SLAC-PUB-1418
(T)
May, 1974

THE CONDUCTIVE STRING: A RELATIVISTIC QUANTUM MODEL OF PARTICLES WITH INTERNAL STRUCTURE*

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(Submitted to Phys. Rev. D)

*Work supported by the U. S. Atomic Energy Commission.

ABSTRACT

A relativistic quantum mechanical model for a one-dimensionally extended composite hadron is studied in detail. The model is suggested by the string model, and has the same spectrum of excitations in the large quantum number limit, but has features which represent departures from the string model as well. The ground state of the system has the character of a conductive medium. Quasi-particle excitations in filled Fermi sea configurations give rise to towers of particles of increasing mass and spin. Lorentz scalar collective excitations with Bose statistics are also supported by the system, and can lead to a Hagedorn-type degeneracy in the spectrum. The fundamental dynamical variables of the theory are canonical Fermi fields, but an internal consistency requirement of the theory demands all physical states must have zero fermion ("quark") number. The theory is relativistically covariant in four-dimensional Minkowski space, without requiring ghosts or tachyons.

I. INTRODUCTION

In recent years considerable effort has been directed toward constructing a consistent theory of strongly interacting particles along the lines suggested by the dual resonance model (DRM). An important aspect of this program has been the development of physical models which give rise to the desirable features of the mathematical DRM, and hopefully yield more information as well.[1-10] The most elegant attempt along these lines is the geometrical description initiated by Nambu.[11] In this case the fundamental physical structure is a massless relativistic string.

The propagation of the string in space-time is determined by an action which was first obtained[11] from the generalization of the action of a point particle. Alternatively, it can be obtained uniquely[12,13] by requiring that, as a function of the string variable and its derivatives, (a) it be Poincaré invariant, (b) it be invariant under the general coordinate transformation of the coordinates of the surface swept out by the string, and (c) the Euler-Lagrange equations for the string variable be of order not higher than two. It has been shown[12,14] that the spectrum as well as the ghost eliminating constraints follow from the parametrization (gauge) invariance of the action. One thus arrives at the <u>gauge theory of the relativistic string</u>.[12]

To quantize the theory, it is necessary to choose a gauge from among various possibilities. If the choice of gauge happens to be at the expense of manifest covariance, one must give a direct proof of the relativistic invariance of the quantized theory. The quantization in one such non-covariant gauge, which has the advantage of being manifestly ghost-free, has been carried out in detail.[15] It was found that the theory is Poincare invariant not in four but in 26 space-time dimensions, and even then it is necessary that the ground state be a tachyon.

However, passage from a consistent classical theory to a quantum theory always involves a certain amount of guess work. Justification for any particular guess is always made a posteriori. We argue, therefore, that the inconsistencies that arise in the quantization of the string clearly indicate that direct passage from the classical theory to the correct quantum description via Poisson brackets is impossible. Stated in another way, the string variables are not the proper variables of the quantized theory. At best, the string picture represents a "high quantum number limit" of the correct theory.

To support this point of view, we elaborate here the construction of an explicit model,[17] which goes to the string picture in a high quantum number limit, but whose quantum spectrum exists without tachyons or ghosts. Further, it carries no extraneous variables other than those demanded by four-dimensional space time symmetry. The resultant trajectories are indefinitely rising. The basic dynamical variables are a pair of fermi fields, whose modes of collective excitation give rise to the physical spectrum. Moreover, the dynamics generating these modes has the remarkable property of permanently trapping the fermions, so that these are not directly observable.

This trapping feature is quite compatible with recent progress in the understanding of deep inelastic lepton-hadron scattering.[16] A variety of empirical observations in such processes can most easily be realized with a quark-parton picture of hadrons. On the other hand, there is as yet no experimental evidence for any fractionally charged objects being produced. Thus a dynamic trapping mechanism will be a desirable attribute in any model. (In our model, we have a $U(1)$ internal symmetry, so our quarklike objects have only integer charges, say "quark number".)

The manner in which trapping occurs in the specific model we discuss is only one aspect of our work which may be of interest to DRM non-specialists. A point which has long been implicit in DRM, and which our approach deals with explicitly, is the many-body nature of the internal dynamics of hadrons. The most important feature is not the traditional statement that this is required to build up an enormously rich spectrum, but rather that we exhibit the relevance of many body collective behavior in a new context for hadronic physics. One may say this behavior is at once responsible for trapping, for the form of the spectrum, and ultimately for the high energy behavior of hadron-hadron scattering.

From a different point of view, our model shares certain features with the so-called "bag" picture, in which fields are confined within finite volumes.[32] Indeed, our model is a mathematical prototype of a one-dimensional bag, but with full Lorentz covariance. Alternately, it may be viewed roughly as the extreme case of a bag with an infinite number of partons distributed uniformly, smoothly, and with no multiple occupancy on the hadron's longitudinal momentum axis.

Still a different concept of the work is as a study in the possible (reducible) timelike representations of the Poincare group.[10]

We have, by construction, that such representations, representing composite hadrons, actually exist in the limit where the number of constituents is infinite. The two-dimensional structure of the dynamics means, of course, it is not "simply" field theory again.

The paper is organized in the following manner. In Section II we describe how the knowledge of the geometrical description is helpful in providing a point of departure from the conventional string formalism. After choosing our dynamical variables, we discuss the general features of the actions which one can write down. In section III, we specialize to one particular model. We cast this model into Hamiltonian form, and discuss the necessity of imposing a charge neutrality constraint on the eigenstates of the Hamiltonian. In Section IV we diagonalize the Hamiltonian by a Bogoliubov transformation and construct its general eigenstates. In Section V, we give a physical interpretation to various operators and show that the Hilbert space of eigenstates of H carries unitary irreducible representations of the Poincare group. Section VI is devoted to a study of the spectrum, and the asymptotic value of the level degeneracy. Finally, in Section VII, we discuss in greater detail the general features of our results and what may be abstracted from them.

II. NEW ACTIONS SUGGESTED BY THE GEOMETRICAL DESCRIPTION

The novel features of the models which we will describe are sufficiently noteworthy to render their connection with the classical string model inconsequential. Nevertheless, it is instructive to see how one might be led to such a model from the knowledge of the geometrical description. It will be recalled[12] that the motion of the relativistic string is most simply described in frames characterized by a pair of coordinate conditions

$$\left(\frac{\partial Y^\mu}{\partial u^\pm}\right)^2 = 0 \qquad (2.1)$$

where $Y^\mu(\theta,\tau)$ is the string variable and $u^\pm = (1/\sqrt{2})\,(\tau \pm \theta)$. These conditions define two null vectors $\partial Y^\mu/\partial u^\pm$, which in 4-space-time dimensions can be expressed in terms of two complex 2-component spinors $\psi_\pm$ as follows:

$$\frac{\partial Y^\mu}{\partial u^\pm} = \psi_\pm^+ \,\sigma^\mu\, \psi_\pm \qquad (2.2)$$

An important feature of the relations (2.2) which should be kept in mind is that these relations are peculiar to 4-space-time dimensions. Although it is possible to define spinors relevant to a space of any dimensionality and signature, the role they play is not quite the same as the two component spinors associated with Minkowski space-time. This can be seen by noting that in the spinor description of 4-vectors the equality

$$\det\begin{bmatrix} X^0 + X^3 & X^1 + iX^2 \\ X^1 - iX^2 & X^0 - X^3 \end{bmatrix} = X^{0^2} - \vec{X}^2 \qquad (2.3)$$

has no direct analogue in any other dimension or signature. By working with 2-component spinors, one is thus selecting the real four-dimensional Minkowski space from among the many possible ones. We shall therefore take the two component spinors as our fundamental dynamical variables.

We then postulate basic algebraic relations in terms of these variables. We shall wind up with a different theory from the conventional scheme; in particular, $[Y^\mu, Y^\nu] = 0$ must be true if Y^μ are the dynamical variables, but cannot hold if ψ are Fermi fields. At the classical level of course, the two points of view are identical; (2.2) is an explicit realization of the light-like gauge conditions.

Having defined our variables, we may now use them to specify the dynamics. In doing this, we shall be guided by the requirement that our theory must maintain the relationship (2.2) in the classical, or large quantum number, limit. The string variables satisfy the equation

$$\frac{\partial^2 Y^\mu}{\partial u^+ \, \partial u^-} = 0 \; . \tag{2.4}$$

Hence, the equations of motion for $\psi_{\pm}$ must imply (2.4). Furthermore they must be invariant under the arbitrary phase transformations which leaves (2.2) invariant. The most general linear equation of motion that $\psi_{\pm}$ can satisfy under such conditions will then be

$$\left\{ i \frac{\partial}{\partial u^{\pm}} - gB_{\pm} \right\} \psi_{\pm} = 0 \; . \tag{2.5}$$

The quantities $B_{\pm}$ are gauge fields; (2.5) is invariant under the transformations

$$\begin{aligned} \psi_{\pm} &\longrightarrow \exp[ib]\psi_{\pm} \\ B_{\pm} &\longrightarrow B_{\pm} + \frac{1}{g}\partial_{\pm} b \; , \end{aligned} \tag{2.6}$$

where b is an arbitrary function of $u^{\pm}$. The parameter g is real, but is otherwise unrestricted. To effect passage into quantum theory, we must construct an action that will generate (2.5), and be invariant under (2.6). The action must necessarily involve $\mathbf{B}_{\pm}$, which is a new additional dynamical variable implied by our construction, and supply gauge invariant equations of motion for it. We shall construct such an action and study its quantization in the next section.

Before concluding this section, we would like to comment on the manner in which one proceeds to quantize a classical theory which involves constraints. If the constraints are not of $0 = 0$ type, one can either quantize the theory as if there were no constraints and then impose the constraints as weak operator conditions on states, or one can use the constraint equations to eliminate the dependent variables and then quantize the independent dynamical variables. On the other hand, if the constraints are satisfied identically at the classical level, they have no bearing on the quantization. In our case it is easy to check that the expressions (2.2) satisfy the constraints (2.1) identically, so that with $\psi_{\pm}$ as dynamical variables there are no classical constraints which are to be carried over to quantum theory.

III. AN EXPLICIT MODEL

We shall now obtain an explicit realization of the general ideas discussed in the previous section. Consider the following Lagrangian density, which is invariant under (2.6),

$$\mathscr{L} = \bar{\psi}[i\Gamma^a\partial_a - i\Gamma^a B_a]\psi - \frac{1}{4} F_{ab}F^{ab}, \quad a, b = 0,1 . \tag{3.1}$$

The field ψ is a four component spinor which is built up from the spinors introduced in (2.2). The Γ^a are 4×4 matrices satisfying

$$\{\Gamma^a,\Gamma^b\} = 2\eta^{ab}; \quad \eta^{00} = -\eta^{11} = 1; \quad \eta^{ab} = 0 , \quad a \neq b \tag{3.2}$$

We shall use the following representation:[19]

$$\Gamma^0 = i\gamma^0\gamma^5; \qquad \Gamma^1 = i\gamma^5 . \tag{3.3}$$

The quantity F_{ab} is defined as

$$F_{ab} = \partial_b B_a - \partial_a B_b, \tag{3.4}$$

where B_a is the field introduced in (2.5).

The Lagrangian is obtained by integrating (3.1) over the range 0 to π. The ψ fields can be chosen to satisfy either of two boundary conditions. We define

$$\psi = \binom{\varphi}{\chi} . \tag{3.5}$$

We may then demand[18]

$$\begin{aligned} &\phi(0) = \chi(0) ; && \varphi(\pi) = -\chi(\pi) ; \\ &\left.\frac{\partial\varphi}{\partial\theta}\right|_0 = -\left.\frac{\partial\chi}{\partial\theta}\right|_0 ; && \left.\frac{\partial\varphi}{\partial\theta}\right|_\pi = \left.\frac{\partial\chi}{\partial\theta}\right|_\pi ; \end{aligned} \tag{3.6}$$

or

$$\phi(0) = \chi(0) \; ; \qquad \varphi(\pi) = \chi(\pi) \; ;$$
$$\left.\frac{\partial\varphi}{\partial\theta}\right|_0 = - \left.\frac{\partial\chi}{\partial\theta}\right|_0 \; , \qquad \left.\frac{\partial\varphi}{\partial\theta}\right|_\pi = - \left.\frac{\partial\chi}{\partial\theta}\right|_\pi \; . \tag{3.7}$$

Application of the variational principle to (3.1) yields Eq. (2.5), plus the following equation for B_a:

$$\partial_b F^{ab} = g j^a \; ; \tag{3.8}$$

where

$$j^a \equiv \bar{\psi}\Gamma^a\psi . \tag{3.9}$$

Equation (3.1) is thus an appropriate Lagrangian density implementing the ideas of the last section. The boundary conditions (3.6) imply that

$$j^1(0) = j^1(\pi) = \left.\frac{\partial j^0}{\partial\theta}\right|_{0=0} = \left.\frac{\partial j^0}{\partial\theta}\right|_{0=\pi} = 0 \; . \tag{3.10}$$

The action (3.1) does not contain a mass term for ψ; there is thus an apparent Γ^5 $(\equiv \Gamma^0\Gamma^1)$ invariance. The attendant axial current is

$$j_5^a = \epsilon^{ab} j_b \; . \tag{3.11}$$

Equation (3.10) thus says that the axial charge density vanishes at the boundary. Similar remarks apply when (3.7) holds. In two dimensions, the axial charge is also the "spin" of the system. Hence, with the above boundary conditions, the "spin" does not leak through the boundary.

We impose the basic algebraic relation:

$$\{\psi_\alpha^\dagger(\theta,\tau),\ \psi_\beta(\theta',\tau)\} = \delta_{\alpha\beta}\ \delta(\theta-\theta')\ . \tag{3.12}$$

The analysis of the theory is most conveniently carried out by introducing the appropriate Fourier expansions of ψ. Using (3.6), for $\tau = 0$,

$$\begin{aligned} \varphi_\lambda &= \frac{1}{\sqrt{2\pi}} \sum_{k>0} \{b_\lambda(k)\ \exp[i(k-\tfrac{1}{2})\theta] + c_\lambda^\dagger(k)\ \exp[-i(k-\tfrac{1}{2})\theta]\} \\ \chi_\lambda &= \frac{1}{\sqrt{2\pi}} \sum_{k>0} \{c_\lambda^\dagger(k)\ \exp[i(k-\tfrac{1}{2})\theta] + b_\lambda(k)\ \exp[-i(k-\tfrac{1}{2})\theta]\} \end{aligned} \tag{3.13}$$

$$\lambda = 1,2.$$

The Fourier coefficients satisfy canonical anti-commutation relations

$$\{b_\lambda(k),\ b_{\lambda'}^\tau(k')\} = \{c_\lambda(k),\ c_{\lambda'}^\tau(k')\} = \delta_{\lambda\lambda'}\ \delta_{kk'}\ , \tag{3.14}$$

all other anti-commutators vanishing. This guarantees Eq. (3.12).

Observe that ψ (as well as φ_λ and χ_λ) carries an SU(2) index, so that under SU(2) transformations the indices 1 and 2 get mixed. In addition, ψ of course transforms as a spinor with respect to the two dimensional Lorentz group O(1,1), which acts directly upon ϕ and χ.

Let us elaborate briefly on the significance of the SU(2) index of the field $\psi(\theta,\tau)$ (or of the coefficients $b_\lambda(k)$, $c_\lambda(k)$, etc.). It will be shown later that the Hilbert space of physical states is constructed by acting on the vacuum state $|0\rangle$ with functions of bilinear products of the form $b^+_\lambda(k)\ c^+_\lambda(k')$, λ, $\lambda' = 1, 2$. The SU(2) index is thus directly related to the spin of the physical Poincare states. This is to be contrasted with the spinor index of a local field, the Dirac field, for example:

$$\psi_\alpha(x,\ t) = \sum_{\pm\lambda} \int \frac{d^3P}{(2\pi)^{3/2}} \sqrt{\frac{m}{E_p}} \left[u_\alpha(P,\lambda)\ b_\lambda(P)\ e^{-iP\cdot X} + v_\alpha(P,\lambda)\ c^+_\lambda(P)\ e^{iP\cdot X}\right] \tag{3.15}$$

In the latter case the index α is eventually summed over and is not directly involved in the construction of the physical Poincare states. In the Lagrangian density (3.17) we have thus given up manifest Poincare invariance and retained only a little group SU(2) symmetry. This is because $\psi(\theta,\tau)$ is not a local field in 4-space-time dimensions, and the direct action of the boost operators on ψ is fairly complicated.

The corresponding expansions for the boundary conditions (3.7) are

$$\psi = \psi_0 + \binom{\varphi}{\chi} \tag{3.16}$$

with

$$\begin{aligned} \varphi_\lambda &= \frac{1}{\sqrt{2\pi}} \sum_{k=1}^{\infty} \{b_\lambda(k)\ e^{ik\theta} + c^+_\lambda(k)\ e^{-ik\theta}\} \\ \chi_\lambda &= \frac{1}{\sqrt{2\pi}} \sum_{k=1}^{\infty} \{c^+_\lambda(k)\ e^{ik\theta} + b_\lambda(k)\ e^{-ik\theta}\}\ . \end{aligned} \tag{3.17}$$

The b_λ and c_λ also satisfy the relations (3.14). ψ_0 satisfies (3.12) without the delta function. In what follows we shall carry out the solution for the case (3.6) explicitly. The case of (3.7) can be similarly analyzed, and we shall indicate how this occurs following the discussion of case (3.6).

In looking for a solution, notice that the action density (3.1) is identical in structure to that of two dimensional spinor electrodynamics (TDED), which is exactly soluble.[20] Since the use of four component spinors, which arise from the additional SU(2) symmetry, leads to the same algebraic relations as those of TDED, we expect that our model is also exactly soluble. However, the usual analysis of TDED proceeds by examining the Green's functions of the theory, and is not suited for our purposes. We are interested more in the spectrum of the theory, and in attempting to construct four-dimensional Poincare generators out of the relevant operators of the theory. We therefore construct the solutions of (3.1) directly by means of operator transformations on the Hamiltonian of (3.1).

We shall work in the axial gauge,

$$B_1 = 0 \ . \tag{3.18}$$

The Hamiltonian in this gauge is

$$H = H_0 + H' \tag{3.19}$$

where

$$H_0 = -i \int_0^{\pi} d\theta \{\bar{\psi}\Gamma^1 \frac{\partial\psi}{\partial\theta}\} \tag{3.20}$$

and

$$H' = \int_0^{\pi} d\theta \left\{ g j^0 B_0 - \frac{1}{2}\left(\frac{\partial B_0}{\partial\theta}\right)^2 \right\} \tag{3.21}$$

The equation (3.8) becomes, with (3.18),

$$\frac{\partial^2 B^0}{\partial \theta^2} = -gj^0 , \tag{3.22}$$

which may be integrated immediately:

$$B^0(\theta,\tau) = -\frac{1}{2} g \int_0^{\pi} d\theta' |\theta - \theta'| \; j^0(\theta',\tau) . \tag{3.23}$$

The new variables B_a which we had to introduce are, in fact, not independent of ψ! The Hamiltonian then takes on the very simple form:

$$H = H_0 - \frac{1}{4} g^2 \iint d\theta \; d\theta' \; j^0(\theta,\tau) |\theta - \theta'| \; j^0(\theta',\tau) . \tag{3.24}$$

As mentioned earlier, we shall pass almost immediately to momentum space, and normal order our operators to have a well-defined theory.

IV. DIAGONALIZATION OF THE HAMILTONIAN

A. Preliminaries Regarding Trapping.

We will diagonalize the Hamiltonian by means of a Bogoliubov transformation.[7,22] The details are considerably simplified if we make use of a consistency condition which our model shares with (TDED) and which arises from the Schwinger term in the commutator $[j^0, j^1]$. Despite the fact that the Lagrangian (3.1) is massless and hence γ^5-invariant, if in the divergence of the axial vector current $\partial_a j_5^a$, the term $\partial j^1/\partial\tau$ is calculated by means of the Heisenberg equations of motion, one finds that

$$\partial_a j_5^a = i[H', j^1] \neq 0 \ . \tag{4.1}$$

The anomaly on the right-hand side can be calculated from the definitions of H' and j^1, with the result that

$$\partial_a j_5^a(\theta,\tau) = \frac{2g}{\pi} \frac{\partial}{\partial\theta} B^0(\theta,\tau) \ . \tag{4.2}$$

This result may be combined with vector current conservation to give

$$[\partial_b \partial^b + \mu^2] \ j^a = 0 \ , \tag{4.3}$$

where

$$\mu^2 = \frac{2g^2}{\pi} \ . \tag{4.4}$$

This shows that the vector current is a massive free field. Integrating eq. (4.3) with respect to θ in the range $(0,\pi)$, and recalling the boundary conditions (3.11), we obtain[23]

$$\left[\frac{\partial^2}{\partial\tau^2} + \mu^2 \right] Q = 0 \ , \tag{4.5}$$

where

$$Q = \int_0^\pi d\theta \ j^0(\theta,\tau) \ . \tag{4.6}$$

The time independence of charge is compatible with (4.5) if and only if Q vanishes. Since Q is not identically zero, it acts as a constraint to select a subset of states from the larger fermionic Fock space:

$$Q|\psi_{\text{physical}}\rangle = 0 \tag{4.7}$$

This superselection rule eliminates all states created by unpaired fermions from the space of physical states, and is perfectly admissible provided the physical states form a complete basis for all simultaneous observables which conserve charge. This is, however, precisely the character of a superselection rule.

Equation (4.7) means, quite simply, that physical states have zero **net** "quark number". In the context of our model, this is a sufficiently strong restriction that we may claim it is a dynamical realization of "trapping", or "containment". This is clearly adequate in our case, because the Fock space in question is to act as a factor in a carrier space for representations of the Poincare group for a <u>single</u> composite particle.

Although it is not part of our main line of development, the question of what constitutes a proper general mathematical definition of containment is an interesting one, and a brief digression may be excusable in view of the importance of this question. We have in mind recent arguments that $Q = 0$ is <u>not</u> a sufficient requirement for confinement in a more general field theoretic context.[24] These arguments are basically classical, and begin from the observation that a state of "charge" zero can consist of plus and minus charges macroscopically separated. A fully <u>local</u> requirement of "charge" conservation must be imposed to prevent this circumstance. Lorentz covariance then implies the full local conserved current, $J_\mu(x)$, must vanish.

In quantum mechanics, however, this last restriction is inconsistent with positivity of the spectrum if the vanishing of the current is imposed weakly on physical states, a la Eq. (4.7). Rather, we must require all matrix elements of $J_\mu(x)$ between physical states to vanish.

However, if the currents transform as an irreducible representation of a symmetry group generated by charges Q^a, it is easily seen that this requirement is satisfied trivially if the physical states are singlets under the group. That is, it may, after all, be sufficient that the charges annihilate the physical states, given a fairly standard set of assumptions on the currents. We reiterate that the entire context of these arguments is quite different from the situation in our model.

B. Construction of the Eigenstates

We now proceed to the diagonalization of the total Hamiltonian H. First, define the "plasmon" operators

$$\rho(p) = \frac{1}{\sqrt{2p}} \int_0^\pi d\theta : [j^0(\theta) \cos p\theta - i j^1(\theta) \sin p\theta] : . \quad (4.8)$$

It is straightforward to verify that

$$[\rho(p), \rho^+(q)] = \delta_{pq} \quad (4.9)$$

Furthermore, they commute with Q. Making use of the plasmon operators and dropping the terms proportional to Q (by (4.7) they do not contribute to eigenvalues of H), the Hamiltonian can be cast into the form

$$H = \sum_{n=1}^{\infty} \sum_{\lambda=1}^{2} (n - 1/2) \, [b_\lambda^+(n) \, b_\lambda(n) + c_\lambda^+(n) \, c_\lambda(n)]$$

$$+ \frac{1}{4} \mu^2 \sum_{n=1}^{\infty} n^{-1} [2\rho^+(n) \, \rho(n) + \rho(n) \, \rho(n) + \rho^+(n) \, \rho^+(n)] \qquad (4.10)$$

where for explicitness we have used the expansion (3.2). Observing the following property of the plasmon operators,

$$[H_0, \rho(n)] = -\rho(n) \, , \qquad [H_0, \rho^+(n)] = \rho^+(n) \, , \qquad (4.11)$$

we construct the plasmon number operator

$$T = \sum_{n > 0} n\rho^+(n) \, \rho(n) \, , \qquad (4.12)$$

so that

$$[T, \rho(n)] = -\rho(n); \qquad [T, \rho^+(n)] = \rho^+(n) \, . \qquad (4.13)$$

This suggests that instead of (3.13), the Hamiltonian should be split in the form

$$H = H_1 + H_2 \qquad (4.14)$$

where

$$H_1 = H_0 - T \qquad (4.15)$$

$$H_2 = H' + T \qquad (4.16)$$

Then clearly,

$$[H_1, \rho(n)] = 0, \quad \text{all} \quad n \, . \qquad (4.17)$$

Moreover, since H_1 is already diagonal, we only have to diagonalize H_2. This must be done in such a way that H_1 remains diagonal.

The transformation which diagonalizes H_2 must therefore be constructed from operators which commute with H_1, i.e., $\rho(n)$ and $\rho^+(n)$.

Consider the Hermitian operator

$$S = -i \sum_{k>0} \phi(k) \, [\rho^+(k)\, \rho^+(k) - \rho(k)\, \rho(k)] \tag{4.18}$$

where $\phi(k)$ is a real function to be determined such that under the transformation

$$H_2 \longrightarrow e^{iS} H_2 e^{-iS} \equiv \hat{H}_2 , \tag{4.19}$$

$\hat{H}_2$ becomes diagonal. By construction

$$\hat{H}_1 = e^{iS} H_1 e^{-iS} = H_1 = H_0 - T . \tag{4.20}$$

Write

$$e^{iS} \rho(n) e^{-iS} = \rho(n) \operatorname{ch} \phi(n) + \rho(n)^+ \operatorname{sh} \phi(n) \tag{4.21}$$

Then, after some algebra, one finds that $\hat{H}_2$ will become diagonal if

$$\bar{\phi}(k) = \frac{1}{2} \tanh^{-1} \left[\frac{\mu^2}{\mu^2 + 2k^2} \right] , \quad \text{all } k . \tag{4.22}$$

With this choice for $\phi(k)$, the transformed Hamiltonian will take the form

$$\hat{H} \equiv e^{iS} H e^{-iS} = H_0 - T + \epsilon_0 + \sum_{k=1}^{\infty} \epsilon(k)\, \rho^+(k)\, \rho(k) \tag{4.23}$$

where

$$\epsilon_0 = \frac{1}{2} \sum_{k=1}^{\infty} \left[\epsilon(k) - k - \frac{\mu^2}{2k} \right] \tag{4.24}$$

is the shift in the ground state energy; and

$$\epsilon(k) = [\mu^2 + k^2]^{1/2}. \tag{4.25}$$

The spectrum of $\hat{H}$ thus consists of "quasi-particle" excitations constituting H_0 as well as the plasmons. Quasi-particles are partons of our model.

Before proceeding, it may be helpful to motivate the choice of S, Eq. (4.18), somewhat further. The form of H_2 suggests we think of the problem in terms of a functional eigenvalue problem, with $\rho(q) = \delta/\delta\rho^+(q)$. Looking at the number of derivatives that appear, a Gaussian in the "coordinate" ρ^+ is clearly indicated as a candidate for the ground state. If we require that the ground state be neutral, and "Lorentz invariant" in the two-dimensional space, we might try the configuration space expression

$$|\Omega\rangle = \exp[i \iint dx\, dx'\, f(x-x') :j_\mu(x)\, j^\mu(x'):]\, |0\rangle\ .$$

In momentum space, this has just the form $e^{-iS}|0\rangle$, with $\phi(k)$ related to the Fourier transform of $f(x-x')$. Hopefully this argument makes the ansatz for S less mysterious than it might otherwise seem.

We next construct eigenstates of $\hat{H}$, using algebraic methods. For consistency, these eigenstates must satisfy the constraint (4.7). Since the operators $\rho^+(k)\, \rho(k)$ commute with both H_1 and Q, we shall construct eigenfunctions of $\hat{H}$ as tensor products of the eigenstates of H_1 and $\rho^+(k)\,\rho(k)$. We first construct neutral parton pair states

$$|F\rangle \equiv |F_{ij}\rangle = \prod_{n=1}^{F} b_i^+(n)\; c_j^+(n)\; |0\rangle\;; \qquad i \neq j\;. \tag{4.26}$$

These states have the property of acting as "vacuum" with respect to the plasmons:

$$\rho(m)\;|F_{ij}\rangle = 0\;; \qquad \text{all} \quad m\,. \tag{4.27}$$

These states are eigenfunctions of $\hat{H}$:

$$\hat{H}|F\rangle = (H_0 + \epsilon_0)\;|F\rangle = (\epsilon_F + \epsilon_0)\;|F\rangle \tag{4.28}$$

where

$$\epsilon_F = 2\sum_{n=1}^{F}\left(n - \frac{1}{2}\right) = F^2 \tag{4.29}$$

Note that

$$Q|F\rangle = 0\;. \tag{4.30}$$

A general eigenstate of $\hat{H}$ can now be constructed as the tensor product of states $|F\rangle$ with states created by plasmon operators:

$$|N_p,P,F\rangle = \prod_{m=1}^{P} \frac{[\rho^+(m)]^{N_m}}{\sqrt{N_m!}}\;|F\rangle \tag{4.31}$$

A general eigenvalue of H is then **obtained as**

$$\hat{H}|N_p,P,F\rangle = \epsilon|N_p,P,F\rangle \tag{4.32}$$

where

$$\epsilon = \epsilon_0 + \epsilon_F + \epsilon_P ;$$

$$\epsilon_P = \sum_{m=1}^{P} N_m \, \epsilon(m) . \qquad (4.33)$$

It can be seen from (4.24) that the ground state energy is finite and negative definite. The fact that the ground state energy of the interacting system is lower than the energy of the free vacuum state indicates the presence of bound states. Henceforth, we define energies with respect to the true ground state by dropping ϵ_0, i.e., we write

$$E = \epsilon - \epsilon_0 \qquad (4.34)$$

The redefinition of the ground state energy is possible since one is not forced to any particular value by such requirements as Poincare invariance. This is to be contrasted with the analogous situation in the conventional string model.

All of the analysis carried out in this section may be applied to the case of (3.7) as well. The results are the same; we merely have additional terms involving ψ^0. For example, the plasmon operator $\rho(p)$ defined in analogy to (4.8) now contains terms involving $(\varphi_\lambda^0 + \chi_\lambda^0)$ and $(\varphi_\lambda^{0+} + \chi_\lambda^{0+})$. This means that states of neutral partons pairs defined as in (4.26) will no longer be annihilated by $\rho(p)$. We must therefore modify our definitions of the ground state. We shall continue to assume the existence of a vacuum state $|0\rangle$, where with

$$\psi_0 \equiv \begin{pmatrix} \varphi_\lambda^0 \\ \chi_\lambda^0 \end{pmatrix} \qquad (4.35)$$

we demand

$$\varphi_\lambda^0 |0\rangle = \chi_\lambda^{0+} |0\rangle = 0 . \qquad (4.36)$$

The presence of such zero mode operators, a peculiarity of the case (3.7), implies an ambiguity in the definition of the ground state. We may define two distinct states

$$|0,0\rangle \equiv \varphi_0^{+\lambda} \chi_{\lambda 0} |0\rangle$$
$$|0,1\rangle_i \equiv \frac{1}{2} \varphi_0^{+\lambda} (\sigma_i)_{\lambda\mu} \chi_0^\mu |0\rangle , \qquad (4.37)$$

both of which satisfy condition (4.7). (The σ_i are 2×2 Pauli matrices.) However, only one of the states can be used to build up the "filled sea" states in analogy to (4.8). An explicit evaluation shows that while those "filled sea" states constructed from $|0,1\rangle_i$ are annihilated by $\rho(p)$, defined by (4.26), those based upon $|0,0\rangle$ are not the ground states of $\rho(p)$. Thus, the proper ground states are those obtained from $|0,1\rangle_i$, and we shall use these for the analysis of Poincare transformations later on.

Before closing this section, it is perhaps incumbent on us to comment briefly on the relation of our solutions to the TDED aspects of our problem, with more traditional solutions. If we throw away the extra SU(2) symmetry of the model and use only two component spinors, we have exactly TDED in a finite domain.

To make contact with the usual TDED in an infinite spatial domain, we remark that even for that problem, use of a finite domain may be viewed as an alternative to Klaiber's infrared regularization procedure, which is necessary for construction of operator solutions to TDED.[20b] We can construct operator solutions as well making use of our S operator, and these pass over in the continuum limit (in momentum) to the usual ones, provided periodic boundary conditions for ψ are adopted. What we have, then, is a solution which gives us a different insight into the physics of TDED, not a different solution to the field equations.

Of particular interest is that the ground state of H $(e^{-iS}|0\rangle)$ contains an indefinite number of quarks (though the number of anti-quarks is necessarily equal to the number of quarks). This is the prototype of the <u>conductive</u> medium (but not necessarily superconductive), and of course the physics of the model can be understood in those terms. In particular, quarks and anti-quarks do not pair off to make bosons localized in space. Rather, there are indefinite numbers of such correlated pairs, with one partner right-moving, the other left-moving. The ground state energy ϵ_0 reflects this correlated state is energetically favorable. We have said the fact ϵ_0 is lower than the non-interacting ground state energy indicates the presence of "bound states"; this is to be understood in the manner discussed here.

We should caution, however, that the fixed time mometum space representations of ψ, Eq. (3.13) or Eq. (3.17), and the analogous representations for the current, give rise to a consistent theory because in the end one obtains a c number Schwinger term in the $[j_0, j_1]$ commutator. That is to say, in other treatments, it is usually <u>assumed</u> that this Schwinger term is a c number, or it is argued this is consistent with free-field behavior at short distances.[25] Our fixed time

time expansions are, in some sense, the equivalents of these assumptions. Because all operator solutions of TDED have these assumptions built in, and TDED is super-renormalizable, it should come as no surprise that "deep inelastic probes" of TDED manifest free-field short distance behavior for the fermions at the end.[26] It is an open question whether different kinds of consistent solutions exist.

V. HILBERT SPACE OF PHYSICAL STATES AND POINCARE INVARIANCE

With eigenstates of $\hat{H}$ at our disposal, it is now a straightforward matter to obtain eigenfunctions and eigenvalues of the operator H. It can be seen from (4.19) and (4.20) that eigenstates of H can be obtained from those of $\hat{H}$ given by (4.32) as follows:

$$|\psi_H\rangle \equiv e^{-iS}\, |N_P, P, F\rangle . \tag{5.1}$$

Then,

$$H|\psi_H\rangle = E|\psi_H\rangle \tag{5.2}$$

It is to be noted that since transformations of the form (4.23) mix creation and destruction operators, they are in general not unitary in the sense that the norm of the transformed states might diverge. In the latter case the transformations connect two inequivalent Hilbert-spaces. In our case, however, the functions $\phi(k)$ given by (4.22) provide damping factors which render the norms of states (5.1) finite.

Our final task is the incorporation of Poincare generators into the model. As defined by (3.1), the model cannot yield such generators, since we had not defined the action of boosts and translations on ψ. The first step is to supply these definitions.

Now, in the classical string picture, the total motion of the system can always be separated into two parts. We have a center of mass motion, parametrized by a pair of variables X_μ and P_μ, the position and momentum; and a relative motion, parametrized by the normal mode oscillations. The variables X_μ and P_μ have definite quantum analogues, of course, and are dynamical variables. The normal mode oscillations, on the other hand, need have no definite quantum analogues. Our arguments regarding the relevant choice of variables applies to them. The action (3.1) must be regarded as only controlling the motion of such relative oscillations. To complete the picture, therefore, one must adjoin to the ψ variables, the pair of variables X_μ and P_μ which commute with ψ. The eigenvalues of P_μ will then be interpreted as the momentum of the physical states.

The normal mode oscillations in the classical picture control the mass of the physical system. Since we expect our picture to have a classical limit, we must demand that the eigenvalues of H, Eq. (4.10), are related directly to the mass of the quantum state. Since P_μ commutes with H (and with ψ), H and P_μ are simultaneously diagonalizable. The degeneracy of H represented by the existence of the SU_2 symmetry in our model may therefore be directly identified as the spin of the state.

To construct the Poincare generators, we make use of the formalism of null-plane dynamics.[27] We shall select as our "time" the variable X^+, and regard the transverse variables X^i, P^j, (i, j labelling the two transverse directions); and X^-, P^+ as canonical pairs. The spin operators of the theory are constructed by means of Noether's theorem applied to (3.1):

$$S^k \equiv \frac{1}{2} \int_0^\pi d\theta :\psi^+ \Sigma^k \psi: \qquad (5.3)$$

where

$$\Sigma^k = \begin{pmatrix} \sigma^k & 0 \\ 0 & \sigma^k \end{pmatrix}, \qquad (5.4)$$

σ^k being Pauli matrices. These operators are compatible with our constraint on physical states since $[Q,S^k] = 0$.

To determine the invariant $(\text{mass})^2$ operator

$$M^2 = 2P^+P^- - \vec{P}_\perp^{\,2} \qquad (5.5)$$

we shall make use of the only operator at our disposal which commutes with the rest frame SU(2) generators, namely, the Hamiltonian operator H:

$$M^2 = f(H) \qquad (5.6)$$

This procedure amounts to determining M^2 in the rest frame of the particle and demanding that the boost generators leave M^2 invariant. Note that insofar as one is dealing with free particle states any function of H is a possible candidate for M^2. Each such choice relates, via (5.5), the null-plane "Hamiltonian operator", P^-, to our dynamical spectrum. For definiteness, we shall set $M^2 = H$ as in the conventional string model. We thus have, as a special case,

$$P^2 = M^2 = H \qquad (5.7$$

where P^2 is the total C.M. momentum operator.

The full set of Poincare generators can now be written down following the work of a number of authors.[28,8] The translation generators are

$$\vec{P}^{\perp}, \qquad P^{+}, \qquad P^{-} = \frac{1}{2P^{+}} [\vec{P}_{\perp}^{\,2} + M^2] \ . \tag{5.8}$$

The Lorentz generators are given by

$$J^3 = \epsilon^{ij} X^i P^j + S^3 \ , \qquad\qquad i = 1,2 \tag{5.9}$$

$$K^3 = \frac{1}{2} \{P^{+}, X^{-}\} \tag{5.10}$$

$$B^j \equiv (K^j - \epsilon^{jk} J^k) = P^{+} X^j \tag{5.11}$$

$$\begin{aligned} C^j &\equiv (K^j + \epsilon^{jk} J^k) \\ &= \frac{1}{2} \{X^j, P^{-}\} + 2X^{-} P^j + \frac{1}{2P^{+}} \epsilon^{jk} [S^3 P^k + \sqrt{M^2}\, S^k] \end{aligned} \tag{5.12}$$

The operator $\sqrt{M^2}$ is well defined because the spectrum of H is positive definite. One can formally write down an integral representation for it:

$$[M^2]^{1/2} = \frac{2}{\sqrt{\pi}} M^2 \int_0^{\infty} dX \, e^{-M^2 X^2} \tag{5.13}$$

Such an operator also appears in the construction of the little group operators of the conventional string model. There, however, M^2 is not positive definite.

The correctness of the above algebra is straightforward but tedious to verify. Unlike the case of the ordinary dual models, there are no ordering problems which necessitate the increase in spatial dimensions or the appearance of the tachyon in the ground state. The construction of the generators in this manner must be subjected to the requirement that they retain their meaning in the presence of external probes. For example, a magnetic field is expected to split the levels which we label as J^3 degenerate.

Finally, we wish to remark on a possible alternative analysis. The basic motivations behind the present work dictate the construction we have presented. In this construction, ψ does not transform covariantly under the Lorentz group. The $\mathscr{L}$ action (3.1) is not defined directly in all of space-time. Now, naively, given any action, the most straightforward way of achieving any invariance is to demand that the dynamical variables transform covariantly under the group of invariance, and make sure that the action itself is a scalar. The relevant generators may then be constructed by use of Noether's theorem. The reader may well consider if one could sever all connections to the string, make the ψ's transform covariantly under the Poincare group, and come up with a spectrum which carries unitary representations of the Poincare group. To properly pursue this course, naturally, will involve the introduction of eight-component spinors, since one has to define conjugate spinors for both φ and X before one can write down invariants.

We have studied this problem, and have found that the resulting equations of motion are still soluble. However, the spectrum now contains ghosts, so that to complete the analysis, we must show that the spectrum of positive norm states is complete by itself. This may be accomplished by a detailed search for compatible constraint equations. We have investigated several relations of this sort, but have not yet reached any definitive conclusions on the feasibility of such an approach.

VI. THE SPECTRUM OF PHYSICAL STATES AND LEVEL DEGENERACY

The particle states are taken to be eigenstates of the Poincare operators labelling the state, i.e.,

$$|P^+, \vec{P}^\perp, M^2, J, \lambda\rangle = e^{ik\cdot X}\,|N_P, P, F\rangle \tag{6.1}$$

The operators P^+, $\vec{P}^\perp$, and M^2 are diagonal in this basis with eigenvalues k^+, $\vec{k}^\perp$, and m^2, respectively. If, as in (5.7), we set $M^2 = H$, then

$$m^2 = \epsilon_F + \epsilon_P \tag{6.2}$$

$$\epsilon_P = \sum_{m=1}^{P} N_m\, \epsilon(m) \tag{6.3}$$

Consider next the action of the other generators. To find the action of J^i, we note that

$$[J^i, \rho(p)] = 0\ , \qquad \text{all} \quad p\ . \tag{6.4}$$

This means that all plasmons are Lorentz scalar excitations, and the action of J^i on the states (5.14) depends entirely on the parton pair states $|F\rangle$. We can go to the rest frame of the states $e^{ik\cdot x}|F\rangle$

by making use of the Galilean boosts B^1 and B^2 and the mass scaling boost $\overleftarrow{K}^3$. In that frame

$$J^3|F_{ij}\rangle = \epsilon_{ij} F|F_{ij}\rangle \tag{6.5}$$

Other members of this multiplet are found by the application of the operators $J^{\pm} = J^1 \pm iJ^2$. We find

$$\begin{aligned} &J^+|F_{12}\rangle = 0 \\ &(J^-)^{2F+n}\,|F_{12}\rangle = 0\ , \qquad n = \text{integer} \geq 1 \end{aligned} \tag{6.6}$$

From these equations it is clear that the state of highest weight in this multiplet is $|F_{12}\rangle$ itself. Therefore, the spin of this multiplet is F.

$$\underset{\sim}{J}^2|F_{12}\rangle = F(F + 1)\ |F_{12}\rangle \tag{6.7}$$

That is, the eigenvalues of the square of the Pauli-Lubanski operator, $W_\mu W^\mu$, a Casimir invariant of the group, are

$$W^\mu W_\mu\ |F_{ij}\rangle = m^2\ F(F + 1)\ |F_{ij}\rangle \tag{6.8}$$

We thus find that for the particular case in which $M^2 = H$, we have

$$J = \sqrt{M^2} = F\ ; \qquad M^2 \equiv f(H) = H \tag{6.9}$$

Therefore in the J-S plane the trajectories are <u>indefinitely rising and parabolic</u>. To obtain <u>linearly rising</u> trajectories one must take

$$M^2 = C\sqrt{H} \tag{6.10}$$

where C is an appropriate dimensional constant.

The physical states corresponding to the boundary conditions (3.7) may be similarly constructed. However, the resultant trajectory function is shifted. Thus, since we used the state $|0,1\rangle_i$ defined in (4.37) as our ground state, our lowest physical state will be endowed with unit spin. The trajectory, while still parabolic, must now pass through spin one for its first state. The mass at which this occurs is arbitrary, since we may now set

$$M^2 = H + m_0^2 , \tag{6.11}$$

where m_0^2 is the mass squared of this spin one state. We shall explicitly set $m_0 \neq 0$ to avoid the presence of massless spin one states.

The arbitrariness in the relationship between M^2 and H can, in principle, be removed by examining the level degeneracy for large M^2. This may be determined by using familiar thermodynamic arguments for a Bose gas.[29] For a fixed eigenvalue of H corresponding to a fixed mass, the degeneracy is effectively the sum of the degeneracies involved in building up the state defined in (4.26), and the degeneracy implied by the free boson Hamiltonian given by the last term in (4.23). The latter degeneracy can be determined to be[26]

$$g(H)\, dH = \exp\{A\sqrt{H}\}\, dH \tag{6.12}$$

where A is a constant. Since there are F ways of forming the base state, F being the maximum spin for the fixed mass value, the total degeneracy is $Fg(H)\,dH$. Let us now suppose that

$$H = f[M^2] \ . \tag{6.13}$$

The total degeneracy, using (4.29), is

$$\begin{aligned} \sqrt{H}\, g(H)\, dH &= \sqrt{f}\, g'(m^2)\, \frac{df}{dm^2}\, dm^2 \\ &= \exp\left\{ A\sqrt{f} + \frac{1}{2}\ln f + \ln \frac{df}{dm^2} \right\} dm^2 \end{aligned} \tag{6.14}$$

If we used (6.9), $f = m^2$, the degeneracy is $\exp\{A\sqrt{m^2} + \frac{1}{2}\ln m^2\}$, while for (6.10), it would be $\exp\{A\sqrt{M^4} + \frac{1}{2}\ln M^4 + \ln M^2\}$, which is larger than the conventional beta function degeneracy. Choice (6.9) gives a Hagedorn spectrum asymptotically, and is thus perhaps preferable to (6.10).

VII. CONCLUSIONS AND SPECULATIONS

Our aim in this work was to give a Poincare invariant description of particles with internal structure. To motivate our work, we reexamined the procedure which leads to undesirable features of the quantized relativistic string model. We pointed out that in the description of a one-dimensional object in space-time there is no compelling reason to consider the string variable as a fundamental dynamical variable. To find physically more relevant dynamical variables, we exploited the intimate connection which exists between null-vectors and spinors in 4-dimensional Minkowski space. In this way we arrived at the description of a one-dimensional particle with internal structure in terms of a spinor gauge theory in a finite spatial domain of the Minkowski space.

To make our ideas more transparent, we constructed an exactly soluble model and worked out its consequences in detail. In addition to being Poincare invariant, ghost free, and tachyon free, this model has a number of other novel features. Its fermionic constituents give rise to "quasi-particle" excitations which constitute the free spin 1/2 partons of our model. Furthermore, because of a charge neutrality constraint which arises from a consistency requirement, only pairs of fermionic constituents can create Poincare states. The model is thus endowed with a dynamical quark trapping mechanism. It is also interesting to note that in this model the string itself, instead of being a mechanical quantity which is identified with the particle, appears as a well-defined, occupied state of the fermionic constituents.

To put the approach that we advocate in its broadest perspective, we wish to elaborate on general features which might be abstracted from our specific considerations. The first point we re-emphasize is that the one-dimensional nature of our treatment reflects the string picture, which in turn idealizes the notion of planar-diagram dominance when the number of internal vertices becomes very large.[30] Nevertheless, the physical hadron sits in four-dimensional Minkowski space.

However, it has been noted elsewhere that while this severe planar approximation may do no violence to basically "soft" physics, such as in Regge limits to scattering, it can give rise to totally misleading results when the process involves more local probes, as in deep inelastic ep scattering.[31] The reason for this is, of course, that the nature of the singularities in Green's functions changes drastically in going from one to three spatial dimensions. It is sensible, therefore, to relax our strict adherence to a one-dimensionally extended object.[31]

One possible way to view a composite hadron extended in any number of space dimensions is as a container trapping fields inside.[32] Another possible way is to notice that some classical field theories exhibit auto-trapping, without introducing a "bag" at all.[33]

A suggestion to be gleaned from our work relates to the very interesting latter cases. The freedom to perform contact transformations in the classical case may help in the solution of a problem by simplifying the equations of motion. Cannonical transformations play a similar role in quantum mechanics, but there is no correspondence between the classical and quantum transformation in any one-to-one fashion for a given problem. We have seen that one obtains different quantum theories by expressing the classical theory in terms of different variables. The possibility is present, we argued, that one version of a classical theory may lead to a consistent quantum theory, while another leads to an inconsistent quantum theory. We speculate that the possibility is also open that more than one version of a classical theory may give consistent, though perhaps inequivalent, quantum theories.[34] Needless to say, this goes beyond anything we can prove from examining a simple model.

A different feature of our model which is worth mentioning again is the fact our containment mechanism arose entirely as a consequence of an internal consistency condition on the theory, at the level of describing the single free particle with internal structure. We should like to suggest absolute containment is so strong a requirement on a theory that detailed dynamical calculations should not be needed to demonstrate the requirement is satisfied or broken by the particular theory. The question may be one of internal consistency. A necessary prerequisite is to decide upon a suitable mathematical statement of confinement.

Finally, as mentioned in the Introduction, one can view the description of free particles with internal structure as a problem in the representation theory of the Poincare group. All the unitary irreducible representations of the Poincare group which are insensitive to the structure of particles are contained in the classic work of Wigner.[35] The manifold on which these representations are constructed does not allow for any operation which would reflect the structure or the internal symmetry of the constituents of the particles. The internal symmetry would then have to be realized in a separate Hilbert space. Realizations of the unitary irreducible representations of the Poincare group on a manifold which allows internal symmetry operations have recently been studied by Günaydin and Gürsey.[36] In particular, they have constructed all the unitary irreducible representations of the Poincare group in an octonionic Hilbert space which allows SU(3) symmetry operations. These Poincare states have the interesting property of being SU(3) color singlets. The appearance of the selection rule in this case can be traced back to the non-associativity of the octonion algebra. In the same spirit, the Poincare states that we have constructed in the manifold of the solutions of our explicit model can be viewed as new unitary representations of the Poincare group for particles with internal structure. Since in our case the selection rule has dynamical origin, we succeed in making a detailed study of the spectrum of states. Our approach thus provides a general method of looking for unitary representations of the Poincare group relevant to particles with internal structure.

We have given little attention to a number of important topics in this work. Among these are the problems of incorporating interactions and realistic internal symmetry groups. We hope to return to these topics in future publications.

ACKNOWLEDGMENTS

In the course of this work one or more of us benefited from discussions with F. Ardalan, G. Domokos, S. Drell, F. Gürsey, L. F. Li, S. MacDowell, S. Mandelstam, Y. Nambu, C. Sommerfield, and C. Thorn. Three of us (C.E.C., L.N.C., F.M.) would like to thank S. Drell, J. D. Bjorken, R. Blankenbecler, and S. J. Brodsky for their kind hospitality at SLAC during the summer of 1973, where this research was initiated.

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FIGURE CAPTIONS

Figure 1. Mass-spin towers in the model with $M^2 = H$ and under conditions (3.6). Arrow (a) represents a double excitation of the plasmon with mode number 1 on the parton-pair sea, with $\mu^2 \ll 1$. Arrow (b) represents a single excitation of the second plasmon mode.

Figure 2. Mass-spin towers in the model with $M^2 = H + m_0^2$ and under conditions (3.7). Arrows (a) and (b) have same significance as figure 1. The lowest state has spin 1 and mass m_0.

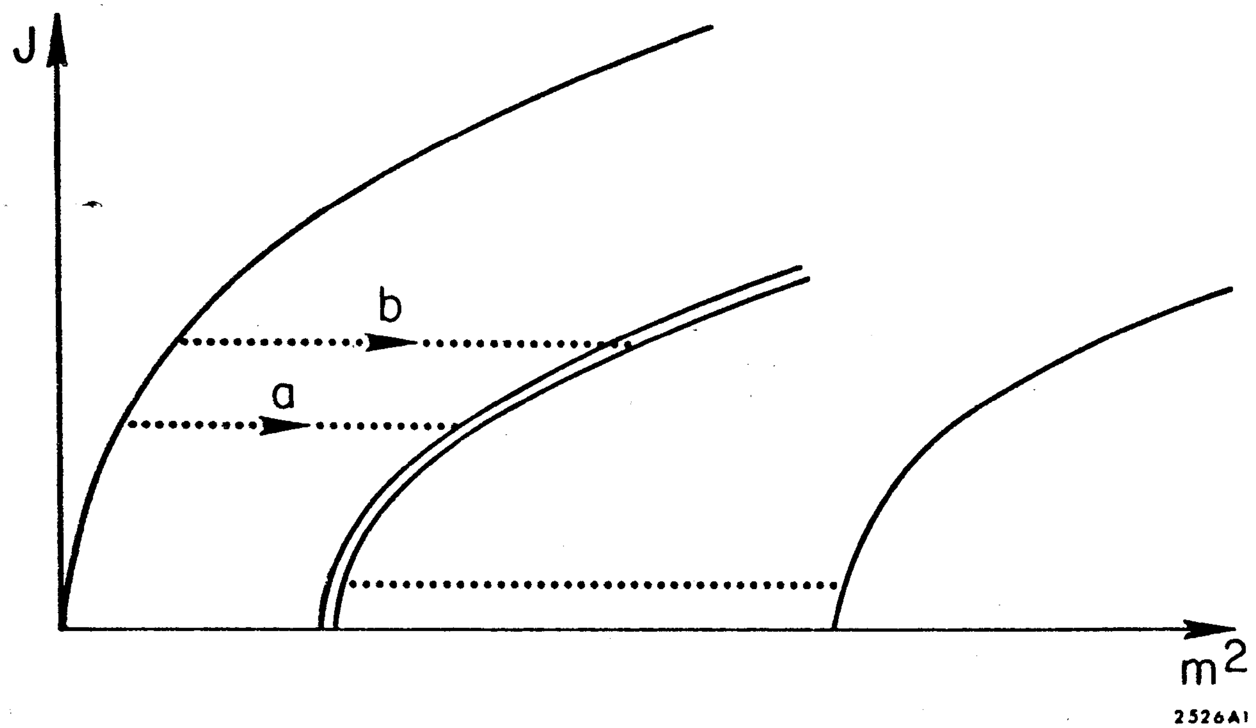

Fig. 1

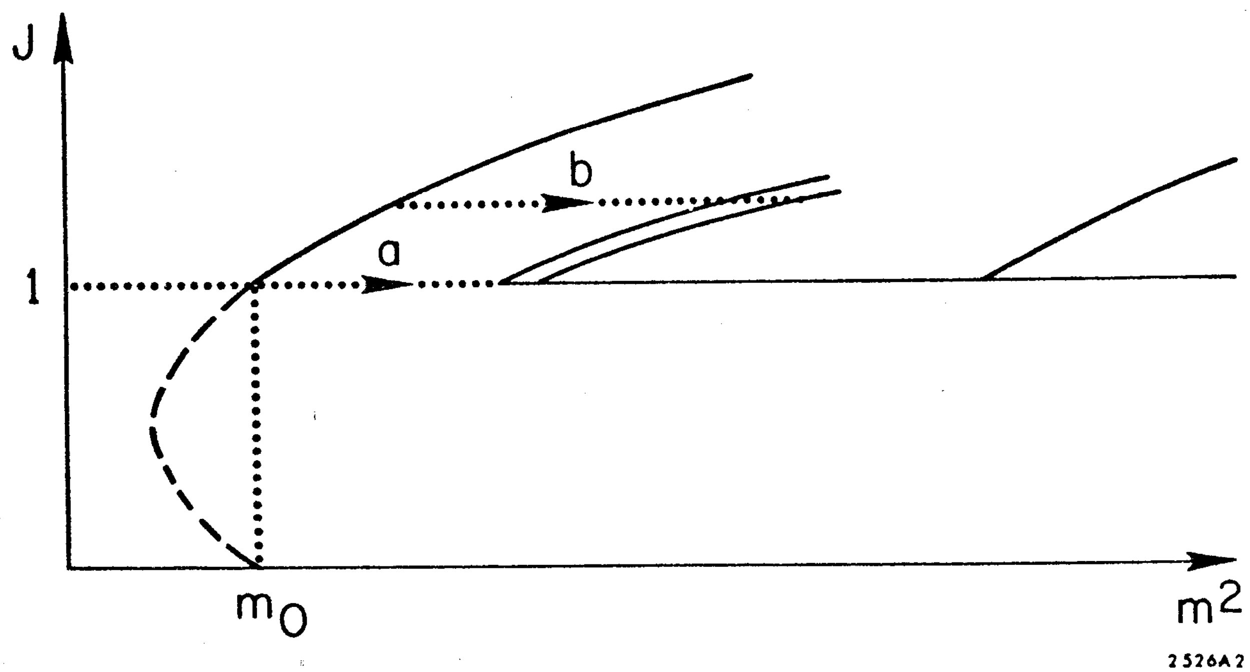

Fig. 2