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DEEP INELASTIC SCATTERING AND THE BREAKING OF DIMENSIONAL SCALING *

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I. INTRODUCTION

The emphasis of the present work is on using final state properties as a guide to the selection of theories of deep inelastic scattering. An important result of our analysis is the introduction of two broad classifications defined as follows: Introduce the moments of the structure function $F_2(\omega, q^2) = \nu W_2$,

$$\int_1^\infty \frac{F_2(\omega, q^2)}{\omega^n} d\omega = C_n \left(\frac{M^2}{-q^2}\right)^{\frac{1}{2}\gamma(n)} + \cdots . \qquad (1)$$

for even n. The C_n are constants and the $\gamma(n)$ are anomolous dimensions of the spin n operators in the operator product expansion of two currents. The restriction to pure power form or fixed point behavior in Eq. (1) is not essential. The results of this paper hold for the more complicated form which arises from the Callan Symanzik equations[1,2] away from a fixed point, ($\beta(g) \neq 0$), including the case of asymptotic freedom.[3] We classify theories by the behavior of $\gamma(n)$ as $n \to \infty$.

Class I: $\gamma(n) \to$ const. as $n \to \infty$

Class II: $\gamma(n) \to +\infty$ as $n \to \infty$

No other possibilities are allowed by Nachtmann's positivity analysis.[4] Bjorken scaling requires $\gamma(n) = 0$ for all n, so a theory with exact Bjorken scaling is in Class I. Asymptotically free theories have $\gamma(n) = 0(g^2 \log n)$ as $n \to \infty$, so they are in Class II.

The results of our analysis are that Class I behavior implies an unacceptable jet structure for deep inelastic final states, whereas no such jet structure need be present for the Class II case. We therefore suggest that the correct theory of deep inelastic scattering is of Class II. Below we outline the method of analysis and the characteristic experimental signals for Class II behavior.

II. CLASS I THEORIES

Introducing the operator product expansion of two currents we write,

$$T\left[J_\mu(x)\, J_\nu(0)\right] \underset{x^2 \to 0}{\longrightarrow} i \sum_n O^n_{\alpha\mu_2\cdots\mu_n}(0)\, \frac{x^{\mu_2}\cdots x^{\mu_n}}{b(n)}\, S^{\alpha\beta}_{\mu\nu} \times$$

$$\times\ \partial_\beta \left[(-M^2x^2)^{\frac{1}{2}\gamma(n)}\, D_F(x^2)\right]$$

$$+ \text{ less singular terms}, \tag{2}$$

where we will carry along explicitly only the operators which contribute to spin averaged matrix elements of $F_2(\omega, q^2)$. The factors $b(n)$, $S^{\alpha\beta}_{\mu\nu}$, and $D_F(x^2)$ are given by

$$b(n) = 2^{\gamma(n)}\,\Gamma(n + \tfrac{1}{2}\gamma(n))/\Gamma(1 - \tfrac{1}{2}\gamma(n)),$$

$$S^{\alpha\beta}_{\mu\nu} = -g_{\mu\nu}\, g^{\alpha\beta} + g^\alpha_\mu\, g^\beta_\nu + g^\beta_\mu\, g^\alpha_\nu,$$

$$D_F = \frac{i}{4\pi^2(x^2 - i\epsilon)}\ .$$

The $O^n(0)$ are local field operators of spin n and dimension $n + 2 + \gamma(n)$. Their spin averaged matrix elements are defined by

$$\langle p|O^n_{\alpha\mu_2\cdots\mu_n}|p\rangle = (i)^n\, \frac{C_n}{M}\left[p_\alpha p_{\mu_2}\cdots p_{\mu_\nu} - \text{trace terms}\right] \tag{3}$$

where C_n is the same constant that appears in Eq. (1).

(a). $\underline{\gamma(n) \equiv 0}$

This is the case of exact Bjorken scaling, which corresponds to free field behavior near $x^2 = 0$. There are no known explicit models which satisfy Bjorken scaling, except for the trivial case of a free field theory, so we must proceed by using general arguments. We first note that in the definition of the structure functions,

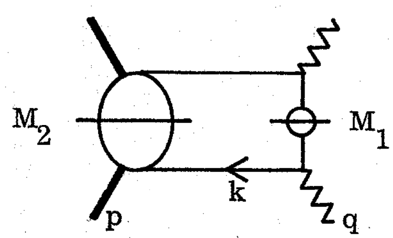
$$W_{\mu\nu} = \left(-g_{\mu\nu} + \frac{q_\mu q_\nu}{q^2}\right) W_1 + \frac{1}{M^2}\left(p_\mu - \frac{p\cdot q}{q^2}\, q_\mu\right)\left(p_\nu - \frac{p\cdot q}{q^2}\, p_\nu\right) W_2$$

$$= (2\pi)^3 \sum_n \langle p|J_\mu(0)|n\rangle\ \delta^4(q+p-n)\ \langle n|J_\nu(0)|p\rangle \qquad (4)$$

only asymptotic states of on shell hadrons appear in the states $|n\rangle$. This remains true when a Fourier transform is introduced and the commutator $[J_\mu(x), J_\nu(0)]$, and time ordered product $T[J_\mu(x) J_\nu(0)]$ are computed as Fourier transforms of the corresponding quantities in momentum space. Therefore only asymptotic states occur between $J_\mu(x)$ and $J_\nu(0)$ in Eq. (2). Since the left hand side of Eq. (2) involves only asymptotic states, so must the right side. The only straightforward way to interpret the factor $D_F(x^2)$ is as the propagator of a finite mass hadron system, where the mass can be ignored in the asymptotic limit. This leads directly to a two jet structure for deep inelastic final states as shown in Fig. (1).

FIG. 1

$W_{\mu\nu} =$



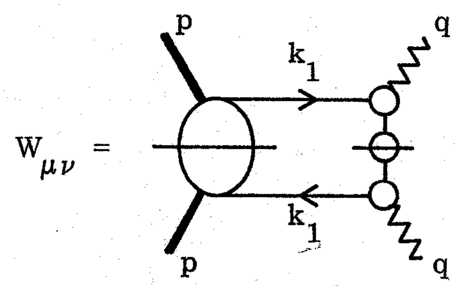
To maintain scaling, the damping in the momentum transfer k^2 must be more rapid than $\left(\frac{1}{k^2}\right)^2$.[5] The masses M_1 and M_2 must also be dominated by finite values. The essential point is that the systems M_1 and M_2 must be composed of hadron asymptotic states, since nonasymptotic states (quarks, partons) never come into the definitions of the structure functions or their Fourier transforms. Whether complicated, non two jet configurations of hadrons could build up the free field factor $D_F(x^2)$ is not completely ruled out, but it would appear unlikely.

(b) $\gamma(n) \neq 0$, $\gamma(n) \rightarrow$ const.

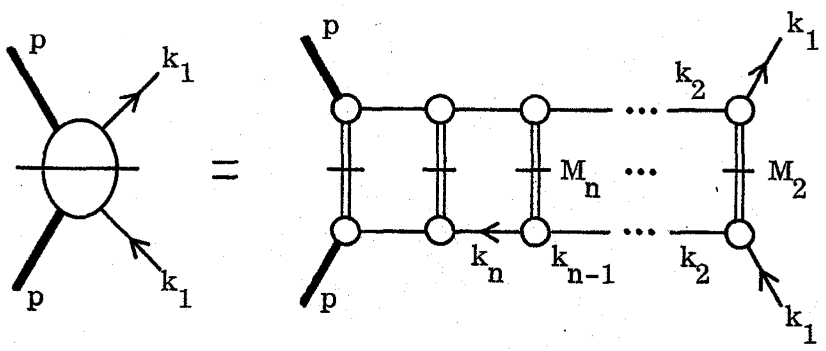
Here much more explicit results can be obtained, since this case is realized in Lagrangian theories involving only spin zero and spin one half particles. The pure power behavior of the structure function moments in Eq. (1) is obtainable in two ways, either at a fixed point $\beta(g_f) = 0$, or at general values of the coupling constant g, if self energy and vertex insertions are omitted from all graphs. If the theory is near a fixed point, only qualitative changes are necessary.

The important result $\gamma(n) \rightarrow 2\gamma_\psi$ for the class of Lagrangian theories described above, was obtained by Callan and Gross,[6] following an earlier conjecture by Parisi.[7] The quantity γ_ψ is defined as the dimension of the fundamental field in the theory with lowest dimension. The final state structure is shown in Fig. (2)

FIG. 2



The dominance of this generalized handbag graph in the Bjorken limit was shown for the case of ϕ^4 theory by Parisi,[7] and can be justified for more general Yukawa theories by the same method, or by using the techniques of Cornwall and Tiktopoulos.[8] There are two differences as compared to the case of exact Bjorken scaling. The first is the presence of vertex functions where the virtual photon enters. The second is that here the damping of the momentum transfer k_1^2 is weaker than $\left(\frac{1}{k_1^2}\right)^2$, whereas the damping must be stronger than $\left(\frac{1}{k_1^2}\right)^2$ for the case of exact Bjorken scaling.[5] The presence of jets in the final states remains. Two jet structure dominates as $\omega \to 1$. As ω increases, a multi-jet structure develops, as shown in Fig. (3). The masses of the systems M_i remain dominated by finite values, but now the momentum transfers are allowed to be large, $\langle \frac{k_1^2}{q^2} \rangle$ = const., $\langle \frac{k_2^2}{k_1^2} \rangle$ = const., etc.

FIG. 3 

The presence of low mass systems produced strongly with large rapidity gaps and large amounts of high $p_\perp$ scattering seems unacceptable for final states of composite hadrons. This motivates the investigation of Class II theories.

III. CLASS II THEORIES

In this section, we take up the case $\gamma(n) \to \infty$. A complete analysis of final state properties has not yet been carried out. Here we concentrate on the single particle inclusive spectrum in the target fragmentation region. We first introduce the useful observation that once the asymptotic region is reached, Mellin transform techniques can be used to calculate the structure function F_2 at q_2^2 from its value at a smaller value, q_1^2.[9] We have

$$F_2(\omega, q_2^2) = \int_1^\omega F_2(\omega', q_1^2)\, g\left(\frac{\omega}{\omega'}, \frac{q_1^2}{q_2^2}\right) \frac{d\omega'}{\omega'} \qquad (5)$$

$$g\left(\frac{\omega}{\omega'}, \frac{q_1^2}{q_2^2}\right) = \frac{1}{2\pi i} \int_{-i\infty}^{i\infty} \left(\frac{\omega}{\omega'}\right)^{n-1} \left(\frac{q_1^2}{q_2^2}\right)^{\frac{1}{2}\gamma(n)} dn \qquad (6)$$

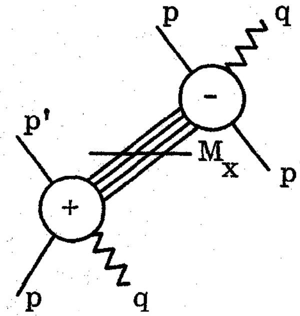
If $\gamma(n) \equiv 0$, Eq. (5) reduces to $F_2(\omega, q_2^2) = F_2(\omega, q_1^2)$, which is just the statement of Bjorken scaling.

We now apply the same method to the calculation of the single particle spectrum. We introduce a function $\mathscr{F}_2$ which describes the process $q + p \to p' +$ missing mass. The functions $\mathscr{F}_2$ and F_2 are related by

$$\int \mathscr{F}_2 \frac{d^3p'}{p_0'} = \langle n' \rangle\, F_2(\omega, q^2) \qquad (7)$$

where $\langle n' \rangle$ is the average multiplicity of hadrons of the same type as p'. An average over the azimuthal angle of p' is implied. We now apply the generalized optical theorem to relate $\mathscr{F}_2$ to the discontinuity of the $3 \to 3$ amplitude as shown in Fig. (4), where $M_x^2 = (q + p - p')^2$. If the momenta p and p' are fixed, we may also use the small x^2 expansion. We use as variables

FIG. 4 $\qquad \mathscr{F}_2 = \mathrm{disc}_{M_x^2}\, T_{33} \quad =$



$x \equiv \frac{p'_-}{p_-}$, $p'_\perp$, ω, q^2, where the direction of the virtual photon defines the +z axis. The same Mellin transform technique can be applied to $\mathscr{F}_2$ as to the structure function itself. The transform is taken in the variable $\bar{\omega} = \frac{2(p-p')\cdot q}{-q^2}$, related to the missing mass by $M_x^2 = -q^2(\bar{\omega} - 1) + \text{const}$. The variable $\bar{\omega}$ is related to ω by $\bar{\omega} = \omega(1-x)$. The formula for $\mathscr{F}_2$ analogous to Eq. (5) for F_2 reads

$$\mathscr{F}_2(x, p'_\perp, \omega, q_2^2) = \int_{\frac{1}{1-x}}^{\omega} \mathscr{F}_2(x, p'_\perp, \omega', q_1^2)\, g\left(\frac{\omega}{\omega'}, \frac{q_1^2}{q_2^2}\right) \frac{d\omega'}{\omega'} \tag{8}$$

The lower limit $\frac{1}{1-x}$ reflects the requirement that the missing mass M_x^2 must be positive.

So far the analysis is general and can be applied to any theory with $\gamma(n) \neq 0$. The characteristic feature of a theory with $\gamma(n) \to \infty$ is that the function $g\left(\frac{\omega}{\omega'}, \frac{q_1^2}{q_2^2}\right)$ of Eq. (6) will be strongly peaked around a value of ω' less than ω, the amount by which it is less depending on the ratio q_1^2/q_2^2. To illustrate, consider the case where $\gamma(n)$ grows linearly, which is the maximum allowed by Nachtmann's positivity analysis.[4] Making the conventional assumption $\gamma(2) = 0$, we take the form $\gamma(n) = \alpha(n-2)$. Equation (6) gives for $g\left(\frac{\omega}{\omega'}, \frac{q_1^2}{q_2^2}\right)$,

$$g\left(\frac{\omega}{\omega'}, \frac{q_1^2}{q_2^2}\right) = \left(\frac{q_2^2}{q_1^2}\right)^{2\alpha} \frac{\omega'}{\omega}\, \delta\left(\log \frac{\omega}{\omega'} - \log\left(\frac{q_2^2}{q_1^2}\right)^{\alpha}\right) \tag{9}$$

The very strong suppression of the structure function near $\omega = 1$, characteristic of Class II behavior, here shows up as the vanishing of the structure function $F_2(\omega, q_2^2)$ for $1 < \omega < \left(\frac{q_2^2}{q_1^2}\right)^{\alpha}$. This literal vanishing is special to the case of linearly growing $\gamma(n)$. For any growth less than linear, say $\gamma(n) \sim n^{\epsilon}$, $\epsilon < 1$, the effect is smoothed out and $F_2(\omega, q_2^2)$ will decrease rapidly as q_2^2/q_1^2 increases, but will not actually vanish.

With regard to the inclusive spectrum, $\mathscr{F}_2$, the lower limit of $\frac{1}{1-x}$ in Eq. (8) causes strong suppression near the maximum allowed value of x, which corresponds to the smallest missing mass M_x^2. In the extreme case of linear growth for $\gamma(n)$, there is vanishing of $\mathscr{F}_2(x, p'_\perp, \omega, q_2^2)$ for $x > 1 - \left(\frac{q_2^2}{q_1^2}\right)^{\alpha}\Big/\omega$.

Again the effect is smoothed out for any growth rate less than linear for $\gamma(n)$. The region of phase space which is being suppressed here is the only region which does contribute for a theory which obeys Bjorken scaling, so the two behaviors are very different.

It is easily checked that there is no simple factorization when the ratio $\mathscr{F}_2/F_2$ is formed, so both Feynman scaling for the inclusive spectrum and Bjorken scaling for the structure functions are broken in an essential way. The experimental signals of Class II behavior are the following:

(a). Strong suppression of the $\omega \to 1$ region, the suppression becoming stronger as q_2^2/q_1^2 increases.

(b). If the assumption $\gamma(2) = 0$ is valid, then there will be enhancement of the large ω region, the enhancement growing as (q_2^2/q_1^2) increases.

(c). Strong suppression of the inclusive spectrum near the region of maximum longitudinal momentum. Feynman scaling cannot be restored by dividing by $F_2(\omega, q_2^2)$.

Qualitatively, the spectrum in longitudinal momentum for a Class II theory will be pushed toward the central region, in complete contrast to the case of a two jet picture for final states. This is the basis of our suggestion that the correct theory is likely to be a member of Class II.

To gain further insight, it is essential to develop models. Without an explicit model, the analysis of final state properties will be limited to the target fragmentation region as discussed above. We close with some speculation on models.

Increasing anomolous dimensions are found in all Lagrangian field theories containing gauge fields. Calculations to lowest order in g^2 show $\gamma(n) = 0(g^2 \log n)$, for any gauge theory, asymptotically free or not.[6] Gauge theories can then form an interesting laboratory for further investigation. However, it is likely that a simple theory in which the fundamental fields create and destroy asymptotic states of particles, while it would have suppression of the $\omega = 1$, $x = x_{max}$ regions, would also suffer from having too much large $p_\perp$ production as in case (b) of Section II. An attractive conjecture is that a gauge theory in which the fundamental fields are confined and the hadrons are all composite would finally achieve the two desirable features of $\gamma(n) \to \infty$, and damped large $p_\perp$ scattering.

There is nothing in the above analysis which particularly favors asymptotically free theories. In fact if phenomenologically a stronger increase than $\gamma(n) \sim \log n$, is indicated, there would be evidence against asymptotic freedom, since at finite coupling constant, a power increase for $\gamma(n)$ could come about by summing terms of the form $(g^2 \log n)^m$, but for the asymptotically free case the coupling constant is renormalized to zero and the lowest order result prevails. To conclude, we strongly suggest Class II behavior, but we regard the question of asymptotic freedom as quite open.

REFERENCES

1. K. Symanzik, Commun. Math. Phys. 23, 49 (1971).
2. C.G. Callan, Phys. Rev. D5, 3202 (1972).
3. D.J. Gross and F. Wilczek, Phys. Rev. D8, 3633; Phys. Rev. D9, 980 (1974).
4. O. Nachtmann, Nucl. Phys. B63, 237 (1973).
5. J. Stack, Illinois preprint ILL-(TH)-73-4 (1973).
6. C.G. Callan and D. Gross, Phys. Rev. D8, 4383 (1973).
7. G. Parisi, Nucl. Phys. B59, 641 (1973).
8. J.M. Cornwall and G. Tiktopoulos, Phys. Rev. D8, 2687 (1974).
9. G. Parisi, Phys. Letters 43B, 207 (1973).