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BEAM-BEAM COUPLING IN SPEAR*

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Summary

The effect of the coupling upon the transverse coherent motion of two separated beams in SPEAR is presented in this paper. This effect has been studied for various parameters such as the separation and beta-values where the beams pass, the currents, energy and transverse betatron oscillation frequencies of the beams.

Introduction

When two single-bunched beams pass each other in a storage ring, they exert forces on each other which produce a coupling between the transverse motion of the beams. In SPEAR it is possible to vary the vertical separation of the two counter-circulating beams by means of an electrostatic beam bump[1] and thus alter the coupling between the beams. This coupling is also a function of the beta-values where the beams pass, the currents, energy and transverse betatron frequencies of the beams. It is possible to vary all of these parameters in SPEAR and to determine experimentally their effect upon the coupling between the beams by measuring the coherent beam response to an externally applied transverse electric field.

The linear theory for the coherent transverse motion of coupled beams is presented and the experimental results for the beam response to an external horizontal oscillating electric field are compared to the response predicated by this linear theory. For values of vertical separation large compared to the horizontal beam dimension, the experimental results agree with the linear theory. However, it is found that when the vertical separation between the beams is reduced to a value comparable with the horizontal beam dimension the non-linearities of the coupled motion become important and the linear theory is no longer adequate.

The deviation of the experimental results from the linear theory is thus a measure of the effect of the non-linear fields upon the coupled coherent motion.

General Equations of Motion

When two separated beams pass each other in a storage ring, each beam produces a transverse deflection of the other beam. In the linear approximation, this deflection results in both a change in the equilibrium positions of the two beams and a shift in the frequency of the transverse coherent oscillations. In the absence of any transverse coherent oscillation we take the equilibrium position at the passage point of beam two (including the static perturbation due to beam one) as $x = 0$, $y = h/2$, while the equilibrium position of beam one is taken as $x = 0$, $y = -h/2$. See Fig. 1.

In the "smooth approximation", the equations of motion for coherent oscillations of the centers of the two beams u_1 and u_2 are

$$u_1'' + \left(\nu_1^2 + \frac{p\nu_1\beta_1\Delta_1}{2\pi}\right)u_1 = \frac{p\nu_1\beta_1\Delta_1}{2\pi}\,u_2 + F\,e^{i\omega t} ,$$

and (1)

$$u_2'' + \left(\nu_2^2 + \frac{p\nu_2\beta_2\Delta_2}{2\pi}\right)u_2 = \frac{p\nu_2\beta_2\Delta_2}{2\pi}\,u_1 ,$$

where we have included a driving term for beam one with magnitude F and frequency ω and ignored the damping terms.

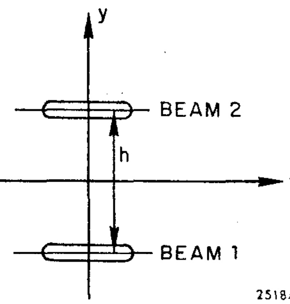

FIG. 1--Equilibrium position of two separated beams at point of passage.

The quantities (ν_1, β_1) and (ν_2, β_2) are the betatron oscillation frequencies and values of beta for beams one and two respectively at the passage point, p is the number of passage points (equal to two for SPEAR), Δ_1 and Δ_2 are the derivatives of the angular deflection for beams one and two respectively due to their passage. The explicit expressions for Δ are given in the appendix where it is seen that $\Delta_1 \propto N_2$ and $\Delta_2 \propto N_1$ with N_1 and N_2 the number of particles in beam one and two, respectively.

The steady state solution to Eq. (1) may be written in the form

$$u_1 = \hat{u}_1 e^{i\omega t} \qquad \text{and} \qquad u_2 = \hat{u}_2 e^{i\omega t} , \tag{2}$$

where the amplitudes $\hat{u}_1$ and $\hat{u}_2$ are given by

$$\hat{u}_1 = \frac{\left(\nu_2^2 + \frac{p\nu_2\beta_2\Delta_2}{2\pi} - \omega^2\right)F}{(\omega^2-\omega_\alpha^2)(\omega^2-\omega_\beta^2)} ;$$

(3)

$$\hat{u}_2 = \frac{\frac{p\nu_2\beta_2\Delta_2}{2\pi}F}{(\omega^2-\omega_\alpha^2)(\omega^2-\omega_\beta^2)} ,$$

with ω_α and ω_β the resonant frequencies.

Case of Equal Betatron Frequencies

One of the special cases we have studied in SPEAR is one in which the two beams have equal betatron frequencies and beta values at the passage points; i.e., $\nu_1 = \nu_2 = \nu$ and $\beta_1 = \beta_2 = \beta$.

The resonance frequencies are then given for a storage ring with two passage points by

$$\omega_\alpha = \nu \qquad \text{and} \qquad \omega_\beta \approx \nu + \frac{\beta}{2\pi}(\Delta_2 + \Delta_1) . \tag{4}$$

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Near the resonance frequency $\omega \approx \omega_\alpha$ the oscillation amplitudes are given by

$$\hat{u}_1 \approx \left(\frac{F}{\omega_\alpha^2-\omega^2}\right)\left(\frac{\Delta_2}{\Delta_1+\Delta_2}\right) \quad \text{and} \quad \hat{u}_2 = \left(\frac{F}{\omega_\alpha^2-\omega^2}\right)\left(\frac{\Delta_2}{\Delta_1+\Delta_2}\right), \tag{5}$$

while near the other resonance frequency $\omega \approx \omega_\beta$ the oscillation amplitudes are given by

$$\hat{u}_1 \approx \left(\frac{F}{\omega_\beta^2-\omega^2}\right)\left(\frac{\Delta_1}{\Delta_1+\Delta_2}\right) \quad \text{and} \quad \hat{u}_1 = \left(\frac{F}{\omega_\beta^2-\omega^2}\right)\left(\frac{-\Delta_2}{\Delta_1+\Delta_2}\right). \tag{6}$$

If we assume the resonance widths as determined by the damping are the same at $\omega \approx \omega_\alpha$ and $\omega \approx \omega_\beta$, and since $\Delta_1 \propto N_2$ and $\Delta_2 \propto N_1$, the amplitudes of the two beams at resonance frequencies are in the ratio of

$$|u_1(\omega_\alpha)| : |u_2(\omega_\alpha)| : |u_1(\omega_\beta)| : |u_2(\omega_\beta)|$$
$$= N_1 : N_1 : N_2 : N_1 \ .$$

At SPEAR[2] we detect the coherent response of the beams at the exciting frequency. This detected response is proportional to the product of the oscillation amplitude $\hat{u}$ and the number of particles in the beam N, and is a function of exciting frequency ω, with the functional dependence displayed in Fig. 2. The labels over the resonance peaks denote their relative heights, and the distance between the peaks $(\omega_\beta - \omega_\alpha) \propto (N_1 + N_2)$.

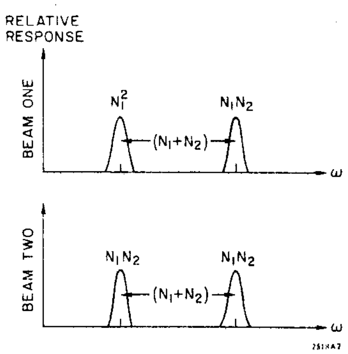

FIG. 2--Coherent response detected versus driving frequency.

For equal currents in each beam, the detected resonance peaks are all equal. If the current in beam one is decreased, the first peak for both beams remains at the same frequency $(\omega_\alpha = \nu)$, but for beam one, the first peak decreases in relative amplitude. The second peak moves toward the first peak and remains relatively large in amplitude for both beams. In the limit of a very weak beam one $(N_1 << N_2)$, the first peak for beam one disappears and both second peaks are shifted by one-half of the amount of frequency shift for the case of both beam currents equal to the current of the strong beam $(N_1 = N_2)$.

Conversely, starting with equal beams and decreasing the current in beam two, we again find that the second peak moves toward the first peak, but in this case, all peaks except peak one of beam one decrease in amplitude.

The experimental results from SPEAR on the positions and relative amplitudes of the resonance peaks for coherent horizontal motion of two separated beams are in excellent agreement with the above results when the beam is driven externally. It is interesting to note that the second resonance peak is often observed even without an externally applied oscillating field.

Case of Equal Beam Currents

Another specific case that has been studied at SPEAR is the case in which the beam currents are equal. For this case, when the difference between the betatron wave numbers is small compared to unity, we can assume that the beta values for the two beams are equal. Under these conditions, the values for the resonance frequencies ω_α and ω_β are given by

$$\omega_{\binom{\beta}{\alpha}} = \frac{1}{2}\left[(\nu_1^2+\nu_2^2) + \frac{2\nu\beta\Delta}{\pi} \pm \sqrt{(\nu_2^2-\nu_1^2)^2 + \frac{4\nu^2\beta^2\Delta^2}{\pi^2}}\right]. \tag{7}$$

For the case where the difference between the tunes is large compared to the tune shift; i.e., $(\nu_2 - \nu_1) >> \beta\Delta/\pi$, the values of ω_β and ω_α become

$$\begin{aligned} \omega_\beta &\approx \nu_2 + \frac{\beta\Delta}{2\pi} + \frac{\beta^2\Delta^2}{4\pi^2(\nu_2-\nu_1)} \ , \\ \omega_\alpha &\approx \nu_1 + \frac{\beta\Delta}{2\pi} - \frac{\beta^2\Delta^2}{4\pi^2(\nu_2-\nu_1)} \ . \end{aligned} \tag{8}$$

The oscillation amplitudes near the resonance $\omega \approx \omega_\alpha$ are

$$\hat{u}_1 \approx \frac{F}{(\omega_\alpha^2-\omega^2)} \quad \text{and} \quad \hat{u}_2 \approx \frac{\beta\Delta}{2\pi(\nu_2-\nu_1)}\left(\frac{F}{\omega_\alpha^2-\omega^2}\right)$$

and near the resonance $\omega \approx \omega_\beta$ (9)

$$\hat{u}_1 \approx \frac{\beta^2\Delta^2}{4\pi^2(\nu_2-\nu_1)^2}\left(\frac{F}{\omega_\beta^2-\omega^2}\right) \quad \text{and} \quad \hat{u}_2 = \frac{-\beta\Delta}{2\pi(\nu_2-\nu_1)}\left(\frac{FF}{\omega_\beta^2-\omega^2}\right).$$

Thus, for the case where $(\nu_2 - \nu_1) >> \beta\Delta/\pi$, we expect to see all resonance response peaks from our detector disappear except for the response of beam one at a frequency ω_α.

This effect has been observed on the coupled horizontal motion at SPEAR, but it should be pointed out that if the tune shift $\beta\Delta/\pi$ becomes too large, it is difficult to obtain values for ν_1 and ν_2 by means of the electric quadrupole lenses that give sufficient tune splitting without being near a destructive machine resonance.

Summary of Experimental Results

Machine experiments have been done at SPEAR to determine the resonance frequencies of the coherent horizontal motion for two separated beams. In these experiments, the following parameters were varied: the currents in the two beams, the vertical separation of the two beams, the values of at the passage points, the separate betatron oscillation frequencies of the two beams and the energy of the two beams. The results of experiments with unequal currents and betatron tunes for the two beams driven externally has been discussed in the previous sections, and the results for the resonance amplitudes and frequencies are in agreement with the theory. In addition, we often observe the second resonance peak without an external driving field.

For most of the experiments, the currents and betatron oscillation frequencies of the two beams were equal. For this case, the frequency shift of the second resonant peak as predicted by the linear theory is given by

$$\omega_\beta - \omega_\alpha = \delta\omega = \frac{\beta\Delta}{\pi} .$$

where the expression for Δ is given in the appendix.

In Fig. 3, the frequency shift $\delta\omega$ is displayed as a function of current in the two beams for various values of vertical beam separation. The lines are theoretical values and the points are the experimental values. For this example,

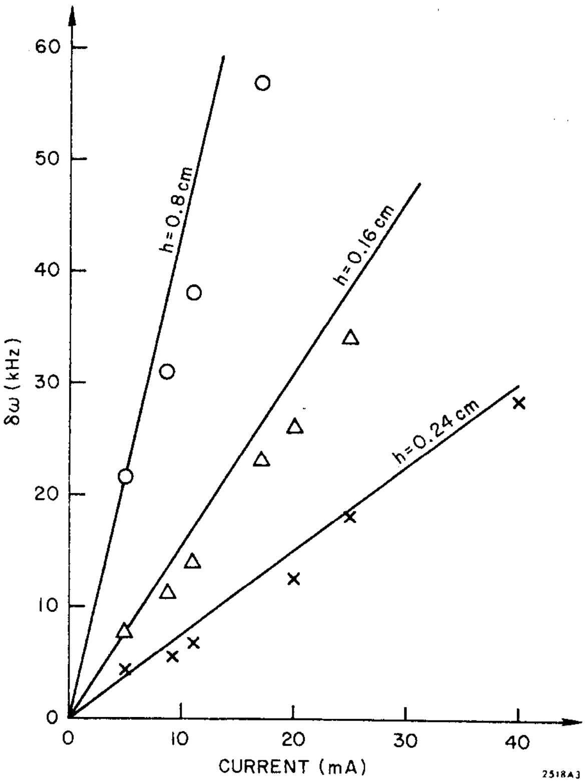

FIG. 3--Frequency shift vs. beam current.

the energy was equal to 1.5 GeV, the values of β_x at the passage points equal to 1.2 m, the betatron wave number =5.23 and the values of σ_x and σ_y = 0.0375 cm and 0.00315 cm respectively. Note that the agreement with theory is best for cases with large beam separation. Figure 4 shows the quantity $\delta\omega/I$ plotted as a function of beam separation h for the same values of energy, β, ν, σ_x and σ_y as those in Fig. 3. It can be seen that the theory breaks down when separation h is smaller than σ_x. The solid line is the theoretical value while the dotted line is the experimental results for two separated 5 mA beams. For values of h less than σ_x, the experimental results do not agree with linear theory and the experimental values for the frequency shift is considerably lower than the theoretical values. These differences have been obtained for several values of beam current, energy, β, and ν, and in all cases the experimental values for the frequency shift are lower than the theoretical values for $h < \sigma_x$, while for $h > 2\sigma_x$ their values agree.

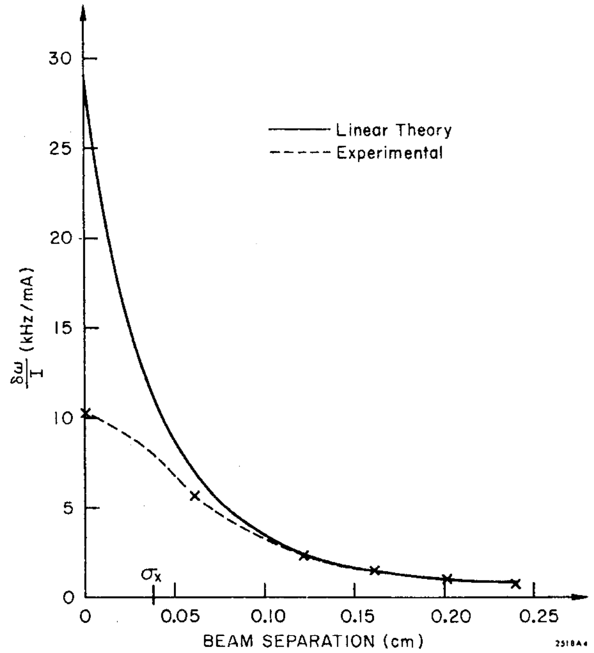

FIG. 4--(Frequency shift/current) vs. beam current.

This discrepancy can be understood if one remembers that the expression for the transverse deflection which has been used in this calculation is valid only for the particles in a beam that are near the beam center. When the separation of the beams is quite large compared to their width, this is a good approximation for all particles; however, as the separation between the beams decreases, the forces become more non-linear and the expression used for the transverse deflection yields a larger value than is actually experienced by many of the particles. Thus the experimental results deviate from the theoretical results obtained from the linear model. This deviation between the experimental and theoretical results is a measure of the amount of non-linearity of the beam-beam interaction for small separation.

Acknowledgments

It is a pleasure to thank the SPEAR Operation Group, who not only aided in the operation of the storage ring but also actively participated in the experimental measurements.

References

1. SPEAR Storage Ring Group (presented by B. Richter), IEEE Trans. Nucl. Sci., NS-20, No. 3, 752 (June 1973).
2. SPEAR Group, "Beam Dynamics Experiments at SPEAR," contribution to this conference.

Appendix

For the case of two beams that are separated in the vertical direction y by amount h, as shown in Fig. 1, the linear portion of the angular deflection of beam one due to beam two is given by

$$\delta\theta_{x1} = -\Delta_{x1}(x_1 - x_2) \qquad \delta\theta_{y1} = -\Delta_{y1}(v_1 - v_2)\;;$$

$$\delta\theta_{x2} = -\Delta_{x2}(x_2 - x_1) \qquad \delta\theta_{y2} = -\Delta(v_2 - v_2) \tag{A-1}$$

with $v_1 = (y_1 + h/2)$ and $v_2 = (y_2 - h/2)$ the vertical deviations from their equilibrium positions. The quantities Δ_x and Δ_y are given by

$$\Delta_{x1} = \frac{2 N_2 r_e}{\gamma(\sigma_x^2 - \sigma_y^2)} \left\{ g\left(\frac{h}{L}\right) - \frac{\sigma_y}{\sigma_x} e^{-\frac{h^2}{2\sigma_y^2}} g\left(\frac{h\sigma_x}{L\sigma_y}\right) \right\} ,$$

$$\Delta_{y1} = \frac{2 N_2 r_e}{\gamma(\sigma_x^2 - \sigma_y^2)} \left\{ -g\left(\frac{h}{L}\right) + \frac{\sigma_x}{\sigma_y} e^{-\frac{h^2}{2\sigma_y^2}} g\left(\frac{h\sigma_x}{L\sigma_y}\right) \right. ,$$

$$\left. + \left(\frac{\sigma_x^2 - \sigma_y^2}{\sigma_x \sigma_y}\right) e^{-\frac{h^2}{2\sigma_y^2}} \left[1 - g\left(\frac{h}{L}\ \frac{\sigma_x}{\sigma_y}\right)\right] \right\} . \tag{A-2}$$

Δ_{x2} and Δ_{y2} are obtained by substituting N_1 for N_2 in the above expressions, where

N_1 and N_2 are the number of particles in beams one and two respectively;

r_e is the classical radius of the electron;

σ_x and σ_y are the horizontal and vertical rms beam sizes of each beam (assumed to be equal);

$L = \sqrt{2(\sigma_x^2 - \sigma_y^2)}\;;$

γ is the relativistic energy parameter; and

$$g(z) = 1 - \sqrt{\pi}\, z\, e^{z^2} \left[1 - \frac{2}{\sqrt{\pi}} \int_0^z e^{-u^2}\, du \right] .$$

Note that deflection of beam one to beam two is proportional to the number of particles in beam two and vice versa.