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DIRECT COUPLING OF HADRONS TO LEPTONS

Ikaros I. Y. Bigi and James D. Bjorken

ERRATUM†

If one assumes that the effective Lagrangian forms a SU(3)-singlet

$$\mathscr{L}_I' = g_{ps} \sum_i (\bar{e}\gamma_5 e)(\bar{q}_i \gamma_5 q_i) \qquad \text{with} \qquad g_{ps} = g_s \approx 3 \times 10^{-3}\ (\mathrm{GeV})^{-2}$$

then one obtains for the decay width

$$\Gamma_{\eta \to e^+e^-} \approx 0.07\ \mathrm{eV} \tag{38}$$

But using an effective interaction Lagrangian which belongs to a SU(3)-octet, e.g.,

$$\mathscr{L}_I' = g_{ps} (\bar{e}\gamma_5 e)(\bar{p}\gamma_5 p + \bar{n}\gamma_5 n - 2\bar{\lambda}\gamma_5 \lambda)$$

we find for the decay width

$$\Gamma_{\eta \to e^+e^-} \approx 2.4\ \mathrm{eV} \tag{39}$$

This width is much larger than found experimentally for decay into muon pairs

$$\Gamma_{\eta \to \mu^+\mu^-} \approx 0.057\ \mathrm{eV}$$

Therefore unless μ-e universality is violated, a pseudoscalar direct coupling is ruled out in such a model.

†Replaces material on page 15 beginning with "Using an effective. . ." and ending with "direct coupling is ruled out."

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DIRECT COUPLING OF HADRONS TO LEPTONS*

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ABSTRACT

We consider the desperate attempt to interpret the recent data on e^+e^- annihilation to hadrons in terms of a direct coupling of electrons to hadrons. Comparable, but not as large, effects are found to be expected in deep-inelastic processes. The possibility of a dependence of $\sigma_{tot}(e^+e^- \rightarrow \text{hadrons})$ upon beam polarization is also discussed, as well as other implications.

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The unexpected behavior of the cross section $\sigma_{e^+e^- \to \text{Hadrons}}$ [1,2] has invited all kinds of radical speculations on its origin. Among these is the supposition that a direct, non-electromagnetic interaction between electrons and hadrons is responsible[2] for the sharp rise with CMS energy $\sqrt{s}$ in the ratio

$$R = \frac{\sigma_{e^+e^- \to \text{Hadrons}}}{\sigma_{e^+e^- \to \mu^+\mu^-}} ,$$

a quantity expected theoretically to be roughly constant.

Such a speculation raises instantly some awkward questions, such as "If electrons interact directly with hadrons, with cross sections $\sim 10^{-32}\text{cm}^2$, should not neutrinos also?" To answer this one seems to require non-universality in the lepton-hadron couplings. This in turn allows one to finesse other awkward questions, such as[3] "Shouldn't this new interaction affect known processes, in particular the decays $\eta \to \mu^+\mu^-$ and $K_L \to \mu^+\mu^-$?" Quite a few questions remain beyond these, and however unlikely the whole hypothesis may be, there still remains the obligation to take a close look at it. This is our purpose here; we shall suppose there is a new interaction between electrons and hadrons given by an approximate local coupling of some hadron operator to a local lepton "current." We find it difficult to avoid this choice of approximately local coupling even given a phenomenological S-matrix viewpoint. There are no low-mass t- or u-channel singularities present in the amplitude for $e^+e^- \to$ hadrons; exchanges in these channels must contain particles carrying lepton number[4], and consist of unknown states of presumably quite high mass (Fig. 1).

Thus the basic interaction should occur in only very few partial waves at present energies, and a low-energy, effective-Lagrangian ("pseudo-potential") description seems the most appropriate.[5]

The paper is arranged as follows: in Chapter I we introduce various types of new, "direct" couplings of quarks (or gluons) to leptons in order to describe deep inelastic annihilation of e^+e^-. In Chapter II we study the consequences this will have for deep inelastic scattering of leptons off nucleons, in particular we give estimates when scaling will be broken, as it eventually has to be in this approach. In Chapter III and IV we check the implications for the decay $\eta \to \mu^+\mu^-$ and for the process $pp \to e^+e^-$ + anything.

I. Electron-Positron Annihilation

In accord with the discussion in the introduction, we introduce a new type of direct or Fermi-like coupling between hadrons and electrons, in addition to the QED coupling via photon exchange, as shown in Fig. 2.

This coupling has the form at low energies

$$\mathscr{L}_I'(x) = \sum_i \bar{e}(x) \Gamma_i e(x) O_i(x) ,$$

where "e" denotes the (canonical) field of mass dimension 3 for the electron, and "Γ_i" any independent matrix of the Dirac algebra.[6] We shall for the sake of simplicity and of historical precedent assume that the hadronic operators $O_i(x)$ can be written as (or are at least algebraically isomorphic to) bilinear products of quark or gluon field operators. For quarks, this choice means:

$$\mathscr{L}_I' = \sum_{i,j} g_i^j (\bar{e}\Gamma_i e)(\bar{q}_j \Gamma_i q_j) ,$$

where "q_j" stands for the quark of type "j" (SU(3) quantum numbers, color, etc.)

Clearly such a Lagrangian is not renormalizable since $[g] = \frac{1}{M^2}$, but this fact shouldn't deter us. One can use the same attitude as with the Fermi coupling in weak interactions, i.e., one uses $\mathscr{L}_I'$ as an "effective" Lagrangian to be used only at sufficiently low energy. At higher energies we would expect this description to fail and nonlocal effects to appear.

The various possible types of Lorentz couplings can be divided into two different classes:

(1) No interference possible with the QED process $\bar{q}q \rightarrow \gamma^* \rightarrow e^+e^-$.
All but vector couplings fall into this class.

(2) Interference with the QED process $q\bar{q} \rightarrow \gamma^* \rightarrow e^+e^-$ can occur.
Vector couplings are in this class.[7]

We first look at class (1), in particular scalar (or pseudoscalar) coupling:

$$\mathscr{L}_I'^{s} = \sum_j g_s^j (\bar{e}\cdot e)(\bar{q}_j \cdot q_j) \tag{1}$$

This ansatz plus the usual parton model assumptions including setting quark masses effectively equal to zero, yields for the total cross section:

$$\sigma_{e^+e^- \rightarrow \text{hadrons}}(s) = \frac{4\pi\alpha^2}{3s} \sum_j e_j^2 + \frac{s}{16\pi} \sum_j g_j^2 , \tag{2}$$

where "e_j" denotes the charge of the "j"-type quark. We assume the color scheme for the quarks[8] and, to make things easy, we also set g equal for all nine quarks. So we get:

$$\sigma_{e^+e^- \to \text{hadrons}}(s) = \frac{8\pi\alpha^2}{3s} + \frac{9 g_s^2 s}{16\pi} \tag{3}$$

In order to fix g, we use the experimental value of $\sigma_{e^+e^- \to \text{hadrons}}$ at $s = 25\ (\text{GeV})^2$ as input:

$$22\ \text{nb} \approx \frac{8\pi\alpha^2}{75} (\text{GeV})^{-2} + \frac{9 \times 25}{16\pi}\, g_s^{\,2}\, (\text{GeV})^2 ,$$

which gives us the following result:

$$g_s \approx \pm 3 \times 10^{-3}\, (\text{GeV})^{-2} . \tag{4}$$

Thus we obtain

$$\sigma_{e^+e^- \to \text{hadrons}}(s) \approx 4.4 \times 10^{-4} \frac{1}{s} + 1.6 \times 10^{-6}\, s\, (\text{GeV})^{-4} \tag{5}$$

$$R = \frac{\sigma_{e^+e^- \to \text{hadrons}}}{\sigma_{e^+e^- \to \mu^+\mu^-}} \approx 2 + (7.2 \times 10^{-3})\, s^2\, (\text{GeV})^{-4} . \tag{6}$$

The curves are shown in Fig. 5a, b.

In the region from 9 to 25 $(\text{GeV})^2$ the total cross section stays fairly constant, because this is the region where the two different processes, QED 1-photon exchange and direct coupling, "cross over." This model implies that at higher energies the total cross section has to rise again, at least until the effective Lagrangian approximation breaks down. What was said here about scalar coupling also applies, cum grano salis, to tensor coupling.

If we assume a vector coupling

$$\mathscr{L}_I^V = \sum_j g_j^V (\bar{e}\gamma_\mu e)(\bar{q}_j \gamma^\mu q_j) \tag{7}$$

we obtain for the total cross section (Fig. 3)

$$\sigma_{e^+e^- \to \text{hadrons}}(s) = \frac{4\pi\alpha^2}{3s} \sum_j e_j^2 + \frac{s}{12\pi} \sum_j g_j^2 + 2\,\mathrm{Re}\,\frac{\alpha}{3} \sum_j e_j g_j \,. \tag{8}$$

Again we use the color scheme and set $g_j \equiv g$:

$$\sigma_{e^+e^- \to h}(s) = \frac{8\pi\alpha^2}{3s} + \frac{3g^2 s}{4\pi} \,. \tag{9}$$

The interference term vanishes for $g_j \equiv g$, since $\sum_j e_j = 0$.

If we take, as above, the experimental value for $\sigma_{e^+e^- \to h}$ (25 GeV^2) as input, we obtain:

$$g_V \approx \pm 2.6 \times 10^{-3}\,(\mathrm{GeV})^{-2} \,. \tag{10}$$

The curves look the same as in the scalar case. Had the interference term not been cancelled, the coupling g would still be of the same order of magnitude. If one assume, for instance, $g_p = g_n = 0$, $g_\lambda \neq 0$, then it follows:

$$\sigma_{e^+e^- \to h}(s) = \frac{8\pi\alpha^2}{3s} + \frac{g_\lambda^2 s}{4\pi} - \frac{2\alpha g_\lambda}{3} \tag{11}$$

with the two solutions for g_λ:

$$g_\lambda^{(1)} \approx -3.5 \times 10^{-3} \text{ (GeV)}^{-2}$$

$$g_\lambda^{(2)} \approx 6 \times 10^{-3} \text{ (GeV)}^{-2} . \tag{12}$$

Both solutions are allowed since $\sigma_{e^+e^- \to h}(s)$ is a positive definite quantity (for $s > 0$) with $g_\lambda^{(1)}$ as well as with $g_\lambda^{(2)}$. But the more interesting case, as shown below, is $g_\lambda < 0$, and we will examine this solution more closely,

$$\sigma_{e^+e^- \to h}(s) \approx 4.4 \times 10^{-4} \times \frac{1}{s} + 9.8 \times 10^{-7} s \text{ (GeV)}^{-4} + 1.7 \times 10^{-5} \text{ (GeV)}^{-2} . \tag{13}$$

Evidently this cross section rises much more slowly with s than the expressions without interference term (5), due to the constant background.

These schemes of direct coupling of leptons to $\bar{q}q$ have the disadvantage that the final state distribution of hadrons in the direct process would be expected to be very similar to the one-photon process. There is some very preliminary evidence that this is not so; the fraction of cms energy found in charged particles seems to decrease from its expected value of $\sim 2/3$ as the cms energy increases from 3 to 5 GeV.

If such evidence holds up, it might be taken as an indication that these schemes of coupling leptons directly to quarks are wrong. An alternative which avoids this question is to couple the leptons to gluons as shown in Fig. 4. If the gluons G_i have $J = 0$, we might have, for example

$$\mathscr{L}_I' = \sum_i h_i (\bar{e} \cdot e)(G^i G_i) , \tag{14}$$

which at high energies yields a constant total cross section:

$$\sigma_{e^+e^- \to h}(s) = \frac{4\pi\alpha^2}{3s} \sum_j e_j^2 + \sum_i \frac{h_i^2}{32\pi} . \tag{15}$$

The qualitative picture we get from this ansatz for the total cross section is the following: there is a constant term associated with the direct coupling of gluons to leptons, which begins to appear and dominate as the QED part of the cross section goes to zero.

There is also a more general picture, which would clearly distinguish contributions in the cross section due to scalar, pseudoscalar or tensor coupling from those due to any pure vector coupling: the total cross section—in contrast to the differential cross section—calculated in a scheme with vector coupling, does not depend on the polarization of the incoming leptons, as long as these are transversely polarized.[9] On the other hand, the scalar contribution to the total cross section depends strongly on the transverse polarization of the incoming leptons:

$$\sigma^{(\mathrm{scalar})}_{(\mathrm{pol})} = (1 - \vec{s}_+ \cdot \vec{s}_-)\, \sigma^{(\mathrm{scalar})}_{(\mathrm{unpol})} , \tag{16}$$

where $\vec{s}_+$ and $\vec{s}_-$ are the transverse spin polarization vectors. If the e^+e^- are each polarized (antiparallel spins) with degree of polarization P, then

$$\sigma^{(\mathrm{scalar})}_{(\mathrm{pol})} = (1 + P^2)\, \sigma^{(\mathrm{scalar})}_{(\mathrm{unpol})} , \tag{17}$$

Transverse polarization is expected to occur naturally in a storage ring where the e^+ and e^- beams get more and more transversely polarized during their lifetimes as a consequence of the spin dependence of synchrotron radiation. Thus, when these lifetimes and the energy of the incoming particles are sufficiently large for P to take on non-negligible values, one should see a change of the total cross section during the storage time, if there is a coupling other than vector coupling present. E.g., for scalar interaction $\sigma(e^+e^- \to \text{hadrons})$ should increase with time. For a pseudoscalar coupling the sign of the effect changes, and $\sigma_{tot} \to 0$ as the beam polarization becomes complete.

II. Deep-Inelastic Electroproduction

The models outlined above are certain to break the scaling behavior observed in deep inelastic electron-proton scattering. We have to check at which energies it happens.

For scalar coupling the process is described by two terms, one given by the one-photon-exchange approximation in QED (= "1 - γ"), the other one by direct coupling (= "0-γ").

$$-x \frac{d\sigma^{ep \to eX}}{dQ^2 dx} = \frac{4\pi\alpha^2}{Q^4} x(1 - y + \tfrac{1}{2} y^2) \sum_i e_i^2 f_i(x) + \frac{y^2}{16\pi} x \sum_j g_j^2 f_j(x) ,$$

$$y = \frac{Q^2}{xs} = \frac{\nu}{E} . \tag{18}$$

Therefore

$$\frac{d\sigma}{dxdy} = \frac{4\pi\alpha^2}{Q^4} s \left[(1-y) x \sum_i e_i^2 f_i(x) + y^2 \left(\tfrac{1}{2} x \sum_i e_i^2 f_i(x) + \frac{Q^4}{64\pi^2\alpha^2} x \sum_j g_j^2 f_j(x) \right) \right] \tag{19}$$

$$\nu W_2 = \nu W_2^{"1-\gamma"} = x \sum_i e_i^2 f_i(x) \tag{20}$$

$$MW_1 = MW_1^{"1-\gamma"} + MW_1^{"0-\gamma"} = \tfrac{1}{2} \sum_i e_i^2 f_i(x) + \frac{Q^4}{64\pi^2 \alpha^2} \sum_j g_j^2 f_j(x) \tag{21}$$ [10]

We immediately see from Eq. (20) and (21) that

(a) the structure function $\nu W_2(-Q^2, \nu)$ gets no contribution from the direct coupling, hence it scales.

(b) $MW_1(-Q^2, \nu)$ is changed by the introduction of direct coupling, and it doesn't scale because

$$\frac{W_1(-Q^2,\nu)}{W_1^{"1-\gamma"}(-Q^2,\nu)} = 1 + \frac{Q^4}{32\pi^2 \alpha^2} \frac{\sum_j g_j^2 f_j(x)}{\sum_i e_i^2 f_i(x)} \tag{22}$$

Making the same assumptions as in Chapter I (i.e. $g \equiv g_i$), we obtain: [11]

$$\frac{W_1(-Q^2,\nu)}{W_1^{"1-\gamma"}(-Q^2,\nu)} = 1 + 2 \times 10^{-3} Q^4 (\text{GeV})^{-4} \tag{23}$$

Therefore at $Q^2 \approx 22$ GeV^2 the scale invariant contribution to $MW_1(-Q^2, \nu)$ extrapolated, for instance, from the SLAC-MIT data, should account only for $\approx 50\%$ of the observed value for $W_1(-Q^2, \nu)$.

The predicted effects on the ratio of νW_2 to W_1 are strong; they can be stated also in terms of the ratio $R = \sigma_L/\sigma_T$

$$R \equiv \frac{\sigma_L}{\sigma_T} = \frac{W_2}{W_1}\left(1+\frac{\nu^2}{Q^2}\right) - 1 \approx \frac{1}{1+2\times 10^{-3}Q^4}\left(1-\frac{4M^2x^2}{Q^2}\right) - 1 \to -1 \tag{24}$$

$$\text{as } Q^2 \to \infty,\ x \text{ fixed}.$$

This result, surprising as it might be at first sight, has to be expected in a model with scalar interactions:

$$\begin{aligned} \sigma_L &= \epsilon_L^{+\mu}\, g_{\mu\nu}\, W \epsilon_L^{\nu} = W \\ \sigma_T &= \epsilon_T^{+\mu}\, g_{\mu\nu}\, W \epsilon_T^{\nu} = -W \end{aligned} \tag{25}$$

since $\epsilon_L^2 = -\epsilon_T^2 = 1$.

For the case of vector coupling one can immediately write down:

$$\begin{aligned} -x\,\frac{d\sigma^{ep\to eX}}{dQ^2dx} &= \frac{4\pi\alpha^2}{Q^4}\, x\,(1-y+\tfrac{1}{2}y^2)\sum_i e_i^2 f_i(x) + (1-y+\tfrac{1}{2}y^2)\sum_j \frac{g_j^2}{4\pi} f_j(x) \\ &\quad - \frac{2\alpha}{Q^2}\, x\,(1-y+\tfrac{1}{2}y^2)\sum_i g_i e_i^2 f_i(x) \end{aligned} \tag{26}$$

$$\nu W_2(-Q^2,\nu) = x\sum_i e_i^2 f_i(x) + \frac{Q^4}{16\pi^2\alpha^2}\, x \sum_j g_j^2 f_j(x) - \frac{Q^2}{2\pi\alpha}\, x\sum_j g_j e_j f_j(x) \tag{27}$$

$$2MxW_1 = \nu W_2 \tag{28}$$

Although Eq. (28) implies $R \equiv \frac{\sigma_L}{\sigma_T} \to 0$ as $Q^2, \nu \to \infty$, x fixed, neither νW_2 nor W_1 scale. In fact (assuming again $SU(3)_{color}$ and $g \equiv g_j$)

$$\frac{\nu W_2(-Q^2,\nu)}{\nu W_2^{"1-\gamma"}(-Q^2,\nu)} \approx 1 + 2.5\times 10^{-3}\,Q^4(\mathrm{GeV})^{-4} - 3.8\times 10^{-2}\,Q^2 \,. \tag{29}$$

The third term is the interference term, which doesn't vanish, unlike in the annihilation process, since $e_i f_i(x) \neq 0$ [11] for the proton. We also used the positive value of g (Eq. 10), since it provides the more interesting case.

At $Q^2 = 25\ (\mathrm{GeV})^2$ $\nu W_2^{"1-\gamma"}(-Q^2,\nu)$, extrapolated from the SLAC-MIT data, should account for only $\approx 65\%$ of the observed value, but at $Q^2 = 10\ (\mathrm{GeV})^2$ $\nu W_2^{"1-\gamma"}(Q^2,\nu)$ would, in this model, give a value too big by a factor ~ 1.15.

Assuming $g_n = g_p = 0$, $g_\lambda \approx -3.5\times 10^{-3}(\mathrm{GeV})^{-2}$ changes the picture also quantitatively. From Eq. (26) we deduce

$$\nu W_2(Q^2,\nu) \approx x\sum e_i^2 f_i(x) + \frac{3Q^4}{4\pi^2\alpha^2}\,\frac{x}{10^6}\,(\mathrm{GeV})^{-4} f_\lambda(x) - \frac{1.2Q^2}{2\pi\alpha 10^3}\,x f_\lambda(x)(\mathrm{GeV})^{-2} \tag{30}$$

$$\begin{aligned}\frac{\nu W_1(Q^2,\nu)}{\nu W_2^{"1-\gamma"}(Q^2,\nu)} &\approx 1 + 4.8\times 10^{-4}Q^4(\mathrm{GeV})^{-4} - 8.8\times 10^{-3}Q^2(\mathrm{GeV})^{-2} \\ &\approx 1.08 \ \text{ for } \ Q^2 \approx 25\ (\mathrm{GeV})^2 \,.\end{aligned} \tag{31}$$

The reason why, in a scheme with an interference term, direct coupling dominates the annihilation process at $Q^2 = -s = -25\ (\mathrm{GeV})^2$, but not deep inelastic scattering at $Q^2 \approx 25\ (\mathrm{Gev})^2$, lies mainly in the fact that Q^2 changes sign going from one process to the other[12], so that the direct coupling term and the interference term which add for Q^2 timelike tend to cancel for Q^2 spacelike.

But at higher values of Q^2, the direct coupling term $\sim Q^4$ dominates and one should see dramatic scale breaking effects[13], provided the effective Lagrangian approach continues to hold. For direct scalar coupling of gluons to leptons, we have instead

$$-x\,\frac{d\sigma^{ep\to eX}}{dQ^2dx} = \frac{4\pi\alpha^2}{Q^4}\,x\left[1-y+\tfrac{1}{2}y^2\right]\sum_i e_i^2 f_i(x) + \frac{y^2}{32\pi Q^2}\,x\sum_\ell h_\ell^2 g_\ell(x)\ , \tag{32}$$

where $g_\ell(x)$ describes the number of gluons of type ℓ and longitudinal momentum xp inside a proton of momentum p.

One can make some qualitative observations immediately:

$$\text{(a)}\quad \nu W_2(\nu, Q^2) = \nu W_2^{\,\text{"}1-\gamma\text{"}}(Q^2,\nu)\ \text{scales,} \tag{33}$$

$$\text{(b)}\quad MW_1(Q^2,\nu) = \tfrac{1}{2}\sum_i e_i^2 f_i(x) + \frac{Q^2}{128\pi^2\alpha^2}\sum h_\ell^2 g_\ell(x)\ \ \text{does not .} \tag{34}$$

Unfortunately, one doesn't know $g_\ell(x)$. However, we do know $\sim 50\%$ of the proton momentum (at $P\to\infty$) is carried by gluons. Hence

$$0.5 \approx \int_0^1 dx\,x \sum_{i=p,n,\lambda}\left(f_i(x)+\overline{f}_i(x)\right) \approx \sum_{\text{gluons}}\int dx\,x\,g_\ell(x)\ . \tag{35}$$

So at least one knows the normalization of $g_\ell(x)$.

Assuming there are 8 (colored) gluons, all coupled in the same way to leptons and with equal distribution functions $g_\ell(x) \sim \frac{1}{4}(1-x)^3$, we can calculate for small x:

$$\frac{W_1(Q^2,\nu)}{W_1^{"1-\gamma"}(Q^2,\nu)} \approx 1 + \frac{Q^2}{13}\,(\text{GeV})^{-2} \tag{36}$$

There is still a lot of work to be done here before one can make very definite quantitative statements.

All these attempts to find an explanation for the annihilation process consistent with deep inelastic scattering is bound to be like a passage through Scylla and Charybdis, especially if one is forced to crude methods such as presented in this paper.

III. Decay of the Eta

Another process in which direct coupling should show up is the decay $\eta \to e^+e^-$ or $\mu^+\mu^-$, which is normally described by the diagram in Fig. 6. Assuming direct coupling and the usual quark representation of the η meson also the diagram in Fig. 7 will contribute where one takes care of $\eta \to \bar{q}q$ by introducing a form factor $|f(m_\eta, 0)|^2 \approx \frac{1}{2} m_\eta m_\pi^2$.[14]

Up to now we have always coupled the leptons to the "current quarks" which are quite different from "constituent quarks"[15]. But since one doesn't know the distribution of the current quarks inside the mesons (as it is the case with the proton) we forget for our estimates about the differences between the two kinds and just use the "constituent quark" - representation of the η meson[16]

$$\eta = \frac{1}{\sqrt{6}}(\bar{p}p + \bar{n}n - 2\bar{\lambda}\lambda)\cos 11^\circ + \frac{1}{\sqrt{3}}(\bar{p}p + \bar{n}n + \bar{\lambda}\lambda)\sin 11^\circ .$$

Using an effective interaction Lagrangian

$$\mathscr{L}_I' = g_{ps} \sum_i (\bar{e}\gamma_5 e)(\bar{q}\gamma_5 q) \quad \text{with} \quad g_{ps} = g_s \approx 3 \times 10^{-3} (\mathrm{GeV})^{-2} \tag{38}$$

we obtain for the decay width

$$\Gamma_{\eta \to e^+e^-} \approx 2.4 \text{ eV} . \tag{39}$$

This width is much larger than found experimentally for decay into muon pairs

$$\Gamma_{\eta \to \mu^+\mu^-} \approx 0.057 \text{ eV}. \tag{40}$$

Therefore unless μ-e universality is violated, a pseudoscalar direct coupling is ruled out.

IV. Lepton Pair Production in Hadron Collisions

The reaction pp $\to \bar{\ell}\ell$ + anything (ℓ = lepton) may provide us with a further test of the direct coupling scheme since—as was first pointed out by Drell and Yan[17]—the process can be described within the quark-parton model by the annihilation of a $\bar{q}q$ pair into a lepton pair (Fig. 8).

If one uses a scalar direct coupling scheme, this ansatz leads to the following enhancement factor to the cross section[18]

$$\begin{aligned} \frac{d\sigma}{dm_{\ell\bar{\ell}}} &\approx \frac{d\sigma^{\text{"}1-\gamma\text{"}}}{dm_{\bar{\ell}\ell}} + \frac{d\sigma^{\text{"}0-\gamma\text{"}}}{dm_{\ell\bar{\ell}}} \approx \frac{d\sigma^{\text{"}1-\gamma\text{"}}}{dm_{\ell\bar{\ell}}} \left(1 + \frac{27}{128} \frac{g_s^2}{\pi^2\alpha^2} m_{\ell\bar{\ell}}^4\right) \\ &\approx \frac{d\sigma^{\text{"}1-\gamma\text{"}}}{dm_{\ell\bar{\ell}}} \left(1 + 3.6 \times 10^{-3} m_{\ell\bar{\ell}}^4 (\mathrm{GeV})^{-4}\right) . \end{aligned} \tag{41}$$

The second term in Eq. (41) rises very rapidly with $m_{\ell\bar{\ell}}$, and for $m_{\ell\bar{\ell}} \gtrsim 10$ GeV it would dominate dramatically the contribution from the first conventional term were the effective Lagrangian still operative at such large masses. But even this prediction is not in contradiction with the existing upper limits, obtained by the CCR group at the ISR.[19] (Fig. 9)

V. Summary

In this paper we discussed a radical, but very simple-minded model which attempts to account for some features of the data in the process $e^+e^- \to$ hadrons. We saw that, at least, we cannot blame the model for not making predictions; for example, it suggests major violations of electroproduction scaling, generally an enhanced cross section, at values of Q^2 comparable to, although somewhat larger than, those for which scaling fails in e^+e^- annihilation ($Q^2 \gtrsim 20$ GeV^2). In addition, for couplings other than vector or axial, the transverse polarization dependence of the total e^+e^- annihilation cross section provides a clear test.

If these tests turn out negative, one can return to the study of more plausible explanations without the disquieting feeling that he forgot about a very simple option.

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REFERENCES

1. H. B. Newman, Invited talk at the Stanford Linear Accelerator Center Summer Institute in Particle Physics, July 1973 (unpublished), and references therein.
2. B. Richter, Invited talk at the Irvine Conference, December 1973.
3. M. Schwartz, private communication.
4. We exclude the possibility of the direct process $e^+e^- \to e^+e^-$ hadrons. Factorization would imply a direct cross section $\sigma(ep) \sim [\sigma(e^+e^-)\sigma(pp)]^{\frac{1}{2}} \gtrsim 10^{-29} cm^2$, which is a little big.
5. In this regard we do not understand the use of Reggeons in recent pre-prints by Greenberg and Yodh [University of Maryland 74-062] and Nanopoulos and Vlassopulos [CERN-TH-1842] in their descriptions of direct lepton-hadron couplings. Pursuing their analogy with ordinary strong interactions, it would seem that employing exchange of Pomeranchuk trajectory when $s/s_0 = s\, G_F^{-1} \sim 2.5 \times 10^{-4}$ is analogous to employing the Pomeranchuk trajectory in pp scattering for incident proton energies $\sim 2.5 \times 10^{-4}$ GeV (inasmuch as for strong interactions $s_0 \sim 1\ GeV^2$). A scattering-length approximation seems somewhat more appropriate.
6. Other terms involving derivative couplings may als be present, but we expect the non-derivative terms to dominate the low energy limit of the production amplitude.
7. The parton form factors discussed by M. Chanowitz and S. Drell, SLAC-PUB-1315 (1973) is a prototype for such a coupling.
8. W. A. Bardeen, H. Fritzsch, and M. Gell-Mann, in Proceedings of the Topical Meeting on Conformal Invariance in Hadron Physics, Frascati, May 1972 (unpublished).

9. See also T. Goldman and P. Vinciarelli, SLAC-PUB-1407 (1974).

10. We define $W_1(\nu, q^2)$ and $W_2(\nu, q^2)$ by

$$\frac{d\sigma}{dxdy} = \frac{4\pi\alpha^2}{Q^4}\, s\left[(1-y)\,\nu W_2(q^2,\nu) + y^2\, x M W_1(q^2,\nu)\right]$$

and not via the decomposition of $W_{\mu\nu} \sim \int d^4x\, e^{iq\cdot x} < p \mid J_\mu(x)\, J_\nu(0) \mid p >$ (which wouldn't make sense for scalar coupling), since one measures $d\sigma/dxdy$ directly, and not $W_{\mu\nu}$. Our deep inelastic notation hopefully standard, see e.g. F. J. Gilman, SLAC-PUB-1338 (1973), lectures presented at the Summer Institute on Particle Interactions at Very High Energies, Louvain, Belgium, 1973.

11. We always use the approximation

$$f_p(x) : f_n(x) : f_\lambda(x) \approx 5:3:1 \quad \text{for } x \gtrsim 0.05\ .$$

See for example J. D. Bjorken, High Transverse Momentum Processes, talk given at the Second Aix-en-Provence Conference (1973).

12. While this paper was in the process of being written up we received a preprint from J. C. Pati and A. Salam, "Are There Anomalous Lepton-Hadron Interactions?", in which the authors follow a similar approach, yet making more specific dynamical assumptions. We agree with their results in general, but disagree in some details. For example, we show that a scalar coupling term can cause rather strong scaling violations in ep → scattering at rather low values of $|q^2|$.

13. Since all the numbers presented here are rather rough estimates, relation (31) should tell us mainly that $\nu W_2(Q^2, \nu)$ in this approach roughly scales (to within ≈ 15%) for values of Q^2 up to 30 $(\mathrm{GeV})^2$.

14. See for example J. J. J. Kokkedee, The Quark Model (1969), p. 56.

15. H. J. Melosh, Ph.D. thesis, CALTECH (1973), unpublished.

16. See for example J. J. J. Kokkedee, The Quark Model (1969), p. 20.

17. S. D. Drell, Tung-Mow Yan, Phys. Rev. Letters **25**, 316 (1970). See also R. W. Fidler, Phys. Letters **46B**, 455 (1973).

18. T. Goldman and P. Vinciarelli (private communication) are presently analyzing this connection in considerable detail.

19. Report by B. G. Pope at the Second Aix-en-Provence Conference.

FIGURE CAPTIONS

Fig. 1: Possible exchange of a very massive particle M carrying lepton number in the t-channel at very high energies.

Fig. 2: Diagram for $q\bar{q} \to e^+e^-$.

Fig. 3: Square of the matrix element for $q\bar{q} \to e^+e^-$ including interfering term.

Fig. 4: Diagram for $e^+e^- \to 2$ gluons.

Fig. 5: a. Estimate for the total cross section $\sigma_{e^+e^- \to \text{hadrons}}$ versus s, assuming scalar coupling.

b. Estimate for the ratio $r = \dfrac{\sigma_{e^+e^- \to \text{hadrons}}}{\sigma_{e^+e^- \to \mu^+\mu^-}}$ versus s in comparison to some data from CEA and Adone.[1]

Fig. 6: Diagram for $\eta \to \gamma\gamma \to e^+e^- \ (\mu^+\mu^-)$.

Fig. 7: Diagram for $\eta \to e^+e^- \ (\mu^+\mu^-)$ assuming direct coupling.

Fig. 8: Diagram for $pp \to \ell\bar{\ell} + X$.

Fig. 9: Limits on lepton-pair production as reported by CCR, along with theoretical estimates. (From J. D. Bjorken, talk given at the Second Aix-en-Provence Conference (1973).)

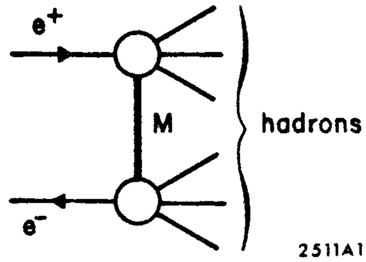

Fig. 1

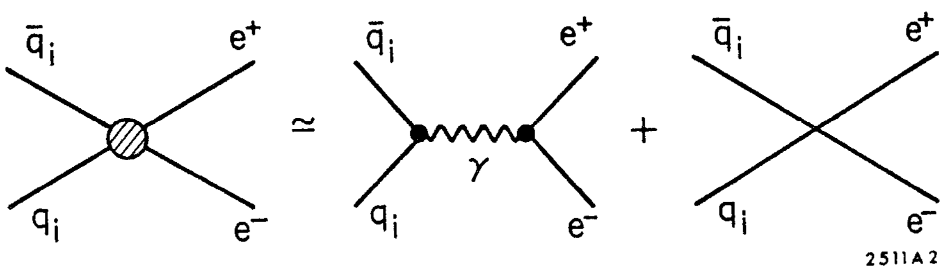

Fig. 2

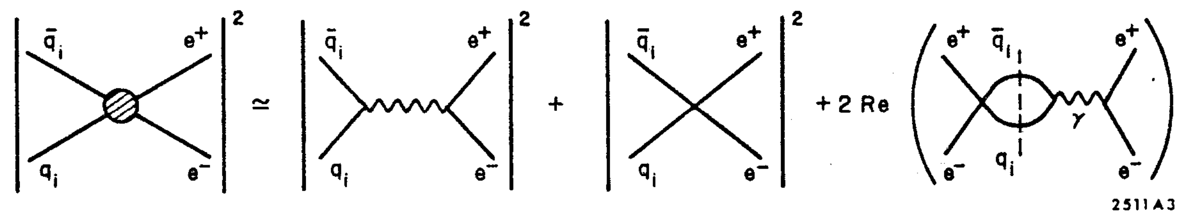

Fig. 3

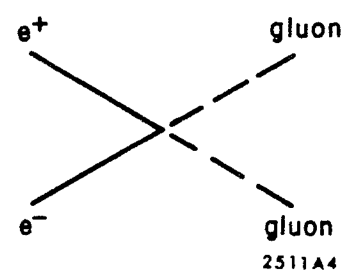

Fig. 4

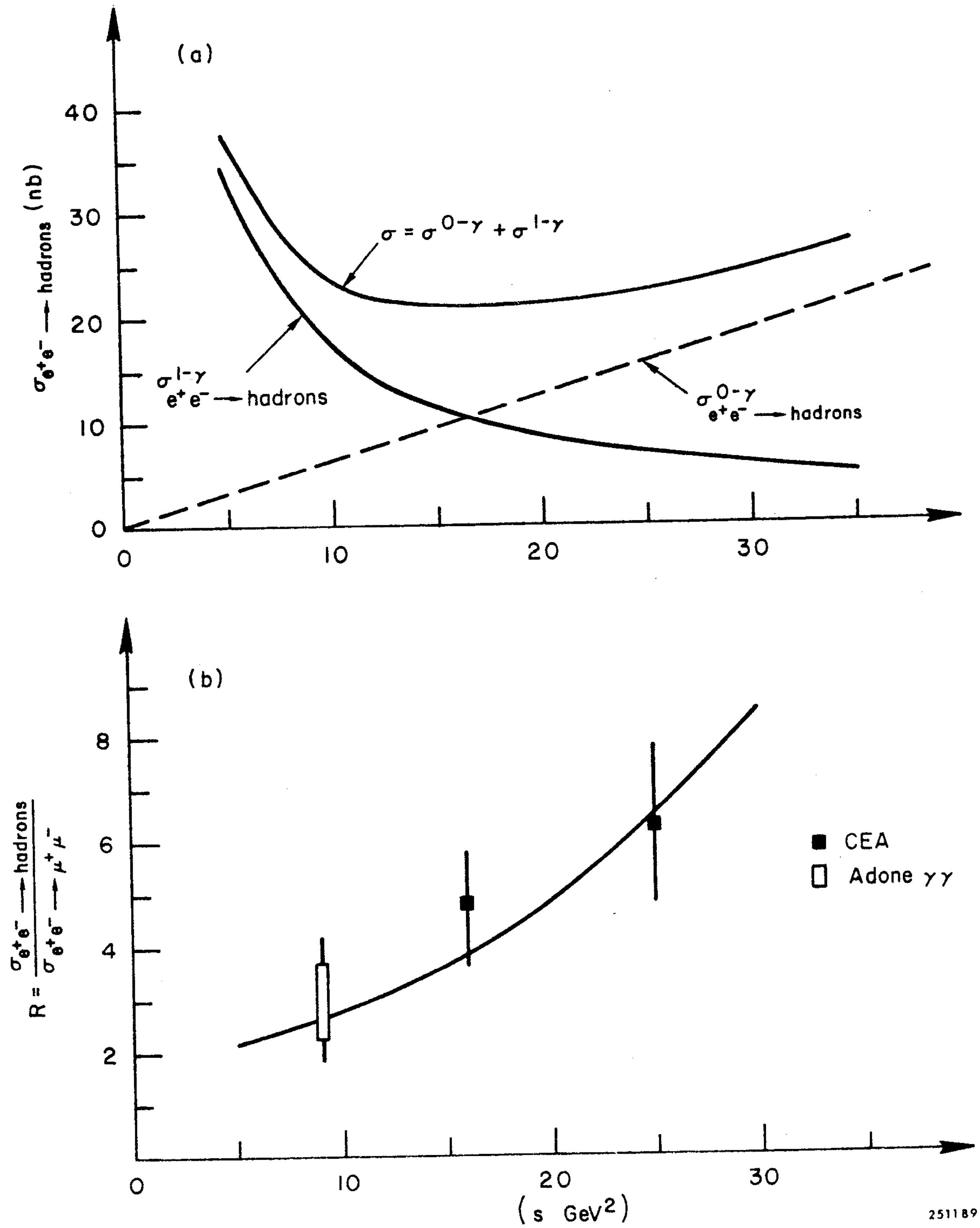
(a)
$\sigma = \sigma^{0-\gamma} + \sigma^{1-\gamma}$
$\sigma^{1-\gamma}_{e^+e^- \to \text{hadrons}}$
$\sigma^{0-\gamma}_{e^+e^- \to \text{hadrons}}$
$\sigma_{e^+e^- \to \text{hadrons}}$ (nb)
0
10
20
30
40
(b)
$R = \frac{\sigma_{e^+e^- \to \text{hadrons}}}{\sigma_{e^+e^- \to \mu^+\mu^-}}$
2
4
6
8
CEA
Adone $\gamma\gamma$
(s GeV^2)
251189

Fig. 5

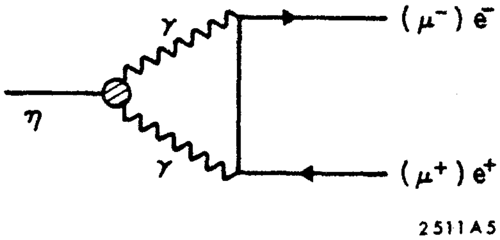

Fig. 6

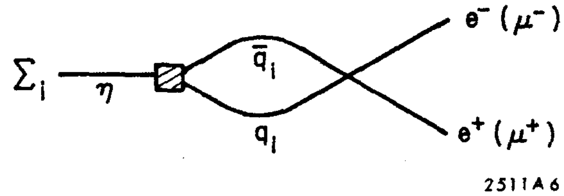

Fig. 7

$$\left|\mathcal{M}_{PP \rightarrow \bar{\ell}\ell + X}\right|^2 \simeq$$

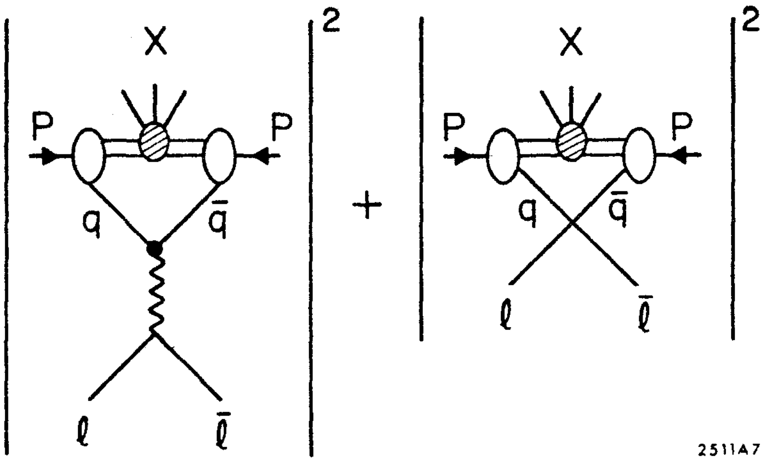

Fig. 8

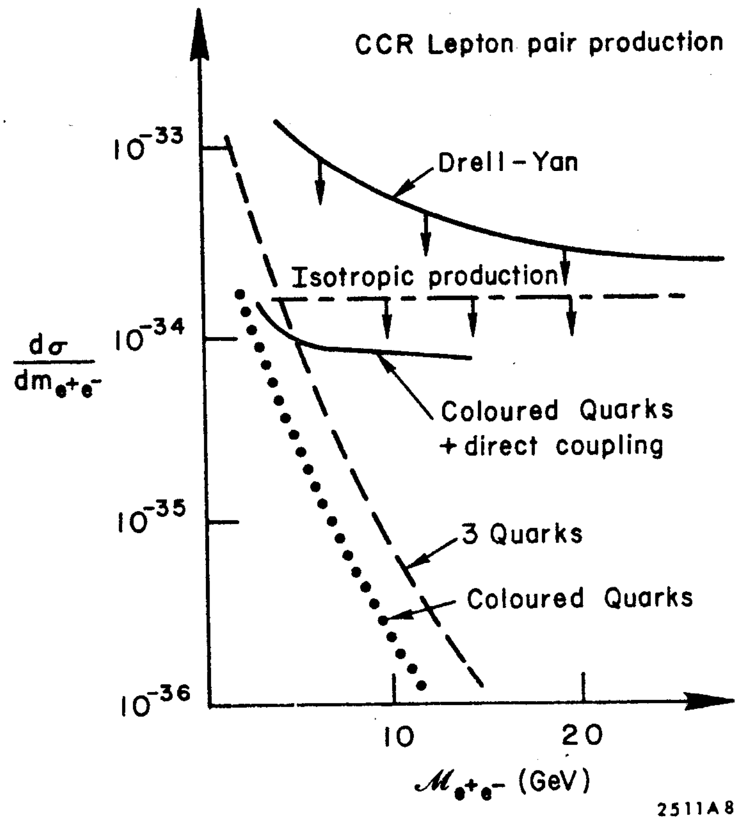
CCR Lepton pair production
10^{-33}
10^{-34}
10^{-35}
10^{-36}
$\frac{d\sigma}{dm_{e^+e^-}}$
Drell-Yan
Isotropic production
Coloured Quarks
+ direct coupling
3 Quarks
Coloured Quarks
10
20
$\mathcal{M}_{e^+e^-}$ (GeV)
2511A8

Fig. 9