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WIDE ANGLE BEHAVIOR OF A DOUBLE SCATTERING DIAGRAM*

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ABSTRACT

Landshoff has recently pointed out that in composite models for hadron-hadron wide angle scattering, processes where the constituents remain close to their mass shells throughout the scattering can dominate the wide angle cross sections. We use a Feynman parametric integral to estimate the phase space for such a process and confirm Landshoff's result. The differential cross section for elastic π-π scattering at fixed c.m. angle is found to fall off as s^{-5}, rather than as s^{-6} which is predicted by the dimensional counting rules.

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Recently Landshoff[1] has shown that the composite hadron models for wide angle elastic scattering allow a double scattering process which violates dimesional counting rules[2]. The fixed angle limit ($s \to \infty$ at constant t/s) of Landshoff's amplitude for $\pi\pi$ scattering is $Cs^{-3/2}$, rather than Cs^{-2} predicted by the dimensional counting rules. Landshoff has derived this result using Sudakov variables[3]. As it is not clear whether it is legitimate to take the fixed angle limit before any of the integrals over Sudakov variables are performed, we rederived his result by more conventional Feynman-parametric method. Here we show how to simply estimate the fixed angle limit of Landshoff's amplitude. The exact calculation is given in the Appendix.

Landshoff's process in its simplest form[4] is given by the Fig.1. The internal (pseudo)scalar elementary constituents have mass $\mu^2 \ll s$, and couple to each other through a 4-point bare vertex. As long as $\mu^2 \gg m^2$, in the fixed angle limit the pion mass can be neglected. This diagram is fully symmetric in s,t and u, where

$$s = (p+p')^2 , \quad t = 4q^2 , \quad u = (p-p')^2 \qquad (1)$$

$$s + t + u = 0 \tag{2}$$

and we will preserve this symmetry throughout our calculation. The amplitude is proportional to [5]

$$M = \int \frac{dz_G \, \delta(1 - Z_G)}{U^2 V^2} \tag{3}$$

where $dz_G = \prod_{i=1}^{4} dz_i \, dz_{i'}$, $z_G = \sum_{i=1}^{4} Z_i$

$$Z_i = z_i + z_{i'}$$

$$U = Z_1 Z_2 Z_3 + Z_1 Z_2 Z_4 + Z_1 Z_3 Z_4 + Z_2 Z_3 Z_4 \tag{4}$$

$$V = \mu^2 - G \tag{5}$$

In calculating G it is convenient to isolate the dependence on the external momenta in new variables x_i defined by

$$z_i = Z_i(\tfrac{1}{2} + x_i)$$

$$z_i = Z_i(\tfrac{1}{2} - x_i), \qquad -1/2 \leq x_i \leq 1/2 \tag{6}$$

Method of Ref. 5 or Symanzik rules[6] lead quickly to

$$G = \frac{\mathcal{Z}}{\mathcal{U}} \left[(x_1 x_4 + x_2 x_3) s + (x_1 x_2 + x_3 x_4) t + (x_1 x_3 + x_2 x_4) u \right] \tag{7}$$

where we have repeatedly used (2) and

$$\mathcal{Z} = Z_1 Z_2 Z_3 Z_4$$

Because of (2) G vanishes whenever any three x_i are equal. This can be understood in terms of the electric network analogy[7]; $x_i = x_j = x_k$ means that the external lines attached to i,j,k, are equipotential with respect to the remaining external line (See Fig.2), and G must depend symmetrically on s,t, and u in this region of the parametric space. The only symmetric combination linear in s is s+t+u=0. Hence, no matter how large are s and t, there will always be a region of x-space where the integrand is finite (of order μ^{-4}). This phase space is characterized by the proximity of any three x_i , and the fourth x_j can be essentially removed from the problem by the redefinition

$$x_4 = X$$
$$x_i = X + y_i \qquad i=1,2,3 \tag{8}$$

In the new variables the amplitude (3) is given by

$$M = \int \frac{dZ_1 \cdots dZ_4}{\mathcal{Z}} \delta(1 - \sum_{i=1}^{4} Z_i)\, I$$

$$I = \int_{-1/2}^{1/2} dX\, J$$

$$J = \int_{-1/2-X}^{1/2-X} dy_1\, dy_2\, dy_3 \frac{1}{[\bar{\mu}^2 - (y_2 y_3 s + y_1 y_2 t + y_1 y_3 u)]^2}$$

$$\bar{\mu}^2 \equiv \frac{U}{\mathcal{Z}} \mu^2$$

and all s, t, and u dependence is contained in the three dimensional integral J. (An exact evaluation of J is given in the Appendix). In s-channel the denominator has form

$$\bar{\mu}^2 - y_2 y_3 s + y_1 y_2 |t| + y_1 y_3 |u| \tag{10}$$

and it vanishes along a two-sheet hyperboloid in y-space. (There the integration contour for J is defined by the Feynman prescription $\mu \to \mu - i\varepsilon$). We are always able to go to a new set of y-axis for which (10) diagonalizes. Such a transformation is explicitely carried out in the Appendix; We simply note that it introduces a finite constant Jacobian c, and even though the new integration volume has a complicated boundary, this is unimportant, because the leading contribution arises from the region of y-space close to the origin. Now J is given by

$$J = c \int_V dy_1\, dy_2\, dy_3 \frac{1}{[\bar{\mu}^2 - i\varepsilon + s y_1^2 + |t| y_2^2 - |u| y_3^2]^2} \tag{11}$$

As the poles of (11) in the complex y -plane are in the second and fourth quadrants, and the integrand away from the origin can be neglected in the fixed angle limit, we can Wick rotate the y -axis, obtaining

$$J = ic \int_V dy_1\, dy_2\, dy_3 \frac{1}{[\bar{\mu}^2 + s y_1^2 + |t| y_2^2 + |u| y_3^2]^2} \tag{12}$$

Within the ellipsoid given by the equation

$$\bar{\mu}^2 = \frac{y_1^2}{s^{-1}} + \frac{y_2^2}{|t|^{-1}} + \frac{y_3^2}{|u|^{-1}} \tag{13}$$

the integrand of J is of order $\bar{\mu}^{-4}$, while outside it vanishes rapidly ($s \gg \bar{\mu}^2$), so that J is proportional to the volume of the ellipsoid (13):

$$J \sim \frac{\bar{\mu}^3}{\sqrt{stu}} \frac{ic}{\bar{\mu}^4} \tag{14}$$

Consequently, the fixed angle limit of the amplitude M is

$$M = \frac{C}{\mu\sqrt{stu}}$$

where $C \sim ic\int dZ_1 \ldots dZ_4\, \delta(1-\Sigma Z_i)(ZU)^{-1/2}$ is a finite constant, in agreement with Landshoff.

A more realistic model, such as the spin-half quarks plus scalar gluons used by Landshoff reduces to (3) in the fixed angle limit because the s^{-2} arising from the gluon propagators is canceled by the s^2 from the fermion traces. However, in such a model we also have to insure the convergence of the overall amplitude by introducing form factors for the external pion vertices.

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I would like to thank Stan Brodsky for stimulating discussions of wide angle scattering and for bringing the above problem to my attention. His unpublished calculation of the above process in the infinite momentum frame provides still another confirmation of Landshoff's result. I would also like to thank Terry Goldman for reading and correcting the manuscript.

APPENDIX

Here we shall evaluate exactly the fixed angle limit of the integral J given by (9). The form of the denominator (10) suggests scaling

$$y_1 \to \sqrt{\frac{s}{tu}}\; y_1$$

$$y_2 \to \sqrt{\frac{u}{st}}\; y_2$$

$$y_3 \to \sqrt{\frac{t}{su}}\; y_3$$

In the $s \to \infty$, fixed t/s, limit the integration in J in terms of the scaled variables goes over all space. (If $X = \pm 1/2$, the integration is restricted to one octant of y-space. This integral can also be evaluated, but as it makes no contribution to the integral I in (9), we shall not evaluate it here). Hence the J of interest is given by

$$\sqrt{stu}\; J = \int_{-\infty}^{\infty} d^3y \; \frac{1}{[\,\bar{\mu}^2 - i\varepsilon - y_2 y_3 + y_1 y_2 + y_1 y_3\,]^2}$$

Next we diagonalize the denominator by the change of variables

$$\bar{y}_3 = 2^{-1/2}(-y_1 + y_2 + y_3)$$

$$\bar{y}_2 = 2^{-1/2}(y_2 - y_3)$$

$$\bar{y}_1 = 2^{-1/2} y_1$$

obtaining

$$\sqrt{stu}\, J = \sqrt{2} \int d^3\bar{y} \frac{1}{[\bar{\mu}^2 - i\varepsilon + \bar{y}_1^2 + \bar{y}_2^2 - \bar{y}_3^2]^2}$$

In the y_3-plane, the poles lie in the second and fourth quadrants. Performing a Wick rotation, we obtain a spherically symmetric integrand. Straightforward integration yields

$$J = \frac{i\sqrt{2}\,\pi^2}{\bar{\mu}\sqrt{stu}}$$

This leads to the following exact expression for the amplitude M from Eq.(3);

$$M = \frac{1}{\mu\sqrt{stu}}\; i\pi^2\sqrt{2} \int_0^1 dZ_1 dZ_2 dZ_3 dZ_4 \frac{\delta(1 - \sum_i^4 Z_i)}{\sqrt{ZU}}$$

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see also ref. 5 and references therein.

FIGURE CAPTIONS

1. A model of π-π wide angle scattering.

2. An electric circuit analogy of the region in the Feynman-parameter space for which the three external legs are "equipotential". Amplitude should depend on s,t, and u symmetrically.

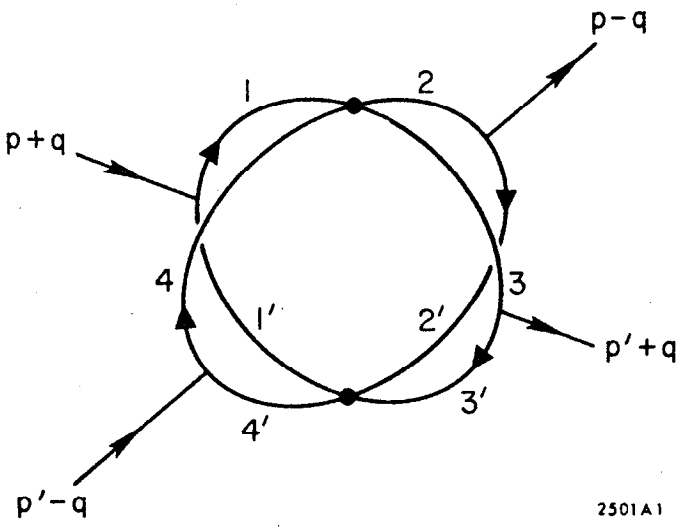

Fig. 1

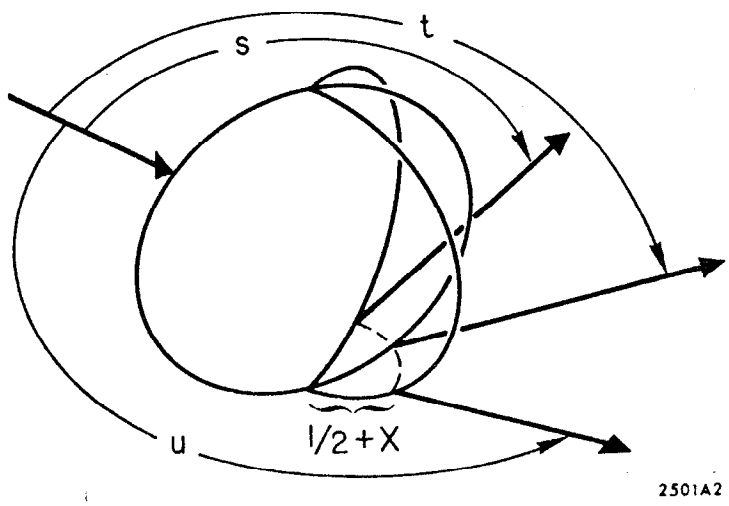

Fig. 2