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RELATION BETWEEN e^+e^- ANNIHILATION INTO HADRONS AND HADRONIC μ-PAIR PRODUCTION*

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ABSTRACT

We propose a relation between the dependence of the differential cross section for $pp \to \mu\mu + X$ on the di-muon mass $\overline{Q^2}$, and the dependence of the total cross section for $e^+e^- \to$ hadrons on the total c.m. energy $\bar{s}$ in the form of a factorization rule. This rule is justified as an extension to the case of a more general di-lepton-di-parton vertex of a factorization rule which is trivially satisfied in the Drell-Yan parton picture of $pp \to \mu\mu + X$ with one-photon mediation of that vertex. The available data from SPEAR and BNL, including the scale-breaking signal, are consistent with this relation. More stringent tests of the relation at higher energies are also considered.

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Spurred by the recent data from the SPEAR-CEA experiments[1] indicating a rise in the ratio $R(s) = (\sigma^{e^+e^- \to h} / \sigma^{e^+e^- \to \mu^+\mu^-})$ for c. m. energies $\sqrt{s}$ greater than 3 GeV, a number of authors have proposed models that attempt to accomodate these surprising results which seem to point at the presence of a new scale or scale-breaking phenomena in lepton-hadron interactions. For some time now, there has also been concern[2,3] generated by the presence of an unexpected "shoulder" (at 3-4 GeV) in the di-muon mass distribution data from the BNL experiment[4] on $pp \to \mu\mu + X$. In this note, we wish to suggest that these two puzzles, which have encouraged some to reject the parton model, may be one and the same. This is not another attempt to solve either puzzle[5]; rather it is an attempt to relate the data of these experiments among themselves in as model independent a way as possible and to do so without sacrificing any of the fundamental principles of the parton picture (impulse approximation[6]). A (presumably complicated) relation may be expected on general grounds,[7] but we give it a simple, explicit form and justification in the parton model approach of Drell and Yan to hadronic μ-pair production[3]. The relation takes the form of a factorization rule connecting the s-dependence of the $e^+e^- \to h$ cross section to the dependence of the differential cross section for $pp \to \mu\mu + X$ on the invariant mass-squared of the μ-pair at fixed total pp energy. We show it to be consistent with the available data[1,4] and describe what may be expected at NAL and ISR. Thus, it would appear that the surprise associated with the discovery of a scale-breaking effect in e^+e^--annihilation[1] had actually been anticipated in hadronic μ-pair production experiments.

Paralleling Drell and Yan, we shall view the pp $\rightarrow \mu\mu + X$ process as the collision of two beams of partons and anti-partons with distribution functions determined by deep inelastic lepto-production scaling functions wherein a parton from one beam and an anti-parton from the other annihilate into a μ-pair, as shown in Fig. 1. Neglecting fermion masses, the resulting cross section, differential in the di-muon mass, and in the longitudinal momentum fraction of the μ-pair, $\xi = 2Q_3/\sqrt{s'}$, where s' is the squared c. m. energy of the p-p system, is given by[8]:

$$\frac{d\sigma}{dQ^2 d\xi} = \sum_i \sigma_i(Q^2) \frac{1}{Q^2} \frac{xy}{x+y} \left[f_i(x) \bar{f}_i(y) + f_i(y) \bar{f}_i(x) \right]. \tag{1}$$

Here, $x = \frac{1}{2}(\xi + \sqrt{\xi^2 + 4\tau})$, $y = \frac{1}{2}(-\xi + \sqrt{\xi^2 + 4\tau})$, $\tau = Q^2/s'$, $\sigma_i(Q^2)$ is the integrated total cross section for the annihilation of a parton and anti-parton of type i into a μ-pair, and $f_i(\bar{f}_i)$ is the probability distribution of partons (anti-partons) of type i in a proton[6]. Note that the total $e^+e^- \rightarrow$ hadrons cross section is

$$\sigma^{e^+e^- \rightarrow h}(s) = \sum_i \sigma_i(s = Q^2) \tag{2}$$

in a "parton model". We must emphasize, at this point, that we are taking the σ_i's to be phenomenological quantities which we do not attempt to calculate in terms of a one-photon mechanism, as in Ref. 3, or any other detailed model[5].

Our factorization rule

$$d\sigma/dQ^2 d\xi \simeq Q^{-2} \sigma^{e^+e^- \rightarrow h}(Q^2)\, G(\tau, \xi), \tag{3}$$

where the definition of $G(\tau, \xi)$ in terms of lepto-production scaling functions is implicit, derives from Eqs. (1) and (2) if any of the following conditions is satisfied:

a) $\sigma_i(Q^2) \simeq \eta_i \sigma^{e^+e^- \to h}(Q^2)$ where η_i is a constant independent of Q^2. This is precisely the case in the approach of Drell and Yan[3] where the parton-anti-parton annihilation proceeds via one photon and η_i is the squared electric charge of partons of type i.

b) $\sigma_i >> \sigma_{j \neq i}$ and $f_{j \neq i} \lesssim f_i$, $\bar{f}_{j \neq i} \lesssim \bar{f}_i$, i. e., one parton-type (i) dominates the annihilation cross section and is, at minimum, about as likely as any other to be found inside the proton.

c) The quantity in square brackets in Eq. (1) is approximately the same for all i. This could certainly not be true in general as the parton distributions inside the proton "sense" the proton quantum numbers[6]. However, it will be true approximately when τ and ξ are sufficiently small so that x and y are restricted to be near zero, i. e. in the region corresponding to the symmetric sea of wee partons[6] ($f_i \simeq \bar{f}_i \simeq f_j$). The connection of the low x region to the Regge (Pomeron-dominated) limit ($\omega \to \infty$) of the scaling functions in lepto-production then enables us to make quantitative statements about the onset of symmetry among the f_i's. The analysis of Ref. 9 of the electroproduction data suggests symmetry between the proton and neutron scaling functions to within 30% for $x < x_0 \sim 0.3$. Consequently, if we were to abstract from these data a general pattern of approach to Pomeron-dominance for anti-parton as well as parton distributions[10] we would conclude that in order to obtain factorization as in Eq. (3) to within 30% it should be sufficient (although perhaps not necessary) to restrict the kinematic domain to:

$$|\xi| \lesssim (x_0 - \tau/x_0) \simeq 3(0.1 - \tau). \qquad (4)$$

Thus, at ISR (NAL), where $s' \sim 1,600\,\mathrm{GeV}^2$ ($800\,\mathrm{GeV}^2$), for $q^2 \lesssim 40\,\mathrm{GeV}^2$, $\tau \lesssim 0.025$ (0.050), and a cut on the longitudinal μ-pair momentum of $|\xi| \lesssim 0.23$ (0.15) would be sufficient to observe the proposed factorization. We wish to remark that this is not really an additional constraint on the experiment since one prefers to look at wide angles (in c. m.) in any case to reduce the accidental rate[4]. On the other hand, in the BNL experiment τ ranged up to ~ 0.3 and the condition given by Eq. (4) could not possibly have been met. Therefore, we must depend upon alternative conditions, such as condition a) or b), to be able to invoke factorization at these energies.

How do we test Eq. (3) ? The most obvious and direct test is clearly to compare $Q(d\sigma/dQ\,d\xi)$ with $\sigma^{e^+e^-\to h}(Q^2)$ at fixed τ and ξ. Such a test requires μ-pair data at significantly different values of s'. Although such data will be available from NAL and ISR, the range of s' values from the BNL experiment is insufficient for this purpose. It is therefore necessary to make supplementary assumptions about the τ-dependence of $G(\tau, \xi)$ if we are to be able to use the BNL data to test Eq. (3) (indirectly). If τ and ξ are sufficiently restricted, it may be reasonable to approximate $G(\tau, \xi)$ as (const.) $\times\, \tau^{-p/2}$. Then, Eq. (3) takes the form

$$Q^{p+1}(d\sigma/dQ\,d\xi) \cong (\text{const.}) \times \sigma^{e^+e^-\to h}(Q^2) \qquad (5)$$

at fixed s'. Let us tentatively assume that $p \cong 2$ with the experimental cuts on τ and ξ which are typical of the BNL experiment. This choice may have some theoretical support[11] and can be justified a posteriori on empirical grounds as leading to consistency between the shapes of the SPEAR-CEA and BNL data via our relation (Eq. (5)). With this assumption, $Q^5(d\sigma/dQ)^{pp\rightarrow\mu\mu+X}$ and R would both be constant if the di-parton-di-lepton vertex were one-photon mediated. A "scale-breaking signal" would then appear as a deviation from constancy.

In Fig. 2 we have plotted $R\,(s = Q^2)$ from several colliding beam experiments[1,12] and $Q^5\,(d\sigma/dQ)^{pp\rightarrow\mu\mu+X}$ at $s' = 56\ \mathrm{GeV}^2$, in arbitrary units, from the BNL experiment versus $Q\ (\equiv\sqrt{Q^2})$ in GeV. The BNL data appear as an error band centered on the data points with a one-standard-deviation half-width[13]. The fall-off in the BNL data beyond $Q \sim 4.1$ GeV occurs because such high μ- pair masses push the process up against its phase-space boundaries as can be seen from a simple application of energy-momentum conservation or by a comparitive analysis of the $s' = 42,\ 48\ \mathrm{GeV}^2$ data[5]. In fact, because of this cut-off effect, the comparison between the SPEAR-CEA and BNL data points beyond $Q \sim 3.8$ GeV ceases to be meaningful.

The BNL data is actually flat for $3\ \mathrm{GeV}^2 < Q^2 < 9\ \mathrm{GeV}^2$ and supports the assumption that $p \cong 2$ in this range. In this same region, the colliding beam data are also consistent with a constant R. As our relation is somewhat trivial when one-photon mediation is a good approximation for these processes, it is thus no surprise that the relation is also satisfied in this regime. However, it is remarkable to find that the onset of the scale-breaking signals in the two processes occurs (within experimental uncertainties) at the same value of $Q^2 = s$, and that both these signals develop in a similar manner. Thus we observe that our relation is consistent with the data over the entire (usable) Q^2 range, including the "scale-breaking part" of the data. This observation reinforces our confidence in the approach which led to Eq. (3), which is based on the assumption that the "new physics", that appears as a new scale or scale-breaking phenomena, resides in the di-parton-di-lepton vertex rather than anywhere else (e. g. – the parton description of hadrons itself).

It is especially important to test our relation against the forthcoming NAL-ISR μ-pair data, in view of the additional phase space available. The availability of data with higher Q^2 and s' values from these experiments will enable us to predict the s-dependence of $\sigma^{e^+e^-\to h}(s=Q^2)$ at SPEAR II in terms of $(d\sigma/dQ\,d\xi)^{pp\to\mu\mu X}$ by means of Eq. (3) for fixed τ and ξ. Conversely, given $G(\tau,\xi)$ in terms of the (anti-)parton distributions (whether determined theoretically or experimentally), the annihilation data will predict the behavior of the μ-pair cross section in all of its variables. Note that in view of our relation and the unexpected character of the annihilation data, it becomes necessary to divide out the experimental annihilation cross section to sensibly abstract $G(\tau,\xi)$ from the μ-pair experiments. We conclude by suggesting that, for the purpose of the testing our relation against these experiments, it may again be convenient (if not any longer necessary) to approximate $G(\tau,\xi)$ as (const.)$\times\tau^{-p/2}$ and use our relation in the form of Eq. (5) for properly restricted ranges of τ and ξ values. Thus, for values of ξ in the cut range given by Eq. (4), $G(\tau,\xi)$ will only depend upon the Regge-determined behavior of $f_i(x)$ $(\bar{f}_i(x))$ near $x=0$. If also $|\xi| < \tau$, Pomeron-dominance implies $p \cong 1$ and that the normalizing constant on the r.h.s. of Eq. (5)

increases as $(s')^{\frac{1}{2}}$.

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3. S. D. Drell and T.-M. Yan, Ann. Phys. (N. Y.) 66, 578 (1971) and references therein.

4. J. H. Christenson et al., Phys. Rev. D 8, 2016 (1973)

5. See, e.g., T. Goldman and P. Vinciarelli, SLAC-PUB-1407 (1974).

6. R. P. Feynman, Photon-Hadron Interactions, Benjamin, Reading, Mass. (1972).

7. Given a trivial form of μ-e universality, each of the (non-interfering) amplitudes for $e^+e^- \rightarrow pp + X_i$ and $pp \rightarrow \mu^+\mu^- + X_i$, where X_i is any specific i-particle state, are related by crossing and analyticity. Furthermore, at sufficiently high energies, every hadronic final state in the e^+e^--annihilation may reasonably be expected to contain at least two protons (due to an increasing mean multiplicity). Hence, the total e^+e^--annihilation cross section may be calculated (at least for energies asymptotic with respect to the thresholds for 2p-production) by integration over the appropriate phase spaces of the modulus squared of the same analytic functions that appear in the calculation of $(d\sigma/dQ)^{pp \rightarrow \mu\mu X}$ for the μ-pair production process, where Q is the di-muon mass. Note that the momentum variable Q^2 directly corresponds to the squared c.m. energy variable s.

8. G. R. Ferrar, Cal. Tech. preprint CALT-68-422 (1974).

9. R. A. Brandt, M. Breidenbach and P. Vinciarelli, Phys. Letters 40 B, 495 (1972).

10. Note that this tends to generate a somewhat larger value for the ratio of anti-neutrino to neutrino total cross section than is indicated by the presently available data, i.e., $\sigma^{\bar{\nu}}/\sigma^{\nu} \sim .46$ instead of the experimental $.38 \pm .05$, unless the anti-parton distributions in nucleons are very strongly suppressed for higher x-values; see , e.g., Ref. 8 or H. Paar and E. Paschos, NAL-PUB 74/29-THY. However, such a suppression implies that there are not enough "hard" anti-partons to account for the magnitude of the BNL data. See also Ref. 11.

11. Drell and Yan (see Ref. 3) obtained this behavior as an approximate result by evaluating their scaling function $f(\tau) = \int d\xi \; G(\tau, \xi)$ including experimental cuts. They used some unusual assumptions about the anti-parton distributions and parton squared charges ($\sim \frac{1}{2} - 1$), where the latter was the result of normalizing to the magnitude of the BNL data. We note, however, that their results are also obtained if, more reasonably, $\bar{f}_i \cong 1/10 \times f_i$ for higher x-values and quark squared charges $(1/9 - 4/9)$ are used.

12. Detailed SLAC-LBL results for R have not yet been released but the data points fall within the shaded area in Fig. 2.

13. The uncertainties are accentuated as we are multiplying the data for $(d\sigma/dQ)^{pp \rightarrow \mu\mu X}$ by Q^5 and the experimental uncertainty in Q was $\approx .2$ GeV. Thus for $Q \approx 3$ GeV, the additional percentage error introduced is $\approx 30\%$.

FIGURE CAPTIONS

1. Parton model diagram for $pp \rightarrow \mu\mu + X$.

2. Comparison of the dependence of the differential cross section for $pp \rightarrow \mu\mu + X$ on the di-muon mass $\sqrt{Q^2}$ with the dependence of the total cross section for $e^+e^- \rightarrow$ hadrons on the total c. m. energy $\sqrt{s} \equiv \sqrt{Q^2}$.

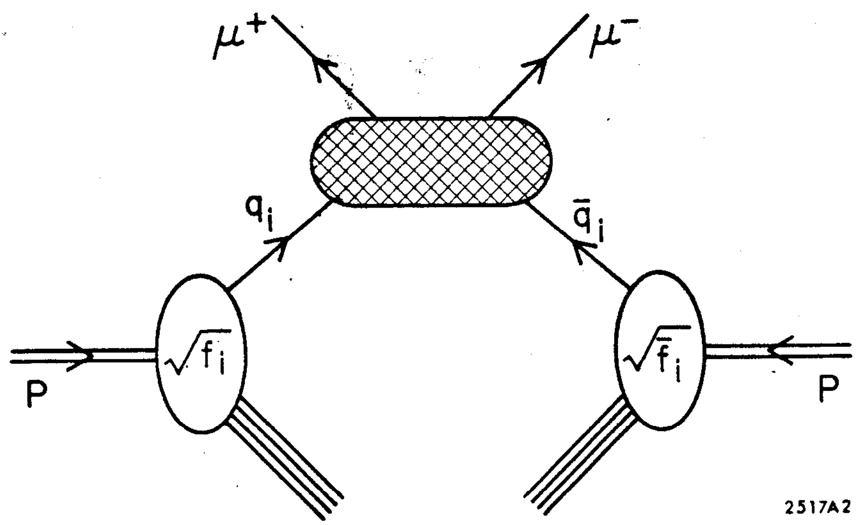

FIG. 1

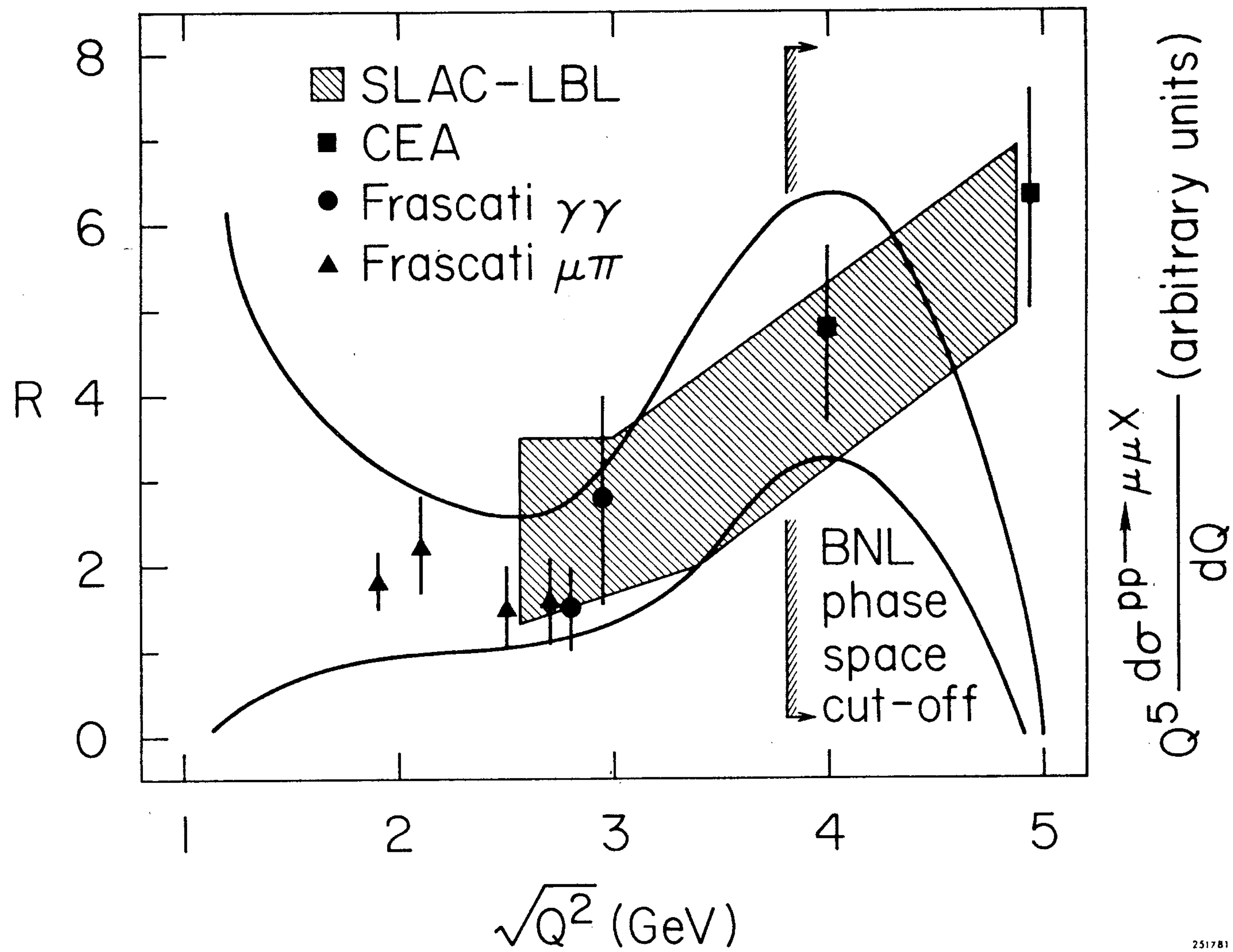
SLAC-LBL
CEA
Frascati $\gamma\gamma$
Frascati $\mu\pi$
R
8
6
4
2
0
1
2
3
4
5
$\sqrt{Q^2}$ (GeV)
BNL phase space cut-off
$Q^5 \frac{d\sigma^{pp\to\mu\mu X}}{dQ}$ (arbitrary units)
251781

FIG. 2