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NON-LOCALITY IN PARTICLE PHYSICS*

Pierre Noyes

Stanford Linear Accelerator Center
Stanford University, Stanford, California 94305

A chapter for

Revisionary Philosophy and Science

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I. INTRODUCTION

The quantum mechanical description of systems of electrons and nuclei - atoms, crystals, metals, molecules, semiconductors, plasmas, ... - has turned out to be enormously successful. Not only can the experimental properties of simple systems be calculated to high accuracy using only four universal constants (c, $\hbar$, m_e, e^2) and the masses of the nuclei $M(A, Z)$, but the extension of the calculations by means of a modest number of empirical constants referring to specific systems allows the quantitative prediction of many of the properties of quite complicated structures. Few physicists doubt that these empirical constants could also be calculated from the basic set given above if a proposal to do so generated sufficient enthusiasm and adequate financial support. These quantum mechanical and empirical ingredients support a detailed physical description of the DNA double helix - the organ of heredity and the instruction tape for protein synthesis within living cells. Biologically important mutation phenomena happen due to the quantum uncertainties in the positions of the hydrogen atoms which zip the two strands of the helix together. There is a significant transition region between the quantal description of particle phenomena and the "classical" physio-chemical descriptions of molecular biology and cell metabolism; physicists usually believe that the two regions can be joined without conceptual conflict.

Yet quantum mechanics was born during a period of raging scientific controversy and philosophical doubt, a time when many physicists were deliberately seeking for acausal physical phenomena, in order to break the chains of "classical" physical determinism.[1] Many philosophers still do not accept quantum physics. Neither do some physicists who cling to Einstein's critique.[2,3] Contemporary physicists who grew up using quantum mechanical paradigms seldom question the validity of those models. As students they may well have been uneasy about how and whether classical and quantum mechanical descriptions join. Yet once they succeed in solving some specific quantum mechanical problem, familiarity starts to breed contempt. Subsequently they are usually content to ignore the basic paradoxes and get on with what they consider to be the main job.

High energy particle physicists are not so fortunate. They study the materialization and disappearance of "particles", usually as "counts" in detectors. They also investigate the "virtual" effects of such particulate degrees of freedom at energies which do not allow the additional particles to be materialized and isolated from the initial system. That new particles could be created from energy, and that they can have measurable effects below the energetic "threshold" for their creation was demonstrated by Wick[4] in a very brief but profound analysis of Yukawa's[5] meson theory of nuclear forces. Both predictions are inescapable consequences of theories which include the Heisenberg uncertainty principle and the mass-energy equivalence of Einstein's special theory of relativity. The successful artificial production of Yukawa particles (pions) in 1948 was one of the great triumphs of experimental and theoretical particle physics, and of accelerator technology.

Theoretical physics has had precious few, if any, comparable triumphs since. Experiments of increasing subtlety, precision, and cost have revealed detailed and intricate systems of ephemeral particles with intriguing characteristics. Many beautiful regularities and partial symmetries have been developed by theorists to describe these results; within broad areas these theoretical structures have demonstrated great predictive power. Yet there is no unified theory exhibiting overall consistency, let alone reliable and quantitative predictive accuracy. The conjuring act by which success appears to be achieved by some practitioners of high energy physics does not lead to that body of "public knowledge" which many see to be the aim of science.[6] Indeed, the tale is told of one theorist (I fear his name is legion) who prepares a separate model to "predict" any conceivable outcome of proposed key experiments, and files the stack in a locked drawer. His task is easy thanks to the ambiguities in basic theories. The time for making the calculations is ample because particle experiments are often major engineering enterprises that can take several years to bring to fruition. When our theorist learns the preliminary results of some experiment from the grapevine, he hurries to his drawer, extracts the "correct" prediction, and mails it off for publication. With luck, his paper can be in print before the results of the experiment are common knowledge.

A scientific community which tolerates the type of behavior just described creates ephemeral theories. There are frantic rushes from one fashion to the next. This situation provides some experimental physicists grim satisfaction and even sadistic delight in shooting down the flimsy structures that pass for predictions. But all experimentalists share to some extent the frustrations of their theoretical brethren, particularly as natural selection weeds out theorists who have not learned the skill of concealing a face-saving ambiguity behind the facade of what appears to be a clear prediction.

In spite of this unhappy situation, there have been surprisingly few attempts to attack the fundamental ambiguities. Epistemological tools developed during the somewhat similar periods when relativity and quantum mechanics were gestating have yet to be effectively employed. The reason is not far to seek. Although particle physics lacks a rigorous paradigm, it has what passes for one in the "local quantum field theory", which with skill can be steered around various ambiguities to arrive at genuinely useful predictive results. Few would challenge the assertion that the elementary version of this theory is quite ambiguous. The basic reason for the ambiguity is quite simple. Local quantum field theory takes over, unexamined, the continuously infinite four-dimensional Minkowski space-time of point events, and uses this framework for the definition of dynamical "field amplitudes" at each point. It is only in this sense that the field theory is "local"; the predictions derived from it definitely are not. Since this is a quantum theory, each field amplitude is subject to the uncertainty principle ($\delta E \delta t \geq \hbar$, $\delta p_i \delta x_i \geq \hbar$). But this means that whenever (as it always must) the theory requires a limit to be taken in which the volume $\delta x_1 \delta x_2 \delta x_3 \delta t$ surrounding a point shrinks to zero, the energy and the momentum carried by the field at that point must go to infinity.

Various clever ways have been found to avoid this apparent disaster. For the interaction of charged particles with the electromagnetic field (quantum electrodynamics or QED) Tomonoga, Schwinger, and Feynman showed that these infinities can be removed by a redefinition ("renormalization") of the charge and mass of the particles, provided the consequences of the theory are calculated to some finite order in a power series in the fine structure constant $e^2/\hbar c \approx 1/137$. For particles such as the electron and muon which exibit no "strong interactions", many of the properties that can be very precisely measured at low energy have been computed and confirmed by experiment[7] to the fantastic accuracy of one part in $(137)^3$. At very high energy still other properties can be computed and measured; conventionally interpreted, these results show that quantum electrodynamics is empirically "local" down to dimensions at least a factor of 10 shorter than the characteristic nuclear dimension of 1.4×10^{-13} cm.

The paradigm can also be used for systems of pions and nucleons and other "strongly interacting particles" called hadrons. This requires some care as the parameter analogous to the fine structure constant $G^2/\hbar c \approx 14$ does not allow a sensible power series expansion. The alternative of expanding in powers of $\hbar c/G^2$ is not available, because no one knows how to construct the "strong coupling limit" with which the series would have to start. So progress has been made by considering situations which introduce a second parameter that cuts down the effective interaction strength. For example, if two hadrons of mass M are so far apart that the uncertainty principle and the mass-energy relation only allow a single pion to be exchanged between them with any great likelihood, the characteristic parameter becomes $f^2/\hbar c = (m_\pi/2M)^2 G^2/\hbar c \approx 0.08$. Thus the leading term in the series has an a priori accuracy of about 10%. Since the next term can rarely be computed unambiguously, discrepancies of 10 to 30% between theory and experiment often count as a "validation" of the theory.

If this were all that was available to test theories of hadrons, one would expect considerably more pressure for fundamental revision than actually exists. But there are other trials that can be carried out to high accuracy which test general features a solution of the field equations must exhibit (if it exists) rather than predictions of numbers for specific dynamical situations. One test connects the probability amplitude for scattering in the forward direction with a specified integral over the probability that there will be scattering at all angles and all energies (total cross section). Such mathematical relationships are called "forward dispersion relations". It is often claimed that any local relativistic field theory in which effects cannot propagate faster than the speed of light (a property often called "causality" in this context) must predict amplitudes which satisfy the forward dispersion relations. Since theorists are really willing to stick their necks out on this prediction, experimental tests have been many and have been pushed to the highest energies available in the accelerator laboratories. Both forward scattering amplitudes and total cross sections show rapid fluctuations in energy (resonances) over the energy region up to a couple of thousand million electron volts and then smooth out, making the tests very restrictive. No "counterinstance" to any forward dispersion relation has been uncovered for any system of particles so far tested. For many physicists this string of successes provides the strongest argument for "local field theory".

Other general properties of particle systems which were first predicted via local field theory and have been shown empirically to hold to very high accuracy provide much of the psychological underpinning for those who resist revisionary thinking in particle theory. Among these were the prediction of antiparticles (both the positron and the anti-proton, for example) electron-positron pair creation, neutrinos, Yukawa's prediction of the meson.... Actually all these predictions can now be viewed as necessary requirements of certain symmetry properties connected with relativistic transformations and the possibility of particle creation which comes from coupling relativity to quantum mechanics (the Wick-Yukawa mechanism already mentioned above, and to be discussed in greater detail below). Another great triumph in the early days of quantum field theory was the proof that particles with integral spin (bosons) must have symmetric wave functions (i.e. wave functions which do not change sign on the interchange of any pair of particle coordinates) and that particles with half-integral spin (fermions) must have wave functions which do change sign (are anti-symmetric) on such an interchange. For this proof Pauli received his Nobel prize. That integral spin particles must obey Bose-Einstein statistics (which is why they are called bosons), and half-integral spin particles must obey Fermi-Dirac statistics had been known (empirically) before, but Pauli provided a fundamental explanation. Further, his proof required that particles be treated as matter fields, not as the relativistic generalization of non-relativistic particle wave functions. Closely connected with this proof is the general proposition (CPT theorem) that the theory must be invariant if particles and antiparticles are interchanged (C = charge conjugation), coordinates are mirrored (P = parity operation) and the direction of all motions is reversed (T = time reversal operation). But Stapp[8] has subsequently shown that once particles have separated from each other sufficiently so that their individual energies and momenta can be measured (an epistemological requirement in S-matrix theory), it is possible to describe both particles and antiparticles as having positive energies, thus avoiding the negative energy states which had forced elaborate constructions onto Dirac and Pauli. Further, since these amplitudes must lead to real probabilities lying between zero and one, he was able to show that very mild assumptions about the structure of these observable amplitudes suffice to establish both the connection between spin and statistics and the CPT theorem. Similarly, the successes of current algebra and the like, which are often cited as evidence for local field currents, could probably be restated in terms of particle wave functions without invoking fields.

This summary of current ideas in particle theory could be extended to provide much evidence that, in spite of ambiguities at the fundamental level, and the somewhat questionable scientific ethics of some of its devotees, the field seems to be enjoying one of Kuhn's[9] periods of normal science rather than a crisis situation. There are no obvious "anomalies", let alone "counter-instances" which might immediately serve as a rallying point for a concerted attack. Thus, if we are to attempt revisionary activity, we know in advance that the going is going to be tough. We should learn from our comrades in Southeast Asia that we must "know our enemy" and attack where he is weak, not where he is strong. The strongest point in the defense of local field theory is obviously QED, so we should leave this to the last, and try to outflank it by finding weaker points. The analysis indicates, as should the title of this paper, that we believe one such weak point could lie in the reliance on locality in the formulation of the theory. S-matrix theorists[10] have already chosen this as a point of attack in their own revisionary efforts, and we should try to seize any ground they have already gained in mounting our own attack. But we believe it possible to go deeper than they have done into the foundations of the theory without giving up (as they tend to) the objective of including both electromagnetism and gravitation within the quantum particulate description.

I was led to the idea of using non-locality as a starting point for theoretical reconstruction in particle physics, not by the analysis presented above, but because I was forced to recognize that the conventional theory already <u>is</u> inescapably non-local. This happened in terms of quite specific research problems, which I will review in the next three sections before turning to the main revisionary discussion. After noting a more conventional operational analysis presented elsewhere[11] we attempt to build up the known structure of particle physics using as the fundamental postulate the principle of rational discrimination. Both methodologies are based on a metaphysical presupposition that the quantum particles and their relationships must, in time, be capable of leading to the physicists who now are discussing the particles. A scientific retrodiction of our past in terms of the particles and their historical consequences which might make this proposition plausible, has been presented elsewhere.[12]

II. THE ETERNAL TRIANGLE EFFECT

Any quantum physicist knows that in order to solve a dynamical problem he must, explicitly or implicitly, describe the system of interest throughout all of space-time. Any attempt to localize space-time regions within this infinite space-time volume can only be fuzzy because of the uncertainty principle, and this "fuzz" can extend to arbitrarily large distances. Yet this elementary fact about quantum mechanics is often ignored by working physicists. One reason is that for many (but not all) macroscopic (compared to atomic) situations the underlying atomic systems can be treated as extended Euclidean volumes (viz. the DNA double helix mentioned in the introduction). In such situations the probabilistic aspects of quantum mechanics can often be kept in the subliminal background of the calculations. A second reason for ignoring non-locality is that the theory makes use of the coordinates of <u>particles</u>, which are treated as space time <u>points</u>. Although the "actual" positions and velocities of the particles can only be computed in terms of probability amplitudes (wave functions), it is taken for granted that the points to which these distributions are referred have themselves a precise meaning. It can therefore be somewhat shocking when a specific problem forces a physicist to accept the <u>necessity</u> of extremely non-local effects within the framework of quantum mechanics; at least this was my own experience.

The specific problem which first forced me to take quantum mechanical non-locality seriously was the quantum mechanical problem of three particles. I had been led to study this problem because my own field of specialization - the "nuclear forces" between two nucleons (i.e. neutron + proton, neutron + neutron, or proton + proton) - had been pretty well worked out experimentally and pretty well correlated theoretically with current ideas in particle physics. It thus seemed that the time had come to try to calculate the properties of systems of three nucleons from first principles. I was encouraged in this task by the fact that Faddeev[13] had constructed a rigorous mathematical theory of the quantum mechanics of three non-relativistic particles interacting via "local potentials", i.e., potentials which depend only on the relative distance between two particles. I knew from the start that I would have to make some modification in the treatment, since we know on general grounds that the Wick-Yukawa mechanism generates a non-local interaction (see below), and I had demonstrated a specific non-local effect in the course of my own work.[14] Further, the details of the non-local interaction are particularly uncertain at short distances. I therefore wanted to recast the problem in such a way that it could be made as insensitive as possible to these sources of uncertainty. Because I understood these problems best in terms of the relative distance, r, between the nucleons, while Faddeev had presented the whole theory in a highly abstract form using a description in terms of their relative momenta, my first step was, naturally, to convert his description from momentum space to coordinate space.

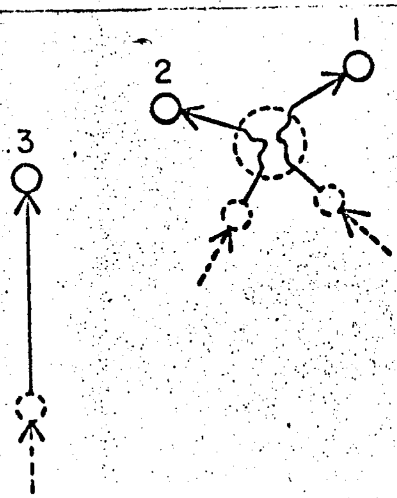
The first step was easy. I showed that the dynamical driving terms in the Faddeev equations which come from a description of the nuclear force contain three pieces: (1) the two nucleon observable amplitudes ("phase shifts") directly available from experiment, (2) a factor multiplying these amplitudes which describes the probability distribution (wave function) of the nucleons at short distance (i.e., $r \lesssim \hbar/m_\pi c$), and (3) a third function which can be constructed from the first two.[15,16,17] But when I tried to put this description into the Faddeev equations, I found that even if the nuclear "potential" is strictly <u>zero</u> for all separations r greater than some <u>finite</u> distance R (e.g. 10^{-13} cm), each pair of particles generates an effect which perturbs the motion of the third particle at arbitrarily large distances ! Superficially this new interaction falls off only like $1/r$, as is very easy to show.[17] At first I thought I had simply made a mathematical mistake in transcribing Faddeev's description into configuration space, and I puzzled over the result for two years before I could make significant progress. A $1/r$ "potential" has the same dependence on distance as the classical electric interaction between two charges (Coulomb potential), so would have observable macroscopic effects. Actually, if we use a little more care, it can be shown that the coefficient of the $1/r$ term is zero, and hence that the leading term in an expansion in powers of $1/r$ goes like $1/r^2$; this term has no need to vanish and in one case (the Efimov effect discussed below) has been rigorously proved to survive. Thus two structureless particles whose "interactions" classically described as a function of their separation vanish outside some finite radius R can affect the motion of a third particle (whose pairwise interactions are similarly bounded) at arbitrarily large distances from the pair ! This rigorously proved result demonstrates the extreme non-locality predicted by ordinary non-relativistic quantum mechanics.

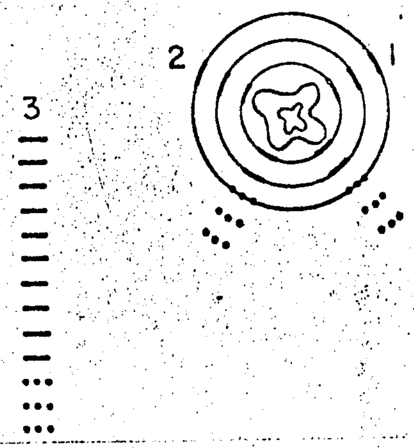
The physical origin of this effect is relatively easy to understand. When two quantum particles scatter, the emerging particles do not come out in

THE THREE BODY PROBLEM

In Classical Physics

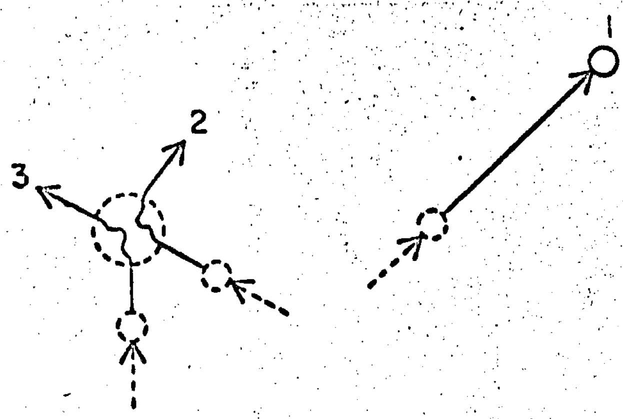
In Quantum Mechanics

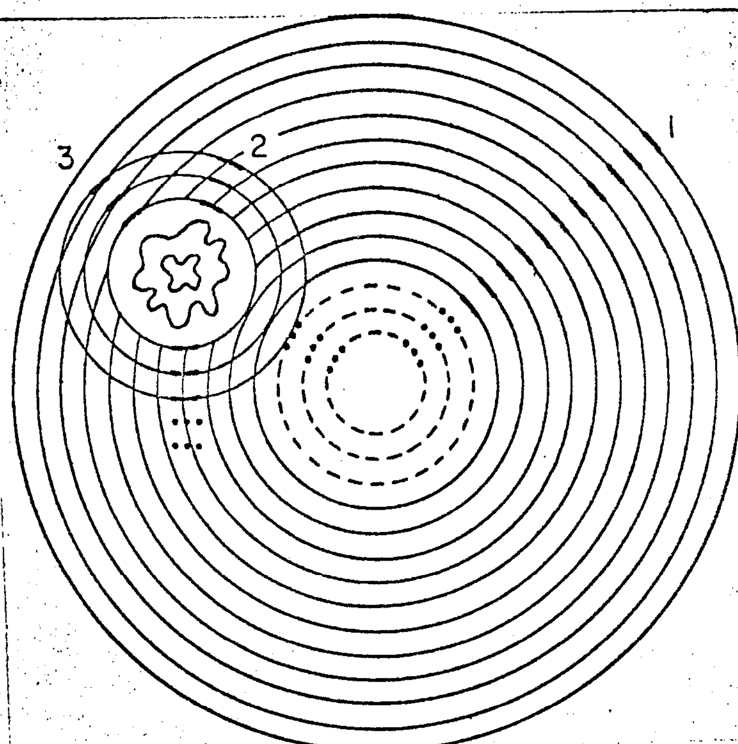




Two particles affect each other's motion only inside a finite region and emerge in definite directions.

The interaction between two waves generates an outgoing circular wave in addition to the plane waves in definite directions.





Interaction with the third particle is unaffected by the distant body. The motion inside the interaction
region is the same as before.

The circular wave from the previous scattering interferes with the new interaction and produces both a new pattern in the interference region and a different distribution of the two particles after scattering. This cannot be predicted from knowledge obtained by studying two particles in isolation. The effect exists no matter how far away the third particle is.

precisely defined directions, as they would classically, but as probability waves. In the Faddeev equations, the interaction between each pair contains not only the "potential" which would exist if they were isolated (and then would be the whole story) but two additional terms representing the scattering of the other two possible pairs. So long as the pair in question are within the range of forces, their energy and momentum are not connected in the way they would be if free, and consequently they can pick up momentum from the outgoing waves in the other two channels, changing the effective interaction. Geometrically, the target presented by this region is proportional to R/y, where y is the distance to the third particle, thus explaining the long-range character of the effect.

Once I had understood this effect, I was struck by an analogy to a well-known phenomenon in behavioral science. If two people in a room with a door come to think that there is a third person outside that door who might enter (in the quantum mechanical analog this is the third particle, out of range but "virtually" present), their behavior changes in ways that could not be readily predicted from their previous communications in the room. This analogy looks superficial, but is in fact profound. If we follow the time-dependent development of a system of three particles (cf. the Figure, next page) which were initially isolated, the first interaction generates probability waves which distort the wave function in subsequent interactions and changes them from those which would occur in a system which contained only two particles. Thus to understand the present state of the system we must know not only the forces momentarily acting between the "isolated" pairs, but also the entire past history of all relevant particles in the system. Similarly, to understand why and how the two people change their behavior, we would need to know their individual histories before entering the room, the histories of their cultures, the evolutionary development of communication systems on this planet.... For obvious reasons, I have dubbed this "the eternal triangle effect", and first so referred to it in an after-dinner speech.[18] I subsequently presented a paper on this subject at an American Physical Society meeting.[19]

As has already been mentioned, Efimov[20] discovered a specific example of this effect, also using a configuration space treatment, in a system of three identical spinless particles with no (relative or total) angular momentum. Actually, as we have seen, the existence of the effect does not require this high degree of symmetry, but assuming such symmetry simplified his original treatment,which he has subsequently generalized.[21] For two spinless particles with no relative angular momentum the cross section (i.e.,the area of a beam of particles which is scattered out when the beam is incident on a target composed of the second particles) goes to a constant value, $4\pi a^2$, at low energy. The constant a, which is called the "scattering length", can be arbitrarily large compared to the range of forces R, and in the limit when there is a bound state of zero binding energy, a goes to infinity. Since this implies an infinite cross section, and the eternal triangle effect is due to the scattering of the third particle from this pair, it is not to surprising that in this limit the three particle system has an infinite number of bound states of arbitrarily large size (the number of such states approaches infinity like $\pi \ln(a/R)$ and corresponds in the limit to the bound state spectrum of a "potential" with radial dependence const. $/r^2$). The Efimov effect provides a rigorously demonstrated example of non-locality generated by finite (and arbitrarily short) range quantum mechanical forces between pairs.

Because of the way in which I arrived at the result, I initially believed that the effect depended on the wave functions of the pairs <u>inside</u> the range of forces, and tried for some time to manipulate the equations into a form that would allow these wave function effects to be observed in three particle systems. I should have been warned by the Efimov effect (which depends only on the measurable two-body scattering length, and not on the details of the force) that this effort might fail. In fact, as we will discuss below, it is impossible using particles which "interact" only at short range to "measure" the wave functions at short distances, no matter how many particles there are in the system. But I was led to that result, which has also been rigorously proved[22] in the case of three particles, by an entirely different line of thought, to which we now turn.

III. FIXED PAST AND UNCERTAIN FUTURE

As can be guessed from the Introduction, I have succeeded in retaining some of the fundamental doubts about quantum mechanics which plague most physics students when they first try to come to grips with the subject. Like many other theorists, my doubts were kept subliminal, or at best preconscious, by the successes of the renormalized perturbation theory of Quantum Electrodynamics. But one paper by Thomas Phipps[23], which eventually got published in an emasculated version by the Physical Review, kept me interested in more than the usual game of shooting down speculations in conflict with experience. He is much concerned with the fact that the conventional route used to pass from classical to quantum mechanics throws away the "constants of the motion" $\underline{X}_k$, $\underline{P}_k$ used to describe the initial state of the system and retains only the "dynamical variables" $\underline{x}_k$, $\underline{p}_k$. To quote a recent communication[24]

"There is nothing to be happy about in a theory that claims to embody a formal 'Correspondence', yet absent-mindedly mislays half the classical canonical variables in the process, then covers its nakedness with a fog of blather about 'mind', which could just as well be the 'God' whose sensorium provided Newton with such convenient cover in circumstances of like embarrasment. I'm pretty absent-minded myself, but when it comes to counting parameters I'll take on any performing horse (or nonperforming physicist)." As he showed[25], the Hamilton-Jacobi equations of classical physics can be interpreted as operator equations acting on a state-vector Ψ with classical physics in the limit $\Psi \rightarrow$ const. and quantum physics in the limit in which the action $S \rightarrow \hbar/i$ = const. The quantum limit concerns us here, but Phipps showed that there are more general solutions ("Class III") in which neither Ψ nor S is constant. Using the one-particle Dirac equation, he showed that these could be related to effects at nuclear dimensions, thanks to the (still unexplained) "coincidence" between half the classical electron "radius" $e^2/2m_ec^2 = 1.4 \times 10^{-13}$ cm and the pion "Compton wavelength" $\hbar/m_\pi c = 1.4 \times 10^{-13}$ cm. Since unpublished results[26] counterindicate normal quantum mechanics, but are consistent with the restrictions on a local classical hidden variable theory placed by Bell's inequality,[27,28] quantum mechanics is obviously empirically frangible. If these results can be made compatible with the empirical corollaries of the Freedman-Clauser result,[29] the theoretical possibility opened by Phipps should be explored with vigor. It would indeed be a start of unquestionably "revisionary" physics.

Lacking clear need to abandon the quantum limit, we use below only the fact that Phipps' prescription supplies the conventional Schroedinger wave function $\phi(\underline{x}_k, t)$ with a phase factor $\exp i \Sigma_k \underline{P}_k \cdot \underline{X}_k$, converting it into his Ψ. Since physicists compute the probability of future events from $|\phi|^2$, which is identically equal to $|\Psi|^2$ (in the quantum limit), the two theories are observationally indistinguishable in that limit. Nevertheless, Phipps' phase factor can still be interpreted as the description of a definite former condition which persists throughout all "virtual" (reversible, uncompleted) processes, yet changes irreversibly (and discontinuously in the quantum limit)when these processes are completed and join the fixed past. This way of looking at quantum mechanics has two conceptual advantages. It removes any subjective element ("the observer") from the physical theory and allows any describable process to take place whether it is "observed" or not. Thus no esoteric element such as the "collapse of the wave function" (other than the non-locality discussed in this article) need enter the theory.

The second conceptual advantage of Phipps' version of quantum mechanics[30,31] is that the irreversible changes in the "constants" of the phase factor define a unique sequence, irreversible as time progresses, independent of the observer. In contrast, both classical statistical mechanics and the conventional interpretation of quantum mechanics rest on laws which are time-reversible at the microscopic level. Classically, the "unidirectionality" of heat flow in time is a statistical prediction applying only to systems of large numbers of particles, while in quantum mechanics irreversibility is a direct consequence of the uncertainty principle. Retrodiction starting from classical microstates leads to hypothetical states that do not correspond to the actual preconditions of systems where the theory is used predictively, while starting from quantum mechanics retrodiction leads into an increasingly chaotic past that loses contact with present experience. These paradoxes can be "avoided" by fiat, as recently suggested[32], by simply accepting that the time-irreversibility arises from the statement of the "boundary conditions" by the physicist who poses the problem. This makes the "observer" as much a part of classical physics as he is of conventional quantum physics. One way to avoid "humanizing" science in this apparently arbitrary way is to attribute irreversibility directly to some unidirectional (in time) cosmological model for the universe.[33] But even in the Einstein models for expanding universes, we must still supply "boundary conditions" either a priori or on the basis of current observations. I find Phipps' alternative of making time-irreversibility a microscopic property of the solution to the equations of motion preferable to either bringing in the "observer" in the disguised form of boundary conditions or of tying local problems of heat flow to an overall cosmology.

In a deeper sense, I accept the necessity of admitting that the theoretical physics we discuss is only possible among physicists at a certain cultural level, which in turn presupposes a long period of both biological and cultural evolution. The advantage of Phipps' approach is that we can now find it easy to understand how the irreversibility of quantum processes and the consequent increasing complexity of systems in time (already mentioned in connection with the eternal triangle effect) could, given time, lead to just such an evolutionary development. We are simply trying to state laws (which conceptually speaking require beings vaguely describable as physicists to state those laws) in such a way that those laws entail an evolutionary development of the physicists who eventually state them.

Although I was familiar with Phipps' ideas for some time, and discussed them casually with colleagues in several institutions, this thinking did not bear fruit until I was questioned by Tom Phipps about the current status of the relativistic quantum mechanical two-body problem. (He had in mind trying to generalize his 1960 covariant treatment of the Dirac equation in a Class III theory.[23]) The usually accepted mechanism for the "strong interactions" (the generalized nuclear force problem) is the materialization of massive particles which transfer momentum from one system to another. Such particles ("mesons"), first predicted by Yukawa,[5] are for most physicists the inescapable consequence of the coupling of quantum mechanics to special relativity, as was argued very simply by Wick.[4] We will describe this process in more detail in the next section, and use it as one of the basic postulates in approaching relativistic quantum mechanics. What Phipps' theory enabled me to do was to see that particle dynamics could be described using <u>only</u> the Wick-Yukawa process. Conceptually, the result is similar to S-matrix theory, and abandons ideas such as "potentials", "forces", "interactions" and "fields". Technically, the possibility of such a theory is obvious once any standard theory of scattering (such as that of Goldberger and Watson) is examined with an eye to grafting on the Phipps phase factor. The result is intuitively obvious[24], and has since been demonstrated in detail;[11] the phase factor represents particles which disappear from the initial state and (<u>non-locally</u>) appear in the final state. Thus the <u>dynamics</u> of calculating the transition matrix can be unambiguously separated from the <u>description</u> of the quantum scattering process.

IV. THE PRIMACY OF PARTICLE NUMBER

The atoms of Leucippus and Democritus had no "natural" or "original" motion; their random collisions were strikingly similar to the nineteenth century model for the kinetic theory of gases. Epicurus assumed that the atoms were falling in straight lines and that it was necessary to postulate that some of them "swerve" in order to initiate the processes which lead to the generation (and decay) of worlds. His random element in atomic theory has been criticized as foreign to the basically materialistic and deterministic focus of this natural philosophy. In recent years we have learned from the success of quantum mechanics that determinism does not inhere in the individual atomic events. The approximate validity of determinism stems from the flow of probability amplitudes from the past up to some event in which the massive particles at least potentially observable in that event manifest their individual particulate behavior. The random character of these individual events is an integral part of quantum mechanics, and supplies a possible answer to the puzzle raised by ancient atomism and its critics.

The conceptual revolution implied by this view is still in process. The existence of physicists (not just philosophers) concerned about the question of "hidden variables" at the experimental and not just the theoretical level[3,27,28,29,34] shows that the issue is by no means settled. But the concept of quantum fluctuations can be used to unify an enormous range of superficially disparate phenomena, and in particular to account for the existence of "forces" between structureless particles. This possibility is counter-intuitive for many people, but Wick[4] showed long ago that once one accepts both special relativity and quantum mechanics, the existence of "short-range forces" is inevitable. The argument goes as follows. Suppose there is a particle with mass m which is finite, and we bring together two other particles (whose masses may be as large as we wish, but also finite) close together. If the distance between them is r, and this close approach persists for a time interval δt, their energy must, because of the Heisenberg principle, be uncertain by an amount $\delta E \gtrsim \hbar/\delta t$. Thus for short enough times the uncertainty of the energy in the whole system must (because of the Einstein relation $\delta E \geq mc^2$ for a particle of mass m) exceed the rest energy mc^2 of the particle we postulated. This particle could, in principle, appear anywhere; during the time δt it can be coherently connected to happenings within a distance less than $c\delta t$, where c is the limiting velocity for particle motion. If we further assume that momentum (but not energy) is conserved (i.e., Newton's third law) when the particle "appears" and "disappears", this means that the initial momentum must be shared among all three particles; when the two large masses separate so far that the uncertainty principle can no longer allow the presence of the mass m, they can emerge with different momenta. Thus momentum is transferred from one system to the other, or, in the language of Newton, there is a "force" between them. The distance over which this force acts can now be calculated easily as $r \leq c\delta t$ (limiting velocity) $\lesssim c\hbar/\delta E$ (uncertainty principle) $\lesssim c\hbar/mc^2 = \hbar/mc$ (mass-energy relation). We have proved that provided only there is some particle of mass m <u>any</u> two systems which can be brought close together, and which are "coupled" to this particle, will experience a "force" of "range" $\hbar/mc$. Further, if the two particles initially present are brought together sufficiently violently so that both can emerge from the collision after losing an amount of energy greater than mc^2 while still conserving momentum, we expect in some instances to find a particle of mass m emerging, along with the two particles in various ways, but <u>total</u> momentum will be conserved between the initial and the final situations.

The actual history of the verification of this prediction was complicated by the fact that the Yukawa particle inferred from the 1.4×10^{-13}cm range of nuclear forces (which has a rest energy $m_\pi c^2$ of 140 million electron volts) was not the first particle discovered intermediate in energy between the electron ($m_e c^2 = 0.51$ MeV) and the proton ($M_p c^2 = 938$ MeV). The first "mesotron" found was the muon, a "heavy electron" now known to have a mass-energy $m_\mu c^2 = 106$ MeV. Three Italian physicists hiding out from the Gestapo in the basement of the University of Rome showed that this particle interacts with nuclei $\sim 10^{-13}$ times more weakly than would be required if it were to serve as the particle predicted by Yukawa to generate nuclear forces.[35] But the Yukawa particle - now called the pion - was eventually discovered and shown to account quantitatively for the longest range part of nuclear forces. Unfortunately from the point of view of simplicity, the same Wick-Yukawa mechanism predicts that at distances of $\hbar/2m_{\pi}c$ we can expect some of the time to encounter two pions in a nuclear system, at distances less than $\hbar/3m_{\pi}c$ we can expect to encounter 3 pions, and so on up to indefinitely larger numbers of particles as we refine our spacial description. Experimentally, the uncertainty principle requires us to use particles of higher and higher energy as we try to "peel the nuclear onion" in to shorter and shorter distances, and indeed as we do so we produce more and more pions. If we use indirect methods to refine our distance measurements, such as electromagnetic fields, we also find (as is required by consistency) phenomena which can be attributed to "virtual" pions, even though we do not use enough energy to produce them as free outgoing particles. Because of the identity of various types of particles, we cannot distinguish at short distance which particles were "initially present" and which were there because of quantum fluctuations. In other words, the Wick-Yukawa mechanism necessarily generates an extremely non-local description of particulate systems at short distance.

These general arguments took concrete form for me when I first heard in a seminar by Ted Bastin at Stanford that the sequence 3, 10, 137, $\sim 10^{38}$ results from a simple hierarchical construction starting from the basis 0,1. This sequence, for a physicist, is the (inverse) numerical sequence of the super-strong, strong, electromagnetic, and gravitational interaction strengths. Current physics leaves the ratios arbitrary, discoverable and corrigible by means of experiment. That they are given by hierarchical construction using more basic elements, represented by $3 = 2^3 - 1$, $10 - 3 = 7$ ($= 2^3 - 1$), $137 - 3 - 7 = 127$ ($= 2^7 - 1$), $\sim 10^{38} - 3 - 7 - 127 = \sim 10^{38}$ ($\sim = 2^{127} - 1$) was tantalizing but not illuminating. Interactions are dimensional concepts, tied in conventional physics to the units of mass, length and time, and hence have no logical connection to "pure numbers" defined by any mathematical sequence. The clue, for me, came in a later statement by Bastin in the same seminar that he viewed "quantization" as quantization of mass rather than of action (quantization of "action" was the historical route to quantum mechanics). If this "pure number" sequence represented numbers of particles, its dimensionless character could be established using non-dimensional concepts. This reminded me of an old paper of Dyson's[36] which showed that quantum electrodynamics changes its character for systems with more than 137 particle-antiparticle pairs. Once this non-dimensional way of describing where an interaction concept fails is suggested, a uniform description of "interactions" might be given in terms of where the number of particles describable by each concept "becomes inoperative", to steal the mortal phrase of Ronald Ziegler.

Dyson's argument[36] was constructed to meet a different problem. He was concerned with the problem of how many terms are meaningful in the "renormalized perturbation theory of quantum electrodynamics" (QED) - a series in powers of the fine structure constant $e^2/\hbar c \approx 1/137$. He noted that if we replace e^2 by $-e^2$ in this series the result should still be meaningful (converge) <u>if</u> the series is absolutely convergent. Physically this amounts to replacing QED by a theory in which like charges attract and unlike charges repel. The original series corresponds, term by term, to including as many electron-positron pairs as there are terms in the series; by the 137th term in e^2 there are 137 pairs. If all particles of the same charge happen to be within their own Compton wavelength ($\hbar/m_e c$), they generate within that volume an electrostatic energy of $\sim 137\ e^2/r = 137\ e^2/\hbar/m_e c = 137\ (e^2/\hbar c) m_e c^2 \approx m_e c^2$. This is highly improbable in the "real world", since all these like charges would repel each other, and the system would rapidly disassemble. But in a theory with $e^2 \rightarrow -e^2$ (accomplished by the imaginary replacement $e \rightarrow ie$) the system would implode rather than explode, with enough gained energy to keep sucking mass out of the quantum fluctuations particle by particle until the whole theory collapsed. Therefore the QED series becomes meaningless after 137 terms. This is a specific application of the Wick-Yukawa mechanism to a problem about the self-consistency of renormalized quantum electrodynamics.

But this same calculation supports a description using different language. We can restate the result as saying that it is not possible to isolate more than 137 individual electrically charged particles (with the universal charge e) within a region as small as their own Compton wavelength. Since the electron is the least massive charged particle, this also says that we cannot meaningfully define what we mean by space-time volumes in regions smaller than (1/137) ($\hbar/m_e c$) by means of electromagnetic measurements. Gravitational definition is still conceptually (though not practically) meaningful down to much shorter distance, but once we try to push it down to distances of the Schwarzschild radius, which we could do by trying to describe $\sim 10^{38}$ protons within their own Compton wavelength, the mathematical singularity discussed by Dyson becomes <u>physical</u> - the particles disappear down a black hole and lose their particulate identity. Thus the pure <u>number</u> 137 is simply the <u>maximum</u> number of charged particles we can identify individually within their own Compton wavelength, while the pure number $\sim 10^{38}$ is the maximum number of gravitating protons we can identify individually within their own Compton wavelength.

There is a technical point about "black holes" which needs explanation here. Dyson points out that the density of "particles", even when there are 137 of them within their common quantum wavelength, is so low that, although the total assemblage has an electrostatic energy of mc^2, the calculation requires the use of the Coulomb potential e^2/r for the individual particles only in an energy region which is well known and does not involve general relativistic effects. This is also true for the corresponding gravitational case where the Coulomb potential is replaced by the Newtonian potential Gm^2/r. For the problem of interest here, the "Schwarzschild", or "black hole", radius can be estimated using only Newtonian gravitational concepts and special relativity.[37] We do not have to invoke "curved space time" or any of the complicated technical apparatus (and postulates about continuous space-time coordinates) of the general theory of relativity in order to get, qualitatively, particulate quantum systems that have such intense gravitational fields that no particle or quantum can escape from them. Perhaps it would be better to call these particulate "black holes" generated by the Dyson mechanism by another name, but they have the most important property of a "black hole", and we think the term should be retained. It will be an interesting technical problem if these ideas ever generate a quantitative theory to see whether all the properties of the "black holes" predicted by the general theory can be reproduced, or whether there will be experimentally detectable quantitative differences.

With this clue, the pure numbers 3 and 10 are also interpretable. The electrostatic interaction we used above was e^2/r, while the corresponding

Yukawa interaction is $f^2 \exp(-m_\pi c/\hbar r)/r$. Since $f^2 = 0.08$, were it not for the exponential the same argument says that we cannot (e.g., by tracing back the trajectories of energing particles to their production volume) meaningfully describe the presence of more than ~ 12.5 pions within their own Compton wavelength; hopefully a more exact calculation including the exponential would bring this down to ~ 10. In any case we do not expect this number to be an exact integer (or $e^2/\hbar c$ to be exactly 1/137) because in some sense quark, pionic, electromagnetic, and gravitational effects all occur within any system.

To understand the number 3 as the maximum number of quarks we can meaningfully define requires a slightly different argument. In the quark model, massive bosons (pions, kaons, ...) are quark-antiquark pairs with quark number zero, while hadronic fermions (protons, neutrons, sigmas, lambdas, ...) are bound states of three quarks. No free quarks have ever been observed, and the quark quantum numbers can be assigned to the individual hadronic systems only in experiments where the two combinations mentioned above can be isolated. Thus the quark theory (if, as it often does, it excludes free quarks as a possibility), has precisely the required character of saying that the only quark numbers which make sense for isolatable individual particles are zero or three.

A number of problems remain before this dimensionless description can be meaningfully connected up to the Bastin approach. One of these is trivial, and was solved during a conference on the chapters in this volume. This is simply to reduce the relativistic quantum particle theory to dimensionless form. As any physicist knows, any dimensional result (i.e. any result that depends on the units in which we express our measurement) can be expressed in terms of three basic units for mass, length, and time. Thus all we need do is to show that the theory requires three fundamental units which can be defined independent of any dimensional considerations. It then is a matter of experimental convenience how we choose to assign numerical values to these units, and is not a matter of any fundamental significance. Our unit of mass is obvious, as the lightest massive particle is the electron (i.e., empirically mass is quantized). All mass ratios with respect to the electron are pure numbers; whether these ratios are real or rational numbers is left open for the theory to decide. Such a question can never be settled by experiment, except for theories which require a specific rational number as an exact consequence (this is how Eddington's theory of the fine structure constant became experimentally counter-indicated). The second fundamental unit is the limiting signal velocity of special relativity. Whitehead[38] has argued that any theory which uses events in space-time volumes (not necessarily at points) as a basic conceptual tool should have such a limiting velocity. That this is also the ratio between electrostatic and electromagnetic units (and hence the "velocity of light") and the square root of the conversion factor between mass and energy is then a requirement of the theory; the theory is therefore frangible if these approximate equalities turn out not to be exactly true. The third fundamental constant comes from the wave-particle duality of quantum mechanics, and is the conversion factor between the (reduced) deBroglie wavelength of the particle ($\bar{\lambda}$) and its momentum (p), namely $\hbar = p\bar{\lambda}$; $\hbar$ is Planck's constant divided by 2π. In fact these last two "natural" units ($\hbar$ and c) are already customarily set equal to unity in high energy particle physics; only the masses of the particles (rather than ratios to the electron mass) are used dimensionally.

The much more difficult problem is to construct a theory in which the hierarchical steps of Bastin's construction lead successively from quarks to pions to electrons to black holes. Starting as I do from existing particle theory, there is no apparent reason why the 10 should not have turned out to be 5 or 15, or the 137 to be 100 or 150, for example. Thus all that can be presented here is a program for constructing, step by step, a theory which is conceptually compatible with Bastin's, but whose success or failure must lie in the uncertain future. This program will be presented in the next section.

V. ATOMS AND THE VOID SUFFICE

Although I believe that the program for creating a Democritean quantum mechanics presented in this section has points of similarity with the self-generating "computer program" type of approach advocated by Parker-Rhodes, Bastin, Amson, and Kilminster[39], the methodology used is different. To me it is obvious that discussions of physics (and metaphysics) such as this can only take place in a culture with a long history of linguistic communication. I therefore find it silly to ignore what is already current experiential "knowledge" in that community, and will make free use of it in what follows. But I would also insist that "circularity" in the argument can be avoided if the physical theory which we construct can be shown to be capable of producing, in time, the community in which this discussion is taking place. Of course to make that statement "scientifically plausible" would take many volumes of detailed argument covering various aspects of physical cosmology, evolutionary biology, the evolution of communication systems, and the class struggles that have led to the current world crisis. An outline of this evolutionary development has been sketched out.[12]

To set the philosophical tone of the discussion, I take exception at the outset with the famous mathematician who said that "God made the integers; all else is the work of man". The integers, like any other part of mathematics, are a human creation, and subject to limitations than come from our finite nature. This is made clear by Godel's Theorem[40], which shows that any postulate system capable of generating the integers necessarily creates an infinite class of (arithmetically true) propositions which cannot be proved to be true within the system. By suitable additional postulates, these undecidable propositions can be made provable, but that process necessarily creates a new infinite class of undecidable propositions. Thus, the indefinitely extendable characteristic of the integers extends to the propositions which they allow us to state, and never closes. This is quite compatible with the methodology I adopt. I recognize from the outset that any system of mathematical or scientific propositions can always be indefinitely extended, and that "closure" can never be more than approximate.

Although I do not aim at "closure" I still aim at the maximum generality I can achieve at any (<u>necessarily</u> finite) stage in the development, and try to find in <u>any</u> aspect of experience "counterinstances" - either factual or conceptual - to the construction which is being developed. That is, part of my methodology is to deliberately try to force contradictions, and to try to meet them. This is how I understand the dialectical process as described by Chairman Mao.[41] But to make the contradictions as clean as possible I also invoke the methodology of the spiritual godfather of the positivists - William of Occam. I try to use his razor to pare away the excresences which have grown up historically around the physical concepts invoked, and more specifically to use an "operational" analysis similar in spirit to that of Mach, Einstein, Heisenberg, and

Bridgman. This approach gets rather technical so only an outline is given here. Details will be presented elsewhere.[11]

I start with the (initially unexamined) concept of "particle detectors" such as are actually employed in high energy particle physics, and which can to some finite "accuracy" be used to assign macroscopic space and time coordinates (which concept also is not initially examined) and hence to define particle velocities. Postulating a limiting signal velocity, and adopting the Einstein convention for the simultaneity of distant events, then gives the proper Lorentz transformations. Postulating homogeneity and isotropy extends this to the Poincaré transformations. To define the mass of particles I assume that there are devices which can change either the energy or the direction of the momentum of a particle; together these define a ratio of charge to mass and either, in conjunction with a velocity measurement,then defines an invariant mass. Adding the concept of a "grating" (or discussing finite aperture diffraction) then allows me to define the deBroglie wavelength, establish the wave-particle duality, and hence the uncertainty principle. This suffices to define covariant free-particle wave functions, and thanks to the Wick-Yukawa mechanism, a complete theory of particle scattering experiments. Since this apparatus entails the concept of (quantized) angular momentum, I can also introduce a dichotomic spin function. The simplest representations of a particle with this internal coordinate are the Dirac spinors, which provides an argument for the spacial coordinates being three-dimensional. It also provides us with the possibility of anti-particles.

Having defined particle wave functions, it is possible now to introduce <u>external</u> electromagnetic fields (defined in terms of macroscopic measurements of charge and current) and hence to exibit explicitly the energy and momentum changing devices invoked above to define mass. Further, this construction yields the Dirac equation, and in the non-relativistic limit the Schroedinger equation, thus recovering the conventional results of quantum mechanics to lowest order in the external field. By assuming that density and velocity distributions computed from the wave functions of <u>charged</u> particles are <u>also</u> the sources of electromagnetic fields in Maxwell's equations, we can then bring the external fields within the framework of description, and justify the use of energy and momentum changes to measure mass. By using "quantum transitions" generated by external field (specifically the photoelectric effect), we can then also explain how a "particle detector" works, and thus close the logical circle to an accuracy of $e^2/\hbar c$. The theory at this level will stand or fall, in the eyes of most physicists, on whether the motion of <u>two</u> charged particles, each acting as the source of the field for the other, can be computed to order $(e^2/\hbar c)^3$ in agreement with experiment and the renormalized perturbation series for QED. A second critical test will be whether the properties of pion and nucleon systems can be computed, not from phenomenological T-matrices but from T-matrices "bootstrapped" from the 3π and 4π systems and the Yukawa coupling constant. Assuming such successes can be achieved (which will take time), it will be interesting to see how far the theory can be pushed into the less well understood regions of very short times and high energies where quarks, neutrinos, and gravitational effects become dominant.

So much for "normal science" and operationalism. In the remainder of this paper I will attempt instead to build up the same picture of particle quantum mechanics starting from the minimum number of abstract postulates, introducing additional ones only when they appear to be required by known experimental "facts". This methodology is not so different as it might seem at first glance - building up a minimal set of postulates in order to achieve agreement with an extant theory is just as much an application of "Occam's Razor" as the reverse procedure of paring them down to a minimal set. In both cases we are still guided by the current status of physics; the requirement that we ultimately be able to explain the development of human culture and science in terms of the theory remains. But the constructive approach is much closer to the methodology of Parker-Rhodes et al.[39] and was deliberately undertaken for the purposes of this volume in the hopes that it might aid their program. I was quite surprised, and a little frightened, to find out how far one can go toward recreating, at least qualitatively, most of the ideas currently being pursued at the frontiers of particle physics using so few assumptions.

The basic postulate adopted here is similar in spirit to that of Parker-Rhodes, Bastin, Amson, and Kilminster.[39] They start with the dichotomic pair (0,1),but I would prefer to leave my starting point even vaguer and claim that a minimal requirement for rational thought be that one can distinguish something from nothing. I believe that this is the idea behind the basic materialistic postulate of Leucippus and Democritus that there are only atoms and the void; I therefore call it the "Democritean" postulate. An immediate corollary is that any refinement of this idea must not allow the construction of an undifferentiated continuum. The idea of the undifferentiated continuum (also called Nirvana), which is the common goal of many mystical traditions,[42] is for me the antithesis of rational thought – although I am quite prepared to admit that the attempt to attain that goal might be a rational activity. Of course we allow discrete approximations that can approach "points in the continuum" in the mathematical sense; but we avoid making use of the limit points themselves.

According to our basic postulate, there must be "particles" which we can distinguish from the void, and which are denumerable. The necessity for distinguishing them requires an additional concept, which I take to be relative motion. For the moment, we will assume that the particles can be distinguished from one another only by their "motions", and that the possibility of zero motion exists for some sets of the particles. But if these motions could be arbitrarily rapid, we could use them to define a continuum background space, a concept we have ruled out above. Therefore we must assume that there is an upper limit to relative motion. Once we accept this we can, roughly speaking, assume that motions, relative to some set of particles that have zero motion, can be ordered between zero motion and the upper limit; for the moment we assume that this ordering does not change. But if the motions do not change, something must, or we would be back in a static Nirvana with no means of distinguishing our particles. What changes when two particles are in constant relative motion with respect to some set of particles that have zero relative motion is the "distance" between the pair. We distinguish two cases: the distance first decreases to zero and then starts increasing, or it keeps on increasing; we save discussion of how long distances can keep on increasing till later. Unless we are willing to introduce "structure" into this description of particles some of which are moving and some of which are at rest, we now have all the ingredients for describing particles in uniform relative motion with respect to each other along a line; further if <u>only</u> relative motions are to be meaningful and there is a limiting velocity, the whole system can be made invariant with respect to which particles are assumed at rest and which in motion only by means of the Lorentz transformations.

At this point the thoughtful reader will note that, although we have introduced relative motion, we have nowhere introduced the concept of direction, so in conventional terms all we have are particles moving along a line. This produces a peculiar result. If we assign numerical values to some description, and then replace all velocities and distances by their negatives, we seem to have a description of the same situation but with different (negative for positive and positive for negative) numbers assigned to each particle. We could either assume that this artificial character of our numerical description has no significance (in which case we would have to describe only the symmetric systems in which this interchange produced no change) or add a new descriptive element. This can take the form of an "internal coordinate" for each particle that tells us whether, relative to some convention for the "direction" of the line, the particle is moving in the positive or negative direction. The simplest interpretation of this "spin coordinate" is that it represents a rotation about the line in a space with at least two directions[11] perpendicular to each other and to the line. If we confined ourselves to a plane we would do no better than on a line ; two directions perpendicular to the line are needed to form non-superposable objects that distinguish "left" from "right". This possibility is a basic empirical requirement.[43] This allows us to extend our concepts to a 3+1 dimensional Minkowski space - simply by requiring that this space be homogeneous and isotropic. Note that the space we invoke was constructed from finite particle motions assumed to be specified to some approximate numerical accuracy; we do not <u>have</u> to assume that this accuracy of measurement can be increased indefinitely (in fact this is ruled out by our anti-continuum corollary to the Democritean postulate).

Given the particles in uniform motion, we note that, once the description is established, all past and future motions are determined; our apparent motions are simply the description of a static four-dimensional world and we are back in Nirvana. So we assume that both directions and velocities can <u>change</u>. At this point, so far as I can see, we need a new concept that does not follow from the Democritean postulate in any obvious way. This is the basic mechanical postulate of Newton's third law, that action and reaction are equal and opposite, or that <u>momentum</u> is conserved. This allows us to introduce a Lorentz invariant descriptive of each particle by noting that the Newtonian concept applies only to the three spacial components $\underline{p}$, and that for Lorentz invariance we must have a time-like component ϵ. The mass in then <u>defined</u> by the scalar invariant $m^2 = \epsilon^2 - p^2$, the same in all coordinate systems. All we need do now is postulate some law that specifies how the motions of the particles affect each other, conserving momentum and mass in a Lorentz-invariant way, and we have a full blown relativistic particle mechanics. But this is again a <u>deterministic</u> system, just as much a static four-dimensional world as the one ruled out above for "free particles", and again in conflict with our basic requirements.

The basic assumption that we make in order to avoid our essay into rational thought collapsing back into Nirvana should be clear from the last section: we assume that the number of particles can change if we try to count them in a small enough volume. In order to make this concept universal, we say that for any (finite) mass, this volume has a radius of $\hbar/mc$ where c is the limiting velocity already introduced and $\hbar$ is a universal constant whose numerical value depends on how we ultimately choose to relate this theory to our experiences. The reason we need a fundamental length at this point is also Democritean; if our particles could be arbitrarily small we would get back a continuum at short distance just as surely as if their number could be arbitrarily large. How many particles we can still distinguish within this volume depends on the means we use to describe them - quark, pionic, electromagnetic or gravitational, and is dimensionless. Our remaining task is to try to derive, again wherever possible from our discriminatory postulate, the necessity for these different aspects of what are, by now, quantum particles satisfying a Lorentz-invariant wave-particle dual description, and whose number can change, thanks to the Wick-Yukawa mechanism.

The fact that we have introduced the concept of changing particle number, and also rejected determinism on the grounds that it would lead back to a static four-dimensional universe, suggests that the means by which particle number changes is random. This basic method of avoiding Nirvana immediately provides a model for time in accord with experience - past events can be considered fixed, but, due to the random character of particulate events, only the probabilities of future occurrences can be predicted from this knowledge of the past. This means that our basic description of particles is statistical. Thanks to Max Born we know how to accomplish such a description: wave functions whose amplitudes satisfy a "causal" law are interpreted as predicting the probability of finding a (the) particle(s) represented by the wave function in one (or more) specifiable space-time volume(s). As has been discussed at length in other contexts,[11,22] these wave functions need only represent the motion of "free particles". The phase velocity of these waves is ϵ/p and hence lies between the limiting velocity and infinity, while the velocity with which the center of a group of waves moves, known at some time in the past to have been (approximately) localized in some finite volume (the "group velocity"), is $d\epsilon/dp = p/\epsilon$, and lies between zero and the limiting velocity. The necessity of constructing such groups of waves to describe localizable particulate events automatically introduces the "uncertainty principle" into our theory. Since we already have the mass-energy relation, this immediately allows us to identify the "Wick-Yukawa mechanism" as the means by which particle number changes, and hence allows us to have scattering, particle production, and particle annihilation (change) without invoking the concept of "interaction".[11] In the limit in which we can ignore effects due to the finite wave lengths of the particles, the wave theory can be reinterpreted as describing particles moving on trajectories perpendicular to the wave fronts, and we recover the relativistic mechanics of a system of particles whose velocities are the "group velocity" of the waves.

The expert will note that, since we have already argued for the necessity of "spin" in order to discriminate direction of motion, the construction sketched in the last paragraph allows us to write down conventional "Dirac spinor wave functions" for one or more particles. This raises an immediate problem. Although our "fixed past-uncertain future" point of view allows us to distinguish a unique meaning for the algebraic sign of the time parameter in our theory (conventionally, negative times are called past and positive times future), the corresponding uncertainty in the conjugate coordinate, the energy, is <u>not</u> resolved. Thus the theory seems to require negative as well as positive energies - which would have disastrous consequences. For instance, if two particles of the same $(\text{mass})^2$ value and equal but opposite momenta came together, they could annihilate each other, leaving behind not just the "undifferentiated continuum" of Nirvana, but quite literally <u>nothing</u>. We conclude that we <u>cannot</u> allow negative energies in the theory. Another reason we cannot allow negative masses is that they would lead to "anti-gravity", for which there is no empirical evidence. The simplest way to avoid them is to assume that (consistent with our Democritean framework), <u>mass</u> is an intrinsically positive concept; then, thanks to the mass-energy relation, only positive energies can occur. We still must require that, whatever our lightest mass is, there is a <u>maximum</u> number of particles of that mass which can be meaningfully described within their own Compton wavelength. Since this ultimate limit refers <u>only</u> to mass, this is the basic <u>gravitational</u> concept in our particle theory. Empirically the lightest massive particle is the electron, and the maximum meaningful number of electrons within an electron Compton wavelength is $\sim 10^{44}$. Also empirically, the maximum number of baryons (protons, neutrons, sigmas, ...) which can be described within a baryon Compton wavelength is $\sim 10^{38}$. The fact that there are <u>two</u> basic numbers here rather than one poses a still unsolved problem - where does the dimensionless mass ratio $M_p/m_e \sim 1837$ come from ? The fact that the sequence 3, 10, 137 ends at 10^{38} rather than 10^{44} shows that it should be read up from quarks (which <u>is</u> the baryon sequence) and does not refer to electrons (leptons). We will see below that there are other reasons for believing that, at some very deep level, there must be <u>two</u> basic types of particle and not just one. The origin of this fact is left as a problem for future research.

Although we have succeeded in arguing that the only interpretation of Dirac spinors which we can allow is one corresponding to particles of positive mass and energy, this does not eliminate the "negative energy states" from the picture. Instead we are now required to assign a <u>second</u> dichotomic variable (in addition to spin). This new quantum number is <u>conserved</u> for systems of particles of one type, and the <u>difference</u> between the numbers of the two types is conserved in mixed systems. Since this is a distinguishing characteristic separate from mass or spin, there is some <u>maximum</u> number of particles which can still be discriminated by this characteristic when they are packed within their own Compton wavelength. If we assume that this number is (approximately) 137, we can identify this new quantum number with electric <u>charge</u>. Why the actual value should be 137 is left for future research; hopefully it can be "derived" along the lines of Parker-Rhodes et al.[39] But whatever this number is, the conservation law already mentioned requires it to be the <u>same</u>

for <u>all</u> massive spin 1/2 particles, independent of their mass. From this point of view, conservation of charge is therefore a reflection of a dichotomic property of spinor wave functions. Since we also have positive mass values, and scattering via the Wick-Yukawa mechanism, Stapp's derivation[8] shows that reasonable assumptions about the conservation of probability suffice to establish "the connection between spin and statistics" and the Pauli exclusion principle for spinors (i.e. that no two half-integral spin particles can occupy the same state) which is crucial for what follows.

Since it is only the <u>difference</u> between the charges which is conserved, our theory allows for electrically neutral systems, and hence for transitions between a state consisting of a positively and a negatively charged particle (e.g., an electron and a positron or a proton and an anti-proton) to such states. One possibility is that such states are massive, of which a specific empirical example would be a neutron and an anti-neutron, but it is also conceivable that the transition leads to two "particles" of negligible mass traveling at the limiting velocity and with their spin aligned along or opposite to their direction of motion. These are obviously <u>neutrinos</u>. We would like to have them available for empirical reasons, but would like to get them out of the framework already established in order to avoid having to apply Occam's sharp tool at a later stage. One possibility is that they are simply spinning neutral black holes. This would explain why, at high enough energy so that the <u>gravitational</u> mass of the relative energy of motion of the electron and positron becomes comparable to the mass energy, the cross section for producing neutrinos becomes enormous (in the phenomenological Fermi theory of "weak interactions" it becomes so large that the probability exceeds one - i.e. the theory has to break down - in this limit), yet at low energy the effect is extremely weak. This would also explain why 10^{13} (the conventional measure for "weak interactions" in the 3, 10, 137, 10^{38} sequence) is not primary; it becomes a derived concept which must eventually be calculated from a more detailed articulation of electromagnetic and gravitational concepts. Another problem is that there are not one kind of lepton but two - muons and anti-muons as well as electrons and positrons - and each carries with it an associated type of neutrino. The ratio of 207 between the muon and electron mass is the only other distinguishing characteristic between the two types of leptons. Hopefully it is not an independent quantity, but can be linked up to the other large mass ratio of baryon to electron mass. Qualitatively, the m_μ/m_e ratio is the m_π/M_p ratio times the M_p/m_e ratio (to the extent that 273 and 207 are approximately the same), so one place to start looking is whether we can understand the m_π/m_e ratio. Before we do this, however, we must say a few words about electromagnetism.

Historically, the first "zero mass particles" used in quantum theory were light quanta (photons) not neutrinos. Like neutrinos, they have two spin states parallel or antiparallel to their momentum, but these states have spin 1 rather than 1/2. Therefore any number of them can be packed into a volume (at least until their energy begins to produce gravitational effects - Wheeler's geons), and as is characteristic of Bose statistics, the more there are, the more likely this is. Outside of their angular momentum and energy they have no other defining characteristics (unlike neutrinos which carry both lepton number and muon or electron number); indeed the type of "quantum" used to describe transitions of charged particle systems depends on mathematical convenience and is not uniquely dictated by the problem. From a Democritean point of view, I would therefore hesitate to call light quanta "particles", and I have a strong urge (already expressed above) to eliminate them from the description altogether. While the program undertaken here must, ultimately, be able to lead to an understanding of classical electromagnetic and gravitational fields, we have already seen that these are rather special limits in a particle theory. As far as "field quantization" goes, Bohr and Rosenfeld showed long ago[44] that, to lowest order in $e^2/\hbar c$, any material system of sources and sinks of the "field" which satisfies the uncertainty principle leads to the same restrictions on the measurability of the fields as does "field quantization". This is a problem which must be tackled to higher order when discussing QED; as was already noted above, that problem is beyond the horizon of this paper.

One of the oldest ideas about the electron is that its mass may not be an intrinsic property, but simply a reflection of the energy due to its electric charge and mass-energy equivalence. Classically, if we assume that the charge is packed into a small enough radius to accomplish this, we find a "classical electron radius" of $e^2/m_e c^2 = 2.8 \times 10^{-13}$cm, 137 times shorter than the Compton wave length of the electron $\hbar/m_e c = (\hbar c/e^2)(e^2/m_e c^2) = 137(e^2/m_e c^2)$. We have also seen that we can define what we mean by up to 137 charged particles of one sign within their own Compton wavelength. Such a system would be highly unstable, but if we combined it with a system of 137 particles of opposite charge the resulting neutral system although electrostatically unstable is not completely ephemeral. It would have a finite lifetime against decay into neutrinos or electromagnetic radiation, and is the most massive system of electrons and positrons we can define. If electrons and positrons have intrinsic mass, this system could not decay purely into radiation, but if they have only electrostatic energy, this system could go into 2 γ-rays, and would have a Compton wave length of

1.4×10^{-13} cm. Such a model predicts the mass of the observed neutral pion, and its decay ($\pi^0 \rightarrow 2\gamma$). Systems with smaller numbers of electrons and positrons are conceivable, but we gain electrostatic energy by adding an electron (or positron) and the appropriate neutrino to such a system; since they are stable against electromagnetic decay (because of charge conservation), they have a longer lifetime. Systems with two electrons or two positrons are presumably electrostatically unstable, thus explaining why the pion is a charge triplet $\pi^+\pi^o\pi^-$. If indeed the pion is composed of 137 e^+e^- pairs (with an electron-neutrino pair or positron-antineutrino pair added for the charged members), the pion would have the requisite odd spacial parity.

Whether or not this model for the pion works, once we have a charge-triplet, pseudoscalar entity of the right mass, a great deal of "strong interaction physics" can be "bootstrapped" out of this. All one has to do is to require the pion to be a bound state of three pions (a particular application of the Wick-Yukawa mechanism, where we need three rather than two because even and odd pion number systems do not freely transform into each other). One approximate way of doing this has recently been presented by Brayshaw[45] and can be interpreted as showing that the probability of the scattering of two pions at low energy is not a free parameter but is determined by the pion mass. Earlier work by Gore[46] and others showed that given the right low energy scattering, and the analytic structure required by the charge-triplet pseudoscalar structure of the pion, the "rho-meson", the dominant resonance in the two pion system,has to follow in terms of this single parameter. If, for each system, the single parameter is indeed determined by the pion mass itself, as Brayshaw's calculation suggests, there may well be a complete low energy dynamics of pionic systems with no explicit "empirical" input. Although this "bootstrap" has yet to become a reality, very modest empirical input allows all the important features of the three-pion system to be computed, and the same is true for the most important feature of the nuclear force.[47]

The construction of a parameter-free theory of "strong-interactions" need not stop with the pions. Provided only we can see why no more than about 10 pions can be meaningfully described within their own Compton wave length, we can "derive" what is conventionally called the (pseudovector) Yukawa coupling constant. But again, given the correct dynamics of two and three pion systems, it has already been shown that this one number plus the existence of spin 1/2 neutrons and protons can be used to couple the nucleonic dynamics to the known pionic dynamics and predict correctly the very complicated structure observed experimentally in the scattering of a pion by a nucleon (including the production of a second pion) and, given that, the scattering of two nucleons (including the production of a pion). Thus all of the "nuclear force" picture up to the point where "strange particles" enter can be brought, at least conceptually, within this basic schema. Bringing strange particles into the act in turn requires only going on up the structural hierarchy to the three quarks of which the neutron and proton, and the "strange" baryons are sometimes thought to be composed.

The entrance of the nucleons (or underlying quarks) at this point in the construction suggests a possible answer to the question of why there are, apparently, two basic masses for spin -1/2 particles - leptonic and baryonic - and not just one unit of mass. We have seen that it is at least conceivable that the "mass" of the electron and positron is simply a reflection of its electromagnetic properties, and not a "mechanical mass". Further, we have seen that there is a possibility of using electrons and positrons to construct the pions and other heavy bosons without introducing any new concepts or constants. The difficulty with this "universe" is that it is not stable, if it starts out electrically neutral. Eventually the electrons and positrons annihilate each other, leading to neutrinos (spinning black holes) and perhaps to ordinary black holes. If the neutrinos are indeed spinning black holes, and there is no overall angular momentum to the universe, they to will end up as spinless black holes. And a universe consisting only of black holes with zero angular momentum strikes me as having so few observable properties that it may well be yet another version of Nirvana. Another way of putting it is that so long as we have spin -1/2 neutrinos we still have a lever for bootstrapping us back to the world of quantized masses, but once these disappear we might have left only the black holes of the general theory of relativity, which can have continuous mass values and hence violate our basic Democritean postulate. We need a basic mass value in the theory, which we can take to be the quark mass. Given that we can then try to construct the sequence: 3-quarks, 10 quark-antiquark pairs (pions and other bosons), 137 charged particles, 10^{38} hadrons, as already suggested. Once we have electric charge, the possibility of systems with charge and <u>only</u> electromagnetic mass occurs naturally and gives us electrons, positrons, and their associated neutrinos. But since we now have a basic mass, the muon might carry the remnant of this quark mass, explaining the large mass ratio to the electron, and some trace of it might even persist in the muon neutrino (provided it is small enough) without doing violence to any currently known experimental facts.

One conceptual loose end remains, and at a level that could bring this whole scheme down in ruins. We found the logical necessity for the existence of electrical charge as a consequence of our basic postulates, once we had succeeded in constructing positive energy spinors, but then went on to use other electromagnetic properties of the conventional theory (basically Coulomb's law) without constructing them.

From the point of view of "classical" physics, it is the long-range gravitational and electrostatic forces which knit the world together and provide the solid base on which all these speculations rest. Newton's laws were brought to the fore by gravitational problems and illustrated by him with examples drawn from the solar system. Electricity was investigated by Coulomb in analogy with Newton's gravitational theory, and both were mathematicized by Laplace in a single fertile paradigm. Rutherford provided the experimental facts for Bohr's "planetary" model of the atom, and Bohr quantized that model, using this mathematical paradigm as the known limit at large distances. Even Schroedinger's equation and Heisenberg's matrix mechanics used the inverse square law as the exemplar of the theory and the test case for comparison with experiment. From that point of view, quantum mechanics provides an explanation of the stability of atoms needed to complete the picture and (with the exclusion principle, electrons and nuclei) the details needed to understand chemistry, rods and clocks, evolution, and the phenomenal world. The ultimate justification of physics as a human creation that leads us back to these concepts by understandable historical and retrodictable stages relies on that consistency.[11] But the link connecting these long-range, classical phenomena with the Democritean picture discussed so far has not been provided.

The missing link in the argument may not be too hard to supply. As has already been noted, electromagnetic "quanta" (photons) differ from the other particles we have been discussing in that they have no quantum numbers intrinsic to their description. Loosely speaking they have "spin one", but the same mathematical property is expressed by the classical equations of Maxwell and Lorentz. We can bring them into the picture developed above by putting a neutrino and an anti-neutrino together. We cannot simply use two neutrinos or two anti-neutrinos, as then the system would have lepton (or muon) number ±2, while photons cannot be allowed particulate quantum numbers. If we fuse a neutrino and an anti-neutrino travelling in the same direction, their spins and lepton (or muon) numbers cancel, so the "spin one" character of photons cannot be intrinsic if this model is to apply. If we give the neutrino and anti-neutrino one unit of relative angular momentum in space, we provide the missing quantum number and the two states needed with respect to the direction of the photon. As already noted, this is all we need for describing other ways photons are used (i.e., we can use these circularly polarized states to construct linearly polarized states or vice versa).

There are a number of advantages in thinking about photons as zero mass states of neutrinos constructed in such a way as to eliminate any quantum numbers which come from their particulate substructure. To begin with this "explains" why classical physics could get started without using particles as a basic and unavoidable concept. The momentum and energy of such a system are not separate but equal (in a dimensional system with c=1), and continuously variable. The zero mass, given a limiting velocity, gives the inverse square law for forces (Coulomb's law) in the static limit. But this continuity breaks down once particles with electromagnetic mass (electrons) or intrinsic mass (baryons) come up over the horizon. Whether this neutrino model of photons can meet the problem of introducing electromagnetism into a Democritean theory remains for the future to decide. In the meantime, we take comfort from the fact that more conventional theorists like Steve Weinberg are (for different reasons, but presumably making use of the same symmetries used above) trying to identify weak and electromagnetic "interactions" as two aspects of related phenomena.

The recent discovery[48,49] of the "Gipsy" resonance[50] and associated higher energy phenomena suggests a specific way to implement this program. Since the gipsy is a sharp resonance in the electron-positron system of unit angular momentum, like the photon, we can start a phenomenology of this angular momentum state by parametrizing the e^-e^+ system as having a bound state of negligible rest mass (the photon), this sharp resonance, and branching ratios to other two-body channels. Since these channels include electron type and muon type neutrino-antineutrino pairs, muon-antimuon pairs, systems of pions, kaons, baryons,...all the strong, electromagnetic, and weak "interactions" can pass through this channel. If we cling firmly to the Democritean idea that there are only particles, the probabilities remain finite, and we can start a unified description without encountering the infinities that plague "field theory".

Putting together a neutrino and an antineutrino to form a spin one photon suggests that the other long-range classical "field" could be constructed in a similar way. Putting them together with zero relative angular momentum would yield a zero-spin zero-mass field congruent with Newton's gravitational theory, were it not for the fact that this state is not reflection-invariant or, using technical terms, is a pseudoscalar rather than a scalar. But if we put together a spin one electron neutrino-antineutrino pair with a spin one muon neutrino-antineutrino pair to form a spin two system of negligible mass, we would have some of the right properties for the spin-two "field" used in Einstein's theory of general relativity. Since putting together two spin one photons would only give another "state" of the Maxwell field, we need two different kinds of neutrinos to make this construction. This is an independent indication of why we need two kinds of neutrinos. So, at least in terms of quantum numbers, our particulate point of view can recover two different long-range effects that mimic the needed aspects of the Einstein and the Maxwell fields. By adding a limiting velocity (special relativity) we get photons with all the symmetry properties needed, and (within a factor of two) the properties of "black holes" needed at the level of precision of the arguments presented in this paper. To construct the full tensor "field" of Einstein's general theory from neutrinos would require at least two kinds, as already noted, while the detailed construction, if successful, might provide an additional clue to the baryon-lepton and muon-electron puzzles which are, from the point of view of this author, the least understood problems confronting modern Democriteans.

Two basic ideas emerge from this discussion as extraordinarily fertile: the atomic or particulate requirement that allows us to start a rational discussion and the random fluctuations of particle number which prevent us from slipping back into a deterministic world; instead we find a fixed past from which we can at present deduce only the probabilities of future events. These basic concepts entail a limiting velocity, wave-particle duality, and a unit of mass, thus removing dimensional constants from the theory. Once we add the basic mechanical postulate of the conservation of momentum, these ideas can be articulated quantitatively, and, by further application of the discriminatory postulate, require two types of limiting particle number which we can identify with the short-range breakdown of the space-time description of the classical electromagnetic and gravitational fields. By invoking technical details about Dirac spinors we arrive at neutrinos and (possibly) the massless quanta of the Maxwell and Einstein fields. But this world is unstable, implying the necessity of an intrinsic unit of mass, which we identify with the baryons. By viewing bosons as assemblages of electrons and positrons we provide the link between the hadronic and the leptonic worlds. This whole construction would not be possible without the empirical knowledge of the classical fields and quantum numbers of the "elementary particles" which guided each step. But the classical and quantum pictures of earlier theories, when applied to known cosmological phenomena, provide a possible route from the particles to the physicists who discuss them.[12] Thus the logical loop "closes" at the point when physicists can logically reconstruct the particles, but necessarily only as an approximation to the fixed past. The novelty which has already emerged by this route, together with the basic fact that at best we can only predict probabilities for future events, warns us to anticipate still more novelty in the future.

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