# SYMMETRIES OF CURRENTS AS SEEN BY HADRONS* 

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#### Abstract

The transformation properties of hadron states and current operators under the $\mathrm{SU}(6)_{\mathrm{W}}$ algebra of currents are reviewed. A transformation from current to constituent quark bases is introduced, and the algebraic properties of certain transformed current operators are abstracted from the free quark model. The resulting theory yields selection rules, relations among widths, and relative signs of amplitudes for both pion and real photon transitions among hadrons. The agreement with experiment found, especially for amplitude signs, lends strong support both to the proposed theory of current-induced-transition amplitudes and to the assignment of hadrons to constituent quark model multiplets. The theory may then be used to classify states and to predict properties of yet unseen decays, thereby providing a new tool in hadron spectroscopy.


## INTRODUC TION

Someday, when we have a real theory of hadrons and their interactions, we will be able to calculate all the current-induced-transitions among them. That is, we will be able to calculate the matrix elements for the vector and axial-vector current induced processes: $\mathrm{V}_{\mu}(\mathrm{x})+$ Hadron $\rightarrow$ Hadron' and $\mathrm{A}_{\mu}(\mathrm{x})+$ Hadron $\rightarrow$ Hadron'. At the particular point $\mathrm{q}^{2}=0$, if we were then willing to invoke the vector dominance hypothesis or the PCAC hypothesis, such amplitudes could be related to those for the purely hadronic processes $\rho+$ Hadron $\rightarrow$ Hadron' and $\pi+$ Hadron $\rightarrow$ Hadron', respectively.

To find ourselves in this happy situation, even at $q^{2}=0$, we must actually solve two problems at once. First, we must know the properties of currents - what symmetry properties do they possess, what are their commutation relations? Second, we need to understand the structure of hadrons - what are their relations to one another, how are the currents flowing within them distributed?

It is progress on these questions which has been most dramatic since the last conference in this series and which forms the principal subject of this talk. These two problems have been attacked by relating them - through a socalled "transformation between current and constituent quarks". The end result is a well-defined theory of vector and axial-vector transition matrix elements within the context of the quark model. We shall review the formulation of the theory and its application to pion and real photon decays of hadrons.

The agreement of the theory with experiment that we shall find lends support both to the theory of current-induced-transitions and to the quark model for hadron spectroscopy. Particularly the success in predicting amplitude

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signs indicates that in a sense, not only do hadrons fall into recognizable quark model multiplets, but the relation between their wave functions is at least roughly given by the quark model as well.

CURRENTS AND QUARKS
In order to formulate a theory of current-induced-transitions among hadrons composed of quarks we need a group theoretic framework for labeling the states and operators involved. For this purpose it is natural to turn to an algebra of charges formed by integrating weak and electromagnetic current densities over all space. We use currents because: (a) it is plausible to work to lowest order in the weak or electromagnetic interaction but to all orders in strong interactions; (b) the symmetries and commutation relations of such currents are relatively well understood; and (c) matrix elements of currents are measured in the laboratory, or if not, in cases of relevance to us they are related by the Partially Conserved Axial-Vector Current Hypothesis (PCAC) to pion amplitudes which are measured.

To start with, consider vector and axial-vector charges:

$$
\begin{align*}
& Q^{\alpha}(\mathrm{t})=\int \mathrm{d}^{3} \mathrm{x} \mathrm{~V}_{0}^{\alpha}(\overrightarrow{\mathrm{x}}, \mathrm{t})  \tag{1a}\\
& \mathrm{Q}_{5}^{\alpha}(\mathrm{t})=\int \mathrm{d}^{3} \mathrm{xA}_{0}^{\alpha}(\overrightarrow{\mathrm{x}, \mathrm{t})}, \tag{1b}
\end{align*}
$$

where $\alpha$ is an $\operatorname{SU}(3)$ index which runs from 1 to 8 and $V_{\mu}^{\alpha}(\vec{x}, t)$ and $A_{\mu}^{\alpha}(\vec{x}, t)$ are the local vector and axial-vector current densities with measurable matrix elements. The vector charges are just the generators of $\operatorname{SU}(3)$. These integrals over the time components of the current densities are assumed to satisfy the equal-time commutation relations proposed by GellMann ${ }^{1}$

$$
\begin{align*}
& {\left[Q^{\alpha}(t), Q^{\beta}(t)\right]=\mathrm{if}^{\alpha \beta \gamma} Q^{\gamma}(t)} \\
& {\left[Q^{\alpha}(t), Q_{5}^{\beta}(t)\right]=\mathrm{if} \mathrm{f}^{\alpha \beta \gamma} \mathrm{Q}_{5}^{\gamma}(\mathrm{t})}  \tag{2}\\
& {\left[Q_{5}^{\alpha}(\mathrm{t}), Q_{5}^{\beta}(\mathrm{t})\right]=\mathrm{if} \mathrm{f}^{\alpha \beta \gamma}{ }_{Q^{\gamma}}(\mathrm{t}),}
\end{align*}
$$

where $\mathrm{f}^{\alpha \beta \gamma}$ are the structure constants of $\operatorname{SU}(3)$. Sandwiched between nucleon states at infinite momentum, the last of Eqs. (2) gives rise to the Adler-Weisberger sum rule. ${ }^{2}$ From this point on, we shall always be considering matrix elements to be taken between hadron states ${ }^{3}$ with $\mathrm{p}_{\mathrm{z}} \rightarrow \infty$.

For the purposes at hand we need a somewhat larger algebraic system then that provided by the measurable vector and axial-vector charges in Eqs. (1), which form the algebra of $\mathrm{SU}(3) \times \mathrm{SU}(3)$ according to EqS. (2). 4 To obtain the larger algebra we adjoin to the integrals over all space of ${ }^{4}$ $\mathrm{V}_{0}^{\alpha}(\overrightarrow{\mathrm{x}}, \mathrm{t})$ and $\mathrm{A}_{\mathrm{Z}}^{\alpha}(\overrightarrow{\mathrm{x}}, \mathrm{t})$, those of the tensor current densities $\mathrm{T}_{\mathrm{yz}}^{\alpha}(\overrightarrow{\mathrm{x}}, \mathrm{t})$ and
$\mathrm{T}_{\mathrm{ZX}}^{\alpha}(\overrightarrow{\mathrm{x}}, \mathrm{t})$. In the free quark model these charges have the form:

$$
\begin{align*}
\int \mathrm{d}^{3} \mathrm{x}_{\mathrm{V}} \mathrm{~V}_{0}^{\alpha}(\overrightarrow{\mathrm{x}}, \mathrm{t}) & \sim \int \mathrm{d}^{3} \mathrm{x} \psi^{+}(\mathrm{x})\left(\frac{\lambda^{\alpha}}{2}\right) \mathbb{1} \psi(\mathrm{x}) \\
\int \mathrm{d}^{3} \mathrm{x} \mathrm{~A}_{\mathrm{z}}^{\alpha}(\overrightarrow{\mathrm{x}}, \mathrm{t}) & \sim \int \mathrm{d}^{3} \mathrm{x} \psi^{+}(\mathrm{x})\left(\frac{\lambda^{\alpha}}{2}\right) \sigma_{\mathrm{z}} \psi(\mathrm{x}) \\
\int \mathrm{d}^{3} \mathrm{x} \mathrm{~T}_{\mathrm{yz}}^{\alpha}(\overrightarrow{\mathrm{x}}, \mathrm{t}) & \sim \int \mathrm{d}^{3} \mathrm{x} \psi^{+}(\mathrm{x})\left(\frac{\lambda^{\alpha}}{2}\right) \beta \sigma_{\mathrm{x}} \psi(\mathrm{x})  \tag{3}\\
\int \mathrm{d}^{3} \mathrm{x} \mathrm{~T}_{\mathrm{zx}}^{\alpha}(\overrightarrow{\mathrm{x}}, \mathrm{t}) & \sim \int \mathrm{d}^{3} \mathrm{x} \psi^{+}(\mathrm{x})\left(\frac{\lambda^{\alpha}}{2}\right) \beta \sigma_{\mathrm{y}} \psi(\mathrm{x})
\end{align*}
$$

where $\psi(x)$ is the Dirac (and $\operatorname{SU}(3)$ ) spinor representing the quark field. When commuted using the free quark field commutation relations, these charges act algebraically like the product of $\operatorname{SU}(3)$ and Dirac matrices $\left(\lambda^{\alpha} / 2\right) \mathbb{I},\left(\lambda^{\alpha} / 2\right) \sigma_{\mathrm{Z}},\left(\lambda^{\alpha} / 2\right) \beta \sigma_{\mathrm{x}}$, and $\left.\lambda^{\alpha} / 2\right) \beta \sigma_{\mathrm{y}}$ respectively. 5 The Dirac matrices $\beta \sigma_{\mathrm{x}}, \beta \sigma_{\mathrm{y}}$, and $\sigma_{\mathrm{z}}$ form the so-called $W$-spin. ${ }^{6}$ They are invariant under boosts in the $z$ direction and the corresponding charges are "good", in the sense that they have finite (generally non-vanishing) matrix elements between states as $p_{z} \rightarrow \infty$. This makes them the correct set of charges to use to label states in terms of their internal quark spin components. If we let $\alpha=0$ correspond to the $\mathrm{SU}(3)$ singlet representation (and $\lambda^{0}$ be a multiple of the unit matrix), then Eqs. (3) consist of 36 charges which close under commutation. They act like an identity operator plus 35 other generators of an $S U(6)$ algebra. We call this algebra the $S U(6)$ Wof currents ${ }^{5}$ because of its origin. $Q^{\alpha}$ and $Q_{5}^{\alpha}$ then essentially ${ }^{4}$ form a chiral $\operatorname{SU}(3) \times \operatorname{SU}(3)$ subalgebra of this larger algebra.

Given such an algebra, we define the smallest representations of it (other than the singlet), the 6 and $\overline{6}$ representations, as the current quark (q) and current anti-quark ( $\bar{q}$ ) respectively. We may build up all the larger representations of $\operatorname{SU}(6)_{\mathrm{W}}$ out of these basic ones.

Can then real baryons be written as three current quarks, qqq, and real mesons as current quark and anti-quark, $q \bar{q}$, with internal angular momentum $L$, as in the constituent quark model ${ }^{7}$ used for hadron spectroscopy? While possible in principle, it is a disaster when compared with experiment. For it leads to $\mathrm{g}_{\mathrm{A}}=5 / 3$, zero anomalous magnetic moment of the nucleon, no electromagnetic transition from the nucleon to the 3-3 resonance ( $\Delta$ ), no decay of $\omega$ to $\gamma \pi$, etc. It would also yield results for masses like $M_{N}=M_{\Delta}$, $\mathrm{M}_{\pi}=\mathrm{M}_{\rho}$, etc. The hadron states we see can not be simple in terms of current quarks. They must lie in mixed representations of the $\operatorname{SU}(6)_{W}$ of currents. Work in past years has shown directly that hadron states are quite complicated when viewed in terms of current algebra. 8

We may restate this complication in terms of the definition of an operator V for any hadron:

$$
\begin{align*}
\text { |Hadron }> & \equiv \text { VIsimple qqq or } q \bar{q} \text { state of current quarks }> \\
& =\text { |simple qqq or } q \bar{q} \text { state of constituent quarks }> \tag{4}
\end{align*}
$$

All the complication of real hadrons under the $\mathrm{SU}(6)_{W}$ of currents (i.e., in terms of current quarks) has been swept into the operator V. On the other hand, real hadrons are supposed to be simple in terms of the "constituent quarks" used for spectroscopy purposes, as indicated by the second equality in Eq. (4). In other words, the transformation V connects the two simple descriptions in terms of current quarks and constituent quarks. 9 It is for this reason that it is sometimes called the "transformation from current to constituent quarks". 10,11

Up to this point we have only managed to restate the problem via Eq. (4). But as often happens, phrasing the problem right is a major way toward the solution. For what we are after in the end are matrix elements of various current operators, $\mathscr{O}$. Using Eq. (4) and assuming V is unitary we may write
< Hadron'| $0 \mid$ Hadron > $=\langle\text { (simple current quark state })^{\prime} \mid \mathrm{V}^{-1} \mathscr{O} \mathrm{~V}$
| (simple current quark state)>.
This has two important advantages. First, we may study the properties of $\mathrm{V}^{-1} \mathscr{O} \mathrm{~V}$ in isolation, and then apply what we learn to the matrix elements of $\mathscr{O}$ between any two hadron states. Second, even though V itself is very complicated and contains (by definition) all information on the current quark composition of each hadron, it is possible that the object $\mathrm{V}^{-1} \mathscr{O} \mathrm{~V}$ for some operators $\mathscr{O}$ may be relatively simple in its algebraic transformation properties.

This last possibility is of course exactly what we shall assume on the basis of calculations done in the free quark model. In that model, Melosh ${ }^{12}$ and others $13,14,15$ have been able to formulate and explicitly calculate the transformation $V$. While one would not take the details of the transformation found there as correctly reflecting the real world, one might try to abstract the algebraic properties of some transformed operators $\mathrm{V}^{-1} \mathscr{O} \mathrm{~V}$, from such a calculation. In cases of interest, this turns out to be equivalent to assuming that the transformed operators $\mathrm{V}^{-1} \mathscr{O} \mathrm{~V}$ have the algebraic properties of the most general combination of single quark operators consistent with $\mathrm{SU}(3)$ and Lorentz invariance.

Thus, while Eq. (3) shows that $Q_{5}^{\alpha}$ itself behaves under the $\operatorname{SU}(6)_{W}$ of currents as simply

$$
\int \mathrm{d}^{3} \mathrm{x} \psi^{+}(\mathrm{x})\left(\frac{\lambda^{\alpha}}{2}\right) \sigma_{\mathrm{z}} \psi(\mathrm{x})
$$

a direct calculation in the free quark model shows that algebraically $\mathrm{V}^{-1} \mathrm{Q}_{5}^{\alpha} \mathrm{V}$ behaves as a sum of two terms: ${ }^{16}$

$$
\begin{align*}
V^{-1} Q_{5}^{\alpha} V & \sim\left(\frac{\lambda^{\alpha}}{2}\right) \sigma_{z} \\
& +\left(\frac{\lambda^{\alpha}}{2}\right)\left[\left(\beta \sigma_{x}+i \beta \sigma_{y}\right)\left(v_{x}-i v_{y}\right)-\left(\beta \sigma_{x}-i \beta_{y}\right)\left(v_{x}+i v_{y}\right)\right] \tag{6}
\end{align*}
$$

where the products of Dirac and $\mathrm{SU}(3)$ matrices are understood to be taken between quark spinors (and integrated over all space). Here $v_{\mu}$ is a vector in configuration space, so that $v_{x} \pm i v_{y}$ raises (lowers) the $z$ component of angular momentum ( $\mathrm{L}_{\mathrm{Z}}$ ) by one unit. The particular combination of Dirac matrices and vector indices in the two terms in Eq. (6) is dictated by the demands that the total $J_{z}=0$ and the parity be odd for the axial-vector charge, $Q_{5}^{\alpha}$, and for ${ }^{2} V^{-1} Q_{5}^{\alpha} V$.

For the vector charge, $\mathrm{Q}^{\alpha}$, we must have

$$
\begin{equation*}
\mathrm{V}^{-1} \mathrm{Q}^{\alpha} \mathrm{V}=\mathrm{Q}^{\alpha}, \tag{7}
\end{equation*}
$$

since we want these charges to be the generators of $\operatorname{SU}(3)$, both before and after the transformation. However, the first moment of the charge density, ${ }^{17}$

$$
\begin{equation*}
D_{+}^{\alpha}=i \int d^{3} x\left(\frac{-x-i y}{\sqrt{2}}\right) V_{0}^{\alpha}(\vec{x}, t) \tag{8}
\end{equation*}
$$

is not a generator and is transformed non-trivially by $V$. One finds in the free quark model that in algebraic properties $\mathrm{V}^{-1} \mathrm{D}_{+}^{\alpha} \mathrm{V}$ behaves as a sum of four terms under the $\operatorname{SU}(6)_{W}$ of currents : 18

$$
\begin{align*}
\mathrm{V}^{-1} \mathrm{D}_{+}^{\alpha} \mathrm{V} & \sim\left(\frac{\lambda^{\alpha}}{2}\right) \mathbb{1}\left(\mathrm{v}_{\mathrm{x}}+\mathrm{i} /_{\mathrm{y}}\right) \\
& +\left(\frac{\lambda^{\alpha}}{2}\right)\left(\beta \sigma_{\mathrm{x}}+\mathrm{i} \beta \sigma_{\mathrm{y}}\right) \\
& +\left(\frac{\lambda^{\alpha}}{2}\right) \sigma_{\mathrm{z}}\left(\mathrm{v}_{\mathrm{x}}+\mathrm{i} \mathrm{v}_{\mathrm{y}}\right) \\
& +\left(\frac{\lambda^{\alpha}}{2}\right)\left(\beta \sigma_{\mathrm{x}}-\mathrm{i} \beta \sigma_{\mathrm{y}}\right)\left(\mathrm{v}_{\mathrm{x}}+\mathrm{i} \mathrm{v}_{\mathrm{y}}\right)\left(\mathrm{v}_{\mathrm{x}}+\mathrm{i} \mathrm{v}_{\mathrm{y}}\right) \tag{9}
\end{align*}
$$

where again the Dirac and $\operatorname{SU}(3)$ matrices are understood to be taken between quark spinors.

We abstract the algebraic properties of $V^{-1} Q_{5}^{\alpha} V$ and $V^{-1} D_{+}^{\alpha} V$ given in Eqs. (6) and (9) from the free quark model and assume them to hold in the real world. We are then able to treat matrix elements of $Q_{5}^{\alpha}$ and $D_{+}^{\alpha}$ between hadron states as follows:
(1) We identify the hadrons with $q q q$ or $q \bar{q}$ states of the constituent quark model where the total quark spin $S$ is coupled to the internal angular momentum L to form the total J of the hadron. The states so constructed fall into $\mathrm{SU}(6)_{W} \times 0$ (3) multiplets. Meson states formed in this simple manner are enumerated in Table I, where candidates are given for the isospin 1 and 0 "slots" for each J ${ }^{P C}$ value from among the observed mesons. ${ }^{7}$ The sad state of meson spectroscopy is reflected in the lack of established states even at the $L=1$ level. The situation for baryons is of course much better, ${ }^{7}$ there being one or more established candidate for every $J^{P}$ value in the $\mathrm{SU}(6)_{X W} \times 0(3)$ multiplets $56 \mathrm{~L}=0, \underline{70} \mathrm{~L}=1$, and $56 \mathrm{~L}=2$.

Table I Meson states of the constituent quark model and possible I = 1 and 0 candidates. ${ }^{7}$

| $\operatorname{su}(6)_{W} \times \quad 0(3)$ <br> Multiplet | SU(3) <br> Multiplet | Quark Spin S | $J^{P C}$ | Candidates $\mathrm{I}=1$ and 0 |
| :---: | :---: | :---: | :---: | :---: |
| $\underline{35}+1$ | $\underline{8}+\underline{1}$ | 1 | $1^{--}$ | $\rho, \omega, \phi$ |
| $\mathrm{L}=0$ | $\underline{8}+\underline{1}$ | 0 | $0^{-+}$ | $\pi, \eta, \mathrm{X}^{0}$ ? |
| $\underline{35}+\underline{1}$ | $\underline{8}+\underline{1}$ | 1 | $2^{++}$ | $A_{2}, \mathbf{f}, \mathbf{f}^{\prime}$ |
| $\mathrm{L}=1$ | $\underline{8}+\underline{1}$ | 1 | $1^{++}$ | $\mathrm{A}_{1}$ ?, D, ? |
|  | $\underline{8}+\underline{1}$ | 1 | $0^{++}$ | $\delta, \in ?, S^{*} ?, E^{\text {l }}$ ? |
|  | $\underline{8}+\underline{1}$ | 0 | $1^{+-}$ | B, ? , |
| $\underline{35}+\underline{1}$ | $\underline{8}+\underline{1}$ | 1 | $3^{-7}$ | $\mathrm{g}, \omega_{3} ?, ?$ |
| $\mathrm{L}=2$ | $\underline{8}+\underline{1}$ | 1 | $2^{--}$ | $\mathrm{F}_{1}$ ?, ?, ? |
|  | $\underline{8}+\underline{1}$ | 1 | $1^{--}$ | $\rho^{\prime} ?, ?$, ? |
|  | $8+1$ | 0 | $2^{-+}$ | $\mathrm{A}_{3} ?, ?, ?$ |

(2) Since very few weak axial-vector transitions are measured, given a matrix element of $Q_{5}^{\alpha}$, we use PCAC to relate it to a measured pion transition amplitude. Application of the golden rule then yields:

$$
\begin{equation*}
\Gamma\left(\mathrm{H}^{\prime} \rightarrow \pi^{-} \mathrm{H}\right)=\frac{1}{4 \pi \mathrm{f}_{\pi}^{2}} \frac{\mathrm{p}_{\pi}}{2 \mathrm{~J}^{\prime}+1} \frac{\left(\mathrm{M}^{\prime 2}-\mathrm{M}^{2}\right)^{2}}{{\mathrm{M}^{\prime 2}}^{2}} \sum_{\lambda}\left|<\mathrm{H}^{\prime}, \lambda\right|(1 / \sqrt{2})\left(Q_{5}^{1}-\mathrm{i} Q_{5}^{2}\right)|\mathrm{H}, \lambda>|^{2}, \tag{10}
\end{equation*}
$$

where $\mathrm{f}_{\pi} \simeq 135 \mathrm{MeV}$. The factors in Eq. (10) are forced on us by PCAC and kinematics - there are no arbitrary phase space factors.

For real photon transitions, matrix elements of $D_{+}^{3}+(1 / \sqrt{3}) D_{+}^{8}$ are directly proportional to the corresponding Feynman amplitudes. The width for $\mathrm{H}^{+} \rightarrow \gamma \mathrm{H}$ is given by ${ }^{17}$
$\Gamma\left(\mathrm{H}^{\prime} \rightarrow \gamma \mathrm{H}\right)=\frac{\mathrm{e}^{2}}{\pi} \frac{\mathrm{p}_{\gamma}^{3}}{2 \mathrm{~J}^{\prime}+1} \sum_{\lambda}\left|<\mathrm{H}^{\prime}, \lambda\right| \mathrm{D}_{+}^{3}+(1 / \sqrt{3}) \mathrm{D}_{+}^{8}|\mathrm{H}, \lambda-1\rangle 1^{2}$.
(3) Given a matrix element of $\mathrm{Q}_{5}^{\alpha}$ or $\mathrm{D}^{\alpha}$ between hadron states which is related to measurements by either Eq. (10) or (11), we transform using V from simple constituent to simple current quark states. The particular matrix element is thus rewritten in terms of $\mathrm{V}^{-1} \mathrm{Q}_{5} \mathrm{~V}$ or $\mathrm{V}^{-1} \mathrm{D}_{+} \mathrm{V}$, and simple current quark states. We know the algebraic properties of all these quantities under the $\operatorname{SU}(6)_{W}$ of currents via abstraction of Eqs. (6) and (9) from the free quark model and our identification of hadrons with quark model states. We may then apply the Wigner-Eckart theorem to each term to express it as a Clebsch-Gordan coefficient ${ }^{19}$ (of $\left.\mathrm{SU}(6)_{W_{W}}\right)$ times a reduced
matrix element. Since the same reduced-matrix element occurs in many different transitions, relations among the corresponding transition amplitudes follow.

## CONSEQUENCES OF THE THEORY

The experimental consequences of the theory outlined in the last section have been considered by a number of authors. ${ }^{12,20-29}$ These consequences fall into the following three categories:
(1) Selection Rules For transitions by pion or photon emission from states (either mesons or baryons) with internal angular momentum $L^{\prime}$ to those with L , one finds 22,23

$$
\begin{align*}
& \left|\left|L^{\prime}-L\right|-1\right| \leq \ell_{\pi} \leq L+L^{\prime}+1  \tag{12a}\\
& \left|\left|L^{\prime}-L\right|-1\right| \leq j_{\gamma} \leq L+L^{\prime}+1 \tag{12b}
\end{align*}
$$

where $\ell_{\pi}$ and $\mathrm{j}_{\gamma}$ are the total angular momentum carried off by the pion and photon in the overall transition.

For example, $\ell_{\pi}$ can be 0 or 2 ( $\ell_{\pi}=1$ is forbidden by parity), but not 4 for a pion decay from $L^{+}=1$ to $L=0$. Thus the decay of the $D_{15}(1670)$, the $\mathrm{J}^{P}=5 / 2^{-} \mathrm{N}^{*}$ resonance with $\mathrm{L}^{\prime}=1$, into $\pi \Delta$ is forbidden in g-wave ( $\ell_{\pi}=4$ ), although otherwise allowed by kinematical considerations. Similarly,only $\mathrm{j}_{\gamma}=1$ is allowed for $\mathrm{L}^{\prime}=0$ to $\mathrm{L}=0$ photon transitions, although $\mathrm{j}_{\gamma}=2$ (and even $\mathrm{j}_{\gamma}=3$ for $\Delta^{\prime} \rightarrow \gamma \Delta$ ) is generally permitted by kinematics. This particular rule is well-known for $\Delta \rightarrow \gamma \mathrm{N}$, where it is just the successful quark model result ${ }^{30}$ that the transition is purely magnetic dipole in character, i. e. the possible electric quadrupole amplitude is forbidden. The inequalities in Eqs. (12) might be regarded as the generalization of these particular results to all L and $\mathrm{L}^{\prime}$ in the present theoretical context.

Note that for $\left|L-L^{\prime}\right| \geq 3$ the lower limit of the inequalities becomes operative in a non-trivial way, forbidding low values of $\ell_{\pi}$ or $\mathrm{j}_{\gamma}$ which would otherwise have been favored kinematically. Unfortunately, the relevant hadron states which would provide an interesting test of this have not yet been found.

Selection rules of a different sort govern the number of independent reduced matrix elements. For pion transitions from a hadron multiplet with internal angular momentum $L^{\prime}$ down to the ground state hadrons with $L=0$, there are at most two independent reduced matrix elements, corresponding to the two terms in Eq. (6). For real photon transitions between the same two multiplets there are at most four independent reduced matrix elements, corresponding to Eq. (9).

In general structure, the theory described above includes various concrete quark model calculations, both non-relativistic ${ }^{31}$ and relativistic. 32 In fact, a one-to-one correspondence exists between the quantities calculated in such models and the reduced matrix elements in the present theory. However, such models are usually much more specific, with parameters like the strength of the "potential", quark masses, etc. fixed. Since the quantities corresponding to reduced matrix elements are expressed explicitly in terms of such parameters, they are computable numerically and the scale of the reduced matrix elements is determined.

Also included in the general structure of the theory are the results following from assuming strong interaction $\operatorname{SU}(6)_{W}$ conservation. 6 For pion transitions, this corresponds in the present theory to retaining only the first term in $\mathrm{V}^{-1} \mathrm{Q}_{5}^{\alpha} \mathrm{V}$. Since this hypothesis fails experimentally, various ad hoc scherfies for breaking $\mathrm{SU}(6)_{W}$ have been proposed. ${ }^{33}$ Such schemes still fall within the general structure of amplitudes presented above ${ }^{34}$ and they are similar in giving relations between amplitudes while not setting their absolute scale. ${ }^{35}$ However, as we shall see below, they are generally more restrictive in that they tie together pion and rho decay amplitudes.
(2) Decay Widths The simplest such set of relations are those for pion transitions from $\mathrm{L}^{\prime}=0$ to $\mathrm{L}=0$ mesons. Here there is only one reduced matrix element (the second term in Eq. (6) has $\Delta \mathrm{L}_{\mathrm{Z}}= \pm 1$ and so can not contribute when $\mathrm{L}^{\prime}=\mathrm{L}=0$ ), so that the amplitudes for $\rho \rightarrow \pi \pi, \mathrm{K}^{*}(890) \rightarrow \pi \mathrm{K}$, and $\omega \rightarrow \pi \rho$ are all proportional. The ratio of the amplitudes for the first two processes may be obtained from $\Gamma(\rho \rightarrow \pi \pi) / \Gamma\left(\mathrm{K}^{*} \rightarrow \pi \mathrm{~K}\right)$, while the amplitude for the latter is obtainable from $\omega \rightarrow 3 \pi$ and rho dominance. Within errors, the ratio of the three amplitudes is that predicted by the theory. ${ }^{36}$

For pion transitions from mesons with internal angular momentum $L^{\prime}=1$ to those with $\mathrm{L}=0$, both terms in Eq. (6) are possible and there are consequently two independent reduced matrix elements which describe all such decays. Rather than performing a fit to all the data, we choose two measured widths as input and thereby determine all the other decay rates. For this purpose we take $\Gamma\left(\mathrm{A}_{2} \rightarrow \pi \rho\right)=71.5 \mathrm{MeV}$, from the latest particle data tables, 37 and $\Gamma_{\lambda=0}(B \rightarrow \pi \omega)=0$. This latter condition, the vanishing of the helicity zero (longitudinal) decay of B $\rightarrow \pi \omega$, is suggested by high statistics experiments 38 which find the transverse decay to be strongly dominant. While probably not exactly zero, we take this as a very reasonable first approximation to the data. Exact vanishing of $\Gamma_{\lambda=0}(B \rightarrow \pi \omega)$ corresponds to only the second term in $\mathrm{V}^{-1} \mathrm{Q}_{5}^{\alpha} \mathrm{V}$, with the algebraic properties of ( $\lambda^{\alpha} / 2$ ) $\left[\left(\beta \sigma_{x}+i \beta \sigma_{y}\right)\left(v_{x}-i v_{y}\right)-\left(\beta \sigma_{x}-i \beta \sigma_{y}\right)\left(v_{x}+i v_{y}\right)\right]$, having a non-zero reduced matrix element. This well illustrates the experimental necessity of a non-trivial transformation $V$; for if $V=\mathbb{1}$, only the term behaving as $(\lambda \alpha / 2) \sigma_{\mathrm{Z}}$ would be present and the predicted helicity structure for $\mathrm{B} \rightarrow \pi \omega$ would be completely opposite that observed.

The results 39 can be seen in Table II. The correct values for $\Gamma\left(\mathrm{A}_{2} \rightarrow \pi \rho\right)$ / $\Gamma\left(\mathrm{K}^{*}(1420) \rightarrow \pi \mathrm{K}^{*}\right)$ and $\Gamma(\mathrm{f} \rightarrow \pi \pi) / \Gamma\left(\mathrm{K}^{*}(1420) \rightarrow \pi \mathrm{K}\right)$ may be regarded as testing the $\operatorname{SU}(3)$ component of the theory, while, for example, the value of $\Gamma\left(\mathrm{A}_{2} \rightarrow \pi \rho\right)$ or $\Gamma\left(\mathrm{K}^{*}(1420) \rightarrow \pi \mathrm{K}^{*}\right)$ relative to $\Gamma(\mathrm{f} \rightarrow \pi \pi), \Gamma\left(\mathrm{K}^{*}(1420) \rightarrow \pi \mathrm{K}\right)$ or $\Gamma\left(\mathrm{A}_{2} \rightarrow \pi \eta\right)$ tests the full theory, including the phase space factors in Eq. (10), since one is relating d-wave pion decays into pseudoscalar vs. vector mesons. As for the other decays in the Table, we note that: (a) other strong interaction decay modes of the $B$ meson very likely exist, as we discuss later, although $\pi \omega$ is certainly dominant; (b) the "real" $\mathrm{A}_{1}$ resonance still remains to be found for comparison with the theory; (c) the now established $\mathrm{I}=1$ scalar meson, $\delta$, only has $\pi \eta$ as a possible strong decay channel, so the total width should almost coincide with that into $\pi \eta$; (d) we have chosen 1300 MeV , the mass where the s-wave $\pi \mathrm{K}$ phase shift ${ }^{37}$ goes through $90^{\circ}$, as the mass of the strange, $\mathrm{J}^{\mathrm{P}}=0^{+}$meson 40 The overall agreement found in Table II between theory and experiment is quite good, with the exception of $\Gamma\left(\mathrm{A}_{2} \rightarrow \pi \mathrm{X}^{\mathrm{O}}\right)$. While mixing of the pseudoscalar mesons is such as to alleviate this discrepancy, reasonable mixing angles do not change the width appreciably from the value in Table II.

Table II Decays of $L^{\prime}=1$ mesons to $L=0$ mesons by pion emission. ${ }^{39}$

| Decay | $\begin{gathered} \Gamma \text { (predicted) } \\ (\mathrm{MeV}) \end{gathered}$ | $\Gamma\left(\underset{(\mathrm{MeV})}{\operatorname{experimental})^{37}}\right.$ |
| :---: | :---: | :---: |
| $A_{2}(1310) \rightarrow \pi \rho$ | 71.5 (input) | $71.5 \pm 8$ |
| $\mathrm{K}^{*}(1420) \rightarrow \pi \mathrm{K}^{*}$ | 27 | $29.5 \pm 4$ |
| $\mathrm{f}(1270) \rightarrow \pi \pi$ | 112 | $141 \pm 26$ |
| $\mathrm{K}^{*}(1420) \rightarrow \pi \mathrm{K}$ | - 55 | $55 \pm 6$ |
| $A_{2}(1310) \rightarrow \pi \eta$ | 16 | $15 \pm 2$ |
| $\mathrm{A}_{2}(1310) \rightarrow \pi \mathrm{X}^{0}$ | 5 | < 1 |
| $\begin{aligned} \mathbf{B}(1235) \rightarrow \pi \omega, \lambda & =0 \\ \lambda & =1 \end{aligned}$ | 0 75 | $\Gamma_{\text {total }}=120 \pm 20$ <br> $\pi \omega$, with $\lambda=1$ strongly dominant, ${ }^{38}$ only mode seen |
| $\mathrm{A}_{1}(1100) \rightarrow \pi \rho, \lambda=0$ | 63 | ?? |
| $\lambda=1$ | 31 |  |
| $\delta(970) \quad \rightarrow \pi \eta$ | 41 | $50 \pm 20$ |
| $K(1300) \rightarrow \pi K$ | 380 | ?, broad |

A more likely source of trouble lies in the theoretical assignment of the $X^{\circ}$ to be dominantly that $\mathrm{SU}(3)$ singlet pseudoscalar meson associated with the octet containing the pion and eta. In any case, an actual measurement of the $\mathrm{A}_{2} \rightarrow \pi \mathrm{X}^{\mathrm{O}}$ decay width, rather than an upper limit, would be an interesting quantity to determine experimentally.

For $L^{\prime}=2$ mesons decaying by pion emission to the $L=0$ states, there are again two independent reduced matrix elements. About the only decay width determined with any certainty is $\mathrm{g} \rightarrow \pi \pi$. The meagre information available on other decays is consistent with the theory within the large experimental errors. ${ }^{23}$

For photon decays of mesons the data are even more sparse, although there are plenty of theoretical predictions. ${ }^{28}$ In fact, only a few decays among $L^{\prime}=0$ mesons are actually measured, where there is just one possible reduced matrix element. Fixing this from $\Gamma(\omega \rightarrow \gamma \pi)$, the predictions ${ }^{41}$ are collected in Table III. What widths have been measured are consistent with the predictions of the theory, although at the limits of the error bars in several cases.

There are a large number of pion and photon transitions among baryons which are predicted by the theory. They are compared with experiment elsewhere. $22,23,28$ Overall there is fair agreement between theory and experiment, with a number of predicted pion widths "right on the nose", but others off by factors of 2 to 3 . In many of these cases there are large experimental uncertainties, as well as the theoretical uncertainty inherent in using the narrow resonance approximation to compute decays of one broad resonance into another.

Table III Decays of $L^{\prime}=0$ mesons to other $\mathrm{L}=0$ mesons by photon emission.

|  | $\Gamma$ (predicted) no mixing (KeV) | $\begin{gathered} \Gamma \text { (predicted) } \\ \theta_{\mathrm{p}}=-10.5^{\circ} \\ (\mathrm{KeV}) \end{gathered}$ | $\begin{aligned} & \Gamma(\text { experimental })^{37} \\ & (\text { KeV }) \end{aligned}$ |
| :---: | :---: | :---: | :---: |
| $\omega \rightarrow \gamma \pi$ | 870 (input) | 870 (input) | $870 \pm 60$ |
| $\rho \rightarrow \gamma \pi$ | 92 | 92 | $\begin{gathered} 30 \pm 10 \leq \Gamma \leq 80 \pm 10 \\ \text { (Ref, } 42 \text { ) } \end{gathered}$ |
| $\phi \rightarrow \gamma \pi$ | 0 | 0 | < 14 |
| $\rho \rightarrow \gamma \eta$ | 36 | 56 | $\begin{gathered} <160 \\ \text { (Ref. } 43 \text { ) } \end{gathered}$ |
| $\omega \rightarrow \gamma \eta$ | 5 | 7 | < 50 |
| $\phi \rightarrow \gamma \eta$ | 220 | 170 | $126 \pm 46$ |
| $\mathrm{x}^{0} \rightarrow \gamma \mathrm{p}$ | 160 | 120 | $0.27 \Gamma\left(\mathrm{X}^{\mathrm{O}} \rightarrow \mathrm{all}\right)$ |
| $\mathrm{x}^{0} \rightarrow \gamma \omega$ | 15 | 11 |  |
| $\phi \rightarrow \gamma \mathrm{X}^{\circ}$ | 0.5 | 0.6 |  |

(3) Relative Signs In the process $\pi N \rightarrow N^{*} \rightarrow \pi \Delta$, the couplings to both $\pi N$ and $\pi \Delta$ of all the $N^{*}$ 's with a given value of $L$ are related by $\left(\operatorname{SU}(6)_{W}\right)$ Clcbsch-Gordan coefficients to the same reduced matrix element(s). The signs of the amplitudes for passing through the various $\mathrm{N}^{*}$ 's in $\pi \mathrm{N} \rightarrow \pi \Delta$ are then computable group theoretically. The correctness of these sign predictions is crucial, for while, for example, one may be willing to envisage a small amount of mixing of the constituent quark states, and corresponding corrections of say, $20 \%$, to amplitudes (and $40 \%$ to widths), this will not change their signs. A wrong sign prediction could well spell the end of the theory:

This in fact seemed to be the case last year ${ }^{44}$ when a comparison of the theoretical predictions 22,45 was made with the amplitude signs observed in an earlier phase shift solution of $\pi \mathrm{N} \rightarrow \pi \Delta$ by the LBL-SLAC collaboration. ${ }^{46}$ Since then a newer solution ${ }^{47,48}$ with much better $\chi^{2}$ has been found - in fact, the new solution is the only one left once additional data in the previous energy "gap" between 1540 and 1650 MeV is used as a constraint. 49

The present situation with regard to amplitude signs for intermediate $\mathrm{N}^{*}$ 's with $\mathrm{L}=1$ in $\pi \mathrm{N} \rightarrow \mathrm{N}^{*} \rightarrow \pi \Delta$ is shown ${ }^{50}$ in Table IV. Aside from an overall phase (chosen so as to give agreement with the sign of the $\mathrm{DD}_{15}$ (1670) amplitude), there is one other free quantity. This is the relative size of the reduced matrix elements of the two terms in $\mathrm{V}^{-1} \mathrm{Q}_{5}^{\alpha} \mathrm{V}$ or, what turns out to be equivalent, the sign of an s-wave relative to a d-wave transition amplitude. In Table IV we have fixed this by using the sign of the $\mathrm{SD}_{31}(1640)$ amplitude. All other signs for $\mathrm{N}^{*}$ 's in the $70 \mathrm{~L}=1$ multiplet are then predicted theoretically. The seven other signs determined experimentally agree with these predictions. The sign of the s-wave relative to d-wave amplitude is such as to show that the reduced matrix element of the second term in $V^{-1} Q_{5}^{\alpha} \mathrm{V}$,

Table IV Signs of resonant amplitudes 50 in $\pi \mathrm{N} \rightarrow \mathrm{N}^{*} \rightarrow \pi \Delta$ for $\mathrm{N}^{*}$ 's in the $70 \mathrm{~L}=1$ multiplet of $\operatorname{SU}(6)_{\mathrm{W}} \times 0(3)$. Amplitudes are labeled by $\left(\ell_{\pi} \ell_{\pi \Delta}\right)_{21,2 J}$ and the resonance mass in MeV.

|  |  |  |
| :--- | :---: | :---: |
| Resonant <br> Amplitude | Theoretical <br> Sign | Experimental <br> Sign $^{48}$ |
| $\mathrm{DS}_{13}(1520)$ | - | - |
| $\mathrm{DD}_{13}(1520)$ | - | - |
| $\mathrm{SD}_{11}(1550)$ | + | $?$ |
| $\mathrm{SD}_{31}(1640)$ | + (input $)$ | + |
| $\mathrm{DS}_{33}(1690)$ | - | - |
| $\mathrm{DD}_{33}(1690)$ | + (input $)$ | + |
| $\mathrm{DD}_{15}(1670)$ | - | + |
| $\mathrm{DS}_{13}(1700)$ | + | + |
| $\mathrm{DD}_{13}(1700)$ | + | + |
| $\mathrm{SD}_{11}(1715)$ | - | + |
| $=$ |  | + |

Table V Signs of resonant amplitudes 50 in $\pi \mathrm{N} \rightarrow \mathrm{N}^{*} \rightarrow \pi \Delta$ for $\mathrm{N}^{*}$ 's in the $56 \mathrm{~L}=2$ multiplet of $\operatorname{SU}(6)_{W} \times 0(3)$ 。 Amplitudes are labeled as in Table IV.

| Resonant Amplitude | Theoretical Sign | $\begin{gathered} \text { Experimental } \\ \operatorname{Sign}^{48} \end{gathered}$ |
| :---: | :---: | :---: |
| $\mathrm{FP}_{15}(1688)$ | - (input) | - |
| $\mathrm{FF}_{15}(1688)$ | + | + |
| $\mathrm{PP}_{13}{ }^{(1860)}$ | - | ? |
| $\mathrm{PF}_{13}(1860)$ | 1 | ? |
| $\mathrm{FF}_{37}(1950)$ | - | - |
| $\mathrm{FP}_{35}(1880)$ | - | ? |
| $\mathrm{FF}_{35}{ }^{(1880)}$ | - | - |
| $\mathrm{PP}_{33}(\quad)$ | + | ? |
| $\mathrm{PF}_{33}($ ) | + i | ? |
| $\mathrm{PP}_{31}(1860)$ | + | ? |

with the algebraic properties of $\left.\lambda^{\alpha} / 2\right)\left[\left(\beta \sigma_{x}+i \beta \sigma_{y}\right)\left(v_{x}-i v_{y}\right)-\right.$ $\left.-\left(\beta \sigma_{\mathrm{x}}-\mathrm{i} \beta \sigma_{\mathrm{y}}\right)\left(\mathrm{v}_{\mathrm{x}}+\mathrm{i} \mathrm{v}_{\mathrm{y}}\right)\right]$, is dominant for $L^{\prime}=1$ to $L=0$ pion transitions of baryons, just as it is for $L^{\prime}=1$ to $\mathrm{L}=0$ pion transitions of mesons.

For $\mathrm{N}^{*}$ 's with $\mathrm{L}=2$, many of the amplitudes have not been seen experimentally. As the overall phase is already fixed, there is just one parameter free. Again this is the relative size of the two possible reduced matrix elements, only now it corresponds to the sign of a p-wave relative to an f-wave pion decay amplitude. We use the $\mathrm{FP}_{15}$ (1688) amplitude in Table V to fix this sign ${ }^{50}$ - it corresponds to the reduced matrix element of the first term in $\mathrm{V}^{-1} \mathrm{Q}_{5}^{\alpha} \mathrm{V}$, behaving algebraically as $\left(\lambda^{\alpha} / 2\right) \sigma_{z}$, being dominant. All other signs (3) which are measured in Table V agree with the theory.

Another reaction where relative signs are predicted is
$\gamma \mathrm{N} \rightarrow \mathrm{N}^{*} \rightarrow \pi \mathrm{~N}$. This involves the theory at both the $\gamma \mathrm{N} \mathrm{N}^{*}$ and $\pi \mathrm{N} \mathrm{N}^{*}$ vertices. Although the situation is more complicated, there are also more amplitudes determined experimentally. An analysis 26,28 of the situation shows that not only are there 15 or so signs correctly predicted, but the information on the $\pi N N^{*}$ vertex so obtained agrees with that from $\pi N \rightarrow N^{*} \rightarrow \pi \Delta$ as to which term in $V^{-1} Q_{5}^{\alpha} V$ has the dominant reduced matrix element.

With our confidence in the theory for giving correct amplitude signs thus established, we may use the theory as a tool to classify new resonances. For example, a $\mathrm{P}_{33}(1700)$ state is seen ${ }^{48}$ in $\pi N \rightarrow N^{*} \rightarrow \pi \Delta$ and other reactions. 37 Does it belong to a state of three constituent quarks with
$\mathrm{L}=0$ or with $\mathrm{L}=2$ ? Both such "slots" are open in the constituent quark model, the former being the partner of the Roper resonance, $\mathrm{P}_{11}(1470)$, and the latter a relative of the third resonance, $\mathrm{F}_{15}(1688)$. Fortunately the
amplitude sign in $\pi \mathrm{N} \rightarrow \mathrm{N}^{*} \rightarrow \pi \Delta$ corresponding to these two choices is opposite. Experiment then allows a determination of the correct assignment: the $\mathrm{P}_{33}(1700)$ belongs with the $\mathrm{P}_{11}(1470)$ and has $\mathrm{L}=0$, as shown in Table VI. We have thus established both non-strange members of a new (although long suspected) quark model multiplet.

Table VI Signs of resonant amplitudes ${ }^{50}$ in $\pi N \rightarrow N^{*} \rightarrow \pi \Delta$ for $N^{*}$ 's in a radially excited $56 \mathrm{~L}=0$ multiplet of $\mathrm{SU}(6)_{\mathrm{W}} \times 0(3)$. Amplitudes are labeled as in Table IV.

| Resonant <br> Amplitude | Theoretical <br> Sign | Experimental <br> $\operatorname{Sign}^{48}$ |
| :--- | :---: | :---: |
| $\mathrm{PP}_{11}(1470)$ | + | + |
| $\mathrm{PP}_{33}(1700)$ | - | - |

Finally note the inelastic reaction $\pi N \rightarrow N^{*} \rightarrow \rho N$. If strong interaction $\mathrm{SU}(6)_{W}$ conservation is assumed, the $\rho \mathrm{NN}^{*}$ and $\pi \mathrm{NN}^{*}$ (or $\pi \Delta \mathrm{N}^{*}$ ) vertices are related since the $\pi$ and $\rho$ are in the same strong interaction or constituent $\mathrm{SU}(6)_{W}$ multiplet. The same result holds in broken $\operatorname{SU}(6)_{1 y}$ schemes. ${ }^{33}$ As far as the transformation from current to constituent quarks is concerned, there is no such relation, for only by using PCAC and vector dominance, respectively, are pion and rho vertices obtainable from axial-vector and vector current amplitudes - amplitudes which are themselves totally unrelated. An examination of the Argand diagrams from the LBL̇-SLAC analysis ${ }^{48}$ shows that the $\pi$ and $\rho$ couple differently to the $\mathrm{N}^{*}$ 's with $\mathrm{L}=1$. This particularly spells trouble for the so-called " $\ell$-broken $\mathrm{SU}(6)_{\mathrm{W}}$ " scheme, 33 as emphasized by Faiman 51 recently.

## SOME APPLICATIONS AND OPEN QUESTIONS

Let us then consider some further problems in meson spectroscopy which can be treated using the theory of pion and photon transitions we have been discussing:
(1) Is the $\rho^{\prime}(1600)$ a $q \bar{q}$ state with $L=0$ or $L=2$ ? In the first case we would have a radial excitation (of the $\rho$ ), while in the second we would be filling out the $\mathrm{L}=2$ multiplet (see Table I). Just as we were able to classify the $P_{33}(1700)$ using amplitude signs, a similar application of the theory permits an unambiguous classification here also. In particular, it turns out that the relative signs of the amplitudes for $\pi \pi \rightarrow \rho^{\prime} \rightarrow \pi \omega$ and $\pi \pi \rightarrow \mathrm{g} \rightarrow \pi \omega$ (or $\pi \pi \rightarrow \rho^{\prime} \rightarrow \overline{\mathrm{K}} \mathrm{K}^{*}$ and $\pi \pi \rightarrow \mathrm{g} \rightarrow \overline{\mathrm{K}} \mathrm{K}^{*}$ ) are the same (opposite) for $\mathrm{L}=0$ ( $\mathrm{L}=2$ ). Amplitude analysis of this kind should be possible given the new generation of spectrometers discussed by Leith ${ }^{52}$ at this conference.
(2) Can we have a $\rho^{\prime}$ state which decays to $\pi \omega$ and not $\pi \pi$ ? This possibility, which is sometimes invoked 53 for a $\rho^{\prime}(1250)$ state, is difficult to understand in the theory of pion transitions discussed above. The ClebschGordan coefficients yield a factor of $2(1 / 2)$ for $\Gamma\left(\rho^{\prime} \rightarrow \pi \omega\right) / \Gamma(\rho \rightarrow \pi \pi)$ if $\mathrm{L}=0(\mathrm{~L}=2)$, while phase space always favors the $\pi \pi$ mode. Thus, without
invoking a very particular mixture of $\mathrm{L}=0$ and $2, \rho^{\prime}$ states should have comparable $\pi \pi$ and $\pi \omega$ decay modes.
(3) Where are the isoscalar $J^{P}=1^{+}$mesons? A direct calculation of the width of an isoscalar partner, H , of the B meson to decay into $\pi \rho$ shows that

$$
\begin{equation*}
\widetilde{\Gamma}(\mathrm{H} \rightarrow \pi \rho)=\widetilde{\Gamma}(\mathrm{B} \rightarrow \pi \omega) \tag{13a}
\end{equation*}
$$

if H is the eighth component of an octet,

$$
\begin{equation*}
\tilde{\Gamma}(\mathrm{H} \rightarrow \pi \rho)=2 \tilde{\Gamma}(\mathrm{~B} \rightarrow \pi \omega) \tag{13b}
\end{equation*}
$$

if $H$ is the appropriate $S U(3)$ singlet, and

$$
\begin{equation*}
\tilde{\Gamma}(\mathrm{H} \rightarrow \pi \rho)=3 \tilde{\Gamma}(\mathrm{~B} \rightarrow \pi \omega) \tag{13c}
\end{equation*}
$$

if $H$ is an ideally mixed combination of singlet and octet. ${ }^{54}$ Here $\widetilde{\Gamma}$ denotes the reduced width, with phase space taken out. As the $H$ is presumably heavier than the B , one should be looking for a broad object - at least 100 MeV wide, and more likely 200 to 300 MeV wide! No wonder it's been hard to find.

On the other hand, the isoscalar partner of the $\mathrm{A}_{1}$, the D , has no decay by pion or kaon emission into the ground state $L=0$ mesons. It can only decay to other $L=1$ mesons by pion emission, and should be relatively narrow, as indeed is seen experimentally 37 for the state at 1285 MeV .
(4) Pion decays among $\mathrm{L}=1$ mesons. The decay $37 \mathrm{D} \rightarrow \pi \delta$ seems to be the dominant decay of the $\mathrm{D}(1285)$, and the recently discovered $\mathrm{A}_{2} \rightarrow \pi \pi \omega$ mode ${ }^{55}$ may well proceed (virtually) via $\mathrm{A}_{2} \rightarrow \pi \mathrm{~B} \rightarrow \pi \pi \omega$. The existence of these pionic transitions among $L=1$ mesons suggests that decays like $\mathrm{B} \rightarrow \pi \delta \rightarrow \pi \pi \eta, \mathrm{B} \rightarrow \pi \mathrm{A}_{1} \rightarrow \pi \pi \rho$, and $\mathrm{D} \rightarrow \pi \mathrm{A}_{1} \rightarrow \pi \pi \rho$ should also occur. While there is insufficient data on other decays to make a definite prediction for the latter three, one expects widths of roughly 10 to 20 MeV . Until these possible modes are investigated one should use caution in assigning the $B$ decay width entirely to $\pi \omega$.

A similar situation holds for pion transitions from $\mathrm{L}=2$ to $\mathrm{L}=1$. The decay $\omega(1675) \rightarrow \pi \mathrm{B} \rightarrow \pi \pi \omega$ seems to have been recently detected, 56 where the $\omega$ (1675) is presumed to be the isoscalar companion of the $\mathrm{g}\left(\mathrm{J}^{\mathrm{PC}}=3^{--}\right)$. Decays like $\mathrm{F}_{1}\left(\mathrm{~J}^{\mathrm{PC}}=2^{--}\right) \rightarrow \pi \delta, \mathrm{g} \rightarrow \pi \mathrm{H}$, etc. should occur with comparable rates.
(5) Bounds on widths. One would like to go beyond symmetry relations and obtain information on the absolute magnitude of some amplitudes. One method of attack is to use the last of the commutation relations in Eq. (2) in the form

$$
\begin{equation*}
\left[Q_{5}^{+}, Q_{5}^{-}\right]=Q^{3} \tag{14}
\end{equation*}
$$

where $Q_{5}^{ \pm}=(1 / \sqrt{2})\left(Q_{5}^{1} \pm i Q_{5}^{2}\right)$. Sandwiching this between $I=1, I_{z}=1$ meson states, $\mathrm{H}^{+}$, with helicity $\lambda$, and assuming no $\mathrm{I}=2$ mesons yields

$$
\begin{equation*}
\sum_{H^{\prime}} I<H^{\prime}, \lambda\left|Q_{5}^{-}\right| H^{+}, \lambda>\left.\right|^{2}=1 . \tag{15}
\end{equation*}
$$

Therefore

$$
\begin{equation*}
1<\mathrm{H}^{\circ}, \lambda\left|\mathrm{Q}_{5}^{-}\right| \mathrm{H}^{+}, \lambda>1 \leq 1, \tag{16}
\end{equation*}
$$

and PCAC then implies a bound on $\Gamma\left(\mathrm{H}^{\prime} \rightarrow \pi \mathrm{H}\right)$ 。 Unfortunately this is not very useful in practice, for it only tells us that $\Gamma(\mathrm{D} \rightarrow \pi \delta) \leq 310 \mathrm{MeV}-$ about a factor of 10 too large; $\Gamma(\mathrm{B} \rightarrow \pi \delta) \leq 135 \mathrm{MeV}$ - roughly the total width and probably also a factor 10 too big; and $\Gamma(\rho \rightarrow \pi \pi) \leq 300 \mathrm{MeV}-$ which is closer to the true width but still not very useful. Equation (16) only assures us that things can't be really wild.
(6) Masses. Information on masses can be obtained 57,58 by using the commutator

$$
\begin{equation*}
\left[Q_{5}^{+}(\mathrm{t}), \frac{\mathrm{d} Q_{5}^{+}(\mathrm{t})}{\mathrm{dt}}\right]=0 \tag{17}
\end{equation*}
$$

where the right hand side is zero under the assumption that there is no $\mathrm{I}=2$ sigma-term. It is clear that masses now enter, since the time derivative is proportional to the commutator with the Hamiltonian. One then probes the structure at a deeper level than when one just uses commutators of charges.

If the transformation $V$ was simply the identity, then it is possible to show that the solution to Eq. (17) is

$$
\begin{equation*}
M^{2}=M_{0}^{2}(L)+M_{1}^{2}(L) \vec{S} \cdot \vec{L}, \tag{18}
\end{equation*}
$$

i.e., states with internal angular momentum $L$ are only split in mass by a spin-orbit term of arbitrary magnitude. 59 When $\mathrm{V} \neq \mathbb{1}$, the situation becomes very complicated. It is clear that $\mathrm{V}^{-1} \mathrm{M}^{2} \mathrm{~V}$ can not be like a single quark operator in algebraic properties, for this would result in $\mathrm{M}_{\pi}^{2}=\mathrm{M}_{\rho}^{2}$, $\mathrm{M}_{\mathrm{N}}^{2}=\mathrm{M}_{\Delta}^{2}$, as Eq. (18) would also have given. Thus we can not abstract some quantities from the free quark model - we do not want its mass spectrum, in particular. 60 While Eq. (17) has been used to derive interesting results for masses in terms of the complicated mixing of representations of current algebra realized by real hadrons, 57,58 it has so far proven difficult to extract much useful information directly from $\mathrm{it}^{61}$ using the transformation $V$. This is an important area of further research.

## CONCLUSION

The theory of pion and photon transitions which we have outlined has had great success in predicting the signs of amplitudes - more than 25 relative signs are correctly predicted in the reactions $\pi N \rightarrow N^{*} \rightarrow \pi \Delta$ and $\gamma N \rightarrow N^{*} \rightarrow \pi N$. There is also at least fair success in predicting the relative magnitude of decay amplitudes, particularly for mesons.

This success lends support both to the theory of current-inducedtransitions we have presented and to the assignment of hadron states to constituent quark model multiplets. In particular, the amplitude signs found to be in agreement with experiment mean that, at least in a rough sense, the relationship between the wave functions of different hadrons is that of the quark
model. At $q^{2}=0$ one sees evidence for a quark picture of hadrons which is just as compelling as that obtained in a very different way as $q^{2} \rightarrow \infty$ in deep inelastic scattering.

Aside from pushing further on questions like masses, the extension 29 to $q^{2} \neq 0$ current induced transitions, the relationship ${ }^{62}$ of $V$ and PCAC, etc., what is most needed is a deeper understanding of why we can get away with such simple assumptions - why can we abstract anything relevant about transformed current operators from the free quark model? Even given that, why can we recognize so clearly the hadrons corresponding to the constituent quark model states? Why aren't the multiplets more badly split in mass and mixed? Most of all, to answer these and other questions we need at least part of the dynamics, at which point we might be able to calculate magnitudes of the matrix elements as well.

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17. The operators $D_{+}^{\alpha}$ have $J_{Z}=+1$. The corresponding operators $D_{-}^{\alpha}$ with $\mathrm{J}_{\mathrm{z}}=-1$, and all their matrix elements, are related (up to a sign) by a parity transformation. Hence we need only consider the properties of ${ }^{\infty} \underset{+}{\alpha}$.
18. Under the chiral $\mathrm{SU}(3) \times \mathrm{SU}(3)$ subalgebra of the $\mathrm{SU}\left(6_{\mathrm{LW}}\right.$ of currents the four terms in Eq. (9) transform as $(8,1)+(1,8),(3,3),(8,1)-(1,8)$, and $(\overline{3}, 3)$ respectively.
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34. The algebraic structure of broken $\mathrm{SU}(6) \widehat{\mathrm{W}_{\mathrm{W}}}$ schemes and their relation to the present theory are discussed in Ref. 24.
35. The relation between various quark model, $\mathrm{SU}(6)_{\mathrm{NV}}$, broken $\mathrm{SU}(6)_{\mathrm{W}}$, and constituent to current quark transformation calculations of pion decay amplitudes is discussed by H.J. Lipkin, NAL preprint NAL-PUB-73/ 62-THY, 1973 (unpublished).
36. See the discussion in Ref. 20, particularly footnote 13.
37. N. Barash-Schmidt et al., "Review of Particle Properties," Phys. Letters 50B, No. 1 (1974).
38. S. U. Chung et al. Brookhaven preprint BNL 18340, 1973 (unpublished); V. Chaloupka et al., CERN preprint, 1974 (unpublished); U. Karshon et al., Weizmann Institute preprint WIS-73/44-Ph, 1973 (unpublished) and references to previous work therein.
39. This is essentially an updated version of results found in F. J. Gilman, M. Kugler, and S. Meshkov, Refs. 22 and 23, and also in J. L. Rosner, Ref. 7. The decay widths are calculated from the expressions in Table I of ref. 23 , with quark model assignments as described there.
40. We have not treated the decay of the $\mathrm{I}=0$ scalar mesons into $\pi \pi$ in Table III because of the unclear situation in assigning the observed states to the quark model multiplet. For recent assessments see J. L. Rosner, Ref. 7 and D. Morgan, Rutherford preprint RL-7-063, 1974 (unpublished).
41. This is an updated version of Table II of F.J. Gilman and I. Karliner, Ref. 28. The decay widths can be calculated from Table I of Ref. 28, with the quark model assignments as described there.
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50. Tables IV and V update the comparison of theory and experiment contained in Table VIII of Ref. 23 (see also Ref. 45). The ordering in angular momentum and isospin Clebsch-Gordan coefficients is the same as in Ref. 23 (see particularly footnote 59) even though there is a changed isospin convention in the new experimental papers, Ref. 48:
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53. A. Bramon, Nuovo Cimento Letters 8, 659 (1973) and references to -previous work therein.
54. In calculating Eqs. (13) we are assuming that the " $\phi$-like" state decouples from $\pi \rho$ in order to relate the $S U(3)$ singlet and octet couplings. The results follow directly from Table I of Ref. 23. See also, J. L. Rosner, Ref. 7.
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57. F.J. Gilman and H. Harari, Phys. Rev. Letters 19, 723 (1967) and Ref. 7.
58. S. Weinberg, Phys. Rev. 177, 2604 (1969).
59. This result has been discussed in a somewhat different context by $C$. Boldrighini et al., Nucl. Phys. 22B, 651 (1970).
60. Note that the properties of the states and operators under $J_{x} \pm i J_{y}$ are at the same level as those under $\mathrm{M}^{2}$, i. e., both depend not just on kinematics but on dynamics. One should not expect to have correct properties with respect to $J_{x} \pm i J_{y}$ (i.e., the angular conditions) unless one also has a reasonable mass spectrum. This presumably applies to the free quark model and any conclusions to be drawn from it (see Ref. 15). I thank R. Carlitz and W.K. Tung for emphasizing this to me. A similar philosophy is expressed by S. P. de Alwis and J. Stern, CERN preprint TH. 1783, 1974 (unpublished).
61. For a recent attempt in this direction, see F. Buccella, F. Nicolo, and A. Pugliese, Nuovo Cimento Letters 8, 244 (1973).
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