SLAC-PUB-1469
(T)
August 1974

RESONANCE PEAKS IN THE INCLUSIVE PION ELECTRO-PRODUCTION SPECTRUM FROM PARTON PICTURE

Chih Kwan Chen

Stanford Linear Accelerator Center
Stanford University, Stanford, California 94305

and

Department of Natural Philosophy
University of Glasgow, Glasgow, Scotland

ABSTRACT

The relative strength of the resonance peaks in the inclusive π electro-production spectrum in the early scaling region are related to that of the total electro-production spectrum through parton picture and Bloom-Gilman duality.

(Submitted to Physics Letters B)

The observed scaling behavior of the deep inelastic eN collision, which can be interpreted as the process $\gamma_v + N \rightarrow$ "anything," is considered to be a strong support of the parton picture [1,2]. Since the scaling behavior seems to hold for relatively small Q^2 ($-Q^2$ is the four momentum square of the virtual photon) and not very large s (the center of mass energy for the system $\gamma_v + N$), which we call the early scaling region, the interpolation between the resonance region and the scaling region becomes important. Bloom and Gilman [3] observe duality between these two regions for the process $\gamma_v + N \rightarrow$ "anything". The correspondence principle is proposed [4] for the process $\gamma_v + N \rightarrow \pi +$ "anything" to relate the inclusive scaling region and the resonance production region via the process $\gamma_v + N \rightarrow \pi +$ "resonance." The purpose of this article is to push the parton picture and Bloom-Gilman duality further to obtain strong correlation between the strength of the resonance peaks for the process $\gamma_v + N \rightarrow \pi +$ "resonance" in the inclusive pion electro-production spectrum and that for the process $\gamma_v + N \rightarrow$ "resonance" in the total electro-production spectrum. This kind of investigation is significant, since the inclusive pion production data in the early scaling region is accumulating rapidly, and the resonance production processes $\gamma_v + N \rightarrow \pi +$ "resonance" play an important role in these regions.

The electro-production of resonances for the process $\gamma_v + N \rightarrow \pi +$ "resonance" in the early scaling region can be represented in parton picture as in Fig. 1(a), where the solid lines represent "partons." In this representation the virtual photon γ_v strike a point-like, massless parton at point A, the hitted parton then emits a pion at point B, and it recombines subsequently with the remaining partons in the initial nucleon to form a resonance. The first step around point A is completely determined by the scaling variable ω in the parton model, the second step around point B, of course, cannot be described by the naive parton model, and it is at this step where we look for the help of Bloom-Gilman duality.

In order to look into the detailed character of Bloom-Gilman duality we consider the process $\gamma_v + N \rightarrow$ "resonance" for a while. In parton picture this process can be described as in Fig. 1(b), where the virtual photon γ_v hits a constituent parton at point A, and the hitted parton then immediately recombines with the remaining partons to form a resonance. The contributions from the i-th resonance with mass M_i and the background underneath it to the structure function νW_2 are denoted as $R(M_i^2,\omega')$ and $B(M_i^2,\omega')$ respectively, where ω' is Bloom-Gilman's scaling variable. Bloom and Gilman first observe that though the prescription of the naive parton model does not apply directly to this case, the strength of each resonance in νW_2 follows in magnitude the smooth scaling curve

$$g(\omega') = \lim(\text{Bjorken})\nu W_2 \ .$$

In a quantitative way we may write

$$R(M_i^2,\omega') + B(M_i^2,\omega') \approx c(M_i^2)\ g(\omega') \ , \qquad (1)$$

where $c(M_i^2)$ is an unknown function of M_i^2. The second observation made by Bloom and Gilman is that the ratio R/B is only a function of M_i^2,

$$\frac{R(M_i^2,\omega')}{B(M_i^2,\omega')} \approx h(M_i^2) . \tag{2}$$

From these two expressions R and B can be written in terms of $c(M_i^2)$, $h(M_i^2)$ and $g(\omega')$. If we compare the strength of two resonances, say the i-th and the j-th ones, we obtain

$$\frac{R(M_i^2,\omega_i')}{R(M_j^2,\omega_j')} \approx \frac{g(\omega_i')}{g(\omega_j')} \cdot \frac{H(M_i^2)}{H(M_j^2)}$$

with

$$H(M_i^2) \equiv c(M_i^2) \left\{1 + \frac{1}{h(M_i^2)}\right\}$$

and

$$\omega_i' \equiv 1 + \frac{M_i^2}{Q^2} .$$

With regard to the electro-production of resonances $\gamma_V + N \rightarrow \pi +$ "resonance," we consider for convenience the dynamics of this process in the frame where the four momenta of the virtual photon and the nucleon are represented as

$$q_\gamma = (0, 0, 0, Q)$$

and

$$P_N = (\sqrt{M_N^2 + P^2}, 0, 0, -P)$$

with

$$P = \frac{M_n}{Q} = \frac{1}{2}\left(\omega' Q - \frac{M_N^2}{Q^2}\right) .$$

We consider the pion inclusive spectrum in this frame. The processes $\gamma_v + N \rightarrow \pi +$ "resonances" are represented by the resonance peaks in the pion inclusive spectrum. From the naive parton model prescription, the momentum of the hitted parton (represented by the line AB in Fig. 1(a)) is $Q/2$ in this photon "rest" frame. Denoting the four momentum of the inclusive pion as k, the inclusive pion spectrum is a function of ω, Q^2, $\vec{k}_T$ and Z, where

$$Z \equiv k_Z / \tfrac{1}{2} Q .$$

The normalized inclusive pion spectrum $E(d^3\sigma/dk^3)/\sigma^{total}$ at the scaling limit is shown [5] in the naive parton model to depend only on ω, Z and $\vec{k}_T$. We write

$$f(\omega, Z) \equiv \lim(\text{Bjorken}) \frac{E}{\sigma^{total}} \cdot \frac{d^3\sigma}{dk^3} .$$

In the early scaling region where the peaks from the processes $\gamma_v + N \rightarrow \pi +$ "resonances" play a significant role in the pion inclusive spectrum, we denote the contributions from the i-th resonance and the background underneath it to the normalized inclusive pion spectrum as $R^{incl}(\omega, Q^2, Z_i)$ and $B^{incl}(\omega, Q^2, Z_i)$ respectively with

$$Z_i \equiv \frac{1}{4s}\left[-(\omega-2)\cdot(s-M_i^2+\mu^2) + \sqrt{\left(\omega^2 + \frac{4M_N^2}{Q^2}\right)\{s-(M_i+\mu)^2\} \cdot \{s-(M_i-\mu)^2\}}\right]$$

and

$$s = M_N^2 + Q^2(\omega - 1).$$

The transverse momentum $\vec{k}_T$ is supposed to be integrated over. The variable Z_i is the position of the i-th resonance peak in the inclusive pion spectrum in the Z-space.

Comparing the dynamics around point B of Fig. 1(a) with that of Fig. 1(b), we see that the only qualitative difference between the two is the mechanism to send the partons into the resonance states, by the emission of a pion in the case of Fig. 1(a) and by absorbing a photon in the case of Fig. 1(b). The binding mechanism of the partons in these two cases are the same. Therefore we expect that the similar duality relations as Eqs. (1) and (2) also holds for the case $\gamma_n + N \rightarrow \pi +$ "resonance". We thus may write

$$R^{incl}(\omega, Q^2, Z_i) + B^{incl}(\omega, Q^2, Z_i) \approx c^{incl}(M_i^2)\ f(\omega, Z_i) \qquad (4)$$

and

$$\frac{R^{incl}(\omega, Q^2, Z_i)}{B^{incl}(\omega, Q^2, Z_i)} \approx h^{incl}(M_i^2)\ . \qquad (5)$$

As in Eq. (3) we obtain from Eqs. (4) and (5) that

$$\frac{R^{incl}(\omega, Q^2, Z_i)}{R^{incl}(\omega, Q^2, Z_j)} \approx \frac{f(\omega, Z_i)}{f(\omega, Z_j)} \cdot \frac{H^{incl}(M_i^2)}{H^{incl}(M_j^2)} \qquad (6)$$

with

$$H^{incl}(M_i^2) \equiv c^{incl}(M_i^2)\ \left\{1 + \frac{1}{h^{incl}(M_i^2)}\right\}.$$

Equations (5) and (6) can be considered as an alternative expression of Bjorken and Kogut's correspondence principle [4]. In order to proceed further we look into the functions $c(M_i^2)$, $h(M_i^2)$, $c^{incl}(M_i^2)$ and $h^{incl}(M_i^2)$. The functions c and h are related to the detailed binding mechanism of the partons. If we fix $s = M_i^2$ and vary Q^2 for the process $\gamma_v + N \rightarrow$ "resonance," the momentum of the hitted parton, which will be $Q/2$ if the partons are free, varies even when the final and the initial partons are not in the free states. Furthermore due to the different momentum distributions of the different species of partons inside the nucleon, varying Q^2 implies varying probabilities for the photon to hit a specific kind of partons. Nevertheless Bloom and Gilman's observations indicate that the functions c and h related to the binding mechanism are irrelevant to what kind of partons and how hard they are hitted. These functions are determined only by the invariant mass of the resonating system, M_i. Now look back at the difference between Fig. 1(a) around the point B and Fig. 1(b) again. The species of partons can be hitted in Fig. 1(a) is restricted if the charge of the inclusive pions is specified, whereas all the partons in the nucleon can be hitted in Fig. 1(b). The degree of disturbance caused by the incoming photon on the hitted parton are also quite different for these two cases. However these differences are exactly those which are irrelevant to the functions c's and h's according to our interpretation of Bloom and Gilman's duality observations. Therefore it is reasonable to postulate that

$$c(M_i^2) = c^{incl}(M_i^2)$$

and

$$h(M_i^2) = h^{incl}(M_i^2) \ .$$

With the help of these equations, we obtain from Eqs. (3) and (6) that

$$\frac{R^{incl}(\omega, Q^2, Z_i)}{f(\omega, Z_i)} \approx \frac{R^{incl}(\omega, Q^2, Z_j)}{f(\omega, Z_j)} \cdot \frac{R(M_i^2, \omega_i')}{g(\omega_i')} \cdot \frac{g(\omega_j')}{R(M_j^2, \omega_j')} \qquad (7)$$

where

$$\omega_i' \equiv 1 + \frac{M_i^2}{Q^2} \ .$$

Equation (7) is our main result. The total cross section for the process $\gamma_v + N \to$ "anything" shows [6] three clear peaks and probably one more in the resonance region. According to Eq. (7), we expect to see similar resonance structure in the process $\gamma_v + P \to \pi^+ +$ "anything." Define

$$r_i \equiv \frac{R_i^{incl}(\omega, Q^2, Z_i)}{f(\omega, Z_i)} \ ,$$

and we see from Eq. (7) that r_i/r_j is independent of ω' but on Q^2 through $\omega_{i,j}'$. The ratios $R(M_i^2, \omega_i')/g(\omega_i')$ are estimated in Ref. [3]. Using this result we obtain the ratios r_2/r_1 and r_3/r_1 for various Q^2 values as listed in Table 1.

The author would like to thank Professor S. D. Drell for the hospitality in SLAC theory group where this work is completed.

REFERENCES

1. R. P. Feynman, Phys. Rev. Letters 23 (1969) 1415.

2. J. D. Bjorken, Phys. Rev. 179 (1969) 1547.

3. E. D. Bloom and F. J. Gilman, Phys. Rev. Letters 25 (1970) 1140; Phys. Rev. D (1971) 2901.

4. J. D. Bjorken and J. Kogut, Phys. Rev. D (1973) 1341.

5. S. D. Drell and T-M. Yan, Phys. Rev. Letters 24 (1970) 855; E. W. Colglazier and F. Ravndal, Phys. Rev. D7 (1973) 1537.

6. M. Breidenbach et al., Phys. Rev. Letters 23 (1969) 935; E. D. Bloom et al., Phys. Rev. Letters 23 (1969) 930.

Q^2 $(GeV/c)^2$	r_2/r_1	r_3/r_1
1.86	0.6	0.5
2.58	0.8	0.6
3.11	0.9	0.5

TABLE 1

Predicted ratios of r_2/r_1 and r_3/r_1 for various Q^2 values

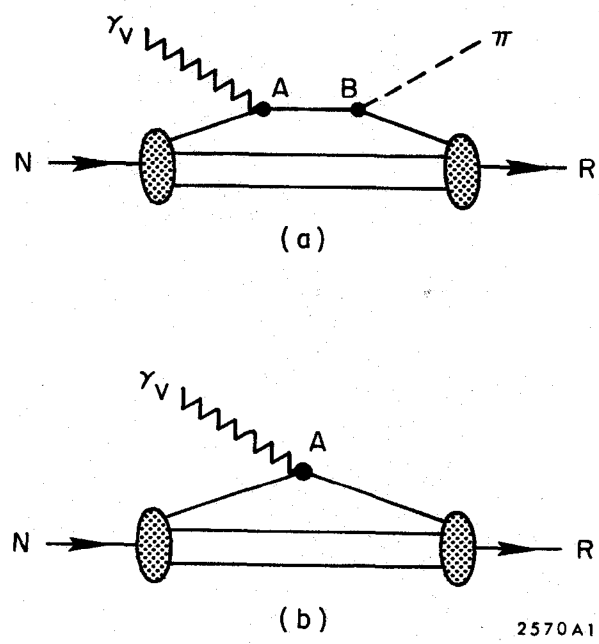

Fig. 1(a): The graphical representation of the process $\gamma_v + N \rightarrow \pi +$ "resonance" in parton picture. The solid lines represent partons, and R represents "resonance."

Fig. 1(b): The graphical representation of the process $\gamma_v + N \rightarrow$ "resonance" in parton picture.