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PRELIMINARY DESIGN CONSIDERATIONS FOR THE STAGE I PEP LATTICE†

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Description and Summary

A general description of the proposed PEP e^+e^- storage ring may be found elsewhere[1,2] and in these Proceedings.[3] In the present paper, we discuss the lattice and its operating characteristics in more detail, show how the design luminosity and other criteria may be met and outline the limits of the operative regions of the beam parameters in several modes of operation.

The relevant design parameters are:

Beam Energy, E (each beam)	5 to 15 GeV
Luminosity per Interaction Regions at 15 GeV, $\mathscr{L}$	$10^{32}\,\mathrm{cm}^{-2}\mathrm{sec}^{-1}$
Number of Superperiods (number of interaction regions)	6
Number of Stored Bunches, N_b (each beam)	3
Available Length per Interaction Region	20 m
Length of Straight Sections	130.416 m
Gross Radius Arcs	220.337 m

Figure 1 depicts schematically the basic elements of the lattice as presently conceived and shows typical betatron and dispersion functions. The 60° arc of each superperiod contains eight cells, of which six are standard while in the two end cells the quadrupoles are independently variable. The structure is symmetric about the interaction points and about the centers of the 60° arcs. The proposed Stage I of PEP lies in a horizontal plane, although a future Stage II option with an added 200 GeV proton ring would involve vertical as well as horizontal bends.[4] Qualitatively speaking, the four independent quadrupoles in the straight sections Q3, Q2, Q1 and QF1 (see Fig. 1) provide matching of the betatron functions from the interaction point to the cells; the two modified cell quadrupoles QD1 and QF2 provide for dispersion matching; and the two sets of standard cell quadrupoles QD and QF allow adjustment of the betatron tunes.

Design Requirements

Luminosity Considerations. In order to define the lattice requirements imposed by the luminosity specification, we consider an idealized situation in which the stored e^+ and e^- beams are equal, and the coupling between horizontal and vertical betatron oscillations is adjusted so that the horizontal and vertical beam-beam tune shifts are equal ($\Delta\nu_x = \Delta\nu_y = \Delta\nu$). It then follows from simplified theory[5,6] that

$$\mathscr{L}_{max} = \frac{\pi}{r_e^2 m_e^2} f_0 N_b (\Delta\nu)^2 E^2 \left(\frac{1}{\beta_y^*} + \frac{1}{\beta_x^*}\right)\left(\epsilon_0 + \frac{\eta^{*2}}{\beta_x^*}\frac{\sigma_E^2}{E^2}\right). \tag{1}$$

Further, if the bending magnets are of equal strength and zero-gradient,

$$\mathscr{L}_{max} = \frac{55\pi}{32\sqrt{3}} \frac{\hbar c/e^2}{r_e m_e^4} \frac{f_0 N_b (\Delta\nu)^2 E^4}{\rho_0} \left(\frac{1}{\beta_y^*} + \frac{1}{\beta_x^*}\right) \left(\langle\overline{H}\rangle + \frac{1}{2}\frac{\eta^{*2}}{\beta_x^*}\right), \tag{2}$$

where $\mathscr{L}_{max}$ is the maximum luminosity at the beam-beam tune shift $\Delta\nu$, $\hbar c/e^2 = 137.0$, r_e is the classical electron radius, m_e is the electron rest energy, f_0 is the orbital frequency, N_b is the number of bunches in each beam, E is the energy of each beam, β_x^* and β_y^* are the horizontal and vertical betatron functions at the interaction point, $\epsilon_0\pi$ is the total transverse emittance (i.e., the sum of the areas in radial and vertical phase space), η^* is the dispersion function at the interaction point, σ_E/E is the relative energy spread due to quantum excitation, ρ_0 is the magnetic radius of the bending magnets and $\langle\overline{H}\rangle$ is defined by

$$\langle\overline{H}\rangle = \frac{1}{2\pi\rho_0} \sum \int_i \frac{1}{\beta_x}\left[\eta^2 + (\beta_x\eta' - \frac{1}{2}\beta_x'\eta)^2\right] ds \tag{3}$$

where $\int_i$ denotes integration over the i-th bending magnet and Σ denotes summation over all bending magnets.

The luminosity is limited by the maximum tolerable beam-beam tune shift which is typically $\Delta\nu_{max} = 0.025$ to 0.08 per interaction point in e^+e^- storage rings.[7,8] It is evident from Eqs. (1) and (2) that the interaction betatron functions should be as small as possible. The minimum value of β_y^* has been chosen to be around 0.2 meters because it is felt that the maximum β-value at the nearest quadrupole, 10 meters from the interaction point, should be not much greater than 500 meters in order to keep apertures and chromaticity within tolerable bounds. The minimum value permitted for β_x^* is considerably larger because of the D-F configuration of the Q3-Q2 doublet. (See Fig. 1.) As will be seen later, useful ranges of β_x^* for the present design lie in the range of ~3 to 5 meters.

Aperture Limitation. Equation (2) shows that for a fixed lattice, luminosity varies as E^4 and therefore falls off very rapidly at low energy. However, it is possible to modify the lattice so that the beam size is kept essentially constant as a function of energy; then luminosity varies only as E^2, as may be seen from Eq. (1). In the PEP design, the apertures are chosen for the design energy and luminosity at 15 GeV and the luminosity is therefore aperture-limited at lower energies with an approximate E^2 dependence.

Beam Enlargement. Several schemes for beam enlargement* have been studied for lower energy operation. Some of the important features of these schemes are described below.

(1) Variable Tune. It can be shown that, approximately, $\epsilon_0 \propto N_c\,\psi_{xc}^{-3}\,E^2$ where N_c is the number of bending cells and ψ_{xc} is the horizontal betatron phase advance per cell.[2]

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* For another discussion of beam enlargement see Ref. 9.

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Thus in order to keep ϵ_0 constant for different energies we wish to make the lattice operable over some range of values of ψ_{xc}. In this design study we have considered only cases with $\nu_x \approx \nu_y$ and also have assumed that the vertical tunes should be just slightly above an integer or half-integer per superperiod.[10,11] This means that for the PEP lattice with superperiodicity of 6, the desirable tunes are, e.g., $\nu_{x,y} \approx 12, 15, 18, 21, \cdots$. In the present PEP design, the horizontal phase advance in the straight insertion is always close to 360°, so that the design values of phase advance per cell are $\psi_{xc} \approx 45^o, 62.5^o, 90^o, 112.5^o, \cdots$.

(2) Energy Disperiosn (High η^*). A variable value of the interaction point dispersion function η^* alters the luminosity limit through the factor η^{*2}/β_x^* [Eqs. (1) and (2)]. This scheme has been used successfully, e.g. at SPEAR.[7]

(3) Mismatched Dispersion Function. In common practice the momentum dispersion function is matched so that it repeats periodically from cell to cell in the standard cells. We may, however, for low energy operation mismatch the dispersion function in the standard cells as described by

$$\eta(s) = \eta_0(s) + \eta_1(s),$$

where $\eta_0(s)$, the matched dispersion function, is periodic from cell to cell, and $\eta_1(s)$, the mismatch component, is oscillatory at the betatron wavelength. As is shown elsewhere, this results in an increase in the function $\langle \overline{H} \rangle$, described roughly by[6]

$$\langle \overline{H} \rangle = \frac{1}{\beta_{xs}} \left(\eta_{0s}^2 + \eta_{1s}^2\right) , \qquad (4)$$

where the subscript s denotes the symmetry point (midpoint) of the bending arc. That is, the emittance is quadratic in the mismatch amplitude.

A fourth method of beam enlargement, mismatching the betatron function β_x, also affects the quantity $\langle \overline{H} \rangle$. This scheme was considered briefly in the PEP study but seemed to offer no obvious advantages. It may be studied further if serious difficulties arise in other schemes.

Another possible method of beam enlargement consists of altering the damping partition number J_x, since $\epsilon_0 \propto \langle \overline{H} \rangle / J_x$. In order to avoid complexity and expense, no special insertion is planned for damping modification as is planned in EPIC.[12] However, it still would be possible to vary J_x by varying the rf frequency from its nominal value; it may be shown that the PEP lattice would require a frequency change of about 2 parts in 10^5, resulting in a maximum orbit shift of less than a centimeter, to reduce J_x to zero.

Since the variable tune and the dispersion-mismatch methods increase the transverse emittance they increase the aperture requirements everywhere in the lattice. The high-η^* method, on the other hand, enlarges the beam in only the few quadrupoles nearest the interaction points, which may be a significant advantage in terms of sensitivity to orbit distortions. However it has been shown that the high-η^* configurations may lead to a longitudinal instability if η^* is too large.[8]

We expect that practical operating configurations will consists of combinations of variable-tune and high-η^*, or of high-η^* and mismatched-η, although other combinations may prove useful. Some of these configurations are discussed in a later section.

First Order Design

Summary of Constraints. The lattice requirements that have been discussed so far imply that the values of β_x^*, β_y^*, η^* and the tunes ν_x and ν_y should be independently adjustable. In addition, the symmetry of the ring requires that the slopes of the betatron and dispersion functions vanish at the interaction regions; i.e., $\beta_x'^* = \beta_y'^* = \eta'^* = 0$. Thus, eight mathematical constraints are imposed on the focusing strengths, requiring at least eight independently variable sets of quadrupoles. A number of other important constraints must also be considered. A twenty meter drift space is required for experiments at each interaction region. The maximum values of the β-functions and η-function must be kept everywhere within reasonable bounds in order to minimize aperture requirements and to reduce chromaticity and other aberrations. Magnetic field values must be kept within conservative limits. Furthermore, sufficient drift space must be reserved for various components such as rf cavities, injection components (septum magnets and kickers), rotated quadrupoles, sextupoles, beam monitors and control devices, electrostatic plates for separating the beams and so on.

The geometry of the ring has been fixed by considerations discussed in Ref. (1). Once the quadrupole locations have been fixed and the quadrupole strengths have been specified to satisfy the eight mathematical constraints mentioned above, it will not always be possible for all configurations to satisfy the additional constraints imposed by aperture and quadrupole strength limits. Hence in the present study we have mapped out the regions of beam parameters within which all of the constraints are satisfied. Some of the results will be described in a later section.

Computational Methods. Solutions for the quadrupole settings under a variety of focusing configurations as specified by the values of β_x^*, β_y^*, η^*, η_s, ν_x and ν_y have been found by use of the magneto-optic programs MAGIC[13] and TRANSPORT.[14] MAGIC employs thin-lens approximations for the transport elements, but has great flexibility and speed, and has been used to survey a wide range of configurations. TRANSPORT has been used to obtain more precise thick-lens solutions in a number of cases, and the beam tracing program SYNCH[15,16] has been used to generate emittances, beam sizes, luminosities, etc., from the TRANSPORT solutions. In all cases which have been checked, the thin-lens MAGIC results* agree with the thick-lens TRANSPORT-SYNCH results to within 5 to 10% in betatron functions, damping rates, emittance and luminosity.

Typical Configurations. The MAGIC program has been used to survey a large number of possible configurations. Some of the interesting results are shown in Figs. 2 through 6.

Figures 2, 3 and 4 show the operative regions in the (β_x^*, η^*) plane at several different tunes in the matched-η mode. The tunes of 18.75, 15.75 and 12.75 have been chosen for illustrative purposes; in practice, of course, ν_x and ν_y would be split to avoid the linear coupling resonance, and neither would be exactly on the quarter integer. The operative regions in these plots are limited by regions in which either the beam size becomes too large (aperture limit) or in which there are no mathematical solutions to the non-linear set of equations which relate the constraints to the variable quadrupole strengths. The luminosity contours are plotted for the nominal energies of 15, 10 and 5 GeV, respectively; the beam-beam tune shifts per interaction region are assumed to be $\Delta\nu_x = \Delta\nu_y = 0.06$. Note that the design luminosities given by $\mathscr{L}_{max} \propto E^2$ are respectively 1.0, 0.44 and 0.11 (in units of $10^{32} cm^{-2} sec^{-1}$) for 15, 10 and 5 GeV. For other values of tune shift and energy the value of luminosity may be scaled according to Eq. (2).

The luminosity contours and boundaries of the operative region in the (β_x^*, η^*) plane may be shifted somewhat by different choices of β_y^*.

* In the computation a pair of thin lenses is used for each quadrupole magnet with a short focal length such as Q2 and Q3. The thin lenses are symmetrically located about the center of the magnet and separated by 1/2 of magnet length.

Table I shows values of some of the beam and machine parameters for three typical matched-η configurations, as obtained by TRANSPORT and SYNCH.

Figure 5 illustrates the operative region in the (η_s, η^*) plane for the mismatched-η mode of operation, for $\nu = 18.75$, $\beta_x^* = 4.0$ m and $\beta_y^* = 0.2$ m. The contours represent the energies at which $\mathscr{L}_{max} = 10^{32}(E/15\ \text{GeV})^2$ (in $cm^{-2}sec^{-1}$) at tune shifts of $\Delta\nu_x = \Delta\nu_y = 0.06$. Note that the full design energy range of 5 to 15 GeV is covered in one continuous region of the (η_s, η^*) plane, but that non-zero values of η^* as well as η-mismatch are required at the lower energies. In the right-hand portion of Fig. 5, the beam size at the interaction point is dominated by the dispersion term for configurations near the lower operative boundary, and by emittance (enhanced by the η-mismatch) near the upper boundary.

Figure 6 shows how the parameters α (momentum compaction) $\sigma_{x\beta}$, σ_{xE}, and the energy for design luminosity vary with the η-mismatch, along the particular path designated as Path A in Fig. 5.

Second-Order Design

Chromaticity. Sextupole magnets will be placed near some of the standard cell quadrupoles in order to control the chromaticity. The uncorrected chromaticities when the tunes are around 18 are $\xi_x \approx -35$, $\xi_y \approx -100$ where the definition is

$$\xi = E_0 \frac{\partial \nu}{\partial E} .$$

Overcorrection of the chromaticities by 10 to 20% will probably be used as at SPEAR, in order to take advantage of the fast damping effect for coherent oscillations.[17] The sextupole strengths required for this correction have been found to be quite modest.

Non-linear Stopbands. The sixfold symmetry of the ring will tend to produce large sextupole Fourier coefficients with indices which are multiples of 6. These will strongly excite the stopbands at tunes 12 and 18. To suppress these coefficients we probably will need additional sextupoles in the straight sections; e.g., near QF1 and Q3.

Damping Variation with Frequency. The damping partition numbers (J_x and J_E) for an off-energy particle are determined by the value of the quantity

$$D = D_0 + D_E \frac{\Delta E}{E_0} + \cdots ,$$

where, for a machine using rectangular zero-gradient bending magnets,[18]

$$|D_0| << 1$$

and

$$D_E \approx \frac{2 \oint \eta^2 G^2 ds}{\oint B^2\, ds} ,$$

in which G is the local quadrupole gradient. The damping rates,

$$\alpha_x \propto J_x = 1 - D \qquad \text{and} \qquad \alpha_E \propto J_E = 2 + D$$

are affected very little by the natural energy spread of the beam because ΔE goes through many synchrotron oscillations during a damping period. However, if the equilibrium closed orbit is shifted by a change in orbital frequency (or equivalently rf frequency), we have

$$D_f \equiv f_0 \frac{\partial D}{\partial f_0} = - \frac{D_E}{\alpha}$$

or

$$D \approx - \frac{D_E}{\alpha} \frac{\Delta f_0}{f_0}$$

where α is the momentum compaction factor. Thus a shift in the rf frequency does affect the damping rate.

In the matched PEP configurations, we typically have $D_E \sim 250$ to 300, with α ranging from ~ 0.004 at tune 18 to ~ 0.02 at tune 12. Hence the frequency change to produce $D \sim 1$ is $\Delta f_0/f_0 \gtrsim 2 \cdot 10^{-5}$. In the unmatched-configurations, however, D_E can be an order of magnitude greater, and the frequency stability required to keep the damping rate constant may be a problem.

Conclusions

The present version of the PEP lattice is capable of reaching the theoretical design luminosity by means of several alternative operating configurations, at least according to first-order theory. Investigation of non-linear effects may further delimit the choice of favorable operating configurations.

Acknowledgments

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References

1. PEP Study Group, A Proposal for a Positron-Electron Colliding Beam Storage Ring Project, ed. J. Rees and W. B. Herrmannsfeldt, Stanford Linear Accelerator Center Report No. 171, Stanford Linear Accelerator Center, Stanford, Calif. (1974).
2. J. Rees and B. Richter, "Preliminary Design of a 15-GeV Electron-Positron Variable-Tune Storage Ring," PEP-NOTE 69, 1973 PEP Summer Study Report (unpublished).
3. J. Rees, "The PEP Electron Positron Ring — PEP Stage I," invited paper to this conference.
4. L. Smith, "The Proton-Electron-Positron Project — PEP," invited paper to this conference.
5. M. Sands, "The Physics of Electron Storage Rings, an Introduction," Proceedings of the International School of Physics "Enrico Fermi", Course XLVI, ed. B. Touschek (Academic Press, 1971); also Stanford Linear Accelerator Center Report No. 121.
6. R. H. Helm, M. J. Lee and J. M. Paterson, "Beam Enlargement by Mismatching the Energy-Dispersion Function," published report to this conference.
7. J. Paterson, "SPEAR, Status and Future Developments," invited paper to this conference.
8. F. Amman, "IEEE Trans. Nucl. Sci. NS-20 No. 3, 858 (1973).
9. M. Bassetti, "Beam Dimension Control in Storage Rings," CNEN, Frascati Laboratories, published report to this conference.
10. M. Bassetti, Proceedings of V International Conference on High Energy Accelerators, Frascati, 709 (1965).
11. B. Richter, "Design Considerations for High Energy Electron-Positron Storage Rings," Proceedings of the International Symposium on Electron and Positron Storage Rings (1966).
12. G. H. Rees, "Aspects of PEP as Compared to EPIC," PEP-NOTE 51, 1973 PEP Summer Study (unpublished).
13. M. J. Lee, W. W. Lee and L. C. Teng, "Magnet Insertion Code," PEP-NOTE 61, 1973 PEP Summer Design Study (unpublished).
14. K. L. Brown, and S. K. Howry, "TRANSPORT/360 A Computer Program for Designing Charged Particle Beam Transport System," Stanford Linear Accelerator Center Report No. 91 (1970).

15. A. A. Garren and A. S. Kenney, "A Computer Program for Synchrotron Design and Orbit Analysis," Internal note LBL 1968 (unpublished).
16. J. S. Colonias, "Particle Accelerator Design; Computer Programs," Book, Academic Press (1974).
17. SPEAR Group, "Fast Damping of Transverse Coherent Dipole Oscillations in SPEAR," published report to this conference.
18. H. Wiedemann, "Scaling of FODO-CELL Parameters," PEP-NOTE 39, 1973 PEP Summer Study (unpublished).

TABLE I

Typical Beam Parameters

		5 GeV	10 GeV	15 GeV	
Total Betatron Tune					
Horizontal	ν_x	12.75	15.75	18.75	
Vertical	ν_y	12.75	15.75	18.75	
Momentum Compaction		0.01668	0.00759	0.00455	
Transverse Damping Time	τ_x, τ_y	0.222	0.0278	0.00823	sec
x-y Coupling Coefficient	κ	0.249	0.294	0.280	
Horizontal Emittance	ϵ_x	1.730×10^{-5}	2.149×10^{-5}	2.297×10^{-5}	cm-rad
Vertical Emittance	ϵ_y	1.072×10^{-6}	1.858×10^{-6}	1.796×10^{-6}	cm-rad
Number of Stored Particles (each beam)	N	1.252×10^{12}	3.055×10^{12}	4.44×10^{12}	
Synchrotron Radiation Power (each beam)	P_{rad}	0.0090	0.353	2.596	MW
Linear Tune Shifts per Interaction Point					
Horizontal	$\Delta\nu_x$	0.06	0.06	0.06	
Vertical	$\Delta\nu_y$	0.06	0.06	0.06	
Luminosity (each interaction point)	$\mathscr{L}$	0.117×10^{-32}	0.462×10^{32}	1.008×10^{32}	$cm^{-2}sec^{-1}$
Cell Parameters					
Horizontal Phase Advance	ψ_x	48.9^o	72.9^o	97.0^o	
Vertical Phase Advance	ψ_y	29.9^o	56.5^o	82.3^o	
Maximum Horizontal Beta	$\beta_{x\,max}$	50.1	45.1	48.2	m
Maximum Vertical Beta	$\beta_{y\,max}$	78.8	53.9	50.2	m
Maximum Momentum Dispersion	η_{max}	6.38	3.34	2.24	m
Interaction Region Parameters					
Horizontal Beta	β_x^*	4.6	4.0	4.0	m
Vertical Beta	β_y^*	0.16	0.20	0.20	m
Momentum Dispersion	η^*	-2.40	-1.20	-0.73	m
Horizontal Beam Size (betatron)	$\sigma_{x\,\beta}^*$	0.0892	0.0927	0.0959	cm
Horizontal Beam Size (dispersion)	$\sigma_{x\,E}^*$	0.0789	0.0789	0.0720	cm
Horizontal Beam Size (total)	σ_x^*	0.1191	0.1217	0.1199	cm
Vertical Beam Size	σ_y^*	0.00414	0.00610	0.00600	cm

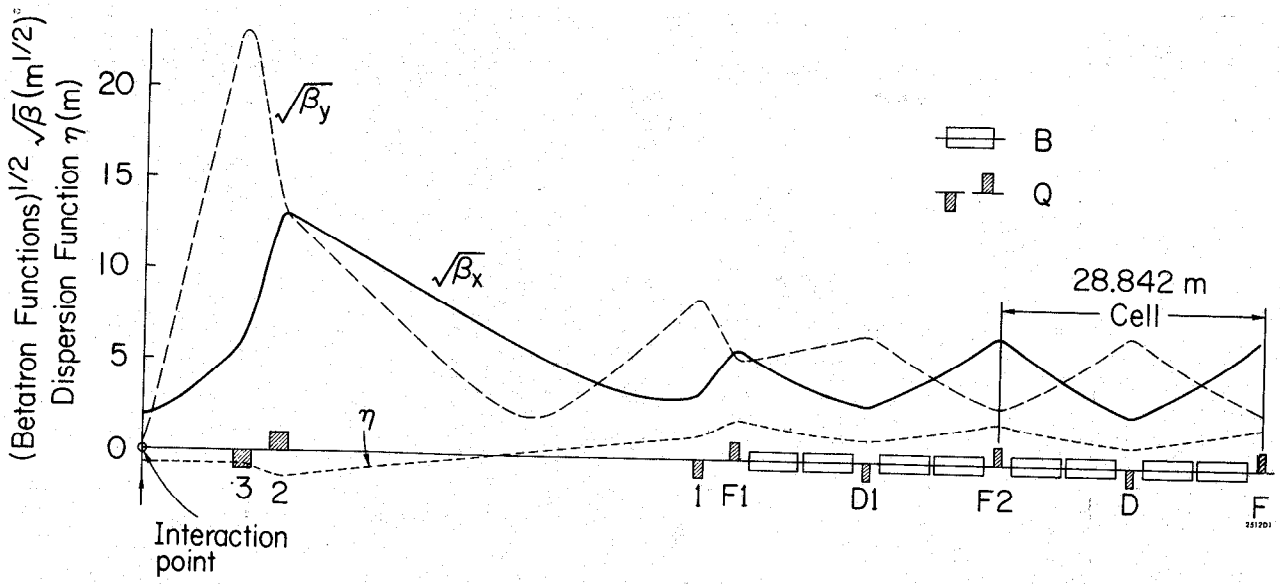

FIG. 1--Schematic of PEP lattice. The typical betatron functions shown are for a matched-η configuration (see text for definition) with $\beta_x^* = 4.0$ m, $\beta_y^* = 0.2$ m, $\eta^* = -0.73$ m, $\nu_x = \nu_y = 18.75$.

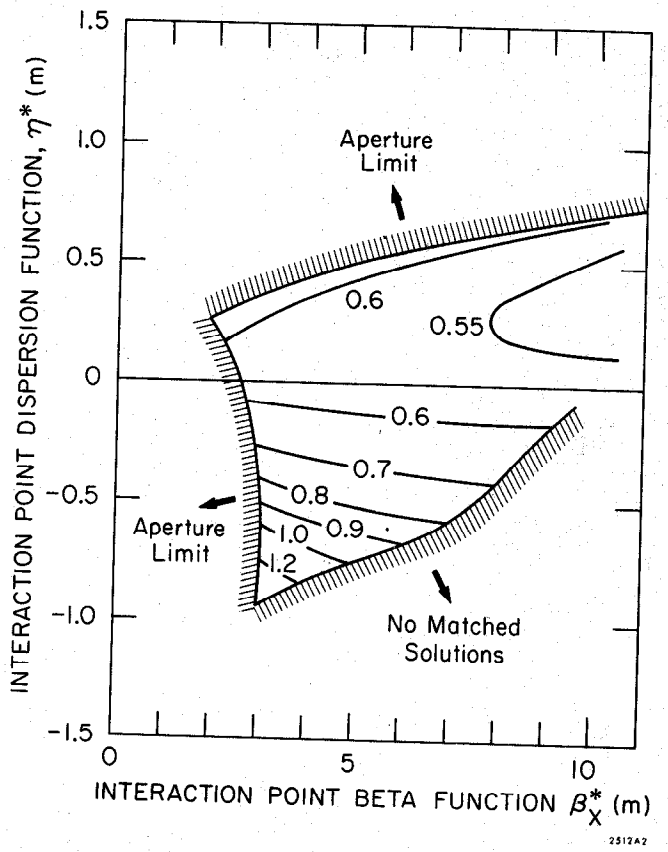

FIG. 2--Matched-η mode operative region in the (β_x^*, η^*) plane with $\nu_x = \nu_y = 18.75$, $\beta_y^* = 0.20$ m. The contours are luminosities in units of 10^{32}cm^{-2} sec^{-1}, computed at energy E = 15 GeV and beam-beam tune shifts $\Delta\nu_x = \Delta\nu_y = 0.06$.

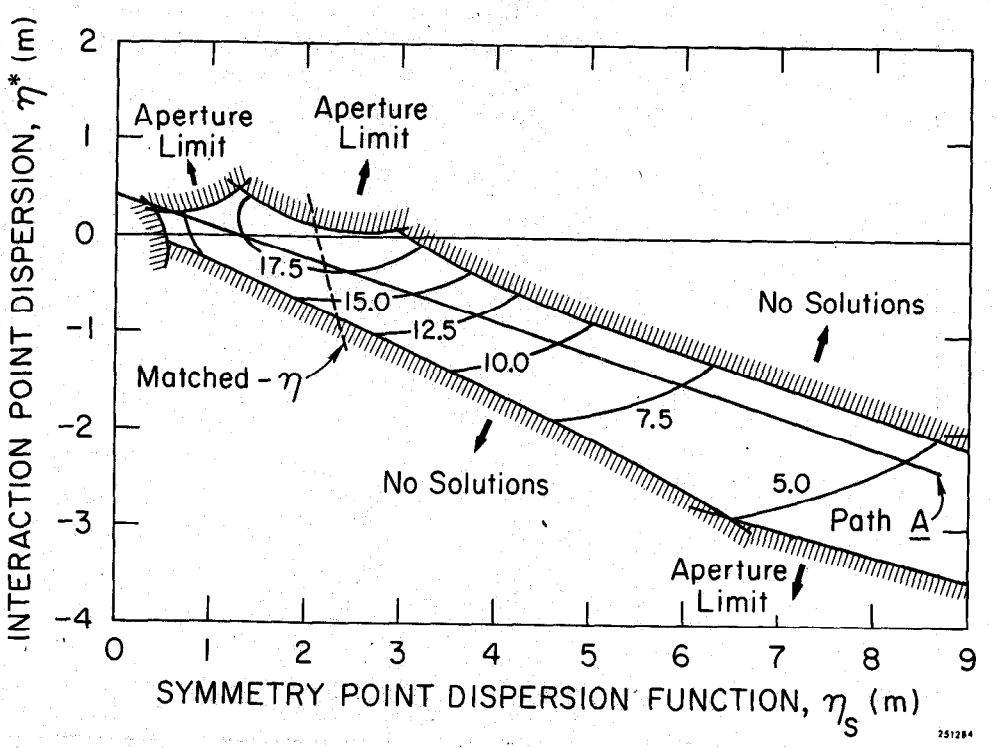

FIG. 5--Mismatched-η mode operative regions in the (η_S, η^*) plane with $\nu_x = \nu_y = 18.75$, $\beta_x^* = 4.0$ m, $\beta_y^* = 0.20$ m. The contours are the energies E_L (GeV) corresponding to luminosities given by $\mathscr{L}_{max} = 10^{32}(E_L/15\ \text{GeV})^2$ (in units of $\text{cm}^{-2}\text{sec}^{-1}$), with $\Delta\nu_x = \Delta\nu_y = 0.06$. The dashed curve labeled "η-matched", near $\eta_S = 2$, represents solutions for which the dispersion function is periodic within the standard cells. The line labeled "Path A" is for reference in Fig. 6.

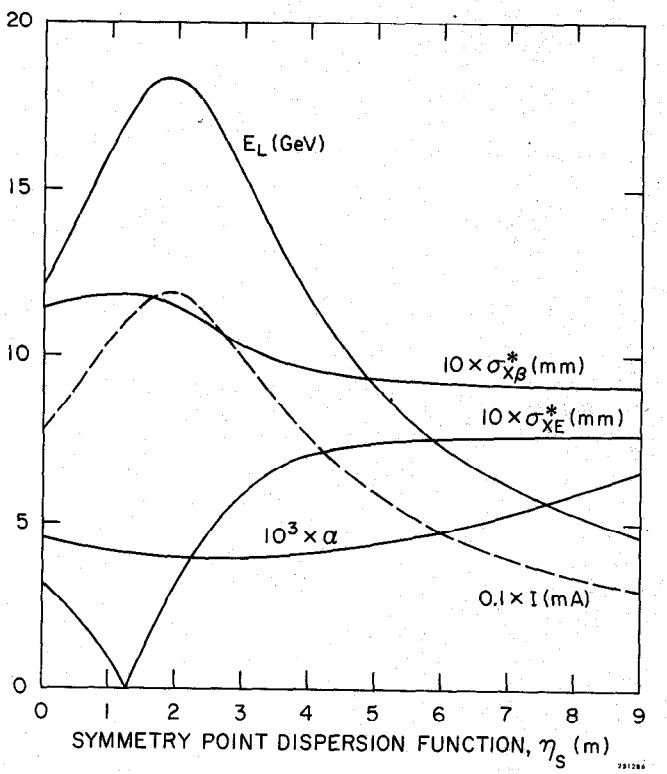

FIG. 6--Variation of beam parameters for different mismatched-η configurations along Path A in Fig. 5 $\sigma_{x\beta}$ is rms beam amplitude due to betatron motion; σ_{xE} is rms beam amplitude due to compaction factor; and I is the current in each beam and $\mathscr{L}_{max} = 10^{32}(E_L/15\ \text{GeV})^2$.