactions[22,5] rather than flat as was once suspected from the behavior of $\alpha_{eff}(t)$ obtained from unpolarized differential cross sections.[2]

2. Other t-Channel Exchanges

A. Rho Meson Exchange

The prominent natural parity t-channel exchanges are expected to be the ρ and A_2 mesons. The projection of the ρ exchange component onto the amplitudes A_i can be readily obtained from the ρ exchange diagram in the $t \to m_\rho^2$ limit. The independently gauge invariant nature of the ρ exchange produces no new constraints. If we assume the simplified Stodolsky-Sakurai form for the $\Delta\rho N$ coupling[23,13] which has proven successful in describing the $\Delta \rightarrow N + \gamma$ process, the matrix element has the following form

$$f^{s(\rho)}_{\lambda_f 0 \lambda_i \lambda_\gamma}$$

$$= g_{\pi\gamma\rho} f_{\Delta\rho N} \bar{u}_\nu(p_f \lambda_f) \epsilon^{\nu\alpha\sigma\varphi} Q_\alpha p_{i\sigma} \left[\frac{-g_{\varphi\varphi'} + \dfrac{Q_\varphi Q_{\varphi'}}{m_\rho^2}}{t - m_\rho^2} \right] \epsilon^{\varphi'\kappa\delta\mu} k_\kappa q_\delta u(p_i \lambda_i) \epsilon_\mu(k\lambda_\gamma) \tag{17}$$

where $Q = p_i - p_f = q - k$.

The projection onto the A_j amplitudes then yields

$$A_1(s,t) \xrightarrow[t \to m_\rho^2]{} \frac{g_{\pi\gamma\rho} f_{\Delta\rho N}}{t - m_\rho^2}$$

$$A_2(s,t) \xrightarrow[t \to m_\rho^2]{} \frac{g_{\pi\gamma\rho} f_{\Delta\rho N}}{t - m_\rho^2} \frac{1}{2} (M_i^2 - M_f^2) \tag{18}$$

$$A_3(s,t) \xrightarrow[t \to m_\rho^2]{} \frac{g_{\pi\gamma\rho} f_{\Delta\rho N}}{t - m_\rho^2} (-t) .$$

It is interesting that only the amplitudes A_1, A_2 and A_3, which have previously played a role, are needed for ρ exchange. A little algebraic manipulation shows that the combinations of A_1, A_2 and A_3, which appear in the unnatural parity amplitudes in Eqs. (11), are such that the expressions in Eq. (18) combine to vanish exactly, leaving the ρ contributing only to the natural parity amplitudes $\bar{f}^{t+}_{\frac{3}{2}\frac{1}{2},01}$ and $\bar{f}^{t+}_{\frac{1}{2}-\frac{1}{2},01}$ as expected.

The appearance of the $(-t)$ factor in A_3 ensures that the ρ contribution to the differential cross section vanishes as $t \to 0$. This also guarantees the trivial satisfaction of the conspiracy equations, Eq. (12) and a secondary role for ρ exchange in the near forward direction.

The dependence of the ρ contribution through amplitude A_2 is reminiscent of daughter trajectory behavior in its lower order dependence on s and the unequal mass factor $(M_i^2 - M_f^2)$.

As in the case of the pion, Reggization is straight forward.[13] Reggeizing $f^{t+}_{\frac{3}{2}\frac{1}{2},01}$ gives the contribution to A_3 and the relations in Eq. (17) give us the contributions to A_1 and A_2.

B. A_2 and B Meson Exchange

The contributions from the A_2 and B meson exchanges may be added to the ρ and π components with the assumptions of exchange degeneracy and SU(3) as has been done by Goldstein and Owens.[13] Since the A_2 and B contributions vanish as $t \to 0$ and vary much more slowly than the pion exchange, they also trivially satisfy the conspiracy equations and their role in the near forward direction should also be of secondary importance.

3. The Background Amplitude.

Finally, let us consider the most interesting dynamical question, that of the nature of the background amplitude. Not only does this component account for the entire cross section in the forward direction, but it most likely also holds the secret of the limited shrinkage behavior of the natural parity component for large t. Where there is general agreement on the treatment of the dynamical contributions we have already discussed, there exists more variety in models proposed to represent the background term.

A. Absorptive Cuts

In models proposed by Namyslowski et al.[11] and Goldstein and Owens[13] this term is represented by absorptive cuts. The Regge exchanges in $f^s_{\frac{3}{2}\frac{1}{2},0-1}$ and $f^s_{\frac{1}{2}-\frac{1}{2},0-1}$ are inserted into absorption integrals to generate the background contributions to these amplitudes. In the absence of $\pi\Delta$ scattering data, the background is normalized to the experimental value in the forward direction. This procedure yields background terms with leading s behavior in the form $s^{\alpha(t)}/\ell n\, s$. This form for A_3 would vary markedly from the low-t form for the amplitude (over a large s range) and would require the B_4 contribution to play an important role. The absorptive cut prescription also has problems with dips for large t.[11,12,13]

B. Poor Man's Absoprtion Model

An alternative method for obtaining the background amplitude is to invoke the "Poor Man's Absorption" (PMA) model originally proposed by P. K. Williams[24] and since discussed in some detail by G. C. Fox[1,12] and M. Glück.[14] In this prescription the background term is obtained by extrapolating the pion exchange amplitude to the pion pole. This procedure is justified by the occurrence of Kronecker delta terms in the calculation of the absorption integrals.[24] It can be seen from the work of Glück that, although there is some variation in the $d\sigma/dt$ behavior in the $|t| \leq \mu^2$ region, the background amplitude in the PMA prescription is equivalent to the low-t theorem value as $t \to \mu^2$.

3. Fixed Poles.

Fox[12] has also noted that the PMA prescription for the background amplitude can be thought of as a fixed pole, $\alpha(t) = 0$. This feature can account for the limited shrinkage phenomena in the natural parity component. The same observation can be made for the representation of the background amplitude by the low-t theorem form modified by a factor depending only on t.

An explicit form for a fixed pole model is given by Bender, Dosch and Rothe[9] who represent $f^{-t-}_{\frac{1}{2}\frac{1}{2}01}$ with a simplified (without signature factors, etc.) pion exchange and they express the background as a fixed Khuri pole. The residue for the fixed pole is fixed by the value of the coupling constants and therefore is equivalent to the low-t theorem form for the amplitude.

C. Conclusions.

It is therefore clear that the variety in the dynamical models which have been suggested for $\gamma N \to \pi\Delta$ occurs mostly in the representation of the background amplitude. A resolution of the correct form for this interesting term should therefore answer some of the most intriguing mysteries associated with the process and will require polarized photon data over a much larger range of s than is presently available.

It should be observed that detailed comparisons with the photon polarization asymmetry data have been attempted thus far only in terms of the absorptive cut model. Much is still to be learned.

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