# ANNIHILATION OF $e^{+} e^{-}$INTO HADRONS ${ }^{*}$ <br> J. D. Bjorken <br> Stanford Linear Accelerator Center, Stanford University, Stanford, California 94305 

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I. Introduction

In the past few years we have seen our confidence in a quark description of hadron structure grow. This stems from two main sources: one is the generally successful description of baryon spectroscopy, and the other is the success of the quark-model light-cone angebra of currents in accounting for data on electroproduction and neutrino reactions. This success led to apparently quite reliable and definite predictions for the behavior of the total cross section for the colliding-beam process $\mathrm{e}^{+} \mathrm{e}^{-} \rightarrow$ hadrons, predictions apparently violated by the recent data from CEA and SPEAR. Likewise the scaling behavior of electroproduction (and of the inclusive spectra of electroproduced hadrons) suggested a similar scaling hypothesis for the inclusive production of hadrons in $\mathrm{e}^{+} \mathrm{e}^{-}$annihilation. This scaling is also apparently violated. With such a situation, it is clear that there lies ahead a period of careful assessment of the theoretical tools we have been using, as well as the facts learned from experiment. We must determine whether the trouble lies in the hypotheses of an underlying quark structure of hadrons or in the overconfidence in the theoretical hypotheses of scaling, or whether the trouble is simply caused by a "background" of additional exotic processes for hadron production in $\mathrm{e}^{+} \mathrm{e}^{-}$collisions.

## II. Status of the Data

Measurement of the total cross section for $\mathrm{e}^{+} \mathrm{e}^{-} \rightarrow$ hadrons at Frascati ${ }^{1}$ showed a large production cross section for multihadron final states for $1.5 \mathrm{GeV}<\mathrm{E}_{\mathrm{CMS}}<3 \mathrm{GeV}$ with, however, a large systematic uncertainty in its magnitude. The hadron production is $\sim 1$ to 2 times larger than the cross
section for $\mathrm{e}^{+} \mathrm{e}^{-} \rightarrow \mu^{+} \mu^{-}$the theoretical standard of reference. That is, $R=$ $\sigma\left(\mathrm{e}^{+} \mathrm{e}^{-} \rightarrow\right.$ hadrons) $/ \sigma\left(\mathrm{e}^{+} \mathrm{e}^{-} \rightarrow \mu^{+} \mu^{-}\right) \sim 1-2$. (The theoretical values for the $\mathrm{e}^{+} \mathrm{e}^{-} \rightarrow \mu^{+} \mu^{-}$cross section have been confirmed experimentally with high accuracy up to energies $E_{C M S} \simeq 5 \mathrm{GeV}$ at SPEAR. )

At higher energies the measurements of the total cross section for $\mathrm{e}^{+} \mathrm{e}^{-} \rightarrow$ hadrons at the Cambridge Electron Accelerator ${ }^{2}$ have been confirmed at SPEAR ${ }^{3}$, and indicate a roughly constant cross section from $\mathrm{E}_{\mathrm{CMS}} \sim 3 \mathrm{GeV}$ to $\mathrm{E}_{\mathrm{CMS}} \sim 4.8 \mathrm{GeV}$ with a magnitude $\sim 20 \mathrm{mb}$. This gives at the highest energy a cross section $\sim 5$ times that for $\mu$-pair production; $\mathrm{R} \approx 5$.

From SPEAR also come a number of other very preliminary results on the spectra of produced hadrons which are briefly summarized below:

1) The inclusive cross section Ed $\sigma / \mathrm{d}^{3}$ p) for production of charged hadrons is well represented (to accuracy $\sim 20 \%$ ) by an exponential, $\exp (\sim E / T)$, with temperature $\mathrm{T} \approx 170 \mathrm{MeV}$, at both $\mathrm{E}_{\mathrm{CMS}} \approx 3 \mathrm{GeV}$ and $\mathrm{E}_{\mathrm{CMS}} \approx 4.8 \mathrm{GeV}$. Above $E \sim 1.2 \mathrm{GeV}$ there may be a break in the curve leading to an excess of a factor $\sim 3.0$ at $\mathrm{E} \approx 1.7 \mathrm{GeV}$.
2) The angular distribution of charged particles (for $|\cos \theta|<0.5$ ) is observed to be uniform, both at low momentum ( $\mathrm{p} / \mathrm{p}_{\max }<0.45$ ) at high momentum, and at both $\mathrm{E}_{\mathrm{CMS}} \simeq 3 \mathrm{GeV}$ and $\mathrm{E}_{\mathrm{CMS}} \approx 4.8 \mathrm{GeV}$. With the angular distribution parametrized as $1+\alpha \cos ^{2} \theta$ it is found typically $\alpha \simeq 0.0 \pm 0.3$ (although this number should be taken only as a general indication of the accuracy of these preliminary conclusions).
3) For hadron momentum $\mathrm{p}<0.6 \mathrm{GeV}$ the inclusive spectrum has been separated (by time-of-flight measurements) into separate pion, kaon, and
nucleon contributions. When $E\left(d \sigma / d^{3} p\right)$ is plotted versus energy these spectra again lie on the same exponential curve, at both $\mathrm{E}_{\mathrm{CMS}} \approx 3 \mathrm{GeV}$ and 4.8 GeV . Integration of these spectra leads to a $\pi: \mathrm{K}: \mathrm{p}$ ratio $100 / 10 / 1$ in rough order of magnitude.
4) Constancy of $\sigma_{\text {tot }}$ and $\mathrm{E}\left(\mathrm{d} \sigma / \mathrm{d}^{3} \mathrm{p}\right)$ with total energy, as well as an assumed full isotropy of the angular distribution and sharp decrease of $\mathrm{E}\left(\mathrm{d} \sigma / \mathrm{d}^{3} \mathrm{p}\right)$ with E , implies that the charged multiplicity $\overline{\mathrm{n}}_{\text {ch }}$ should also be constant. More direct measurement indicates a slow rise between $\mathrm{E}_{\mathrm{CMS}} \approx 3 \mathrm{GeV}$ and 4.8 GeV , with $\bar{n}_{\mathrm{ch}} \approx 4$ and the increase $\Delta \overline{\mathrm{n}}_{\mathrm{ch}} \approx 0.5$.
5) Constancy of $\sigma_{\text {tot }}$ and $E\left(d \sigma / d^{3} p\right)$ with total energy, along with isotropy of the angular distribution and proportionality $E d \sigma / d^{3} p \sim e^{-E / T}$, also would imply that the total mean energy found in the final-state charged hadrons is a constant independent of $E_{\text {CMS }}$. Another way of saying this is that the fraction of initial energy found in charged hadrons is a decreasing function of $\mathrm{E}_{\mathrm{CMS}}{ }^{*}$ More directly measured than through the above argument, experiment indicates that this fraction decreases from $\approx 2 / 3$ at $E_{C M S} \approx 3 \mathrm{GeV}$ to $\approx \frac{1}{2}$ at $\mathrm{E}_{\mathrm{CMS}}=$ 4.8 GeV. The precise numbers associated with this "energy crisis" depend slightly upon production models inasmuch as the solid angle subtended by the apparatus is $\approx 2 \pi$.
6) While the above results, so characteristic of pure hadronic processes, invite an interpretation of the data in terms of the two-photon mechanism $e^{+} e^{-} \rightarrow e^{+} e^{-}+$hadrons, there is some experimental evidence that less than $\sim 10 \%$ can be due to the two-photon mechanism. Luminosity monitors in the SPEAR apparatus near $\theta=0^{\circ}$ sensitive to $\mathrm{e}^{+}$and $\mathrm{e}^{-}$show no coincidences with the multihadron reactions, except for a few coplanar low-mass $\mathrm{e}^{+} \mathrm{e}^{-}$and
$\mu^{+} \mu^{-}$pairs at the expected theoretical rate. Improved measurements of the two-photon contribution should be available in the coming year from SPEAR and the new $\mathrm{e}^{+} \mathrm{e}^{-}$rings DORIS at DESY.
7) For exclusive channels, measurements at high energy exist only at Frascati ${ }^{4,5}$. At $\mathrm{E}_{\mathrm{CMS}} \approx 2.1 \mathrm{GeV}$ the total cross sections into $\pi^{+} \pi^{-}, \mathrm{K}^{+} \mathrm{K}^{-}$ and pp final states are comparable, and the form factors are of the same order $\left|\mathrm{F}_{\pi}\right|^{2} \approx\left|\mathrm{~F}_{\mathrm{K}}\right|^{2} \approx 0.02,\left|\mathrm{~F}_{\mathrm{peff}}\right|^{2} \approx 0.014$.
III. General Comments About the Data
8) Exotic explanations: While it is probable that the multihadron production proceeds through materialization of a single time-like virtual photon, it is important to check out other alternatives. The two-photon hypothesis has already troubles with experiments. But even if that were not the case the observed cross section is at least an order of magnitude larger than expected in this case from considerations using the vector dominance model. The hypothesis of direct coupling of electrons to hadrons ("zero-photon process") via a new coupling of a semiweak strength must face the following questions:
(a) Will there be a large violation of scaling behavior in electroproduction at comparable values of $Q^{2} \sim 10-25 \mathrm{GeV} ?$ (b) Why is there no such coupling for neutrinos? (c) If there is a similar coupling for muons, will it affect the decays $\eta \rightarrow \mu^{+} \mu^{-}, \eta \rightarrow \mathrm{e}^{+} \mathrm{e}^{-}$and $\mathrm{K}_{\mathrm{L}} \rightarrow \mu^{+} \mu^{-}$and will it affect level shifts in mumesic atoms?

Greenberg and Yodh ${ }^{6}$, and Nanopoulos and Vlasspoulos ${ }^{7}$, have ascribed fully hadronic properties to the electron and argued that one is observing "diffraction scattering"; they suggest a sharp forward peak in the angular distribution
of hadrons. Inasmuch as any object which is exchanged in the t-channel or u-channel amplitude for $\mathrm{e}^{+} \mathrm{e}^{-} \rightarrow$ hadrons carries lepton-number, this process is very different (and most probably much more local in impact parameter space) than an ordinary hadronic process. Any resemblance of a zero-photon process to ordinary hadron physics is purely coincidental.

Within the context of the single-photon exchange mechanism, there also exist "exotic" explanations, such as production of heavy-lepton pairs decaying predominantly into multihadron states, or for that matter any pairs of charged non-hadronic objects (especially $J=1$ bosons) which, however, decay predominantly into hadrons. For such a hypothesis, good tests are the ultimate constancy of $\bar{n}$ with increasing $\mathrm{E}_{\text {CMS }}$, and (at high $\mathrm{E}_{\mathrm{CMS}}$ ) the strict scaling behavior of the inclusive distribution.

Hereafter, we shall not consider exotic alternatives. Thus we assume that what is measured are essentially squared matrix elements of the hadron electromagnetic current operator between the vacuum and the final hadron states in question.
2) Implications of $\sigma_{\text {tot }} \sim$ Constant

If $\sigma_{\text {tot }}$ continues to remain constant at $\sim 20 \mathrm{mb}$ as $\mathrm{E}_{\mathrm{CMS}}$ continues to rise, the hadronic vacuum polarization contribution to the photon propagator becomes large, and perturbative quantum electrodynamics breaks down. This is discussed in more detail in Section IV. However, in any case a large value of $R$ can give rise to measurable corrections to the $\mu^{+} \mu^{-}$production and Bhabha scattering cross sections. For example, were $\sigma_{\text {tot }}$ to remain constant up to $\mathrm{E}_{\mathrm{CMS}} \sim 50 \mathrm{GeV}$, then the correction to the Bhabha cross section at $90^{\circ}$ for $\mathrm{E}_{\mathrm{CMS}} \sim 8 \mathrm{GeV}$ is $\sim 3 \%$.

## 3）Senile Scaling？

Many theorists have entertained the idea that scaling behavior for $\sigma_{\text {tot }}$ exists（ $\mathrm{R}=$ const）but that the approach to scaling is slow（＂senile scaling＂）， at least a factor $\sim 20$ slower than the approach to scaling in electroproduction． However，it is also possible that scaling behavior does exist for $2 \mathrm{GeV}^{2} \lesssim$ $\mathrm{Q}^{2} \lesssim 10 \mathrm{GeV}^{2}$（e．g． $\mathrm{R} \sim$ constant $\sim 2$ and an inclusive spectrum which exhibits scaling behavior）．This is，after all，the main region of space -1 ike $Q^{2}$ for which scaling behavior has been established．

It is a testament to the power of colliding beam in reaching high $Q^{2}$ efficiently that we could have moved past（in the sense of experiments）the ＂scaling＂region $2<q^{2}<10 \mathrm{GeV}^{2}$ without scarcely recognizing it．In the corresponding time－like region，only the Frascati data on $\sigma_{\text {tot }}$ exists，with large scatter in the experimental points，and there is not yet any measurement of the inclusive spectrum．

4）The＂Energy Crisis＂：As mentioned above，there is observed a larger fraction of energy in neutral particles than what is expected naively from equal production of $\pi^{0}, \pi^{+}$，and $\pi^{-}$，namely，（neutral energy）／（charged energy）$\approx$ 0．5．At $\mathrm{E}_{\mathrm{CMS}} \approx 3 \mathrm{GeV}$ the data are in rough agreement with this estimate． However the ratio is nearly unity（with errors $\sim 20 \%$ ）at $\mathrm{E}_{\mathrm{CMS}} \approx 4.8 \mathrm{GeV}$ ． Before drawing far－reaching conclusions，it is important to observe that in the pure annihilation process $\bar{p} \rightarrow$ mesons + no baryons，there is also an excess ${ }^{8}$

$$
\frac{\text { neutral energy }}{\text { charged energy }} \overline{p p}^{\sim} \sim 0.7
$$

for $10 \lesssim \mathrm{~s} \lesssim 15 \mathrm{GeV}^{2}$.

This much of an excess may be accounted for by supposing that inclusive production of $\eta$ is comparable to the inclusive production of kaons (or pions) inasmuch as over $80 \%$ of the energy in $\eta$-decay products is found in neutrals. However the energy dependence of the experimental effect remains unexplained, especially since the $K / \pi$ ratio (at fixed hadron energy) does not depend much on $\mathrm{E}_{\text {CMS }}$, and since the K must be produced in pairs. This makes an explanation. based upon an energy dependence of $\eta$-production as a consequence of threshold effects more difficult to support.

However, after inclusion of the effect of $\eta$ production, the magnitude of the residual "energy crisis" is rather small and conceivably may even be removed by the more refined analysis of the data now in progress.
5) Particle ratios: Everywhere it has been measured the ratio of produced $\pi / \mathrm{K} / \mathrm{p}$ at the same hadron energy is of order unity. This might be considered a natural consequence of a statistical-hydrodynamical picture of the production process. ${ }^{9-12}$ However, if generalized to include inclusive production at large final energies, this (together with "duality," or "correspondence" ideas ${ }^{13,14}$ ) would suggest that the ratio of exclusive $\bar{p} \bar{p}$ and $\pi \pi$ cross sections should likewise be of order unity everywhere, i. e. $F_{\pi}\left(q^{2}\right)$ and $G_{M p}\left(q^{2}\right)$ should have the same $q^{2}$ dependence. This is not in accord with theoretical ideas (or experimental trends) suggesting $F_{\pi} \gg \mathrm{G}_{\mathrm{Mp}}$ as $q^{2}$ becomes large. ${ }^{*}$. For example,

[^0]taking $F_{\pi} \sim\left(1-q^{2} / m_{\rho}^{2}\right)^{-1}$ and $G_{M p} \sim\left(1-q^{2} / 0.7\right)^{-2}$ leads to a $\bar{p} p / \pi \pi$ ratio falling by a factor $\sim 5$ from $\mathrm{q}^{2} \sim 10$ to $\mathrm{q}^{2} \sim 25$, and by a factor $\sim 25$ from the Frascati energy of $q^{2} \simeq 4.4 \mathrm{GeV}^{2}$ (where $\bar{p} p / \pi \pi \sim 1$ ) to the highest energy at SPEAR.
6) Inclusive scaling behavior: As will be described later, a variety of theoretical ideas led to the conjecture that $\left(\mathrm{E} / \sigma_{\text {tot }}\right)\left(\mathrm{d} \sigma / \mathrm{d}^{3} \mathrm{p}\right)$ should exhibit scaling behavior (with $\sigma_{\text {tot }} \sim 1 /$ s), i. e.
\[

$$
\begin{equation*}
q^{2} E \frac{d \sigma}{d \mathrm{E}}=\mathrm{f}\left(\frac{\mathrm{E}}{\mathrm{E}_{\max }}\right) \tag{1}
\end{equation*}
$$

\]

when $\mathrm{q}^{2} \mathrm{E}(\mathrm{d} \sigma / \mathrm{dE})$ is plotted versus $\omega=\mathrm{E} / \mathrm{E}_{\mathrm{max}}$, approximate scaling behavior is in fact observed for $\omega \gtrsim 0.45$. Furthermore, the shape of the structure function at $\mathrm{s} \sim 10 \mathrm{GeV}^{2}$ (where $\mathrm{R} \sim 2$ and (neutral energy)/(charged energy) $\sim 0.5$ ) is in rough accord with theoretical expectations, ${ }^{15} \sim 2(1-\omega)$. These results might suggest a two-component picture in which the rise in $R$ is a consequence of a new process which creates only hadrons of low momentum. However, the scaling hypothesis also generally assumed a non-isotropic angular distribution (type of $1+\cos ^{2} \theta$ ) of energetic hadrons. No trace of such angular dependence is found, even at the higher energy, making a two-component explanation more difficult to support, unless a large longitudinal contribution (not present in the electroproduction process) is present. However, some caution may be exercised here inasmuch as the data is preliminary and this measurement is quite delicate, depending upon good understanding of detection efficiency as a function of angle.*

[^1]
## IV. Total Cross Section of $\mathrm{e}^{+} \mathrm{e}^{-} \rightarrow$ Hadrons Annihilation: Theoretical Expectations

It is well known that in the one-photon approximation the total cross section for $\mathrm{e}^{+} \mathrm{e}^{-}$annihilation into a pair of noninteracting particles with spin $\frac{1}{2}$ or 0 at high energies (i.e. the beam energy $E \gg$ all particle masses) is proportional to $1 / s$ where $s=q^{2}=4 \mathrm{E}^{2}$. Namely, for the case of annihilation of the $\mathrm{e}^{+} \mathrm{e}^{-}$pair into a fermion pair with $\operatorname{spin} \frac{1}{2}$ (for instance, $\mathrm{e}^{+} \mathrm{e}^{-} \rightarrow \mu^{+} \mu^{-}$)

$$
\begin{equation*}
\sigma_{\mathrm{ann}}=\frac{4 \pi \alpha^{2}}{3} \frac{1}{\mathrm{~s}} \tag{2}
\end{equation*}
$$

and for $\mathrm{e}^{+} \mathrm{e}^{-}$annihilation into a boson pair with spin 0

$$
\begin{equation*}
\sigma_{\mathrm{ann}}=\frac{\pi \alpha^{2}}{3} \frac{1}{\mathrm{~s}} \tag{2'}
\end{equation*}
$$

There are ${ }^{16-21}$ a number of arguments and proofs, based on various approaches stating that such a $\mathrm{s}^{\mathbf{- 1}}$ type of asymptotic dependence of the total amihilation cross section for $\mathrm{e}^{+} \mathrm{e}^{-} \rightarrow$ hadrons will survive inclusion of strong interactions provided only that the bare particles contained in the hadron electromagnetic interaction Lagrangian have spins 0 or $\frac{1}{2}$. We will consider these arguments successively.

## 1. Dimensional Considerations and the Use of Wilson's Small Distance Expansion of Operator Products

From dimensional analysis, given that at large masses of the virtual photon $\mathrm{q}^{2}$ the theory does not contain any dimensional constants, it evidently follows that $\sigma_{\text {tot }} \sim 1 / s$. The same result can be obtained ${ }^{18}$ using Wilson's method. ${ }^{22}$ The total cross section $\sigma_{\text {tot }}$ may be represented as

$$
\begin{equation*}
\sigma_{\text {tot }}=-\frac{8 \pi^{2} \alpha^{2}}{3 q^{4}} \int d^{4} x e^{i q x}<0\left|\left[j_{\mu}(x), j_{\mu}(0)\right]\right| 0> \tag{3}
\end{equation*}
$$

where $j_{\mu}(x)$ is the electromagnetic current of hadrons. At large $q^{2}$, according to the causality condition, only the values $x \lesssim 1 / q_{0}=1 / \sqrt{q^{2}}$ (in the c.m.s.) are of importance. According to Wilson, the behavior of $\sigma_{\text {tot }}$ at $q^{2} \rightarrow \infty$ is determined by the dimension of the current operator product. Because of charge conservation, the dimension of the current operators cannot change as a consequence of the strong interaction, and $\langle 0|\left[j_{\mu}(x), j_{\mu}(0)\right]|0\rangle \sim x^{-6}$ and $\sigma_{\text {tot }} \sim 1 / \mathrm{s}$. Note that in the case of deep inelastic electroproduction, analogous arguments based again on the essential space-time region lead ${ }^{23}$ to a behavior of the total cross section for absorption of the virtual photon with mass $\sqrt{Q^{2}}$ on a nucleon $\sigma_{\gamma N}\left(Q^{2}, \nu\right) \lesssim 1 / Q^{2}$. This is in accordance with the ep scattering data.

We may note here an interesting relation obtained by Crewther ${ }^{24,25}$ based on the assumption of the field operator behavior at small distances discussed above. This relation connects the low-energy parameter the $\pi^{\circ} \rightarrow 2 \gamma$ decay constant S, with the high-energy parameters and is of the form

$$
3 S=K R^{\prime}
$$

Here $R^{\prime}$ is the quantity analogous to $R=\sigma\left(e^{+} e^{-} \rightarrow \mathrm{hadr}\right) / \sigma\left(\mathrm{e}^{+} \mathrm{e}^{-} \rightarrow \mu^{+} \mu^{-}\right)$and differing from $R$ only that instead of the vector hadronic current it contains the axial current. K is the constant determining the value of the difference of the electron-scattering cross sections for electrons polarized parallel and antiparallel to the beam direction in deep inelastic electron scattering from longitudinally polarized protons. ${ }^{16}$ While $R^{\prime}$ cannot be directly measured
experimentally, it is believed to be equal to $R$, because the relation $R^{\prime}=R$ is a necessary conditions for Weinberg's first spectral-function sum rule to converge. ${ }^{26}$

## 2. The Proof based on Consideration of the Schwinger Term in the Current Commutator. ${ }^{17}$

Using only Lorentz-invariance, spectral conditions, and current conservation one may obtain the following sum rule

$$
\begin{array}{r}
\int_{0}^{\infty} \sigma_{t o t}\left(q^{2}\right) q^{2} d q^{2}=-16 \pi^{3} i \alpha^{3} \int d^{3} x x_{i}<0\left|\left[j_{0}(x), j_{i}(0)\right]\right| 0>x_{0}=0  \tag{4}\\
i=1,2,3
\end{array}
$$

which relates the integral of $\sigma_{\text {tot }}$ with $q^{2}$ to the equal-time commutator of the time-component and space-component of the currents (the so-called Schwinger term). This commutator quadratically diverges in theories where $\mathrm{j}_{\mu}(\mathrm{x})$ is the current of charged fermions with $\operatorname{spin} \frac{1}{2}$ and/or of bosons with spin 0 . Hence it follows that the integral on the left-hand side of (4) also quadratically diverges *,
i. e. $\sigma_{\text {tot }}(\mathrm{s}) \sim 1 / \mathrm{s}$.

* However, it should be noted that, in a theory for which the electromagnetic current is only contributed by quarks, the right-hand side may be related to a matrix element of the form

$$
\lim _{\epsilon \rightarrow 0}\langle 0| \psi^{t}(x+\epsilon) \gamma \cdot \epsilon \psi(x)|0\rangle
$$

which in turn is related to an integral over the spectral functions appearing in the Kallen-Lehmann representation for the quark propagator. Only physical states with quark quantum numbers contribute to such a spectral function; hence the cut-off must be chosen larger than the mass of the physical quark. Thus this argument may not be at all relevant at present energies.

## 3. The Parton Model ${ }^{19-21}$

In the parton model, the $\mathrm{e}^{+} \mathrm{e}^{-}$- pair first annihilates into a parton-antiparton noninteracting pair which then subsequently evolves into various hadronic states.

The possibility for such a description is based on the fact, previously mentioned, that the important times $\tau$ over which a parton must propagate as a free particle (inorder to use the free-field calculation for $\sigma_{\text {tot }}$ ) are $\tau \sim 1 / q_{0}$ compared to the parton life-time $T \sim 1 / M_{\text {eff }}$. It is supposed that the value of the parton effective mass does not increase with energy (or increases, but slowly) and therefore $\tau \ll T$. Some evidence that a parton of momentum $q_{0}$ suffers negligible "dressing" interactions during the time interval $\tau \sim q_{0}^{-1}$ comes from the scaling behavior of the electroproduction data, where there apparently is negligible dressing of the struck parton over a time-interval $\tau^{\prime} \sim \omega \mathrm{m}^{-1}$, where $\omega$, the dimensionless scale variable, is typically $\sim 2$ to 20. One should recognize that the dynamics of the parton model is not clearly understood, and that the different kinematics in electroproduction and annihilation processes may lead to different behavior. But setting such doubts aside gives the prediction

$$
\begin{equation*}
R=\frac{\sigma_{\text {tot }}}{\sigma\left(\mathrm{e}^{+} \mathrm{e}^{-} \rightarrow \mu^{+} \mu^{-}\right)}=\sum_{\mathrm{i}, \mathrm{~s}=\frac{1}{2}}^{\mathrm{N}_{\mathrm{F}}} \mathrm{Q}_{\mathrm{i}}^{2}+\frac{1}{4} \sum_{\mathrm{k}, \mathrm{~s}=0}^{\mathrm{N}_{\mathrm{B}}} \mathrm{Q}_{\mathrm{k}}^{2} \tag{5}
\end{equation*}
$$

where $Q_{i}$ and $Q_{k}$ are the charges of spin- $\frac{1}{2}$ partons and -0 partons respectively. Hence it follows for the usual three quarks $R=2 / 3$, for colored quarks ${ }^{27} R=2$ and for the model with three quartets of fractionally charged quarks ${ }^{28} R=10 / 3$. All of these values contradict the experimental data.

In the Han-Nambu model ${ }^{29,30}$ with three triplets of quarks with integer charges $(0,-1,-1),(1,0,0),(1,00)$, one obtains $R=4$, in approximate agreement with the available experimental data. Furthermore, ${ }^{31}$ at low energies $\left(Q^{2} \ll 15 \mathrm{GeV}^{2}\right)$ colored degrees of freedom are expected to be "frozen out." That is, all hadron states are color singlets and only the color-singlet piece of the electromagnetic current is operative, leading to the prediction $R=2$, also in accord with experiment. Such agreement requires that half of the $\mathrm{e}^{+} \mathrm{e}^{-} \rightarrow$ hadron events at $\sqrt{\mathrm{s}} \sim 4.5 \mathrm{GeV}$ are "colored" states, some of which decay into usual hadrons with an accompanying $\gamma$-quantum emission and others of which into lepton pairs as well.

The excess energy found in $\gamma$-rays, or even neutrinos, might be welcome, again considering the situation regarding the "energy crisis". However, such a "color thaw" predicts a similar increase ( $\sim$ a factor 2 ) in deep-inelastic electroproduction structure functions relative to the value predicted by scaling at sufficiently high $Q^{2}$ and high $\nu$. On the other hand, at NAL the preliminary data on $\mu^{+} \mathrm{p} \rightarrow \mu^{+}+$hadrons at 150 GeV and $Q^{2} \sim 30 \mathrm{GeV}^{2}$ show in fact a $30 \%$ decrease.

One may of course consider more complicated models with larger numbers of quarks to get agreement of the experimental values of $R$ with theory. (For example, three quartets of integer charge partons give $R=6$.) But a criterion for any of these models to be taken seriously is that they must also successfully connect many other phenomena together as well.

In a number of papers ${ }^{32-34}$ attempting to explain the behavior of the total cross section of $\mathrm{e}^{+} \mathrm{e}^{-} \rightarrow$ hadrons in the framework of the parton model, it is assumed that partons have a form factor of resonant character, such as
$\left[q^{2}-(\Lambda+i \Gamma)^{2}\right]^{-1}$. While this assumption contradicts the main idea of the parton approach - the point-like nature and the local structure of weak electromagnetic currents, it is up to experiment to tell us whether, after probing nuclei, nucleons, partons, we must yet pass through another level of hadron substructure before arriving at constituents (Weisskopf names these partinos) which rival the leptons in their point-like properties. But even with an assumption of parton structure it is not totally simple to fit the data on $\mathrm{e}^{+} \mathrm{e}^{-}$annihilation and electroproduction: one needs an anomalous magnetic moment as well as a finite size ${ }^{34}$. There is also no explanation of the large violation of scaling of the inclusive process.

We now discuss variants which lead to asymptotic behaviors of $\sigma_{\text {tot }}$ other than $\sigma_{\text {tot }} \sim 1 / \mathrm{s}$.

## 4. Theory with Charged Strongly Interacting Vector Bosons

It has been known for a long time that the cross section of $\mathrm{e}^{+} \mathrm{e}^{-}$annihilation into a pair of noninteracting vector bosons with zero anomalous magnetic moment $\kappa$ at $\mathrm{E} \rightarrow \infty$ behaves as a constant (for $\kappa \neq 0, \sigma \sim \mathrm{E}^{2}$ at $\mathrm{E} \rightarrow \infty$ ). Hence it is natural to ascribe the experimentally observed constancy of $\sigma_{\text {tot }}$ to the charged vector boson contribution to the electromagnetic current of hadrons. The lack of decrease of $\sigma\left(\mathrm{e}^{+} \mathrm{e}^{-} \rightarrow \mathrm{V}^{+} \mathrm{V}^{-}\right)$at $\mathrm{E} \rightarrow \infty$ is a consequence of the growth with energy of vector-boson electromagnetic interactions (nonrenormalizability of the vector boson electrodynamics). Thus it is very important to determine whether taking exact account of strong interactions can result in a cutoff of the electromagnetic interactions of vector bosons and in a decrease of $\sigma_{\text {tot }}$. If in the presence of strong interactions one still has $\sigma_{\text {tot }}(\mathrm{E}) \xrightarrow[\mathrm{E} \rightarrow \infty]{ }$ Const, it would
then mean that strong interactions do not cut off electromagnetic interactions of vector bosons, with deep consequences for the theory. A study of this problem has been made by considering the Schwinger term in the current commutator. This gave the following result ${ }^{35}$ (at $\kappa=0$ )

$$
\begin{equation*}
\sigma_{\text {tot }}=\frac{\text { const }}{s^{\gamma}} \quad 0 \leq \gamma \leq 1 . \tag{6}
\end{equation*}
$$

The case $\gamma=0$ corresponds to the bare mass of $V$-boson remaining finite (up to logarithmic terms) even with the strong interactions included.

Thus the theory with charged strongly interacting vector bosons may in principle describe the experimentally observed behavior of $\sigma_{\text {tot }}(\mathrm{s})$.

In papers ${ }^{36,37}$ a study was made of the process $\mathrm{e}^{+} \mathrm{e}^{-} \rightarrow$ hadrons assuming that the vector bosons $V^{ \pm}$are partons. Definite results were obtained, in particular, $\sigma_{\text {tot }} \sim$ const at $E \rightarrow \infty$ and an angular distribution of energetic hadrons ( $1+\cos ^{2} \theta$ ). When discussing these results one should take into account that with such an approach the main hypothesis of the parton model (the assumption on the absence of parton "dressing" over the important time scale $\tau \sim q_{0}^{-1}$, is even more dangerous owing to the more singular behavior of the theory at short distances.
5. Vector Dominance Theory (VDM) based upon Field Algebra ${ }^{38}$

This theory assumes that the electromagnetic current of hadrons is proportional to the field of neutral vector mesons

$$
\begin{equation*}
j_{\mu}(x)=-\frac{\mathrm{m}_{\mathrm{V}}^{2}}{\mathrm{~g}_{\mathrm{V}}} \mathrm{~V}_{\mu}(\mathrm{x}) \tag{7}
\end{equation*}
$$

For $\sigma_{\text {tot }}(\mathrm{s})$ there was obtained the exact result ${ }^{39-41}$

$$
\begin{equation*}
\sigma_{\text {tot }}(s)=\frac{\pi \alpha^{2}}{s^{2}} \frac{m_{V}^{2}}{g_{V}^{2}} f\left(q^{2}\right) \tag{8}
\end{equation*}
$$

where $f\left(q^{2}\right)$ is a decreasing function of $q^{2}$. Eq. (8) evidently disagrees with the experimental data so that VDM in the form of Ref. 38 apparently had to be rejected. Some attempts have been made ${ }^{42-47}$ to save VDM by introducing into the theory a mass spectrum of vector mesons $\rho\left(\mathrm{m}_{\mathrm{V}}^{2}\right)$. A fast enough increase of $\rho\left(\mathrm{m}_{\mathrm{V}}^{2}\right)$ with $\mathrm{m}_{\mathrm{V}}^{2}$ may reasonably describe the experimental data on $\sigma_{\text {tot }}(\mathrm{s})$, but in this case the theory loses the original attractive features of conventional VDM. However, since the origination of VDM ideas, dual models have created a new motivation for introduction of a large number of $J=1$ particles. ${ }^{48,49}$
6. Description of $\sigma_{\text {tot }}$ using a Model Involving Production of a Large Number of Different Boson Resonances ${ }^{50}$

With this approach one may get a good description of the experimental data on $\sigma_{\text {tot }}$. But this method uses a large number of unknown constants and requires a number of assumptions of form factors of boson resonances.

Summing up various theoretical descriptions of experimentally observed behavior of the $\mathrm{e}^{+} \mathrm{e}^{-} \rightarrow$ hadron annihilation total cross section it should be said that now there is no satisfactory explanation of the $R(E)$ increase with energy especially if it continues also at higher energies (excluding, perhaps, a theory with charged vector bosons, which however requires further theoretical and experimental investigations).
7. What is the Maximum Energy for which $R(E)$ may Continue to Grow?

The answer to this question may be obtained with the help of the KallenLehmann representation for the Green's function of the photon $D\left(q^{2}\right)$, from which the strict inequality follows 51

$$
\begin{equation*}
\frac{1}{\pi} \int_{0}^{\infty} \frac{\operatorname{Im} D\left(q^{2}\right)}{\left|D\left(q^{2}\right)\right|^{2} q^{4}} d q^{2} \leq 1 \tag{9}
\end{equation*}
$$

Supposing that the annihilation $\mathrm{e}^{+} \mathrm{e}^{-} \rightarrow$ hadrons proceeds through one-photon exchange and neglecting the lepton contribution to $\operatorname{Im} D\left(q^{2}\right)$ we get from (9)

$$
\begin{equation*}
\frac{\alpha}{3 \pi} \int \frac{\mathrm{R}(\mathrm{~s})}{\mathrm{s}} \mathrm{ds}<1 \tag{10}
\end{equation*}
$$

Inthe proof of Eq. (10) it was supposed only the weakness of electromagnetic interaction of leptons, but not of hadrons (one-photon approximation). Therefore the inequality (10) is also correct if the electromagnetic interaction of hadrons increases with energy (for example in final states of $\mathrm{e}^{+} \mathrm{e}^{-}$annihilation there may be photons emitted by hadrons).

If we take for $R(s)$ the linear growth with the same slope as at existing energies $R(s) \approx s / 5 m^{2}$, then from (10) for the maximal permissible $\overline{\bar{S}}=\overline{\bar{S}}$ max we get

$$
\begin{equation*}
\sqrt{S_{\max }}=80 \mathrm{GeV} \tag{11}
\end{equation*}
$$

In fact one may expect the increase of $R(s)$ to stop at considerably lower energies. As was previously discussed, the rapid increase of $R(s)$ with $s$ means that strong interactions do not cut off the growth of hadron electromagnetic interactions with energy. Extending this result to the virtual processes one would conclude
that there should be large violations of isotopic spin invariance in hadron interactions. Since experimentally it is not the case, then one may naturally expect that the effective parameter $(\alpha / \pi) R(\mathrm{~s})$ is bounded by the magnitude of isotopic invariance violation, i.e.

$$
\begin{equation*}
\sqrt{S_{\max }} \lesssim 10 \mathrm{GeV} \tag{12}
\end{equation*}
$$

According to this estimate the linear increase of $R(s)$ has to already cease at the energies expected in SPEAR and DORIS.

## V. Inclusive Annihilation and Electroproduction

The differential cross section of the inclusive process involving production in $\mathrm{e}^{+} \mathrm{e}^{-}$annihilation of one observed hadron $h$ with energy $E^{\prime}$ and emission angle $\theta$ (in c.m.s.) has, in the one-photon approximation, the form
$\frac{d \sigma}{d E^{\prime} d \Omega}=\frac{2 \alpha^{2}}{q^{4}} \frac{m \nu}{\sqrt{q}^{2}}\left(1-\frac{m^{2} q^{2}}{\nu^{2}}\right)^{\frac{1}{2}}\left[2 \bar{W}_{1}\left(q^{2}, \nu\right)+\frac{\nu^{2}}{q^{2} m^{2}}\left(1-\frac{q^{2} m^{2}}{\nu^{2}}\right) \sin ^{2} \theta \bar{W}_{2}\left(\nu, q^{2}\right)\right]$.

Here $m$ is the hadron mass, $\nu=\mathrm{E}_{\mathrm{cms}} \mathrm{E}^{\prime}$ and $\overline{\mathrm{W}}_{1}\left(q^{2}, \nu\right), \overline{\mathrm{W}}_{2}\left(q^{2}, \nu\right)$ are the functions depending upon the invariants $q^{2}, \nu$ analogously to the functions $W_{1}$, $\mathrm{W}_{2}$ in the case of electroproduction. From (13) for $\omega=2 \nu / \mathrm{q}^{2}=2 \mathrm{E}^{\mathrm{I}} / \mathrm{E}_{\mathrm{cms}}$ it follows

$$
\begin{equation*}
\frac{\mathrm{d} \sigma}{\mathrm{~d} \omega}=\frac{4 \pi \alpha^{2}}{\mathrm{q}^{2}} \omega \mathrm{~m}\left(1-\frac{4 \mathrm{~m}^{2}}{\mathrm{q}^{2} \omega^{2}}\right)^{\frac{1}{2}}\left[\overline{\mathrm{~W}}_{1}\left(\mathrm{q}^{2}, \nu\right)+\frac{1}{6} \omega \frac{\nu}{\mathrm{~m}^{2}}\left(1-\frac{4 \mathrm{~m}^{2}}{\mathrm{q}^{2} \omega^{2}}\right) \overline{\mathrm{W}}_{2}\left(\mathrm{q}^{2}, \nu\right)\right] \tag{14}
\end{equation*}
$$

If there is scale invariance then the functions $\bar{W}_{1}\left(q^{2}, \nu\right)$ and $\bar{W}_{2}\left(q^{2}, v\right)$ have the form ${ }^{52,20}$

$$
\begin{align*}
& \overline{\mathrm{W}}_{1}\left(\nu, q^{2}\right)=\frac{1}{\mathrm{~m}} \overline{\mathrm{~F}}_{1}(\omega) \\
& \mathrm{W}_{2}\left(\nu, q^{2}\right)=\frac{\mathrm{m}}{\nu} \overline{\mathrm{~F}}_{2}(\omega) \tag{15}
\end{align*}
$$

The experimental data discussed in Section II show that in the process $\mathrm{e}^{+} \mathrm{e}^{-} \Rightarrow \mathrm{h}+$ all at $10<\mathrm{q}^{2}<25 \mathrm{GeV}^{2}$ and $\omega<0.5$ scale invariance is absent. On the other hand, in nucleon electroproduction at $\left|\mathrm{q}^{2}\right|<15 \mathrm{GeV}^{2}$ and $\omega=-2 \nu / q^{2}>1.5$ scale invariance is established with reasonable accuracy. Thus we start the consideration of inclusive processes with a discussion of the relation between invariant functions in the processes of inclusive annihilation and electroproduction.

The relation between the cross section for inclusive $\mathrm{e}^{+} \mathrm{e}^{-}$annihilation involving one-photon production and the cross section of electron production on protons follows from consideration of the scattering amplitude for a virtual quantum with initial momentum $q_{1}$ and final momentum $q_{2}$ on a proton with initial momentum $\mathrm{p}_{1}$ and final $\mathrm{p}_{2} \mathrm{~T}_{\mu \nu}\left(\mathrm{s}, \mathrm{u}, \mathrm{t} ; \mathrm{q}_{1}^{2}, \mathrm{q}_{2}^{2}\right)$ where $\mathrm{s}, \mathrm{t}, \mathrm{u}$ are usual Mandelstam variables: $s=q_{1}^{2}+2 \nu+m^{2}, u=q_{1}^{2}-2 \nu+m^{2}, t=\left(q-q_{2}\right)^{2}$. The electroproduction cross section is expressed by the functions

$$
\begin{equation*}
\mathrm{W}_{\mu \nu}\left(\mathrm{q}^{2}, \nu\right)=\frac{1}{2 \mathrm{i}}\left[\mathrm{~T}_{\mu \nu}\left(\mathrm{s}+\mathrm{i} \epsilon, \mathrm{u}, 0, \mathrm{q}^{2}, \mathrm{q}^{2}\right)-\mathrm{T}_{\mu \nu}\left(\mathrm{s}-\mathrm{i} \epsilon, \mathrm{u}, 0, \mathrm{q}^{2}, \mathrm{q}^{2}\right)\right] \tag{16}
\end{equation*}
$$

at $q^{2}<0, s \geq m^{2}$. The inclusive annihilation cross section involving production of a proton with momentum $p$ is expressed by the function ${ }^{53}$

$$
\begin{align*}
\overline{\mathrm{W}}_{\mu \nu}\left(\mathrm{q}^{2}, \nu\right)= & \frac{1}{2 \mathrm{i}}\left[\mathrm{~T}_{\mu \nu}\left(\mathrm{s}, \mathrm{u}+\mathrm{i} \epsilon, 0, \mathrm{q}^{2}+\mathrm{i} \epsilon_{1}, \mathrm{q}^{2}-\mathrm{i} \epsilon_{1}\right)-\right. \\
& \left.-\mathrm{T}_{\mu \nu}\left(\mathrm{s}, \mathrm{u}-\mathrm{i} \epsilon, 0, \mathrm{q}^{2}+\mathrm{i} \epsilon_{1}, \mathrm{q}^{2}-\mathrm{i} \epsilon_{1}\right)\right] \tag{17}
\end{align*}
$$

at $u \geq m^{2}$.
It follows from (16), (17) that in general $\bar{W}_{\mu \nu}\left(q^{2}, \nu\right)$ cannot be obtained from $W_{\mu \nu}\left(q^{2}, \nu\right)$ by analytic continuation due to the different signs of the imaginary additions to the mass squares of the two photons. (The presence of different discontinuities in $s$ and $u$ in (16), (17) is not a difficulty in analytic continuation owing to the crossing-symmetry relation.) The impossibility of getting $\overline{\mathrm{W}}_{\mu \nu}\left(\nu, q^{2}\right)$ from $\mathrm{W}_{\mu \nu}\left(\nu, q^{2}\right)$ using analytic continuation may be seen by considering for instance the diagrams of Figures 1 and 2. For electroproduction, the function $\mathrm{W}\left(\nu, \mathrm{q}^{2}\right)$ corresponding to the graph of Figure 1 takes the form

$$
\begin{equation*}
W\left(\nu, q^{2}\right)=\phi^{2}\left(q^{2}\right) f(s) \tag{18}
\end{equation*}
$$

while the function $\overline{\mathrm{W}}\left(\nu, q^{2}\right)$ corresponding to the diagram crossing-symmetric to Figure 1 is equal to

$$
\begin{equation*}
\overline{\mathrm{W}}\left(v, q^{2}\right)=\left|\phi\left(q^{2}\right)\right|^{2} f(u) \tag{19}
\end{equation*}
$$

It is well known that the problem of determining the modulus of the function $\phi\left(q^{2}\right)$ on the cut by its values outside the cut is mathematically incorrect since its solution is unstable with respect to small changes of $\phi\left(q^{2}\right)$ off the cut. The problem will be even more complicated if we shall consider a sum of terms of type (18), (19) with different intermediate states, i.e.

$$
\begin{align*}
& W\left(\nu, q^{2}\right)=\sum_{i} \phi_{i}^{2}\left(q^{2}\right) f_{i}(s)  \tag{18}\\
& \bar{W}\left(\nu, q^{2}\right)=\sum_{i}\left|\phi_{i}\left(q^{2}\right)\right|^{2} f_{i}(u) . \tag{19'}
\end{align*}
$$

It is evident that one cannot express (19') through (18) with the help of dispersion relations. An analogous situation arises, as was shown ${ }^{54,55}$ for the diagram of Figure 2, in the cases when the particle masses at the virtual lines $M, M^{1}$ and the values $q^{2}$ are such that the particles $M, M^{\dagger}, \mu^{\prime}, \mu$ are simultaneously able to be on the mass shell. That is, the discontinuity in $s$ (or in $u$ ) and the discontinuity in $q^{2}$ may be simultaneously non-zero. A physical example is the inclusive spectrum of pions resulting from the process $\mathrm{e}^{+} \mathrm{e}^{-} \rightarrow \omega+$ hadrons and the decay $\omega-3 \pi$. Calculation ${ }^{54,55}$ of the diagram of Figure 2 shows that the situation does not become better even in the scaling limit $\left(\left|q^{2}\right| \rightarrow \infty, \nu \rightarrow \infty\right.$, $\left|q^{2}\right| \geq \nu=$ const): the functions $\bar{W}(\omega)$ and $W(\omega)$ in this region also are not analytic continuations of each other.

It follows from consideration of these two examples that a direct connection between $\bar{W}\left(\nu, q^{2}\right)$ and $W\left(\nu, q^{2}\right)$ as an analytic continuation or the expressing of one function through another one using dispersion relations is in principle impossible. This does not exclude, however, more complicated relations of the type of sum rules which relate integrals from both functions to each other. *

[^2]A direct connection between $\bar{W}$ and $W$ may, of course, arise in definite models. We mention two such model relations. For the pseudoscalar meson theory with a cut off of transverse momenta (one of the variants of the parton model), in the scaling region Drell, Levy and Yan have obtained the following expression between the functions $\bar{F}_{1}(\omega)$ and $\bar{F}_{2}(\omega)$ determined in (15) when $\mathrm{h}=\mathrm{p}$ (proton and functions $\mathrm{F}_{1}(\omega), \mathrm{F}_{2}(\omega)$ for the electroproduction on protons ${ }^{20}$

$$
\begin{align*}
& \bar{F}_{1}(\omega)=-F_{1}(\omega) \\
& \bar{F}_{2}(\omega)=F_{2}(\omega) . \tag{20}
\end{align*}
$$

Thus, in the Drell-Levy-Yan model the values of $\bar{F}_{1}(\omega)$ and $\bar{F}_{2}(\omega)$ for inclusive annihilation with proton production at $0 \leq \omega<1$ are obtained by analytic continuation from the values of $F_{1}(\omega), F_{2}(\omega)$ for the electroproduction on protons at $1 \leq \omega \leq \infty$. (In the case of inclusive annihilation to a boson with spin 0 the signs in relations (20) are reversed.)

Summing up terms of order $\mathrm{g}^{2} \mathrm{ln}^{2}$ in the scaling region in neutral vector and neutral pseudoscalar theory, Gribov and Lipatov ${ }^{56}$ have found an interesting connection between $\bar{W}$ and $W$

$$
\begin{align*}
\bar{W}\left(\omega, q^{2}\right) & =-\frac{1}{\omega} \mathrm{~W}\left(\frac{1}{\omega}, q^{2}\right) \\
W & =2 \mathrm{~m}^{2} \mathrm{~W}_{1}=\omega \nu \mathrm{W}_{2} \tag{21}
\end{align*}
$$

(within the Gribov-Lipatov model, scaling is absent and the limit (15) does not exist). It should however be noted that in the Gribov-Lipatov model the relation (17) holds only in the case when the particle target is also only one virtual
particle interacting with photon (i. e. it is absent, for instance, in the quark model or in the pseudoscalar symmetrical theory).

There is some reasoning ${ }^{57}$ that in the scaling limit a simple relation between functions $\overline{\mathrm{W}}$ and W near the point $\omega=1$ exists. The argumentation is based on the consideration of diagrams of ladder type with exact propagators in the vertical lines and exact form factors for the vertices (the diagram of Figure 2 is the simplest diagram of this class). Under this approximation it was shown that if $F(\omega)$ at $\omega \rightarrow 1$ has the form

$$
\begin{equation*}
\mathrm{F}(\omega)_{\omega \rightarrow 1} \rightarrow \mathrm{~A}(\omega-1)^{\mathrm{p}} \tag{22}
\end{equation*}
$$

then

$$
\begin{equation*}
\bar{F}(\omega) \Rightarrow \mathbf{A}(1-\omega)^{\mathrm{p}} \tag{23}
\end{equation*}
$$

It is known that in deep inelastic electroproduction the behavior (22) at $\omega \rightarrow 1$ follows from the parton model ${ }^{58}$ or from the consideration based on the hypothesis of smooth joining of the resonance region with the scaling region ${ }^{14 \mathrm{~b}}$ that $p=2 n-1$, where $n$ is the power of decreasing of elastic form factor $G\left(q^{2}\right) \sim$ $\left(1 / q^{2}\right)^{n}$.

Turning to the data mentioned at the beginning of this section, one should first of all note that for kinematical reasons alone, scaling in $\mathrm{e}^{+} \mathrm{e}^{-}$annihilation at present energies can only be tested for pions. A necessary condition for scaling behavior is that the energy of an emitted hadron in the c.m.s. frame be large compared to its rest mass. ${ }^{*}$ In terms of the scale variable $\omega$ this

[^3]means $\omega \gg m / \sqrt{q^{2}}$ in order for scale invariance to be valid.
Thus there are the following possibilities to interpret the lack of scaling in inclusive annihilation:
a) Scaling exists both in electroproduction and $\mathrm{e}^{+} \mathrm{e}^{-}$annihilation but the approach is much more slow for $\pi$ than $p$.
b) Scaling exists both in eh $\rightarrow e+$ all and in $\mathrm{e}^{+} \mathrm{e}^{-} \rightarrow h+$ all but the approach is much more slow in case of inclusive annihilation (especially at small $\omega$ ). This is clearly the case for $h=K, p$ on kinematical grounds alone, as mentioned above.
c) Scaling exists in ep $\rightarrow e+$ all but is absent in $\mathrm{e}^{+} \mathrm{e}^{-} \rightarrow \mathrm{h}+$ all.
d) Scaling is absent both in inclusive annihilation and in electroproduction. It means in the latter case that scaling will disappear at larger values of $q^{2}$ and $\nu$.

## VI. Pros and Cons for Scaling Behavior of the Incl usive Spectrum

Many arguments were advanced for the presence of scaling behavior of the inclusive spectrum. The first is based on the parton model. However, the hypothesis of "parton fragmentation," Eq. (1) (which leads to the scaling), is just that - a hypothesis. There is some support from the evidence for scaling of inclusive hadron spectra in electroproduction, but data exists only at relatively low $Q^{2}$.

Another motivation for inclusive scaling in annihilation is the similarity in kinematical and diagrammatic structure of the cross sections for $\mathrm{e}^{+} \mathrm{e}^{-} \rightarrow \mathrm{h}+$ all and $e^{-}+h \rightarrow e^{-}+$all. This problem was discussed in Section V. It should be clear, however, that the electroproduction structure-functions $\nu \mathrm{W}_{2}$ and $\mathrm{W}_{1}$
(related to $\langle\mathrm{p}| \mathrm{j}_{\mu}(\mathrm{x}) \mathrm{j}_{\nu}(0)|\mathrm{p}\rangle$ are dynamically much more related to $\sigma_{\text {tot }}\left(\mathrm{e}^{+} \mathrm{e}^{-} \rightarrow\right.$ hadrons $)$, which is proportional to $\langle 0| \mathrm{j}_{\mu}(\mathrm{x}) \mathrm{j}_{\mu}(0)|0\rangle$. That is, each of these quantities are controlled by the behavior of current commutators near the light cone. (There is also a difference between these two quantities: in $\sigma_{\text {tot }}$ essential are the small distances along the light cone $\mathrm{z} \sim 1 / \mathrm{q}_{0}$ and $\nu \mathrm{W}_{2}, \mathrm{~W}_{1}$ the distance along the light cone of order of inverse hadrons mass $z \sim \omega / m, \omega>1$ ). The third argument advanced for inclusive scaling is that ${ }^{59}$ the inclusive functions $\bar{W}_{1}$ and $\bar{W}_{2}$ for $e^{+} e^{-} \rightarrow h+$ all are related to the function

$$
\begin{align*}
\overline{\mathrm{W}}_{\mu \nu}\left(\nu, q^{2}\right) \simeq \int & \mathrm{d}^{4} x d^{4} y d^{4} z e^{\mathrm{ip}(\mathrm{x}-\mathrm{y})} \mathrm{e}^{\mathrm{iqz}}  \tag{24}\\
& \times<0 \mid R\left\{\mathrm{j}_{\mu}^{\left.(\mathrm{z}) \eta_{\alpha}(\mathrm{x})\right\} R\left\{\bar{\eta}_{\beta}(\mathrm{y}) j_{\nu}(0)\right\} \mid 0>(\hat{\mathrm{p}}-\mathrm{m})} \alpha \beta\right.
\end{align*}
$$

where $\eta(x)$ is the source of the hadronic field ${ }^{*}$ and $R$ denotes retarded commutator. Consideration of the exponential factor in the above leads to the argument that the region of importance in the integrand is again in the neighborhood of the light cone $z^{2} \sim 1 / q^{2} \sim 0$. However the function multiplying these exponential factors is very complicated, and the possibility of compensating oscillatory factors cannot be excluded. Furthermore, from light-cone dominance alone, without additional essential assumptions, one cannot obtain scale invariance for the inclusive hadron spectrum in $\mathrm{e}^{+} \mathrm{e}^{-}$annihilation. This holds for electroproduction as well, which is governed by an expression similar to Eq. (24). ${ }^{23}$

There exist as well arguments against inclusive scaling in $\mathrm{e}^{+} \mathrm{e}^{-} \rightarrow \mathrm{h}+$ all. One such argument is based on consideration of the weak coupling approximation

[^4]in the framework of quantum field theory, when $\mathrm{g}^{2} \ll 1$ but $\mathrm{g}^{2} \ln \left(\left|\mathrm{q}^{2}\right| / \mathrm{m}^{2}\right) \sim 1 .{ }^{56,60}$ Under this approximation $\mathrm{W}_{\mu \nu}\left(\nu, \mathrm{q}^{2}\right)$, both for the electroproduction and for the annihilation, are not scale invariant. This argument is not quite convincing since the growth of $W\left(\omega, q^{2}\right)$ with $\left|q^{2}\right|$ at fixed $\omega$ appearing in the theory with vector gluons is an implication of the rise of the interaction strength at small distances and a reflection of the well-known fact of the inconsistency of the theory under this approximation (the limiting procedure $\left|q^{2}\right| \rightarrow \infty$ is impossible). In a theory with massless Yang-Mills gauge fields, with asymptotic freedom, one may expect this difficulty to be absent and scale invariance to appear in this approximation (up to logarithmic terms). We still, however, cannot give mass to Yang-Mills particles in theories with non Abelian group symmetry other than using the Higgs mechanism, which in turn abolishes the asymptotic freedom and apparently has to destroy the scale invariance.

Another argument against the existence of the scale-invariant limit (15) for functions $\bar{W}_{1}\left(\nu, q^{2}\right), \bar{W}_{2}\left(\nu, q^{2}\right)$ at $q^{2} \rightarrow \infty, \omega=$ const is based on the consideration of the implications of scale transformation in strong interactions in the presence of anomalous dimensions ${ }^{18,61}$. The assumptions based on this approach and the physical consequences resulting from it will be considered below in Section VII. We will show here only one of the results, the sum rules for the functions $\bar{W}_{1}, \bar{W}_{2}$ :

$$
\begin{align*}
& \int_{0}^{1} d \omega \omega^{j+1} \bar{W}_{1}\left(\omega, q^{2}\right)=f_{i}(j)\left(q^{2}\right)^{\rho(j)}  \tag{25}\\
& \int_{0}^{1} d \omega \omega^{j+3} \bar{W}_{2}\left(\omega, q^{2}\right)=f_{2}(j)\left(q^{2}\right)^{\rho(j)-1}
\end{align*}
$$

where $f_{1}(j), f_{2}(j), \rho(j)$ are some unknown functions of $j$ and, generally speaking, $\rho(\mathrm{j}) \neq 0$. Obviously, (25) may be put into agreement with (15) only if $\rho(\mathrm{j}) \equiv 0$.

But in any case both of these arguments against scaling lead to similar conclusions for electroproduction and $\mathrm{e}^{+} \mathrm{e}^{-}$annihilation while the observed scaling behavior appears to be different in the two cases.

We can conclude that the quality of the arguments regarding inclusive scaling in $\mathrm{e}^{+} \mathrm{e}^{-}$annihilation is much poorer than either the arguments for scaling of $\sigma_{\text {tot }}$ ( $\mathrm{e}^{+} \mathrm{e}^{-}-$hadrons) or of the electroproduction structure functions $\nu \mathrm{W}_{2}$ and $W_{1}$.

A number of authors ${ }^{62,63}$ have tried to interpret the lack of scaling of $\sigma_{\text {tot }}$ by first considering the inclusive spectrum and its scaling properties. As noted above, we cannot expect inclusive scaling for values of $\omega \lesssim m / \sqrt{q^{2}}$. Then by integrating the inclusive spectrum to obtain $\sigma_{\text {tot }}$ one also obtains a nonscaling piece $\sigma_{\text {tot }}$ (at low energies as well).

However, arguments regarding scaling behavior of the inclusive spectrum should be regarded as less reliable than the arguments used for scaling of $\sigma_{\text {tot }}$. While scaling of inclusive spectra is a sufficient condition for obtaining scaling of $\sigma_{\text {tot }}$, it is not a necessary one. The comparable situation in electroproduction is to try to obtain scaling behavior for the structure functions $\mathrm{W}_{1}$ and $\nu \mathrm{W}_{2}$ by postulating scaling behavior for the inclusive hadron spectrum, and then determining $\mathrm{W}_{1}$ and $\nu \mathrm{W}_{2}$ by integration over hadron momenta. This seems to be a much more speculative line of argument than the usual direct discussion using properties of current commutators on the light cone.

## VII. Models

In this section we discuss predictions of different models for multiple and inclusive processes in $\mathrm{e}^{+} \mathrm{e}^{-} \rightarrow$ hadron annihilation. We consider such physical parameters of multiple processes as 1) average multiplicity; 2) distribution in multiplicity; 3) inclusive energy distributions; and 4) angular correlations. We shall consider the following models: 1) parton model; 2) a model based on the scaling hypothesis in strong interactions; 3) a model with strongly interacting charged vector bosons; 4) a model using light-cone dominance (and some other assumptions); and 5) statistical and hydrodynamic models.

## 1. The Parton Model

The parton model (see e.g. Refs. 19, 64, 65) has no precise formulation, but embodies the notions that (1) hadrons are composed of a number of pointlike constituents, and that (2) for a certain class of processes (again imprecisely defined, but including deep-inelastic electroproduction and neutrino processes) a high-energy incident hadron may be considered, for the purpose of computing cross sections for which all final hadron states are summed, as a beam of massless (or fixed low mass), point-like, non-interacting constituents - the partons. The electromagnetic and weak interactions of these partons are given by the elementary point couplings, in analogy to those for leptons.

These hypotheses allow one to compute the cross sections for such processes as

$$
\begin{align*}
& \mathrm{e}^{\mp} \mathrm{p} \rightarrow \mathrm{e}^{\mp}+\text { hadrons }  \tag{a}\\
& \mu^{ \pm} \mathrm{p} \rightarrow \mu^{ \pm}+\text {hadrons }  \tag{b}\\
& \nu \mathrm{p} \rightarrow \mu^{-}+\text {hadrons, etc. }  \tag{c}\\
& \gamma \mathrm{p} \rightarrow \gamma+\text { hadrons (with final } \gamma \text { possessing high } \mathrm{p}_{\perp} \text { ) }  \tag{d}\\
& \mathrm{pp} \rightarrow \mu^{+} \mu^{-}+\text {hadrons, etc. }  \tag{e}\\
& \mu \mathrm{p} \rightarrow \mu \gamma+\text { hadrons, etc. (with final } \mu \text { and } \gamma \\
& \quad \begin{array}{l}
\text { possessing high } \mathrm{p}_{\perp} \text { ) }
\end{array} \tag{f}
\end{align*}
$$

given only the momentum distribution of the parton "beam" which replaces each incident hadron. Also eminently reasonable, considering the physically intuitive connection with the more general scaling ideas discussed in Part VI, is the inclusion of the processes

$$
\mathrm{e}^{+} \mathrm{e}^{-} \rightarrow \text { hadrons }
$$

in the above list, with the parton-model prediction given by Eq. (1). However, with the data in apparent disagreement with the prediction, one is invited to reconsider the question and ask whether there is a difference between $\mathrm{e}^{+} \mathrm{e}^{-} \rightarrow$ hadrons and the other processes listed above. There is one evident difference: in all other cases the partons are already present in the initial state, while in $\mathrm{e}^{+} \mathrm{e}^{-}$annihilation they are not. Thus perhaps the parton in the nucleon continually interacts with its environment in such a way as to maintain free-field properties, while when a parton pair is created from a single virtual photon, there is no time to establish the environment. In the annihilation process, the parton-antiparton pair recede from the point of creation at the speed of light. The environment is created at no greater speed, and it will in this case be
more difficult to establish the equilibrium between parton and environment necessary to maintain the supposed massless free-field behavior.

Several different theoretical schemes are being developed which consider that the environment of the nucleon contains some kind of space-dependent classical field, such as a Higgs scalar field which in the neighborhood of a nucleon cancels a large bare quark mass present for an isolated parton. There may be also massless Yang-Mills fields present which serve to confine the partons - in particular, quarks. At least in some of these works, the authors have been partially motivated by the status of the colliding-beam data and the above line of argument. Hereafter, however, we shall not consider further this possibility and instead regard $\mathrm{e}^{+} \mathrm{e}^{-}$annihilation on the same footing as the other deep inelastic processes. In all cases except $\mathrm{e}^{+} \mathrm{e}^{-}$annihilation, there is no clear disagreement with the parton picture (except, perhaps, as mentioned above in Section IV.2, the very preliminary data on deep inelastic scattering from NAL). There is no confirmation of the parton picture either, except in cases (a)-(c) which may be obtained in a more general framework than the parton model. In cases (d), (e), and (f) the experimental yields have been larger than predicted by the parton model and may be due to some kind of "background. "

A more speculative extension of parton-model ideas, which is logically independent of the preceding considerations, concerns the properties of hadron final states in the deep inelastic processes. A parton is, roughly speaking, a quantum of the bare Hamiltonian $H_{0}$. After being struck or created as in $\mathrm{e}^{+} \mathrm{e}^{-}$ annihilation, it must evolve into quanta of the full Hamiltonian H, i. e. hadrons. The hypothesis made here, supported to some extent by calculations in cutoff
field theory models ${ }^{64}$ (but violated by other more honest field-theory calculations 58,56 ), is that if the struck or created parton has momentum $\rho_{0}$ large and $\mathrm{p}^{2} \sim 0$ ) then the typical momentum of a hadron emerging in the direction of the original parton will be $\sim \mathrm{xp}_{\mu}$ with the distribution of momentum given by

$$
\begin{equation*}
x \frac{d N}{d x}=\frac{x}{\sigma} \frac{d \sigma_{h, q}}{d x}=g_{h, q}(x) \tag{26}
\end{equation*}
$$

This "parton-fragmentation" hypothesis has some weak support from electroproduction data, and leads as well to the scaling behavior of $\bar{W}_{1}$ and $\bar{W}_{2}$ discussed in Section VI. For partons with spin $\frac{1}{2}$ it also gives at sufficiently high energy, the relation $-2 \overline{\mathrm{~W}}_{1}=\omega \overline{\mathrm{W}}_{2} \nu$ leading to the $1+\cos ^{2} \theta$ angular distribution. This is in apparent disagreement with the data as we have discussed earlier.

The power of the "parton-fragmentation" hypothesis embodied in Eq. (26) lies in its universality: the final hadron spectra in all deep-inelastic processes depend only upon a relatively small number of functions $\mathrm{g}_{\mathrm{hq}}(\mathrm{x})$, the number being proportional to the number of different partons. In this way, once inclusive hadron spectra in deep inelastic electroproduction and/or neutrinoprocesses are measured, the hadron spectra in $\mathrm{e}^{+} \mathrm{e}^{-}$annihilation are largely determined. If the isotopic spin of the partons does not exceed $\frac{1}{2}$, various isospin restrictions also exist: for example

$$
\begin{equation*}
\frac{\mathrm{dN}_{\pi} \mathrm{oq}}{\mathrm{dx}}=\frac{1}{2}\left(\frac{\mathrm{dN}_{\pi^{+} q}}{\mathrm{dx}}+\frac{\mathrm{dN}_{\pi-q}}{\mathrm{dx}}\right) \tag{27}
\end{equation*}
$$

leading to the prediction in $\mathrm{e}^{+} \mathrm{e}^{-}$annihilation that the $\pi^{\circ}$ inclusive spectrum should be equal to the charged spectrum. There exist in the literature a large number of such relations between inclusive spectra.

Another consequence of the parton-fragmentation hypothesis is the prediction of two-jet structure in $\mathrm{e}^{+} \mathrm{e}^{-}$annihilation. ${ }^{20,21}$ That is, energetic hadrons should be emitted nearly parallel to the axis defined by the direction of emission of the parton-antiparton pair. The transverse momentum of the hadron (relative to the jet axis) is expected to be limited, of order $300-400 \mathrm{MeV}$ as in hadron-hadron collisions. Indeed, the entire configuration of hadrons then might look like those found in, say, a $\pi \pi$ collision with the same center-of-mass energy. In particular, the rapidity distribution of emitted hadrons at sufficiently high energy (measured always along the jet axis, event by event) could have a plateau structure, similar to what is found in hadron-hadron collisions. If the partons have quark quantum numbers, one cannot expect two groups of leading particles separated by a rapidity gap, inasmuch as charge conservation would imply each have fractional charge. For this reason something like a plateau may be expected to exist. ${ }^{19,69}$ In terms of Eq. (26) this means $\mathrm{g}(0) \neq 0$ and for a plateau $\mathrm{g}(0)<\infty$. A consequence of this hypothesis is the prediction that the mean multiplicity of hadrons $\bar{n} \sim \ln q^{2}$ at sufficiently high energies. However just to obtain a jet structure requires secondary hadron momentum $\gg<p_{\perp}>\sim 0.4 \mathrm{GeV}$. Thus present energies are a little low for jet studies, a statement supported by more detailed investigations. The next generation of experiments with $\mathrm{E}_{\mathrm{CMS}} \sim 8 \mathrm{GeV}$ should be sufficient to provide a good test of jet structure. However, even if jets were in the future to miraculously emerge from the present chaos, to find a central plateau will
require very high $\mathrm{E}_{\mathrm{CMS}}$, comparable to the CERN ISR. Thus the logarithmic growth ofmultiplicity strictly need not set in until very high energies. However, given that the hadron inclusive distributions are expected to be similar to those found in strong interaction processes, one would likewise expect the multiplicity to be close to that found in strong interactions (roughly logarithmic).

The jet structure has obvious strong implications for the nature of twoparticle (or higher) correlations as well. These correlation functions have been studied in some detail by various authors, in particular by Gatto and Preparata, 70 who use a Mueller-Regge formalism which is quite compatible in its structure with the parton-fragmentation picture (including plateau structure) we have discussed. Again, strong correlations expected from jet structure only emerge at energies somewhat larger than those now available.

The most speculative application of parton-model ideas regards the production of high $p_{\perp}$ hadrons in hadron-hadron collisions. To the preceding hypotheses must now be added hypothesis about the strong parton-parton interactions. Despite the very large uncertainty involved in such guesswork, there remains a tenuous relation between the hadron-hadron process at high $p_{\perp}$ and $e^{+} e^{-}$annihilation. The ratios of $\pi: K: p$ at high $p_{\perp}$ in the hadron collisions and for $\omega$ near 1 in the $\mathrm{e}^{+} \mathrm{e}^{-}$annihilation should be closely related if these hadrons are "fragments" of the same kind of parton. Indeed, in the pp collisions the $K / \pi$ and $\overline{\mathrm{p}} / \pi$ ratios are quite large. In $\mathrm{e}^{+} \mathrm{e}^{-}$collisions the ratios increase with increasing momentum in a way quite similar to that observed for the ratios in pp collisions. This agreement with expectations should not, however, be taken as serious evidence in support of the parton-model hypothesis.

## 2. Model Based on Scaling Hypothesis in Strong Interactions ${ }^{18,61}$

The initial point of this approach is the assumption that strong interactions are invariant relative to scale transformations $\mathrm{x} \rightarrow \lambda \mathrm{x}$ at small distances. Here the operators of different fields transform as $\phi_{i} \rightarrow \lambda^{-\Delta_{i}} \phi_{i}$ where $\Delta_{i}$ is the dimension of $\phi_{i}$ (as a rule anomalous, not coinciding with usual canonical dimension of the field $\phi$ ). Different Green's functions transform according to the number and form of the field operators contained in them. The second, very important assumption used in this approach is that at small distances, i.e. at large (and all of one order of magnitude $K^{2}$ ) momenta of all the external particles off mass shell, the unitarity conditions for the Green's functions $G$ and vertex part $\Gamma$ expressed via $G$ and $\Gamma$, are saturated by a number of terms of order unity (i. e. $K^{2}$ independent). As a consequence, it follows that the anomalous dimensions in $G$ and $\Gamma$ necessarily appear.

From these assumptions applied to the consideration of the imaginary part of the photon polarization operator due to hadrons, the following picture for annihilation of $\mathrm{e}^{+} \mathrm{e}^{-} \rightarrow$ hadrons appears. The heavy virtual quantum decays first into a small number of virtual hadronic fragments. Then each of these fragments in turn decays into some fragments with lesser masses and this holds unless the fragment masses will be of order of the real hadron masses. Since in each decay appears a number of fragments of order unity, then

$$
\begin{equation*}
\mathrm{m} \sim \mathrm{c}^{-\mathrm{L}} \sqrt{\mathrm{~s}} \tag{28}
\end{equation*}
$$

where $I$ is the number of the decays, $m$ is the value of order of hadron mass, and $c>1$. Hence $L \sim \ln \sqrt{s / m^{2} / l n} c$. The appearing fragment energy after the $n$-th decay in the rest system of the parent fragment seems to be of order $\widetilde{E}_{n+1} \sim m_{n}$, and the energy in c.m.s. $E_{n+1} \sim b_{n+1} E_{n+1}$, where $b_{n+1}$ is the Lorentz factor. Thus the mean energy of the real hadrons will be of order

$$
\overline{\mathrm{E}} \sim(\mathrm{~b} / \mathrm{c})^{\mathrm{L}} \sqrt{\mathrm{~s}} \sim \sqrt{\mathrm{~s}}\left(\mathrm{~s} / \mathrm{m}^{2}\right)^{-\delta}, \quad \delta=-2 \ln \frac{\mathrm{~b}}{\mathrm{~d}} / \ln \mathrm{c} \quad 0<\delta<\frac{1}{2} .
$$

From the power-like behavior for $\overline{\mathrm{E}}$ following from the equality $\overline{\mathrm{E}} \overline{\mathrm{n}}=\sqrt{\text { s }}$, there follows power-like behavior for the average multiplicity $\overline{\mathrm{n}} \sim\left(\mathrm{s} / \mathrm{m}^{2}\right)^{\delta}$. Such behavior is natural for the model consideration where it is assumed that the asymptotic behavior of all the Green functions is of power-law character. With this approach it is also easy to find the form of the dependence of the n-particle production cross section $\sigma_{n}\left(q^{2}\right)$ on $n$ and $q^{2}$. Since we expect power-like dependence of $\sigma_{n}\left(q^{2}\right)$ on $q^{2}$, then $\sigma_{n}\left(q^{2}\right)$ must be of the form

$$
\begin{equation*}
\sigma_{n}\left(q^{2}\right)=\frac{1}{\left(q^{2}\right)^{2}} \times\left(\frac{n}{\left(q^{2}\right)^{b}}\right) \tag{29}
\end{equation*}
$$

The values $a$ and $b$ are determined from conditions

$$
\begin{align*}
& \sigma\left(q^{2}\right)=\sum_{n} \sigma_{n}\left(q^{2}\right) \approx \int d n \sigma_{n}\left(q^{2}\right) \sim \frac{1}{s}  \tag{30}\\
& \bar{n} \approx \int \operatorname{dn} n \sigma_{n}\left(q^{2}\right) / \sigma\left(q^{2}\right) \sim\left(\frac{q^{2}}{m^{2}}\right)^{\delta} . \tag{31}
\end{align*}
$$

(In the model under consideration $\sigma \sim 1 / s$; see Section IV, 1.)

From (29)-(32) we set $\mathrm{b}=\delta, \mathrm{a}=1+\delta$, i. e.

$$
\begin{equation*}
\sigma_{n}\left(q^{2}\right)=\frac{1}{\left(q^{2}\right)^{1+\delta}} \times\left(\frac{n}{\left(q^{2}\right)^{\delta}}\right) \tag{32}
\end{equation*}
$$

It is rather difficult to say anything definite on the angular and energy distribution within this model.

The tree-like diagrammatic structure of the fragmenting quanta in this model appears to lead to fractional charge (or triality) in final states if the original pair of quanta produced have fractional charge (or triality). The problem of how such quantum numbers are neutralized has not been addressed.

## 3. Model with Charged Strongly Interacting Vector Bosons

In its physical implications this model seems to be close to the model discussed in the preceding section as one may expect in it, as well as in the preceding one, power-like asymptotic behavior. Since the rise of $R=$ $\sigma\left(\mathrm{e}^{+} \mathrm{e}^{-} \Rightarrow\right.$ hadron $) / \sigma\left(\mathrm{e}^{+} \mathrm{e}^{-} \Rightarrow \mu^{+} \mu^{-}\right)$in such a model is due to the photon-charged vector boson interaction, and the angular distribution of the produced (free) vector bosons is proportional to $1+\cos ^{2} \theta$, then one may expect for the fast hadron angular distribution the dependence $1+\alpha \cos ^{2} \theta$ with $\alpha \lesssim 1$. Besides in the charged vector boson model the hadron distribution seems to be of the two-jet form. Up to now there has been no quantitative consideration of this theory (except for $\mathrm{e}^{+} \mathrm{e}^{-} \Rightarrow$ hadron annihilation total cross section behavior). Thus what is mentioned above is of a qualitative character.

## 4. Light-Cone Dominance Model (LCDM) ${ }^{59,71}$

In the case of electroproduction, when using this model, a number of results were obtained which indicated that the LCDM appeared to be equivalent to the parton model. In the case of $\mathrm{e}^{+} \mathrm{e}^{-}$annihilation, in order to obtain physical results in LCDM it is necessary (besides assuming the light-cone dominance to be relevant in $\mathrm{e}^{+} \mathrm{e}^{-}$annihilation) to make some essential additional assumptions. These refer to the type of singularity of (24) as $q^{2} \rightarrow 0$ and to dimensions of the operators at which the singularity appears as a factor. After making these assumptions the result ${ }^{71}$ appears to be unfortunately rather ambiguous: depending on the supposed dimension of the operators, the multiplicity behaves either like a power $\overline{\mathrm{n}} \sim\left(\mathrm{q}^{2} / \mathrm{m}^{2}\right){ }^{\delta}, 0<\delta \lesssim 1 / 2$, or logarithmically $n \sim \ln \left(q^{2} / m^{2}\right)$ or a constant.

## 5. The Statistical and Hydrodynamical Models

Of the various models proposed, those of statistical character seem to give the best account of the approximate exponential fall of the inclusive spectrum ( $\left.\mathrm{E} / \sigma_{\text {tot }}\right) \mathrm{d} \sigma / \mathrm{d}^{3} \mathrm{p}$ with energy and the universality of this spectrum with respect to $\pi, \mathrm{K}$ and p . Indeed, in the statistical model assuming that all particles in the interaction region are emitted at the same temperature $T_{k}$, the particle distribution of the $i$-th kind particle ( $\pi, \mathrm{K}, \mathrm{p}$, etc.) is described by formula ${ }^{72}$

$$
\begin{equation*}
\frac{\mathrm{E}}{\sigma_{\text {tot }}} \frac{\mathrm{d} \sigma}{\mathrm{~d}^{3} \mathrm{p}}=\mathrm{g}_{\mathrm{i}} \mathrm{~A}_{\mathrm{i}}\left[\mathrm{e}^{\mathrm{E} / \mathrm{T}_{\mathrm{k}}} \pm 1\right]^{-1} \approx \mathrm{~g}_{\mathrm{i}} A_{\mathrm{i}} \mathrm{e}^{\mathrm{E} / \mathrm{T}_{\mathrm{k}}} \mathrm{E} \gg \mathrm{~T}_{\mathrm{k}} \tag{33}
\end{equation*}
$$

where $g_{i}$ are spin and isospin weights of $i-t h$ kind particles, the sign + refers to Fermi, and - to Bose particles. The constants $A_{i}$ are with reasonable accuracy equal for $\pi, K, p$. The equality follows from baryon conservation (neglecting antihyper on production). This requires that the sum of chemical potentials of nucleons and antinucleons is zero $\mu_{\mathrm{N}}+\mu_{\overline{\mathrm{N}}}=0$, i. e. $\mu_{\mathrm{N}}=-\mu_{\overline{\mathrm{N}}}$. On the other hand, due to the charge and isotopic symmetry the spectra of $p$, n , and $\overline{\mathrm{p}}, \overline{\mathrm{n}}$ have to coincide. Hence it follows that $\mu_{\mathrm{p}}=\mu_{\mathrm{p}}=0$, i. e. $\mathrm{A}_{\pi}=\mathrm{A}_{\mathrm{p}}$. Making use of strangeness conservation, the equality $A_{K}=A_{\pi}$ may be proved analogously.

Although (33) describes well the experimentally observed energy distribution in the region of comparably small momenta, it is unlikely that such a description should work in the region of large momenta where by analogy with $p_{1}$ distribution in hadronic collisions one may expect a power -like rather than an exponential decrease. Another point in favor of the latter argument is that in the case of exponential decrease of $d \sigma / d^{3} p$ there is no smooth joining of the inclusive spectrum at its endpoint with the exclusive channels falling as a power, as might be expected ${ }^{14}$ and is in fact the case in electroproduction. ${ }^{13}$ If a power-like fall of the energy spectrum at large momenta will be observed, this will not imply that the statistical model is of no use but will only restrict its region of applicability. Such a situation is quite natural since in measuring particles with large momenta, we select those which were produced in the initial act and had no time to suffer a sufficient number of collisions. (Here an analogy with the problem of neutron moderation in a medium may be useful. If we have a point-like source of fast neutrons and observe slow ones, then their radial distribution due to the moderation in the media atoms is a Gaussian character.

But if we are interested in fast neutrons with an energy of the order of initial one, then there will be more of them than are given by the diffusion theory owing to free flights without interaction.) If this consideration is valid, then as the beam energy increases the region of applicability of the statistical region has to increase.

In the model under consideration, the energy dependence of multiplicity must be of a different character in the region of high and superhigh energies (see Ref. 72). In the region of high but not superhigh energies, if the process of thermodynamical expansion of the fluid is not too long and the total multiplicity is not very large, one should expect a realization of the Pomeranchuk regime. ${ }^{73}$ Statistical equilibrium described by ideal gas formulae occurs when the volume of the system $V$ is proportional to the number of particles: $V_{c}=$ $n V_{\pi}, V_{\pi} \sim m_{\pi}^{-3}$. Because the total energy $\sqrt{s}=V_{c} T_{k}^{4}$ and $T_{k} \sim m_{\pi}$, it follows that n is proportional to $\sqrt{\mathrm{s}}$ and the mean energy per particle is independent of energy. At superhigh energies one enters the Landau hydrodynamical regime ${ }^{74}$ which occurs when the hydrodynamical pressure in the process of expansion of the fluid is of importance. The boundary between the two regimes seems to lie ${ }^{72}$ at $n \sim 10$. The hydrodynamical expansion of fluids proceeds adiabatically. If, in accord with Landau, we take in this process the ultrarelativistic equation of state of matter $p=\epsilon / 3$ ( $\epsilon$ is the energy density), then the entropy will be proportional to

$$
\begin{equation*}
\mathrm{S} \sim \mathrm{VT}^{3} \tag{34}
\end{equation*}
$$

The total number of particles produced ${ }^{74}$ is $n \sim S$. As the entropy is conserved in the expansion process, we may apply (34) in the initial moment and use energy
conservation, $\mathrm{E}_{\mathrm{CMS}}=\mathrm{V}_{0} \mathrm{~T}_{0}^{3}$, to get

$$
\begin{equation*}
\mathrm{n} \sim \mathrm{E}_{\mathrm{CMS}}^{3 / 4} \mathrm{v}_{0}^{1 / 4}=\mathrm{s}^{3 / 8} \mathrm{v}_{0}^{1 / 4} \tag{35}
\end{equation*}
$$

where $\mathrm{V}_{0}$ is the system volume at the initial moment. It is usually supposed ${ }^{74}$, $72,75,9,10$ that $V_{0}=$ const. $\sim 1 \mathrm{f}^{3}$. Then the mean multiplicity

$$
\begin{equation*}
n \sim E_{C M S}^{3 / 4} \sim\left(q^{2}\right)^{3 / 8} \tag{36}
\end{equation*}
$$

It seems to be also reasonable, however, proceeding from dimensional considerations or from the estimate of characteristic distances in this process (see IV. 1) to take $V_{0} \sim\left(q^{2}\right)^{-3 / 2}$. Then

$$
\begin{equation*}
\mathrm{n} \sim \text { const. } \tag{37}
\end{equation*}
$$

Note that in the case of deep inelastic electroproduction the determination of the effective initial volume based on an estimate of characteristic distances in the electroproduction process results also in other values of the volume and multiplicity than usually accepted. In electroproduction at large $\left|q^{2}\right|$ (in the lab. system) the transversal distances $r^{2} \sim\left|q^{2}\right|^{-1}$, and longitudinal distances $\mathrm{z} \sim \nu / / \mathrm{q}^{2} \mathrm{Im}$ so that the initial volume in the lab. system is $\mathrm{V}_{0 \mathrm{~L}} \sim \nu /\left(\mathrm{q}^{2}\right)^{2} \mathrm{~m}$. In the c.m.s. at $\nu \gg \mathrm{m}^{2}$

$$
\begin{equation*}
\mathrm{V}_{0} \sim \frac{\nu}{\left(\mathrm{q}^{2}\right)^{2}} \frac{\mathrm{E}_{\mathrm{CMS}}}{\nu}=\frac{\mathrm{E}_{\mathrm{CMS}}}{\left(\mathrm{q}^{2}\right)^{2}} \tag{38}
\end{equation*}
$$

Thus, with our estimate of the volume in deep inelastic electroproduction one should expect a multiplicity proportional to

$$
\begin{equation*}
\mathrm{n} \sim \frac{\mathrm{E}_{\mathrm{CMS}}}{\sqrt{\left|q^{2}\right|}}=\frac{\sqrt{q^{2}+2 \nu+\mathrm{m}^{2}}}{\sqrt{\left|\mathrm{q}^{2}\right|}} \tag{39}
\end{equation*}
$$

unlike the usually accepted ${ }^{75} \mathrm{n} \sim \mathrm{E}_{\mathrm{CMS}} / \nu^{1 / 4}$. In papers 9,10 calculations were made of angular and energy distribution of emitted hadrons in the hydrodynamical model. In doing so a more general equation of state than $p=\epsilon / 3$ was considered. ${ }^{10}$

It should be emphasized that all the calculations in the statistical and hydrodynamical model are of a phenomenological character. In particular, the question of the total cross section behavior in the annihilation of $\mathrm{e}^{+} \mathrm{e}^{-} \rightarrow$ hadrons remains beyond the scope of this model. The statistical and hydrodynamical approach is now used by the experimental groups in their data analysis and its success and limitations will be much better determined after further analysis has been carried out.
VIII. Impact of $\mathrm{e}^{+} \mathrm{e}^{-}$Annihilation Data on Quantum Electrodynamics

As mentioned earlier, a monotonically rising ratio $R=\sigma\left(e^{+} e^{-} \rightarrow\right.$ hadr $) /$ $\sigma\left(\mathrm{e}^{+} \mathrm{e}^{-} \rightarrow \mu^{+} \mu^{-}\right)$leads to major modifications to the photon propagator and hence a breakdown of perturbative quantum electrodynamics when the center-of-mass energies are such that $R \gtrsim 137 \pi$. Even at low energies there can be a measurable effect in Bhabha scattering and the process $\mathrm{e}^{+} \mathrm{e}^{-} \rightarrow \mu^{+} \mu^{-} .76,64$ Taking into account the contribution of hadrons to vacuum polarization, the general form of the photon propagator is given (for space-like $Q^{2}=-q^{2}$ )

$$
\begin{equation*}
D\left(Q^{2}\right)=\frac{-1}{Q^{2}}\left[1-\frac{\alpha}{3 \pi} Q^{2} \int_{0}^{\infty} \frac{\mathrm{dsR}(\mathrm{~s})}{\mathrm{s}\left(\mathrm{~s}+\mathrm{Q}^{2}\right)}\right]^{-1} \tag{40}
\end{equation*}
$$

If (i) $R(s) \approx\left(s / M^{2}\right)^{n}, \quad 0<n<1$, then one obtains

$$
\begin{equation*}
\mathrm{D}\left(\mathrm{Q}^{2}\right)=\frac{-1}{Q^{2}}\left[1-\frac{\alpha}{3 \sin \pi n} R\left(Q^{2}\right)\right]^{-1} \tag{41}
\end{equation*}
$$

If (ii) $R(s) \sim \begin{cases}s / M^{2} & s<\Lambda^{2} \\ \Lambda^{2} / M^{2} & s>\Lambda^{2}\end{cases}$
(the present data indicate that $\mathrm{M}^{2} \approx 50 \mathrm{GeV}^{2}$ ), then, for $\Lambda^{2} \gg Q^{2}$ one obtains up to terms of order $1 / \ln \left(\Lambda^{2} / Q^{2}\right)$

$$
\begin{equation*}
\mathrm{D}\left(\mathrm{Q}^{2}\right)=\frac{-1}{Q^{2}}\left[1-\frac{\alpha}{3 \pi} R\left(Q^{2}\right) \ln \frac{\Lambda^{2}}{Q^{2}}\right]^{-1} \tag{42}
\end{equation*}
$$

For time-like $q^{2}$ a power-law increase, as in the first case, leads to an extra phase factor $e^{-i \pi n} . \operatorname{Re} D\left(q^{2}\right)$ is of special experimental interest (because it alone interferes with the lowest order) and

$$
\begin{equation*}
\operatorname{Re} D\left(q^{2}\right)=\frac{1}{q^{2}}\left[1+\frac{\alpha}{3}(\operatorname{Ctg} \pi n) R\left(q^{2}\right)\right]^{-1} \quad(0<n<1) . \tag{43}
\end{equation*}
$$

In the second case, again neglecting terms of order $1 / \ln \left(\Lambda^{2} / q^{2}\right)$, ReD remains unchanged compared to (42).

With regard to connecting the problem of $\mathrm{e}^{+} \mathrm{e}^{-}$annihilation with the general problems of quantum electrodynamics, we mention (see also Ref. 76) an interesting possibility of solving the difficulties of quantum electrodynamics suggested by Landau and Pomeranchuk ${ }^{77}$ many years ago. If at very high energies $s \gg \Lambda^{2}$ the total cross section of one photon annihilation $\mathrm{e}^{+} \mathrm{e}^{-}$in all particles $\sigma \sim 1 / \mathrm{s}$ and $\mathrm{R}=\sigma\left(\mathrm{e}^{+} \mathrm{e}^{-} \rightarrow\right.$ hadrons or some other particles, besides e and $\mu$ if any exist $) / \sigma\left(\mathrm{e}^{+} \mathrm{e}^{-} \rightarrow \mu^{+} \mu^{-}\right)=\mathrm{R}_{0}=$ const, then the photon Green function

$$
\begin{equation*}
\mathrm{D}\left(\mathrm{Q}^{2}\right)=\mathrm{Q}^{-2}\left\{1-\frac{\alpha}{3 \pi}\left[R_{0} \ln \frac{\mathrm{Q}^{2}}{\Lambda^{2}}+\ln \frac{\mathrm{Q}^{2}}{\mathrm{~m}_{\mathrm{e}}^{2}}+\ln \frac{\mathrm{Q}^{2}}{\mathrm{~m}_{\mu}^{2}}\right]\right\}^{-1} \tag{44}
\end{equation*}
$$

has an unphysical pole at $\ln \left(Q^{2} / \Lambda^{2}\right) \approx \frac{3 \pi}{\alpha}\left(R_{0}+2\right)^{-1}$. To overcome this difficulty Landau and Pomeranchuk supposed that the pole position is at energies such that the gravitational interaction becomes important, i. e. $Q^{2} \sim 1 / \kappa$, where $\kappa=6.10^{-39} / \mathrm{m}^{2}$ is the gravitational constant. This pole position corresponds to $R_{0}=12$, a value which may be reached in the next generation of experiments in SPEAR and DORIS. If the Landau and Pomeranchuk suggestion is correct, the increasing of $R$ must stop at this value.

The experimental implications are the following (see also Ref. 84):

1) For Bhabha scattering, the angular distribution will be modified according to Eq. (41) or (42). Comparison of small angle (small $q^{2}$ ) with large angle (large $q^{2}$ ) data can yield a measurable effect in the next generation of experiments with $\mathrm{s} \sim 60 \mathrm{GeV}^{2}$. In this case at a $90^{\circ}$ scattering angle, the Bhabha cross section is dominated by space-like photon exchanges with $Q^{2} \sim$ $30 \mathrm{GeV}^{2}$ and $\mathrm{R} \sim 5$. The correction, for case (i) $\mathrm{D}\left(Q^{2}\right)$ is $\sim(1 \%)(\sin \pi n)^{-1} \gtrsim 1 \%$, and to the cross section $\sim 2 \%$. For case (ii), with $\Lambda \sim 80 \mathrm{GeV}$, the effect on the cross section is $\sim 4 \%$.
2) The ratio $\sigma\left(\mathrm{e}^{+} \mathrm{e}^{-} \rightarrow \mu^{+} \mu^{-}\right) / \sigma\left(\mathrm{e}^{+} \mathrm{e}^{-} \rightarrow \mathrm{e}^{+} \mathrm{e}^{-}\right)$at $\theta=90^{\circ}$ is most sensitive to the modification of the photon propagator for the time-like annihilation graph. In case (i) the contribution is very sensitive to the exponent $n$, and for $n=1 / 2$ vanishes. However, for case (ii), and $s \sim 60$ we may expect $R \sim 12$, and, according to Eq. (42) roughly a $10 \%$ correction to the $\mathrm{e}^{+} \mathrm{e}^{-}-\mu^{+} \mu^{-}$cross section.

Thus precision measurements of the pure electrodynamics processes are
capable of yielding some information on the ratio $R$ at energies which cannot be obtained directly. Consequently it is most important to push the precision of these measurements even beyond the quite impressive values obtained at Frascati and SPEAR.
IX. Form Factors of $p, \pi, K$, in the Time--Like Region

The simplest of exclusive processes in the annihilation of $\mathrm{e}^{+} \mathrm{e}^{-}$into hadrons is the $\mathrm{e}^{+} \mathrm{e}^{-} \rightarrow \overline{\mathrm{h}}$ process, measurement of which gives us some information on the form factor $F_{h}\left(q^{2}\right)$ of hadron $h$ in the time-like region $q^{2}>0$ and at $q^{2}<0$ allows one to determine the analytic properties of $F\left(q^{2}\right)$. As is well known, (see, e.g. Ref. 78) the analytic properties of the form factor resulting directly from the causality condition are the following: $F\left(q^{2}\right)$ is an analytic function of $q^{2}$ in the whole complex $q^{2}$ plane, with a cut along the real axis from $M^{2}$ to infinity, where $M$ is the mass of the lowest hadronic state with quantum numbers of the photon and $\overline{\mathrm{h}} \mathrm{h}$. Thus, a check of these properties is a direct test of the microcausality condition. * On the other hand, the use of analyticity allows one to make some predictions on the behavior of $F\left(q^{2}\right)$.

At present we experimentally know rather well the proton (and neutron) form factors in the region of space-like $q^{2}$ up to $q^{2} \approx-25 \mathrm{GeV}^{2}$. Here the

* The test of whether $F\left(q^{2}\right)$ is an analytic function of $q^{2}$ at large $\left|q^{2}\right|$ corresponds to that of causality at small distances. There is a difference in this method of testing causality from use of the dispersion relations for forward scattering at high energies, where such a statement cannot be definitely made.
proton form factors are approximated by the dipole fit $F\left(q^{2}\right) \approx 1 /\left(1-q^{2} / m_{0}^{2}\right)^{2}$, $\mathrm{m}_{0}^{2}=0.7 \mathrm{GeV}^{2}$. At $\mathrm{q}^{2}>0$ the $\mathrm{e}^{+} \mathrm{e}^{-} \rightarrow \overline{\mathrm{p}} \mathrm{p}$ process has been measured only at $q^{2}=4.4 \mathrm{GeV}^{2}$, with the result ${ }^{4}$

$$
\begin{equation*}
\left\{\left|\mathrm{G}_{\mathrm{M}}\right|^{2}+\left(2 \mathrm{~m}^{2} / \mathrm{q}^{2}\right)\left|\mathrm{G}_{\mathrm{E}}\right|^{2}\right\} \mu^{-2} \approx 0.014 \tag{45}
\end{equation*}
$$

( $\mathrm{G}_{\mathrm{M}}\left(\mathrm{q}^{2}\right)$ and $\mathrm{G}_{\mathrm{E}}\left(\mathrm{q}^{2}\right)$ are magnetic and electric form factors of the nucleon, $\mathrm{G}_{\mathrm{M}}(0)=\mu, \mathrm{G}_{\mathrm{E}}(0)=1$ ). If one puts at $\mathrm{q}^{2}=4.4 \mathrm{GeV}^{2}, \mathrm{G}_{\mathrm{M}}=\mathrm{G}_{\mathrm{E}}{ }^{*}$, then $\left|\mathrm{G}_{\mathrm{M}} / \mu\right|=0.10 \pm 0.01,\left|\mathrm{G}_{\mathrm{E}}\right|=0.27 \pm 0.04$. These values are considerably (by a factor 5-10) larger than those obtained by using in the $q^{2}>0$ region the same dipole formula as for $q^{2}<0$. For $\pi$ - and $K$-mesons the form factors in the time-like region have been measured up to $q^{2} \approx 9 \mathrm{GeV}^{2} .5$ Here it appears that at $q^{2}>1.5 \mathrm{GeV}^{2}$ the form factors of $\pi$ - and K -mesons are nearly equal and at the point $q^{2}=4.4 \mathrm{GeV}^{2}\left|\mathrm{~F}_{\pi}\right|^{2} \approx\left|\mathrm{~F}_{\mathrm{K}}\right|^{2} \approx 0.02$, i. e. of the same order of magnitude as the proton effective form factor. At $q^{2}>$ $1.5 \mathrm{GeV}^{2}$ the $\pi$ and $K$ form factors decrease as $1 / q^{2}$, i. e. much more slowly than the proton form factor at $q^{2}<0$. The $\pi$ meson form factor data in the space-like region obtained indirectly by measuring the pion electroproduction correspond to comparable small $q^{2},\left|q^{2}\right|<1.2 \mathrm{GeV}^{2}$. Thus in the following we will not consider then. There is, in principle, ${ }^{79}$ an experimental possibility of directly measuring $\pi$ and $K$ meson form factors at $q^{2}<0$ and large $\left|q^{2}\right|$.

* The equality $G_{M}=G_{E}$ at $q^{2}=4 m^{2}$ follows from expressing $G_{E}$ and $G_{M}$ via Pauli form factors $G_{E}=F_{1}+\left(Q^{2} / 4 \mathrm{~m}^{2}\right) \mathrm{F}_{2}, \mathrm{G}_{\mathrm{M}}=\mathrm{F}_{1}+\mathrm{F}_{2}$.

It is possible, at present or in the near future, taking into account the rapid progress in the colliding beam physics, to say samething on the analytic properties of the form factors and on their asymptotic behavior at $\left|q^{2}\right|>\infty$ ? We will start with the consideration of bosonic form factors, in particular with the pion form factor, where all the values of $q^{2}>4 \mathrm{~m}_{\pi}^{2}$ on the cut are in the physically observed region. In the $e^{+} \mathrm{e}^{-} \rightarrow \overline{\mathrm{h}}$ process what is measured is not a form factor but its squared modulus. The knowledge of the squared modulus on the cut is insufficient to determine the analytic function $F_{\pi}$ in the whole complex plane. For this, one must also know the position of all of its zeroes. It is, however, possible, proceeding from the above analytic properties of $\mathrm{F}_{\pi}\left(q^{2}\right)$ (and from the assumption that $\mid$ en $\mathrm{F}_{\pi}\left(q^{2}\right) \mid$ rises in the complex plane as $\left|q^{2}\right| \rightarrow \infty$ slower than $\left.\left|q^{2}\right|^{\alpha}, \alpha<1 / 2\right)$ to get a strict inequality ${ }^{80}$

$$
\begin{equation*}
\int_{4 \mathrm{~m}_{\pi}^{2}}^{\infty} \mathrm{dq}^{2} \frac{\ln \left|\mathrm{~F}_{\pi}\left(\mathrm{q}^{2}\right)\right|^{2}}{q^{2} \sqrt{q^{2}-4 m_{\pi}^{2}}} \geq 0 \tag{46}
\end{equation*}
$$

The experimental data available are insufficient to check the inequality (46). A large uncertainty appears from large $q^{2}>4 \mathrm{GeV}^{2}$, where the experimental accuracy is poor, and from small $q^{2}, 0.08<q^{2}<0.3 \mathrm{GeV}^{2}$, where there are no experimental data at all. But if one supposes the inequality to hold then one may obtain a restriction on the rate of decrease of $\left|F_{\pi}\left(q^{2}\right)\right|$ at large $\left|q^{2}\right|$. For instance, using the experimental data on $\left|F_{\pi}\left(q^{2}\right)\right|$ at $0.3<q^{2}<4 \mathrm{GeV}^{2}$ it may be shown that a fall of the form factor like $\left|F_{\pi}\right| \sim\left(1 / q^{2}\right)^{2}$, beginning with $q^{2}=4 \mathrm{GeV}^{2}$, is in disagreement with (46) (if only at $4 \mathrm{~m}_{\pi}^{2}<\mathrm{q}^{2}<0.3 \mathrm{GeV}^{2}$ $\left|F_{\pi}\left(q^{2}\right)\right|^{2}$ is not very large, so that on the average there $\left.\left|F_{\pi}\right|^{2}<4\right)$. There
are also a number of other strict inequalities connecting integrals from $\left|F\left(q^{2}\right)\right|^{2}$ at $q^{2}>0$ with the values of $F\left(q^{2}\right)$ at $q^{2}<0 .{ }^{81}$

In the case of the proton form factor, where there is a lot of experimental information at $q^{2}<0$ and rather poor data at $q^{2}>0$, another question is reasonable: what may be said regarding the form factor behavior at $q^{2}>0$ proceeding from the data at $q^{2}<0 ? F\left(q^{2}\right)$ is an analytic function of $q^{2}$, so that for given $F\left(q^{2}\right)$ at any part of the real axis at $q^{2}<0$, where $F\left(q^{2}\right)$ is real, then $F\left(q^{2}\right)$ is determined, in principle, in the whole complex plane. But, in fact, the problem of determining the function on the cut by its values away from the cut is unstable, because small oscillating additions away from the cut may give a large contribution on the cut. Thus it is really impossible to write dispersion relations expressing $F\left(q^{2}\right)$ at $q^{2}>0$ via integrals of $F\left(q^{2}\right)$ at $q^{2}<0$. (For such integrals to have sense, the experimental data must be fantastically accurate.) Instead, one may write sum rule type relations connecting integrals of $F\left(q^{2}\right)$ at $q^{2}>0$ and $q^{2}<0$. Taking into account that at $q^{2}>0$ only $\left|F\left(q^{2}\right)\right|$ is measured, it is convenient to this end to consider the function

$$
\begin{equation*}
\phi(z)=f(z) \ln F(z) / z \sqrt{z_{0}-z}, \quad z=\frac{q^{2}}{4 m^{2}}, \quad z_{0}=\frac{m_{\pi}^{2}}{m^{2}} \tag{47}
\end{equation*}
$$

where $f(z)$ is an analytic function in the complex $z$ plane with the cut at $z<0$. Considering the integral of $\phi(z)$ along the contour consisting of both edges of the cuts provided by $F(z)$ and $f(z)$, and of the large circle at $\infty$, and supposing that $F(z)$ has no zeroes in the complex plane, it is easy to get the sum rule ${ }^{82}$

$$
\begin{equation*}
\int_{z_{0}}^{\infty} d x f(z) \frac{\ln |F(z)|^{2}}{z \sqrt{z-z_{0}}}=2 \int_{0}^{\infty} d \rho \operatorname{Im} f(\rho) \frac{F(-\rho)}{\rho \sqrt{z_{0}+\rho}} \tag{48}
\end{equation*}
$$

In the case of the nucleon form factor, $f(z)$ may be chosen so that the contribution from the nonphysical region $4 \mathrm{~m}_{\pi}^{2}<\mathrm{q}^{2}<4 \mathrm{~m}^{2}$ to the left-hand part of (48) is small. In spite of the fact that the experimental information on the proton form factor for $q^{2}>0$ now available is very limited, even now (using the sum rule) we may make some physical conclusions. It seems to be possible to assert ${ }^{82}$ that the dipole fit for $F\left(q^{2}\right)$ at $q^{2}<0$ may be put into agreement with the data at $q^{2}>0$ only in the case when the form factor has not less than two zeroes in the complex plane, i. e. that in the $\bar{p} p$ system there are not less than four broad resonances with the photon quantum numbers ( $\rho, \rho^{\prime}$, and two yet unknown). Another possibility to make the data at $q^{2}<0$ and $q^{2}>0$ agree is to assume exponential behavior for the proton form factor at $\left|q^{2}\right| \rightarrow \infty$. Thus, for instance, the dependence

$$
\begin{equation*}
F\left(q^{2}\right)=\frac{1}{1-\frac{q^{2}}{a}} e^{-b\left(\sqrt{4 m_{\pi}^{2}-q^{2}}-4 m_{\pi}^{2}\right) / 2 m} \quad a=0.33 \mathrm{GeV}^{2}, b=1.28 \tag{49}
\end{equation*}
$$

describes well the experimental data at $q^{2}<-3 \mathrm{GeV}^{2}$ and $q^{2}=4.4 \mathrm{GeV}^{2}$. It can be hoped that as new experimental data on form factors at $q^{2}>0$ appears, our understanding of this problem will improve in an essential way.

## X. Conclusion

We have already learned a great deal from the $\mathrm{e}^{+} \mathrm{e}^{-} \rightarrow$ hadron annihilation process and no doubt will learn more from it in the near future. We have
learned that most likely the "orthodoxy" suggested by a number of theoretical approaches (i.e. $\sigma_{\text {tot }}$ falls as $1 / q^{2}$ and scaling appears in the inclusive processes), and considered quite reliable, does not exist at present energies. Perhaps the most direct interpretation of the data is simply that because $\sigma_{\text {tot }}$ does not exhibit scaling behavior, the behavior of the current product over the very short time scale $\Delta t \sim\left(q^{2}\right)^{-1 / 2}$ is not as the free field (with spins $1 / 2$ and 0 ) one. In terms of the parton model this means that in $\mathrm{e}^{+} \mathrm{e}^{-}$annihilation partons interact or fragment on a time scale small compared with $\left(q^{2}\right)^{-1 / 2}$, hence rapidly become a fluid containing many partons.

And no matter whether this behavior continues at higher energies, or somehow the expectations of "orthodoxy" will come true thanks to $\mathrm{e}^{+} \mathrm{e}^{-}$annihilation we will understand much in strong and electromagnetic hadron interaction at small distances. And if at higher energies in the $\mathrm{e}^{+} \mathrm{e}^{-}$annihilation new surprises will appear, then we must revise much of our attitudes regarding the laws of Nature at small distances. As a rule, reviews of this type are concluded by suggestions to experimentalists on what they should think about. But there is no need for that in this case. Everybody knows that experimentalists are to measure as much as possible, both at present and at higher energies, and that theorists are not to stop being surprised.

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Fig. 1


Fig. 2


[^0]:    * It must be remarked, however, that experimentally we know the proton form factor mainly at $q^{2}<0$ and the pion form factor at $q^{2}>0$. Thus, if the asymptotic behavior of the form factors is different in the time-like and spacelike regions, then it will be possible to have $F_{\pi} \sim G_{M p}$ asymptotically. This problem will be discussed in more detail in Section IX.

[^1]:    * Scale breaking due to the low $\mathrm{E}_{\mathrm{CMS}}$ will tend to reduce the anisotropy in the angular distribution. Crude estimates lead to a distribution $1+\frac{1}{2} \cos ^{2} \theta$ at $\mathrm{E}_{\mathrm{CMS}} \sim 3 \mathrm{GeV}$.

[^2]:    * Some examples of such relations for exclusive reactions will be considered below in Section IX.

[^3]:    * For pions $\mathrm{E}_{\pi}$ should also be large compared to the typical momentum $\sim 0.4 \mathrm{GeV}$ characteristic of finite hadron size.

[^4]:    * Eq. (24) is written for the case when the selected hadron is a fermion.

