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THE CLASSIFICATION AND DECAYS OF RESONANT PARTICLES*

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ABSTRACT

The classification of resonant particles and some currently popular symmetries for describing their decays are reviewed. Experimental developments are brought up to date. The successes of SU(3), Regge trajectories, and the quark model are noted. Recent models for decays, based on quark pair creation or on transformation between two kinds of quarks, are discussed and their relation to each other and to $SU(6)_W$ is mentioned. Sections are devoted to duality, whose most specific predictions remain to be confirmed, and to the scalar mesons, whose spectrum and decays may be relevant to considerations of broken scale invariance. Suggestions are made for future experimental work.

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CONTENTS

I. INTRODUCTION

At the time of Mendeleev, the elements were arranged into sequences which described their reactions but had no deeper basis. It remained for Bohr to propose a simple model of the atom which explained the properties of Mendele periodic table.

Elementary particle physics today is at the Mendeleev stage. The wealth of resonances can be classified according to relatively simple rules. Other rules, only slightly less simple, describe their decays. It remains for us to construct a self-consistent theory yielding these rules: a Bohr model of the hadrons. Within a generation, one can hope such a model will indeed exist.

The quark model will form the basis of our "periodic table" of the particles Since the quarks have not been seen, however, there will always remain a suspicion that they may be discarded in the end. We shall thus attempt to discuss only those algebraic properties that could hold even if real quarks were not seen.

Some very simple and compelling rules for classifying particles and their decays can be expressed in the language of symmetries. The symmetries, in turn, are a convenient point of contact with dynamical theories. For example, symmetries of resonance decays can indicate what features of specific quark models should be taken seriously.

Symmetries also allow the systematic experimental testing of dynamical models. There is the obvious job of filling missing gaps, just as was necessary to confirm the periodic table of the elements. Furthermore, once a symmetry is considered as established (we may rank SU(3) in this category) it allows one to draw on a broader base of experiments in testing theories. This is essential in hadron physics, where the spectroscopic data are relatively meager in comparison with studies of atoms or nuclei.

The Rutherford experiments on large-angle scattering of α-particles were a basic key to the Bohr theory. In fact, Bohr was largely ignorant of spectroscopy until a month before he wrote his first paper on the hydrogen atom (see, e.g., Klein's biography of Ehrenfest, Klein, 1970). Perhaps large-angle scattering experiments will similarly provide the key to hadron structure. But any models of the hadrons will have to reproduce their level structure and decays just as the Bohr theory — and later, quantum mechanics — explained atomic levels and transitions so successfully. At present we cannot ask that theories reproduce hadron level structures exactly. But certain gross features — mostly those expressible in terms of symmetries — would be pleasing to see emerge. We shall try to stress as many of these overall patterns as possible without straining the data beyond credibility.

There have been some new experimental developments in resonance physics in the past few years. These have been well-reviewed quite recently, both for the mesons (Diebold, 1972) and for the baryons (Lovelace, 1972). We would like to recall briefly some of the salient points discussed in these reviews, in the parallel sessions of the 1972 Batavia conference, and more recently in the literature. This "mini-review" is the subject of section II.

The success of the group SU(3) is without question in particle physics. The hadrons fall into SU(3) multiplets, and their decays obey SU(3). The symmetry is not exact: certainly not for masses, and probably not for couplings. Nonetheless it appears to be a tremendous help in classifying resonances and their decays, as discussed in section III.

The hadrons exhibit some regularities associated with spins and parities which are treated in section IV. The lowest levels have a parity alternation suggestive of orbital-excitation sequences. States of a given spin are associated with similar states with higher masses and spins spaced by intervals of two units. These families (Regge trajectories) are well-known and we simply update the evidence for them.

The quark model is the most compelling scheme at present for classifying resonances. The quarks are spin-1/2 members of an SU(3) triplet. All mesons behave as if constructed of $q\bar{q}$; all baryons look like qqq. The details of this picture are presented in section V. The "box score" for filling quark model multiplets is given there. Any alternative scheme which explained the same data with the same number of (or fewer) assumptions would be welcome. In particular a picture of particles as made of <u>each other</u> is <u>not</u> inconsistent with our view of the quark model. It is only that this "bootstrap" view of the hadrons has not been as useful a guide to further experiments or to the overall pattern of the particles.

As in the case of atomic and nuclear spectra, the understanding of the pattern of levels is only a first step in the physics of resonant particles. Selection rules, transition intensities, and interference phenomena play an important role in determining hadron structure. Some recent progress in understanding the strong decays of resonances indeed has been made. This progress, bearing on the concept of what we mean by quarks, is described in section VI. Symmetries such as $SU(6)_W$, and the much-discussed Melosh transformation (Melosh, 1973), are treated in this part.

The notion of "duality" — the matching of direct-channel and particle exchange descriptions of scattering — has rich consequences for resonances. Some of the consequences, in fact, are <u>too</u> rich, and <u>exotic</u> resonances must occur. The whole situation is brought up to data in section VII.

Broken scale invariance has some bearing on the 0^+ resonances, allowing for a tenth scalar meson. Some of the simplest considerations are presented in section VIII.

Where does one go from here? There remain some important experimental resonance questions. Some of them are very old but were never answered satisfactorily. Now they can be, with carefully chosen experiments. Others are new questions that have arisen in the context of our greater understanding of the existing pattern of resonances and couplings. A pair of sections is devoted to prospects for answering the old and new questions.

In section IX we treat elastic and inelastic two-body meson-baryon scattering in the resonance region. Elastic πN scattering is in good shape. It needs to be complemented by information on specific inelastic channels such as $\pi N \rightarrow K\Lambda$, $K\Sigma$, ηN, $\pi\Delta$, etc. Hyperon physics is harder but some suggestions can be made for overcoming the difficulties.

New accelerator facilities will be able to study resonances in colliding e^+e^- beams, in multiparticle spectrometers, and in old-style experiments (like $\pi^- p \rightarrow \pi^+\pi^- n$) which get cleaner as the energy increases. Some of these high-energy tests are discussed in section X.

In the last section (XI) we conclude that resonance physics is making progress, worth doing, and telling us something. Some useful formulae are presented in the Appendices.

II. WHAT'S NEW?

The present section is a selected sample of some new developments on resonances. These topics are chosen subjectively, with an eye to their possible bearing on interesting theoretical questions. Hence not everything new is mentioned, and not everything mentioned is really all that new. Specific points are covered in more detail by (Diebold, 1972; Lovelace, 1972; and Rosner, 197

A. Mesons

1. The A_2 is unsplit

Why is this important? Many people (myself included) believed for several years that the A_2 was a doubled resonance. This was bad for the quark model. It made classification schemes very complicated.

In 1971 an attempt was made to duplicate the conditions of the original CERN experiment, but no splitting was seen (Bowen, 1971). It now appears that the original splitting may have resulted from an apparent statistical fluctuation, amplified by suitable cuts in the data. No other recent experiment has seen the splitting, and CERN itself is less confident of the effect (Kienzle, 1972

The A_2 experience led to considerable improvement in our understanding of production mechanisms (Michael, 1971; Rosner, 1971b). It showed how muc could be learned about a given resonance when the power of several serious counter experiments was focussed on it. As we shall see, several other resonances could benefit from such a detailed study.

2. The B has $J^{PC} = 1^{+-}$ and dominantly transverse $\omega\pi$ decay

A review of the B was presented by (Chung, 1972) at the NAL conference. Most experiments have agreed for a long time that there is an $\omega\pi$ resonance around 1235 MeV, in which the ω's emerge dominantly (but not completely) with helicity ± 1. More recently, ambiguities in the J^P assignment of the B have been eliminated, to the extent that 1^+ is now favored by a large margin.

The B decays to $\omega\pi$ via both S and D waves. The exact amount of D wave is still somewhat in question, but there seems to be general agreement that it is responsible for at least six percent of the $\omega\pi$ decay width. In the absence of D waves, the ω helicity states would be equally populated. In terms of normalized helicity amplitudes F_λ (Appendix A), pure S wave would entail $|F_0|^2 = |F_1|^2 = |F_{-1}|^2 = 1/3$. Recent values for $|F_0|^2$ include

$$|F_0|^2 = \left\{\begin{matrix} 0.12 \pm 0.04 \\ 0.16 \pm 0.04 \end{matrix}\right\} \text{two analyses of (Chung, 1973)*} \qquad \text{(II. 1a)}$$

and

$$|F_0|^2 = 0.01 \pm 0.07 \quad \text{(Karshon, 1973)} \qquad \text{(II. 1b)}$$

These are shown in Fig. 1. The world average corresponds to $|F_0|^2 = 0.13 \pm 0.05$, or dominantly transverse $B \to \omega\pi$, and only 6% D wave. More details are mentioned by (Rosner, 1973c).

The significance of the new analyses is that they help to tie down a parameter of great importance in discussing $SU(6)_W$. The naive use of this symmetry predicts $|F_0|^2 = 1$, in complete disagreement with data. On the other hand, the transformation between current quarks and constituent quarks proposed by (Melosh, 1973), and related models for decays discussed in section VI, allow $|F_0|^2$ to be a free parameter. It is then the <u>only</u> free parameter (aside from an overall scale) describing nearly <u>all</u> decays of the lowest-lying positive parity mesons: the 0^+, 1^+, and 2^+ families. As we shall see, a satisfactory description emerges of these decays. The D-wave decay is related to other D-waves such as $A_2 \to \rho\pi$ or $f_0 \to \pi\pi$, while the S-wave decay is related to $\delta \to \eta\pi$, etc.

Another aspect of the recent analyses is that they find no evidence for the claim of a $J^P = 1^-$ state under the B (mentioned by Diebold, 1972).

*These supersede an earlier claim by (Ott, 1972) for the absence of D-waves. The analysis of (Afzal, 1973) is consistent with these values of $|F_0|^2$.

3. High-mass vector mesons

Theorists have wanted a higher-mass copy of the $\rho(J^{PC} = 1^{--})$ for some time, at various masses and for a variety of reasons. Finally, a candidate has been found at a mass ($\sim$1500 MeV) that makes quark modelists happy. Certain dual models wanted a ρ' under the f_0, which now seems unlikely (see above and the discussion on $\pi\pi$ scattering below).

Various versions of the $\rho'(\sim 1500)$ have been reported. Two mentioned last year included: (a) a direct-channel effect seen at ADONE in $e^+e^- \rightarrow \pi^+\pi^-\pi^+\pi^-$, $m \sim 1600$ GeV, $\Gamma \sim 350$ MeV;* and (b) a $\rho^0\pi^+\pi^-$ resonance with $m = 1430 \pm 50$ MeV and $\Gamma = 650 \pm 100$ MeV, seen in $\gamma p \rightarrow \rho^0\pi^+\pi^- p$ at 9.3 GeV.** There seems to be some controversy as to just how low the $\pi\pi$ coupling of this state is (see the mini-review in Lasinski, 1973). At any rate, no convincing evidence has yet been presented for $\rho' \rightarrow 2\pi$, and theorists are beginning to wonder why.

In the $\rho^0\pi^+\pi^-$ mode, the $\pi\pi$ system seems to have I = J = 0 (the "ϵ"). The $\rho\epsilon$ system appears to have ℓ=0. One amusing conclusion from sections V and VI will be that such a ρ' must be an L = 0 (not an L = 2) quark-antiquark state. In the quark model any ρ' which couples to photons should indeed be an S-wave $q\bar{q}$ state (see, e.g., Dalitz, 1968).*** The SLAC-Frascati ρ' is thus a good candidate for the radial excitation of the ρ. The L=2 state expected nearby in mass (see, e.g., Gilman, 1972) then remains to be discovered. An improved fit to high-energy $\pi\pi$ scattering indeed follows from assuming the existence of a rather narrow ρ'(Hyams, 1973) with $\Gamma \lesssim 200$ MeV, $\Gamma_{\pi\pi}/\Gamma \simeq 1/4$, lying under the g(1680). The properties of this object seem to be rather different from those observed in photon-induced properties, but its existence is still highly tentative.

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*(Ceradini, 1973).
**(Davier, 1973).
*** This point has been raised more recently by (Raitio, 1973).

4. The $A_1(1070)$ and $A_3(1640)$: nonresonant?

Analyses of $\pi^- p \rightarrow \pi^- \pi^+ \pi^- p$ (Ascoli, 1973, Antipov, 1972) indicate trouble for the resonant interpretation of the A_1 and A_3. The $\pi^+\pi^-$ systems are approximated by I=0 and I=1 "ϵ" or f_0 and "ρ" states ($\ell_{\pi\pi} = 0$ or 2 and $\ell_{\pi\pi} = 1$, respectively). One can then parametrize the 3π system by $\epsilon\pi$, $f_0\pi$, and $\rho\pi$, and present the data as if for two-body final states.

Broad peaks are found in $\rho\pi$ ($J^P = 1^+$, $\ell_{\rho\pi} = 0$), extending from $\rho\pi$ threshold to ~ 1.4 GeV, and $f_0\pi$ ($J^P = 2^-$, $\ell_{f\pi} = 0$), around 1.6 GeV. The phase variation with respect to other partial waves which would be expected if these peak were resonant is not found, however. This looks suspiciously like the nonresonant Deck effect (Fig. 2a) (Deck, 1964; Berger, 1968).

One suggestion (Wright, 1972) with regard to the A_1 is to tuck it neatly on an appropriate unphysical sheet so that the expected phase variation will not occur. This example is important whether or not it's relevant here; nearby thresholds will always influence our extraction of pole parameters.

It is also possible that the true "resonant" A_1 is hiding under a big non-resonant background. If so, the mechanisms of Figs. 2a and 2b could coexist. The "resonant" A_1 would have to be produced rather weakly, however, and not to have too strong a coupling to the S-wave $\rho\pi$ system.

Certain symmetries mentioned in section VI yield predictions for $A_1 \rightarrow \rho\pi$ decay widths ranging from $\lesssim 100$ to 400 MeV. Part of the indeterminacy comes from our poor knowledge of 0^+ meson partial decay widths, which are discussed next.

5. New information on scalar mesons

There have been two recent large analyses of pion-pion scattering. One is based on $\pi^+ p \rightarrow \pi^+ \pi^- \Delta^{++}$ (Protopopescu, 1973) at 7 GeV/c. The other comes from the CERN-Munich data on $\pi^- p \rightarrow \pi^+ \pi^- n$ at 18 GeV/c (Estabrooks, 1973; Hyams, 1973).

The new developments concern the $I=Y=0$ phase shifts. These behave a illustrated in Fig. 3. There is rapid variation just below $K\bar{K}$ threshold, whic probably is due to a resonance (the S*) around 997 MeV. There is also a res nance around 700 MeV — the ϵ — and possibly a third state (Carroll, 1972; Estabrooks, 1973) under the f_0, coupling mainly to $\pi\pi$.*

Three $I=Y=0$ states with $J^{PC}=0^{++}$ cannot be accommodated in the quark model, which predicts only two with nearby masses. The third state can be a "dilaton", (see, e.g., Carruthers, 1971a). In section VIII we shall construct one model for the 0^+ states which incorporates such a resonance.

The $K\pi$ situation has also advanced of late. An analysis below $M_{K\pi} \simeq 1$ G (Barbaro-Galtieri, 1973) rules out the possibility of all but a very narrow S-wave $K\pi$ state under the K*(890) $\left[\Gamma_{K_N} < 7 \text{ MeV}\right]$. This analysis is based o a large sample of 12 GeV/c K^+p bubble chamber events. It is then likely that a $J^P=0^+$ $K\pi$ resonance exists somewhere between 1100 and 1400 MeV, bu its parameters are not yet satisfactorily determined. A lower-lying 0^+ $K\pi$ resonance would have been surprising, indicating mass patterns other than th seen in 0^-, 1^-, 1^+, 2^+, 2^-, and 3^- multiplets. [The masses of the 0^+ mesons are discussed further in section VIII.]

To complete the picture of advances in the 0^+ meson area, we mention a very recent result from a (now dispersed) Chicago-Wisconsin-Argonne group (Conforto, 1973). The $\eta\pi$ mass spectrum in $\pi^-p \rightarrow \eta\pi^-p \rightarrow \gamma\gamma\pi^-p$ at 4.5 GeV/ shows a bump of 3.7σ with fitted mass 980 ± 1 MeV/c^2 and width 60^{+50}_{-30} MeV/c (Fig. 4). This peak corresponds to that currently labelled $\delta(970)$ by the Particle Data Group (Lasinski, 1973). Its J^P is consistent with 0^+, in accord with earlier studies. The new experiment is the first in which this resonance has been produced in $\pi^-p \rightarrow \delta^-p$, aside from an early missing-mass study

*See (Lipkin, 1973f) for one interpretation of this state.

(Focacci, 1966) whose observation of an extremely narrow state ($\Gamma < 5$ MeV) has not been confirmed. Its production cross section ($1.8 \pm 0.8\ \mu b$) is considerably below the estimate ($\sim 40\ \mu b$) of (Fox, 1973).

The $\delta(970)$ decay is important from a theoretical standpoint. The $\delta(970)$ is probably our best source of information on the couplings of 0^+ mesons, since (unlike the $I = Y = 0$ states) it is not affected by arbitrary mixing assumptions. These couplings are of great importance in testing schemes of broken $SU(6)_W$ (section VI) and broken scale invariance (section VIII). Hence even though the experiment is clearly very difficult, it is worthy of attention and confirmation.

6. Heavy mesons

In 1966 broad bumps in $\sigma_T(\bar{p}p)$ were observed (Abrams, 1970). At the same time, missing-mass experiments in $\pi^- p \rightarrow X^- p$ showed narrow structures dubbed, R, S, T, U, ... (Focacci, 1966). These latter experiments were extended to higher and higher missing masses (Maglić, 1969) and there seemed no end to narrow structures (X's). It seemed hard to reconcile such bumps with the broad ones in $\sigma_T(\bar{p}p)$.

Recently the Rutgers group has analyzed the $\bar{p}p$ bumps in terms of detailed annihilation channels. It is found that no particular multiplicity or configuration is responsible for the bumps (Alspector, 1973). At the same time, no narrow structures appear, no matter what selections on multiplicity are made. The missing-mass studies by the Northeastern-Stony Brook group (Bowen, 1973) also fail to confirm the narrow resonances found earlier in similar experiments. On the other hand, some support for a narrow $\bar{p}p$ bump now comes from total cross section measurements! A new very low-energy study of $\sigma_T(\bar{p}p)$ and $\sigma_T(\bar{p}d)$ (Carroll, 1973b) at Brookhaven indicates the existence of a bump at the mass of the old S(1930) with narrow ($\lesssim 30$ MeV) width. The situations are compared in Table I. Other data are reviewed by (Smith, 1973).

While missing-mass studies seem to have difficulty in agreeing with one another, one can hope to gain information on high-mass resonances by various partial-wave techniques. The measurement of $\bar{p}p$ total cross sections and other elastic scattering parameters, and the study of $\bar{N}N \rightarrow$ (meson pairs), will be an important step in this direction. Recently (Parsons, 1973; Hovjat, 1973) some high-statistics differential cross sections for $\bar{p}p \rightarrow \pi^-\pi^+$ and $\bar{p}p \rightarrow K^-K^+$ have been obtained at CERN. These will be an important ingredient of phase shift analyses once the corresponding polarized target data have been taken. Broad, overlapping high-mass mesons may be sorted out from one another by multiparticle partial wave analyses like those of (Ascoli, 1973). Some particularly clean two-body channels — like $K\bar{K}$ and $\eta\pi$, to be discussed in section X — are probably ideal for studying heavy mesons. All these methods, as opposed to missing-mass spectrometry, have the potential of making precise statements about spins and parities.

7. $N\bar{N}$ annihilations at rest: do S-waves dominate?

It is usually assumed that nucleon-antinucleon annihilation at rest takes place dominantly from S-wave states. This conclusion (Day, 1959) is intuitively reasonable since for all higher waves the relative $N\bar{N}$ wave function vanishes at the origin with a slope (outside the range of the strong interactions, that is) which should be characteristic of the electromagnetic interaction.

A recent experiment suggests that the process $\bar{p}p \rightarrow \pi\pi$ at rest is not pure S-wave (Devons, 1971). The process $\bar{p}p \rightarrow \pi^o\pi^o$ must occur via odd $\bar{p}p$ angular momentum states with $J = \ell \pm 1$ (e.g., ${}^3P_0$, ${}^3P_2$, ${}^3F_2$, ${}^3F_4$, ...) in order to satisfy C and P invariance. It was found that

$$\frac{\text{Rate } (\bar{p}p \rightarrow \pi^o\pi^o)}{\text{Rate } (\bar{p}p \rightarrow \pi^+\pi^-)} = 0.15 \ (\pm\ 30\%)$$

'his indicates a ratio

$$\frac{(\bar{p}p)_{\ell \text{ odd}} \to \pi\pi \atop I=0}{\bar{p}p \to \pi\pi} \simeq 0.4\ (\pm 25\%) \qquad \text{(II. 2)}$$

Another $\bar{N}N$ experiment (Gray, 1971) uses the tail of the deuteron wave unction to dig below threshold in the reaction

$$\bar{p}n \to \text{(resonance)} \to \text{mesons} \qquad \text{(II. 3)}$$

n this manner the momentum spectrum of the spectator proton in

$$\bar{p}d \to \text{(mesons)} + p_s \qquad \text{(II. 4)}$$

ıllows reaction (II. 4) to be utilized as a missing-mass experiment. A peak in he $p(p_s)$ distribution corresponding to

$$M_R \sim 1795 \text{ MeV}, \quad G = +, \quad \Gamma < 8 \text{ MeV} \qquad \text{(II.5)}$$

.s seen: the signal seems to occur only for an even number of pions.

The statistical significance of the effect in (II. 5) is not overwhelming, and he authors admit the possibility of a bias due to the size of their bubble chamber. Nonetheless the effect is interesting and needs to be confirmed. If such a narrow resonance really existed, it would be very puzzling. Why wouldn't it fall apart immediately into many pions?

Very recently the ratio

$$\frac{\bar{p}n \to \pi^-\pi^o}{\bar{p}p \to \pi^+\pi^-} = 0.68 \pm 0.07 \qquad \text{(II.6)}$$

has been reported in annihilations at rest (Gray, 1973). Again, this is in conflict with S-wave dominance, which would require the ratio to be 2.* Moreover, the reaction $\bar{p}n \to \pi^-\pi^o$ is <u>only</u> seen clearly when the spectator proton is invisible. This effect, as well as all the others mentioned here, could be symptomatic of narrow resonant structure around $\bar{N}N$ threshold. We shall

*The evidence for S wave dominance of the bulk of $\bar{N}N$ annihilations at rest is discussed by (Bizzarri, 1972).

discuss this possibility further in section IX in the context of suggestions for further low-energy $\bar{N}N$ studies.

B. Baryons

1. A missing N* discovered

Several new analyses see an $N^*(1730, 3/2^-)$. This is an interesting and important resonance.

Five years ago nearly all the nonstrange members of a $\underline{70}$, L=1 multiplet of $SU(6) \times 0(3)$ had already been discovered: all, that is, except this one (Harari, 1968b). What took us so long?

Theorists said this resonance should be hard to find (see, e.g., Moorhouse 1971). It was expected to couple very weakly to πN and strongly to $\pi\Delta$. This is exactly what was found. Its elasticity is $\sim .1$ (Ayed, 1972) and it shows up in $\pi N \to \pi\Delta$ (Herndon, 1972).

Quite independently, a similar resonance also now shows up in the first of two solutions in an analysis of $\pi N \to K\Sigma$ (Langbein, 1973).

Even the mass of the new resonance is close to theoretical expectations (Feynman, 1971). It is mainly quark-spin 3/2 and the prominent $N(1520, 3/2^-)$ is mainly quark-spin 1/2 (Faiman, 1972).

2. "SU(3)-inelastic" reactions

The resonant contributions in such processes as $\bar{K}N \to \pi\Lambda$ are related to those in $\bar{K}N \to \bar{K}N$ by an SU(3) Clebsch-Gordan coefficient, and comparison of the two processes tells us about the quality of SU(3). This is not true when comparing $\gamma N \to \pi N$ or $\pi N \to \pi\Delta$ with $\pi N \to \pi N$, however: such a comparison instead can provide valuable information on higher symmetries such as $SU(6)_W$. We refer to processes of the latter type as "SU(3)-inelastic". Much recent progress has been made in analyzing the data on such channels. As will be mentioned in section VI, these analyses generally agree with relativistic quark models such as those of (Feynman, 1971), and with various broken-$SU(6)_W$ symmetry schemes. They rule out "naive" $SU(6)_W$ in much the same way as did the data on $B \to \omega\pi$ (see above).

In a massive compilation of low-energy data on $\pi N \to \pi\pi N$, the LBL-SLAC group has performed an isobar fit whereby resonant amplitudes for $\pi N \to \pi\Delta$, $\pi N \to \rho N$, and $\pi N \to \epsilon N$ are obtained simultaneously. (See Herndon, 1972; Cashmore, 1973a,b.) It is still in progress and alternative solutions may exist. Nonetheless the magnitudes and phases of the resonant amplitudes allow many checks of $SU(6)_W$ to be made, with the result that this symmetry almost certainly fails.

The data available to (Herndon, 1972) span the ranges 1.3 GeV $\leq E_{c.m.} \leq$ 1.54 GeV and 1.65 GeV $\leq E_{c.m.} \leq$ 2 GeV. The relative phases of resonant contributions in $\pi N \to \pi\Delta$ within <u>each</u> of these two ranges are consistent with nearly all the <u>broken</u>-$SU(6)_W$ schemes to be discussed in section VI. Data in the gap exist, but are currently the subject of private smaller-scale analyses whose results we should hope to see in the next year.

The photoproduction of single pions in the resonance region, $\gamma N \to \pi N$, has been analyzed by (Moorhouse, 1973c). The magnitudes and phases of many new resonant amplitudes have been obtained, with results which support both the quark model and more abstract symmetry approaches based on the transformation (Melosh, 1973) between two different kinds of quarks. The data and their agreement with these theories are discussed in detail in section VI.

There has been other recent experimental interest in SU(3)-inelastic reactions. An independent analysis of $\pi^+ p \to \pi\pi N$ (Kernan, 1973) uses cuts on the Dalitz plot instead of an isobar fit. The same results as (Herndon, 1972) are generally obtained for $\pi N \to \pi\Delta$, but the authors disagree with the conclusion that much resonant contribution to $\pi N \to \rho N$ is occurring, and ascribe much of this process to one-pion-exchange instead. A study has also been made by a Dubna group of $\pi N \to \pi\pi N$ at a single low energy (Bunyatov, 1972). The reaction $K^- p \to \pi\pi\Lambda$ has recently been re-analyzed by (Prevost, 1973). This reaction is capable of providing information on $\bar{K}N \to \pi\Sigma(1385)$, but the statistics are not competitive with $\pi N \to \pi\pi N$, and the solutions do not look particularly stable yet.

At low energies, the decay $\Lambda(1520, 3/2^-) \to \pi\Sigma(1385)$ has been seen quite clearly (Mast, 1973). This decay indicates that $\Lambda(1520, 3/2^-)$ cannot be a pure SU(3) singlet.

3. New data relevant to SU(3) for baryons

a. $\Xi^*(1530)$ widths. In the past year several new numbers have been quoted, shown in Table II. The question of whether these numbers are accurate enough to demonstrate SU(3) breaking depends on how one assigns errors, but the pattern is clearly visible in Fig. 5. It has been suspected for a long time that $\Delta \to N\pi$ is slightly too large and $\Xi^* \to \Xi\pi$ slightly too small for exact SU(3) (Tripp, 1968). These possible discrepancies do not exceed the order of mass splittings among the members of the multiplets, as can be seen by comparing the results of one fit with experiment (Samios, 1973) (Table III).

b. Decays $\underline{8} \to \underline{10} + \underline{8}$. From the SU(3)-inelastic reactions mentioned above, one can in principle compare rates for resonance decays to $\Delta\pi$ and to $\Sigma(1385)\pi$. These comparisons have not met with success (Barbaro-Galtieri, 1972; Samios, 1973). On the other hand, they make use of $\Sigma(1385)\pi$ decays which are still poorly determined, and often ignore important mixing effects (whose significance has been discussed by Faiman, 1972). The successes in analyzing $\pi N \to \pi\Delta$ must now be matched by ones for $\bar{K}N \to \pi\Sigma(1385)$, and one will then have some useful SU(3) tests to make.

c. Decays $\underline{8} \to \underline{8} + \underline{8}$. Here there has been no substantial change since 1970 (Plane, 1970; Samios, 1970). There have been some small adjustments in resonance parameters (Barbero-Galtieri, 1972) as a result of trying to restore harmony among discordant hyperon-resonance analyses (see, e.g., Lovelace, 1972). The newly fitted f/d ratios are shown in Table IV, together with $SU(6)_W$ or quark model predictions. (These will be discussed in more detail in section VI.) In what follows we shall often take $f+d=1$.

It is interesting to see how various partial widths influence the determination of values of f. This can be seen in Fig. 6 (Samios, 1973), which plots $\chi \equiv \left[\Gamma_{exp} - \Gamma_{SU(3)}\right]/\Delta\Gamma$ against f for decays in various multiplets. For example, in Fig. 6e, taking the $\Sigma\pi/N\bar{K}$ branching ratio of $\Lambda(1817, 5/2^+)$ very seriously leads to very tight constraints on f. Errors assigned by (Barbaro-Galtieri, 1972) are larger, leading to a very different value of f. The experimental phases of $\Sigma(1915, 5/2^+)$ relative to other resonances in $\bar{K}N \to \pi\Lambda$ and $\bar{K}N \to \pi\Sigma$ support a value of $f < 1/2$, as discussed in detail by (Levi-Setti, 1969).

Table IV indicates that SU(3) is very well obeyed in the $\underline{8} \to \underline{8} \times \underline{8}$ decays of baryon resonances. Certain tests are more significant than others, since the demonstrated existence of decimets and second octets of $1/2^-$ and $3/2^-$ resonances (to be mentioned in detail in section V) makes agreement with SU(3) for $1/2^-$ and $3/2^-$ decays fortuitous. Theoretical expectations (though not concrete data) also indicate that the $7/2^-$ and $5/2^+$ multiplets are capable of being affected by serious mixing. The $5/2^-$ states are expected to remain much purer, and the SU(3) test here is thus of greatest significance among those in Table IV and Fig. 6.

While SU(3) seems to hold, the value $f \simeq -0.13$ or -0.16 for $5/2^-$ decays is rather far from that expected from $SU(6)_W$, namely -0.5 . The experimental rate for $\Sigma(1767) \rightarrow \Sigma\pi$, which is very low at present, is a major source of this discrepancy, as one may see in Fig. 6d. If $SU(6)_W$ is to be proven right, this rate must increase considerably.

d. Decays $\underline{10}(7/2^+) \rightarrow \underline{8} \times \underline{8}$. In the past couple of years there have been analyses of $\pi N \rightarrow K\Sigma$ (Kalmus, 1971, and Langbein, 1973) allowing the extraction of the $K\Sigma/\pi N$ branching ratio of $\Delta(1931, 7/2^+)$. The decays of this resonance are combined with those of $\Sigma(2031, 7/2^+)$ in an analysis by (Samios, 1973) giving $\chi^2/\text{d.f.} = 7.6/4$ and shown in Fig. 7.

The world average used by (Samios, 1973) — $\Gamma[\Delta(1930) \rightarrow K\Sigma] = 3.7 \pm 0.7$ MeV — does not include the number of (Langbein, 1973), which is about a factor of 4 smaller. This last number is difficult to understand, since it comes from analyzing both $I_s = 1/2$ and $I_s = 3/2$ data. (Kalmus, 1971), analyze just the $I_s = 3/2$ data (a reasonable exercise for learning about $I = 3/2$ resonances!). This is an interesting discrepancy and needs to be resolved.

The decay $\Sigma(2031, 7/2^+) \rightarrow \Sigma\pi$ has been on the verge of disagreeing with SU(3) for some time (see Fig. 7). It should be remeasured in other processes, perhaps $K^o_L p \rightarrow \Sigma\pi$. The crucial number of interest is the $\Sigma\pi/\Lambda\pi$ branching ratio, which cannot be altered by mixing the decimet with an octet expected nearby in mass (Faiman, 1971).

4. Elastic pion-nucleon scattering

There have been two major analyses of elastic πN scattering in the past two years (Almehed, 1972; Ayed, 1972). In addition a fit to higher energy data was performed using a Regge amplitude as the starting point (Wagner, 1972a).

The phase shift analyses have helped immensely to confirm and restrict parameters of nonstrange resonances below 2 GeV. These are related to one another in various symmetry schemes (Lovelace, 1972; section VI), so that information on them is welcome.

Above 2 GeV a pattern of resonances is emerging which supports general ideas of Regge trajectories, the quark model, and duality. These states are listed in Table V, along with their significance. A number of possible Regge recurrences are being found. In addition, there is growing evidence for new multiplets, such as $\underline{70}$, L=2 and $\underline{56}$, L=3 (the dimensions are those of SU(6), to be discussed in section V). These new multiplets would be welcome in the quark model (section V), and are features of certain exact duality solutions for baryons (section VII). The resonances are expected to be the first on their Regge trajectories. Finally, some of the observed states fit naturally as "radial excitations" of lower-mass ones. The assignments of states as Regge recurrences or radial excitations are highly speculative, and require data on inelastic channels to confirm them. A Regge recurrence or a radial excitation should have similar SU(3) properties to its lower-lying partner.

Some confirmatory evidence for a prominent negative-parity Δ resonance near 2200 MeV comes from an experiment studying backward π^+p elastic scattering (Baker, 1972). The cross section shows a deep dip at a mass compatible with that of the G_{39} state of (Wagner, 1972a): See Table V and Fig. 8. This same experiment shows peaks at higher mass corresponding to ones observed earlier in backward π^-p scattering (Kormanyos, 1967).

At lower energies, recent measurements of $d\sigma/dt$ for $\pi^-p \rightarrow \pi^0 n$ have been performed by UCLA (Berardo, 1972; Blasberg, 1972), MIT (Yamamoto, 1972), and Berkeley (Nelson, 1973a). The last showed some disagreement with the forward dispersion relation calculations of (Hohler, 1971), as mentioned by (Lovelace, 1972) (Fig. 9). A new dispersion calculation has been performed by (Carter, 1973); its results are also shown in Fig. 9. The agreement is much better; the change is of course due to the real part, as the imaginary part is determined from total cross section differences and is well known by now.

It has already been mentioned that the data of (Nelson, 1973a) disagree substantially with the phase shifts of both (Almehed, 1972) and (Ayed, 1972) above $E_{c.m.} \simeq 1950$ MeV. These data are thus expected to provide important constraints in future solutions.

The first measurement of πN charge-exchange polarization in the resonance region has been carried out at Berkeley (Shannon, 1973). Values are preliminary, and hence are not shown here, but they indicate that substantial re-adjustment of phase shift solutions — particularly those of (Almehed, 1972) — will be needed above $E_{c.m.} = 2$ GeV. This was already clear from the charge-exchange differential cross sections. Below 2 GeV, the polarizations generally agree nicely with both (Almehed, 1972 and Ayed, 1972). It is encouraging that when πN phase shift analyses agree with one another, they also agree with the data! The polarization measurements thus provide a necessary check on the stability of phase shift solutions, and on the physical assumptions common to various analyses.

5. Z*'s

According to the three-quark picture of baryons, the KN system shouldn't have resonances. It has many fewer than $\bar{K}N$ or πN, indeed. Figure 10 shows the time development of all "established" meson and baryon resonances, and Fig. 11 is the corresponding picture for "exotics".

A $J^P = 3/2^+$ Z*(1900) K^+p resonance has seemed possible for some time (Lovelace, 1972), but a recent analysis (Cutkosky, 1973), can do without it satisfactorily. The behavior of the corresponding partial wave amplitude is explicable purely as an opening of the $K\Delta$ channel.* More recently, the evidence has been growing for a $Z_0^*(\sim 1800)$ (I=0). Its J^P is probably $1/2^+$, but could also be $1/2^-$. To resolve the question (Dowell, 1972) one would need polarization measurements of $K^+n \to K^0p$ around the mass of the resonance.

*See, e.g., (Griffiths, 1972).

The K^+N total cross sections in I=1 and I=0 channels have recently been remeasured at Brookhaven. The I=0 cross section, shown in Fig. 12, rises rapidly before the inelastic threshold, supporting the possibility of a very broad Z_0^* ($\Gamma \gtrsim 600$ MeV) with mass ~1800 MeV and elasticity close to one if J=1/2 (Carroll, 1973a). *

The Z* question is particularly intriguing at present since the strong-coupling theory (Goebel, 1966) predicted just those Z*'s which may actually have been seen (I=1, $J^P=3/2^+$, belonging to an SU(3) $\underline{27}$, and I=0, $J^P=1/2^+$, belonging to an SU(3) $\underline{10}$). The strong-coupling theory is very far from the quark model and it is hard to see how the two could coexist.

C. Theoretical Developments

A glance at Fig. 10 shows that the number of resonant particles is not growing very rapidly any more. The emphasis has shifted to answering detailed questions about the structure of decays of known resonances. These have already provided considerable insight into SU(3), and attention has now turned to higher symmetries.

As we shall see in section V, the quark model has been very successful in classifying the hadrons. This success can be stated in algebraic terms, so that one need not relay on a specific dynamical model. The mathematical building blocks used in classification have been termed "constituent" quarks (Gell-Mann, 1972a, b).Nonetheless, models sometimes make predictions for decays (transition matrix elements) as well as for levels. To what extent are these predictions obtained in abstract approaches?

The problem in applying the quark model directly to decays has usually been that one must rely on concrete (and, most likely, incorrect) wave functions. Alternatively, decays in which a hadron A emits a pion and becomes another

*For recent K^+n elastic data see (Giacomelli, 1973).

hadron B are related via PCAC to the matrix element of the axial charge Q_5^i between A and B (here P* is the magnitude of the c.m. 3-momentum):

$$\Gamma(A \to B\pi_i) \sim P^* \frac{\left(M_A^2 - M_B^2\right)^2}{M_A^2} \left| \langle B | Q_5^i | A \rangle \right|^2 \qquad (II.7)$$

The kinematic factor is discussed, for example, by (Horn, 1966). The axial charge Q_5^i is the spatial integral of a local density

$$Q_5^i = \int d^3x \, \mathscr{F}^i_{05}(x) \qquad (II.8)$$

where, in the free quark case,

$$\mathscr{F}^i_{05}(x) = q^+(x) \gamma_5 \frac{\lambda_i}{2} q(x) \qquad (II.9)$$

The Q_5^i, along with vector charges, satisfy an algebra (Gell-Mann, 1962b).

One might hope to evaluate the matrix elements $\langle B | Q_5^i | A \rangle$ by using the quark-model classification for states A and B. However, this leads to a number of predictions which are violated by experiment as we shall see in Section VI. Moreover, there is no guarantee that the quarks q(x) in Eq. (II.9) are the same as those used to classify resonances. (See, e.g., Dashen, 1966; Gell-Mann, 1972a). The "current quarks" of Eq. (II.9) are then related to the "constituent quarks" by a transformation V. This transformation has been constructed by (Melosh, 1973) for the case of the free quark model.

Local densities constructed from the two types of quarks generate two inequivalent SU(6) algebrae, known as $SU(6)_{W,\,\text{constituents}}$ and $SU(6)_{W,\,\text{currents}}$. The physical states seem to be approximately pure representations of $SU(6)_{W,\,\text{constituents}}$. They are almost definitely <u>mixed</u> representations of $SU(6)_{W,\,\text{currents}}$; the mixing is necessary to avoid such "bad" predictions as $G_A/G_V = -5/3$, the vanishing of nucleon anomalous magnetic moments, and so on (references are given in Section VI). The transformation

V "undoes" this mixing. Let the physical state $|A>$ belong to a pure representation of $SU(6)_{W,\,constituents}$ and thus to a mixed representation of $SU(6)_{W,currents}$. Define the state $|\tilde{A}>$ as that state belonging to the same pure representation of $SU(6)_{W,\,currents}$ as the representation of $SU(6)_{W,\,constituents}$ to which $|A>$ belonged. Then V connects the two:

$$|\tilde{A}> = V|A> \tag{II. 10}$$

If we knew V, we could clearly evaluate the matrix elements $<B|Q_5^i|A>$.

A powerful application of the Melosh transformation has recently been used by (Gilman, 1973a-e). Instead of guessing at the form of V, which was essentially the approach of the early mixing schemes mentioned above, one may evaluate the matrix element of the axial charge by casting V onto Q_5 itself:

$$\begin{aligned} <B|Q_5^i|A> &= <B|V^{-1}V\,Q_5^i\,V^{-1}V|A> \\ &= <\tilde{B}|\,\tilde{Q}_5^i\,|\tilde{A}> \quad , \end{aligned} \tag{II. 11}$$

where

$$\tilde{Q}_5^i = V\,Q_5^i\,V^{-1} \tag{II. 12}$$

The advantage of this approach is that $\tilde{Q}_5^i$ is not much more complicated than Q_5^i in its transformation properties, at least in the free quark case. These algebraic properties are then assumed to hold for interacting quarks as well. Two important differences between $\tilde{Q}_5^i$ and Q_5^i are the following:

(1) The "strength" of $\tilde{Q}_5^i$ is not determined. This avoids such predictions as

$$G_A/G_V = -5/3 \quad .$$

(2) $\tilde{Q}_5^i$ has both $\Delta L_z = \pm 1$ and $\Delta L_z = 0$ pieces. In contrast, Q_5^i has only a $\Delta L_z = 0$ piece. Here L is the internal quark orbital angular momentum.

Both Q_5^i and $\widetilde{Q}_5^i$ transform as a W-spin = 1 octet member of a 35-dimensional representation of $SU(6)_{W,\,currents}$. (The W-spin is an abstract SU(2) subgroup, related to quark spin, which will be described in section VI.) The calculation of pionic decays then reduces to evaluating $SU(6)_W$ and angular momentum Clebsch-Gordan coefficients, allowing for both $\Delta L_z = 0$ and $\Delta L_z = \pm 1$ transitions. Older "naive" $SU(6)_W$ calculations were based on the $\Delta L_z = 0$ term alone. The ratio of $\Delta L_z = \pm 1$ to $\Delta L_z = 0$ transitions is arbitrary for each decaying multiplet.

Evidence for $\Delta L_z = \pm 1$ transitions comes from a number of quarters.

a. $\underline{B \to \omega\pi \text{ decays}}$. The helicity λ of the ω is equal to the ΔL_z, since the final state has L=0 and the initial (by assignment of the B) has $\vec{J} = \vec{L}$. Experimentally $\lambda = \pm 1$ predominates, as mentioned earlier.

b. $\underline{\pi N \to \pi\Delta}$. The phase-shift analyses mentioned above indicate that $\Delta L_z = \pm 1$ transitions are very important. In the case of $\underline{70}$, L=1 resonances, whenever the π and Δ can appear in two different partial waves or in a partial wave different from that of the initial πN system, the sign of the resonant amplitude is sensitive to which value of ΔL_z dominates, and favors $\Delta L_z = \pm 1$ dominance. (When $\ell_{\pi N} = \ell_{\pi\Delta}$, the sign is independent of ΔL_z. We shall see this in section VI.) For $\underline{56}$, L=2 decays, one test indicates $\Delta L_z = 0$ may dominate; other tests are as yet unavailable.

c. $\underline{\gamma N \to \pi N}$. The signs of resonant amplitudes have been discussed using the Melosh transformation approach by (Gilman, 1973d). These confirm the dominance of $\Delta L_z = \pm 1$ in $\underline{70}$, L = 1 decays.

Two earlier approaches should also be noted here.

(i) The relativistic quark model of (Feynman, 1971) implies that $\Delta L_z = \pm 1$ transitions are not only important, but occur with a definite ratio to $\Delta L_z = 0$ ones The ratio is different for each decaying multiplet, and contains details of wave functions. In both 70, L=1 and 56, L=2 pionic decays, this model predicts resonant signs in $\pi N \rightarrow \pi\Delta$ which would follow from $\Delta L_z = \pm 1$ dominance.*

(ii) A model for introducing $\Delta L_z = \pm 1$ transitions with the same algebraic structure for pionic decays as that studied by (Gilman, 1973b, e) was discussed by (Colglazier, 1971a, b). It is a covariant generalization of the picture suggested by (Micu, 1969), in which hadronic decays occur via the creation of quark-antiquark pair. The kinematic factors associated with this picture are ambiguous; which is not the case with Eq. (II. 7). Nonetheless, the phase tests in $\pi N \rightarrow \pi\Delta$ are identical to those of (Gilman, 1973b), and have been discussed by (Faiman, 1973b) in some detail.

The transformation between current quarks and constituent quarks thus has led to renewed interest in detailed spectroscopic questions associated with hadronic decay amplitudes, and to a fundamental understanding of how current algebra and particle classification are interrelated.

D. Conclusions

So, what's new? In a purely experimental context, perhaps not as much as in previous years (Dalitz, 1966b; Goldhaber, 1966; Ferro-Luzzi, 1966; Lovelace, 1967). "Bump-hunting" is nearly at an end, but decay amplitudes are of renewed theoretical interest and are receiving the corresponding experimental attention. As the field changes direction (and becomes harder!) some suggestions are perhaps worth making as to what things would enhance our theoretical appreciation of resonance patterns. The remainder of this review thus seeks to outline the theories of current urgency, and to suggest simple ways in which future experiments can check them directly.

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*Quark models with both $\Delta L_z = 0$ and $\Delta L_z = \pm 1$ transitions have been considered earlier by other authors. (See section VI.B.4.)

III. THE SU(3) SITUATION

A. Absence of Exotics

As a comparison of Figs. 10 and 11 shows, the established resonant particles all fall into the following multiplets:

	I	Y	Possible SU(3) dimension
Mesons, baryons	0	0	$\underline{1}$ or $\underline{8}$
Mesons, baryons	1	0	$\underline{8}$ or $\underline{10}$
	1/2	±1	
Baryons	3/2	1	$\underline{10}$
	0	-2	

(III. 1)

The situation is very different from that of nuclei or of atoms, where more and more complex systems correspond to the higher states. Equation (III. 1) sugge that all hadronic levels are excited states of the <u>same</u> set of fundamental buildi blocks. In contrast, nuclei can contain large numbers of neutrons and protons, and to <u>each</u> (N, Z) value there corresponds an intricate set of energy levels.

Hadronic states lying outside the (I, Y) values in Eq. (III. 1) have come to be known as <u>exotic</u>.* There may exist a couple of candidates for exotic baryons as mentioned in section II. Nonetheless, the overall distinction between exotic and non-exotic channels is quite striking, and is based on a wide variety of dat

1. <u>Total cross sections</u>

The πN (I = 1/2, 3/2) and $\bar{K}N$ (I = 0, 1) systems both have many pronounced bumps in σ_T, while the KN (I = 0, 1) systems have much less pronounced effect (see Lasinski, 1973 for graphs of these cross sections).

*(Goldhaber, 1967).

2. Resonant circles in partial-wave amplitudes

A resonant partial-wave amplitude moves counter-clockwise in the complex plane as energy increases. This motion is most rapid near the resonant energy. In πN scattering nearly every such amplitude exhibits such resonant behavior at some time or another (Lovelace, 1972). In KN scattering only a select few amplitudes (if any) have even the faintest possibility of being resonant. This is reflected in measurable data by rapid variations in the form of $d\sigma/dt$ and of the polarization in πN scattering. No such variations are seen in the KN system.

Partial-wave analyses of the $\pi\pi$ and $K\pi$ systems, similarly, show no hint of resonances in exotic channels (Diebold, 1972; Rosner, 1970b). The non-exotic channels, on the other hand, continue to yield new information.

3. Effective-mass plots

One can compare such channels as $\pi^-\Sigma^+$ and $\pi^+\Sigma^+$ in production processes. No evidence exists for resonances in such exotic channels as $\pi^\pm\Sigma^\pm$, $\pi^+\Xi^o$, $\pi^-\Xi^-$, etc. On the other hand, these channels are considerably harder to study than the Y=2 KN ones; there is no available reaction $A+B \to \Sigma^\pm + \pi^\pm$, for example. The exception would perhaps be in a study of $\Sigma\pi$ scattering via

$$\Sigma^+ + p \to \Sigma^+ + \pi^+ + n \quad ,$$

or (III. 2)

$$\Sigma^- + p \to \Sigma^- + \pi^- + \Delta^{++} \quad ,$$

still considerably less efficient than the direct KN channels.

No exotic meson resonances have been found in effective-mass plots (Rosner, 1970b)* A suggestion was made several years ago that such exotics might be observable in their couplings to baryon-antibaryon systems (Rosner, 1968). As yet, this suggestion remains unproven. It is discussed in section VII.

*See also (Rosenfeld, 1968).

B. Existence of SU(3) Multiplets

The assignment of resonant particles to complete multiplets of SU(3) (Gell-Mann, 1961; Ne'eman, 1961) is an assumption which has been well borne out in the cases most easily accessible to experiment. We wish to discuss in this subsection the evidence for the general validity of such a scheme. We shall necessarily treat only the very gross features of such assignments. Many detailed reviews have appeared over the years, most recently by (Samios, 1973).

1. Baryons

The "complete" baryon multiplets of SU(3) are listed in Table VI. The masses of the $1/2^+$ and $3/2^+$ states provide confirmation of the mass formula (Gell-Mann, 1961; Okubo, 1962):

$$m = m_0 + aY + b\left[I(I+1) - Y^2/4\right] , \qquad \text{(III. 3)}$$

while the other cases listed in Table VI are consistent with Eq. (III. 3).* Equation (III. 3) is obtained by assuming that the SU(3) violating part of the Hamiltonian transforms as an I=Y=0 member of an octet. It is remarkable that such an assumption describes mass splitting of the order of 30% to such a high accuracy. The success of Eq. (III. 3) is of course contingent on confirmation that $J^P(\Omega^-) = 3/2^+$.

The J^P of the Ξ states assumed to belong to the $5/2^{\pm}$ and $3/2^-$ octets is not known, and states with other J^P values are expected nearby if all SU(3) multiplets are complete. Moreover, the existence of two $N(3/2^-)$ states (section II. B.1) suggests that there are two $3/2^-$ octets, and there seems to be a $3/2^-$ singlet as well. Such states may mix with one another. Whenever such mixing is possible, additional assumptions or data are needed to predict masses.

*This distinction between the $1/2^+$ and $3/2^+$ states on one hand and the $5/2^{\pm}$ and $3/2^-$ states on the other is based largely on J^P and mixing uncertainties in the latter three cases.

A number of baryonic resonances belong to <u>incomplete</u> SU(3) multiplets. In nearly all cases, such incomplete multiplets contain a N or Δ member.* This fact reflects the relative ease of phase shift analyses in $\pi N \to \pi N$, as compared with $\bar{K}N \to \bar{K}N$, $\pi\Lambda$, or $\pi\Sigma$ channels. Analyses of the latter reactions have not yet agreed on a stable set of parameters for the low-spin resonances (Lovelace, 1972; Barbero-Galtieri, 1972). Consequently, we list in Table VII the non-strange members of incomplete SU(3) multiplets, and indicate the number of predicted and observed hyperons to be associated with them if they belonged to complete octets (N) or decimets (Δ).

One may guess the masses of missing hyperons by adding 150 MeV per unit of negative strangeness to the N masses in Table VII. (Mixing effects are thus neglected.) The result is shown in Fig. 13,** from which several conclusions may be drawn.

a. <u>Λ and Σ resonances below 1.8 GeV</u>. Only the $1/2^+$ states remain to be confirmed; these would be partners of the Roper resonance. Above 1.8 GeV, especially for the Σ states, the situation is expected to be more complicated. Here symmetries higher than SU(3) are essential in providing a guide to expectations (Faiman, 1972) but independent checks of SU(3) will be difficult.

b. <u>Ξ states around 1.8 GeV</u>. The detailed study of $\Xi(1820)$ (see Table VI) may reveal a <u>mixture</u> of a few spin-parities. The region above 1.9 GeV should have a number of levels. The reaction

$$K^- + p \to K^+ + \Xi^{*-}$$
$$\hookrightarrow \Xi\pi,\ \Xi\pi\pi,\ YK,\ \ldots \qquad \text{(III. 4)}$$

in which a multiparticle spectrometer would be triggered on a forward K^+ (to ensure baryon exchange), could provide considerable insight into the missing Ξ^* levels in Fig. 13.

*The only exceptions are unitary singlets.
**Figure 13 contains <u>all</u> observed states, including unitary singlet Λ's.

c. Reason for simplicity of N and Δ states. Each such state would be associated with a unique SU(3) multiplet. A Λ can belong to $\underline{1}$ or $\underline{8}$; a Σ or Ξ to $\underline{8}$ or $\underline{10}$.

d. Reasons for studying hyperons. First, one wishes to establish that more resonances belong to complete SU(3) multiplets. This is not true for the majority of observed states. Secondly, hyperons sometimes provide the only way of getting information regarding symmetries higher than SU(3). An exampl is the ratio of f-type to d-type couplings in decays of octet baryons to the $1/2^+$ baryon octet and pseudoscalar mesons. These f/d ratios may be compared witl predictions of $SU(6)_W$. (See Table IV and section VI.) Thirdly, it is desirable to confirm effects seen in non-strange resonances, assuming SU(3) to hold. Th reactions $\pi N \to \pi\Delta$ and $\pi N \to \rho N$, to be discussed at greater length in section VI can be compared with $\bar{K}N \to \pi\Sigma(1385)$, $\bar{K}N \to \bar{K}\Delta$, and $\bar{K}N \to \bar{K}^*(890)N$. Resona phases in such reactions provide important information about $SU(6)_W$ and its breaking.

e. Couplings as sources of further information. The study of the mass spectrum alone, with detailed mechanisms for level splitting and mixing, canno possibly succeed without additional information from couplings (Faiman, 1972). For example, we shall see that branching ratios and phases of resonant amplitudes (Kernan, 1966) are an important tool in deciding possible SU(3) assignments (Meyer, 1971).

2. Mesons

Some well-established mesons nonets(octets accompanied by a unitary singlet) are shown in Table VIII. The 0^{-+} system forms an octet weakly mixed with a singlet, with

$$(0^{-+}) \qquad m_K^2 \simeq \left(3m_\eta^2 + m_\pi^2\right)/4 \tag{III.5}$$

while the 1^{--} and 2^{++} systems form nearly "ideal" nonets (Okubo, 1963) defined by the relations

$$(1^{--}) \qquad m_\rho^2 \simeq m_\omega^2 \; ; \qquad m_\phi^2 - m_{K^*}^2 \simeq m_{K^*}^2 - m_\rho^2 \qquad \text{(III.6)}$$

and

$$(2^{++}) \qquad m_{A_2}^2 \simeq m_f^2 \; ; \qquad m_{f'}^2 - m_{K^{**}}^2 \simeq m_{K^{**}}^2 - m_{A_2}^2 \qquad \text{(III.7)}$$

Deviations from these formulae may be ascribed to mixing effects differing slightly from those just mentioned, so that SU(3) mass formulae by themselves are useless for the mesons without being supplemented by coupling information. On the other hand, higher symmetries such as those possessed by the quark model have allowed one to understand the patterns (III.5) - (III.7), as we shall see below.

The meson nonets which lack at least one member are shown in Table IX. One can guess at the position of the missing members and the result is shown in Fig. 14. Here our rule is to expect "ideal" nonets with one I=Y=0 member degenerate with the I=1, Y=0 member, a "strange" member about 100 MeV higher, and another I=Y=0 member still $\sim$ 100 MeV higher. Figure 14 shows that the situation regarding SU(3) for the mesons is rather good below 1.6 GeV, except for I=Y=0 states of negative G-parity. The only conclusive decay mode of such states is $\rho^0\pi^0$; this appears in the channel $\pi^+\pi^-\pi^0$ and is affected by background from $\pi^\pm\rho^\mp$ (which can be associated with I=1 as well as I=0 resonances). Hence, as in the case of the baryons, it is expected that the failure to find complete SU(3) multiplets in all cases is basically an experimental difficulty, and does not reflect any deep property of nature.

Just as in the baryons, where N and Δ states provided the most solid information, the I=1, Y=0 states are singled out in the case of the mesons as particularly useful starting points around which to expect octets or nonets.

These states cannot undergo octet-singlet mixing, and their definite G-parity means that (in contrast to the Q's or L) they have a definite charge-parity. This feature is of significance when one wishes to classify them further according to the quark model.

C. Couplings and SU(3)

One does not need full multiplets to apply SU(3) to couplings. The decays of resonances have been analyzed in some detail and the basic conclusion is that SU(3) for couplings is probably about as good as for masses, i.e., to within 30%. Depending on the choice of centrifugal barrier factor describing resonance decays, SU(3) can be made better or worse nearly at random (Barbaro-Galtieri, 1972).

The simplest assumption regarding SU(3) for resonance couplings is that one has an elementary effective Lagrangian leading to a matrix element behaving as p^{ℓ}, where p is the magnitude of the final c.m. 3-momentum in the decay. This leads to a centrifugal barrier factor $\Gamma \sim p^{2\ell+1}/M^2_{\mathrm{Resonance}}$, associated with zero interaction radius.* At present, the data are consistent with the zero-radius form, and we shall use this in most subsequent discussions while recognizing that the results thus obtained should not be trusted to greater accuracy than fractional mass splittings in a multiplet.

In SU(3) the partial widths are assumed to be given by

$$\tilde{\Gamma} = \{cg\}^2 \tag{III.8}$$

or

$$\tilde{\Gamma} = \left\{c_D g_D + c_F g_F\right\}^2 \quad , \tag{III.9}$$

the second case holding for $8 \to 8 \times 8$ baryon decays (meson decays into meson pairs are pure D or pure F by C invariance). $\tilde{\Gamma}$ is the partial width before

*Other possibilities are discussed by (Quigg, 1970).

correction for barrier factors. The couplings are assumed universal, and the Clebsch-Gordan coefficients can be found, for example, in (Lasinski, 1973).

1. Baryon decays and resonant amplitude magnitudes

Baryon resonance formation data often yield most directly an inelastic amplitude at resonance. For final states in the same multiplets as the initial ones, the overall couplings are the same and only the Clebsch-Gordan coefficients are different, e.g.:

$$\Gamma_{tot} t_{res} \sim \left\{ c_D g_D + c_F g_F \right\} \times \left\{ c'_D g_D + c'_F g_F \right\} . \tag{III. 10}$$

These values, together with those of Eqs. (III. 9), allow one to obtain information on the baryon decays shown in Figs. 5 - 7.

The values of f/d for $8 \rightarrow 8 + 8$ are based on the normalization of (Gell-Mann, 1961): in terms of the constants introduced in Eqs. (III. 9),

$$\frac{f}{d} = \frac{g_F}{g_D} \frac{\sqrt{5}}{3} \quad ; \tag{III. 11}$$

we have also chosen $f + d = 1$ in Figs. 5-7. These values will be of interest when we come to discuss higher symmetries such as $SU(6)_W$.

2. Signs of baryon resonant amplitudes

The signs of resonant amplitudes may be utilized by referring to Fig. 15, which gives a number of such signs in terms of the representations in the intermediate state and the f/d values if this state is an octet. Such signs are determined in SU(3) since the resonant amplitude is related by Clebsch-Gordan coefficients to an elastic one (whose imaginary part must be positive, by unitarity).

Several examples of the usefulness of Fig. 15 may be given.

a. Reaction $\pi N \rightarrow K\Sigma$. The resonant contributions to $\pi^+ p \rightarrow K^+ \Sigma^+$ all belong to $\underline{10}$, and are expected all to be of the same sign. Experimentally this is indeed the case (Kalmus, 1971; Langbein, 1973), aside from one possible exception discussed in section IX. B. 3.

b. Reactions $\bar{K}N \to \pi\Lambda$, $\pi\Sigma$. The observed signs of resonant amplitudes now all agree with favored SU(3) assignments (Lasinski, 1973).

c. $\pi N \to K\Lambda$. All resonant contributions of octets with $-1/3 < f/d < \infty$ are expected to be of the same sign. So far all observed octets appear to have this range of f/d values, and this is the range expected for unmixed states in $SU(6)_W$. (See section VI.)

One analysis of this channel (Wagner, 1971a) shows prominent $1/2^-$ and $1/2^+$ resonances near 1700 MeV contributing with the same sign. As stressed by the authors, important gaps in the data exist, allowing for wildly different results (Deans, 1971). The channel $\pi N \to K\Lambda$ is an important one for the futu study of low-spin N* resonances between 1.6 and 2 GeV.

d. Unique resonant sign in $\bar{K}N \to \eta\Lambda$. Arguments like those just presente suggest a unique resonant sign in $\bar{K}N \to \eta\Lambda$ (at least if octets with $-\infty < f/d < -1/$ so far not observed, do not exist.) This seems true (Rader, 1973).

e. Importance of $\pi N \to \eta N$. The data on this channel are sketchy but resonance fits have nonetheless been made (Deans, 1971; Lemoigne, 1973). W the signs of the resonant amplitudes are in general accord with expectations based on Fig. 15, one should wait for data on $\pi^- p \to \eta n$ of sufficient detail to allow a genuine phase shift analysis. These data, including polarization measurements, are anticipated in the near future from the Rutherford Laborat

3. Meson couplings to hadrons

One can test SU(3) for hadronic couplings of meson resonances in the deca of the 1^{--}, 2^{++}, and 3^{--} states (Diebold, 1972; Graham, 1972; Samios, 1973). Satisfactory agreement is obtained in all cases. As an example, Fig. 16 show the fits to $2^+ \to 0^- 0^-$ and $2^+ \to 1^- 0^-$ decays obtained by (Samios, 1973).

For the 3^{--} states, which do not yet form a complete nonet, the couplings of the unseen member can be predicted (Graham, 1972).

For the remaining mesonic states making up incomplete SU(3) multiplets (Table IX), we do not yet know whether the couplings obey SU(3). The 0^{++} states are closest to being testable, since the widths of the $K\pi$ and $\eta\pi$ resonances are directly correlated without any mixing assumptions. For a K_N (0^{++}) state of 1.1 GeV SU(3) predicts

$$\frac{\Gamma(K_N \rightarrow K\pi)}{\Gamma(\delta \rightarrow \eta\pi)} = \frac{9}{4} \times 1.07 \cong 2.4 \left(p^{2\ell+1}/M^2 \text{ factor}\right) \tag{III.11}$$

where the first term in Eq. (III.11) comes from the symmetry and the second from the factor $p^{2\ell+1}/M^2$. The factor of Eq. (II.7), based on PCAC and motivated recently by studies of the transformation between two types of quarks (section VI), would multiply Eq. (III.11) by an additional factor of $(M^2_{K_N} - m^2_K)^2/(M^2_\delta - m^2_\eta)^2$, leading to the prediction

$$\frac{\Gamma(K_N \rightarrow K\pi)}{\Gamma(\delta \rightarrow \eta\pi)} = \frac{9}{4} \times 2.40 \cong 5.4 \text{ (PCAC factor)} \tag{III.12}$$

4. Meson couplings involving photons

The photon is usually assumed to be a pure member of an octet, entailing the prediction

$$\frac{\Gamma(\eta \rightarrow \gamma\gamma)}{\Gamma(\pi^o \rightarrow \gamma\gamma)} = \frac{1}{3}\left(\frac{m_\eta}{m_{\pi^o}}\right)^3 \cong 22 \quad , \tag{III.13}$$

if the η is a pure octet member. Experiments indicate that this ratio is in fact somewhat larger:

$$\frac{\Gamma(\eta \rightarrow \gamma\gamma)}{\Gamma(\pi^{0} \rightarrow \gamma\gamma)} = \frac{374 \pm 60 \text{ eV}}{7.8 \pm .9 \text{ eV}} \simeq 50 \pm 13 \qquad \text{(III. 14)}$$

The value for $\Gamma(\eta \rightarrow \gamma\gamma)$ is new and comes from a measurement using the Primakoff effect at Cornell (Browman, 1973).

One can resolve the discrepancy between Eqs. (III. 13) and (III. 14) by mixing η with a unitary singlet (e.g., η'), whose intrinsic coupling to $\gamma\gamma$ is a free parameter (Harari, 1968b). This mixing is constrained by the observed masses, and depends on whether a linear or quadratic mass formula is used. In brief, the new data on $\Gamma(\eta \rightarrow \gamma\gamma)$ do not change a previous conclusion: the experimental bound (Binnie, 1972; Harvey, 1971; Dalpiaz, 1972)

$$\Gamma(\eta' \rightarrow \gamma\gamma) \lesssim 40 \text{ keV} \qquad \text{(III. 15)}$$

rules out a quadratic mass formula with one sign of the mixing angle. The sign preferred by the quark model* (in which the η has relatively more nonstrange quarks than would a pure octet member) is allowed, as is either sign for a linear mass formula.

A simple model for radiative decays of mesons involving vector dominance assumptions has recently been considered by (Kotlewski, 1973). These authors correlate experimental values and bounds for twelve mesonic radiative decays in terms of five parameters: three SU(3)-invariant couplings and two mixing angles. As they use the old value $\Gamma(\eta \rightarrow \gamma\gamma) = 1$ keV (see Lasinski, 1973 for references), their analysis should be performed again once the data have settled down. One of the points illustrated by their calculation is that whereas

*See the discussions by (Suura, 1972) and (Okubo, 1969).

the quark model predicts

$$\frac{\Gamma(\rho^o \to \pi^o \gamma)}{\Gamma(\omega \to \pi^o \gamma)} = \frac{1}{9} \quad , \tag{III. 15}$$

this prediction is not true in a general SU(3)-invariant calculation, for which the SU(3)-singlet coupling to $\pi^o \gamma$ is unspecified. Hence the measurement of $\rho^o \to \pi^o \gamma$ is mainly a test of symmetries higher than SU(3). Equation (III. 15) follows, for example, in chiral SU(3) × SU(3), to be discussed in section VI.

5. Summary of SU(3) for couplings.

It is an open question of whether SU(3) holds for couplings besides those of the highest-spin resonances at any given mass. As stressed by (Meyer, 1971), this is still very much in doubt, probably as a result of experimental complexity. We have seen that, where testable, SU(3) seems to work at least as well for couplings as for masses.

IV. OTHER EMPIRICAL REGULARITIES

A. Parity Alternation

A regularity which will be of interest when we come to discuss symmetries higher than SU(3) is that the observed levels in Figs. 13 and 14 fall into very rough groups of alternating parity, with period of about 2 GeV^2 in m^2.

The effects of SU(3) breaking may first be taken into account — roughly — by subtracting 150 MeV per unit of absolute value of strangeness for each observed level (solid or wavy line in Figs. 13 and 14). One may then construct histograms for baryon and meson states showing the distribution of levels of each parity as a function of mass. The results are shown in Figs. 17 and 18.

For the baryons, the rough pattern is (+, -, +), and for the mesons (-, +, -). It remains to be seen whether this oscillatory pattern is borne out at higher energies. One consequence would be an oscillation in $\sigma(e^+e^- \rightarrow \text{hadrons}$ if this process is mediated by intermediate states consisting of single resonances (Fig. 19a). Such resonances must have $J^P = 1^-$ and would therefore be expected to be prominent only every $\sim$2 GeV^2 in $(\text{mass})^2$. The cross section would then behave as shown symbolically in Fig. 19b.

The period of $\sim$2 GeV^2 in Figs. 17 and 18 is suggestive of a similar one observed for Regge trajectories. Recent phase shift analyses confirm a pattern of approximate linearity in the J vs m^2 plot. The trajectories containing the nucleon, the Λ, and the Δ, containing three members each, are the basis for this claim (see Fig. 20). In addition, meson-exchange processes indicate that the intercepts of the ρ and ω trajectories at $m^2 = 0$ fall roughly on a straight line with observed recurrences at $J^P = 1^-$ and 3^- (see Fig. 21). We do not place any higher-mass states on the trajectories of Figs. 20 and 21 since their J^P values are not confirmed. The intercepts of baryonic trajectories at $m^2 = 0$

also are determined to some extent by baryon-exchange processes, and are consistent with Fig. 20.

The slopes in Figs. 20 and 21 are all about 0.9 to 1.0 GeV^{-2}. Many other particles have single recurrences corresponding to this slope. They are listed in Table X. (Some more speculative Regge recurrences have already been noted in Table V.)

The pseudoscalars seem to be on trajectories of somewhat lower slope. We would guess that this is associated with the low masses expected for the 0^- mesons if they are Goldstone bosons of spontaneously broken chiral $SU(3) \times SU(3)$. (See, e.g., Gell-Mann, 1968.) No such depression is expected for 2^- mesons. Our guess is that the slope between the 2^- and 4^- states (if the latter are ever discovered) will turn out to be the same as all the others, namely, around 0.9 GeV^2.

B. Optical Considerations

The prominent resonances appear to be produced with roughly constant impact parameter $b \simeq 1$ f, defined by $b = \ell / P^*$. Here ℓ is the orbital angular momentum, and P^* is the magnitude of the 3-momentum in the c.m. The value of $b \simeq 1$ f also figures in the dominant contributions to imaginary parts of non-Pomeranchuk trajectory exchanges at energies above the resonance region. (See, e.g., Davier, 1971.) The suggestion that the two phenomena are related is one of the aspects of duality, which will be discussed in more detail in section VII.

A related question is the relative momentum at which any two particles form their first resonance above threshold. Any meson-meson or meson-baryon pair which can form a resonance (i.e., in a non-exotic channel) does so below a relatively low c.m. three-momentum: about 350 MeV/c in meson-meson systems and 250 MeV/c in meson-baryon systems. The distribution of these cases for established decays is shown in Fig. 22 (Rosner, 1972b).

The high-momentum tail in Fig. 22b is partly a result of our incomplete knowledge about the channels shown: for example, low-energy $\bar{K}N$ scattering in the I=1 channel is very poorly studied as yet, and there may exist a resonance considerably lower than the one shown (Carroll, 1973b).

The fact that the peak in meson-baryon systems occurs for a lower 3-momentum than in meson-meson systems may indicate optics at work. We note that, on the average, "first resonances" are formed in a P-wave. Then writing

$$1 \cong \ell_{av} = P^* b_{eff} \tag{IV.1}$$

we find

$$b_{eff} \cong 0.6 \text{ f} \qquad \text{(meson-meson)} \tag{IV.2}$$

and

$$b_{eff} \cong 0.8 \text{ f} \qquad \text{(meson-baryon)} \tag{IV.3}$$

The latter value is not far from that associated with the dominant resonances in meson-baryon scattering for <u>all</u> ℓ and P^*, as mentioned above. Equations (IV.2) and (IV.3) indicate that hadrons do not have to get too "close" to one another to begin forming resonances. Moreover, they indicate that mesons are "smaller" than baryons, a fact familiar from total cross section measurements.

The crucial test of the quark-graph mnemonic shown in Fig. 22 would be the existence of <u>exotic</u> ($qq\,\bar{q}\bar{q}$) resonances in baryon-antibaryon systems not far above threshold. These proposed exotic resonances are discussed further in section VII.

C. Summary

We have discussed two schemes: the Regge picture, in which resonances lie on straight-line trajectories $J \sim m^2$, and the optical picture, in which the dominant resonances in a given channel have $J \sim \sqrt{m^2}$. The resonances on the leading trajectory thus become "ultraperipheral" with respect to lowest-spin pairs of decay products such as πN. One might expect them instead to have appreciable decays to excited states so as to preserve the relation $b \simeq 1$ f.

The importance of inelastic states in sustaining high-rising Regge trajectories has been stressed for some time (Mandelstam, 1968). Now, with the recent experimental progress in partial-wave analyses of three-body states, this suggestion can be checked.

V. THE QUARK MODEL

Many properties of the states described in sections II-IV can be described economically in terms of the fundamental building blocks (Gell-Mann, 1964; Zweig, 1964) called quarks. We shall be brief to avoid overlap with the recent review of (Lipkin, 1973a).

A. States of Nonrelativistic Quarks (Dalitz, 1966b, 1967, 1968, 1969)

The quark model may be characterized by some simple rules whose deepe basis is not understood at present. These rules are the following.

1. Quarks belong to the $\underline{3}$ of SU(3). This guarantees that all SU(3) representations can be constructed of quarks. They act as elementary units of I_3 and Y (Fig. 23).*

2. Mesons are $q\bar{q}$, baryons are qqq. This "explains" why mesons belong only to $\underline{1}$ and $\underline{8}$, and baryons to $\underline{1}$, $\underline{8}$, and $\underline{10}$; see section III.

3. Quarks have spin 1/2. Rule 2 then describes mesons with integral spin and baryons with half-integral spin, as observed.

4. Quarks undergo orbital excitations. The parity of an excited system is then

$$P = (-)^{L+1} \quad \text{(mesons)} \tag{V. 1}$$

$$P = (-)^{L} \quad \text{(baryons)**} \tag{V. 2}$$

and its total angular momentum is

$$\vec{J} = \vec{L} + \vec{S} \tag{V. 3}$$

where $\vec{S}$ is the quark spin: S=0, 1 for mesons and 1/2, 3/2 for baryons. Mesons and baryons then must have well-defined J^P values determined by L and S. There is evidence for both meson and baryon levels of L=0, 1 and 2

*Quartets of SU(4) have also been proposed (Maki, 1964; Hara, 1964).

**This rule depends to some extent on simple assumptions about couplings, to be discussed presently.

with $m^2 \sim L + \text{const}$ (Fig. 24). The parity alternation in Figs. 17 and 18 is what allows the levels to be distinguished from one another: the lowest L=0 and L=1 levels thus can be identified fairly easily.

5. Quarks obey special statistics. This assumption is needed to explain the order of the multiplets of different L and their symmetry in (quark spin) $\times$ SU(3). We shall, unfortunately, sidestep such a question (see, e.g., Greenberg, 1964; Han, 1965; Lipkin, 1973a,b), and ask instead how the data can tell us what the symmetry of these multiplets is.

Quarks should behave as fermions in the sense that in a $q\bar{q}$ pair (a meson) the charge-parity of the neutral Y=0 members should be given by the usual expression

$$C = (-)^{L+S} \tag{V.4}$$

Equation (V.4) together with (V.1), says that mesons must have

$$CP = (-)^{S+1} \tag{V.5}$$

We introduce the definitions

$$P = (-)^J \quad (\text{"normal" parity}) \tag{V.6}$$

$$P = (-)^J \quad (\text{"abnormal" parity}) \tag{V.7}$$

Mesons of normal parity must have S=1 since $J = L \pm 1$ (compare (V.6) and (V.1)), and hence their CP must be even. Hence the sequence

$$J^{PC} = 0^{+-},\ 1^{-+},\ 2^{+-},\ 3^{-+},\ \ldots \tag{V.8}$$

is not allowed for $q\bar{q}$ states. The only other mesonic state which is forbidden in the quark model has

$$J^{PC} = 0^{--} \quad . \tag{V.9}$$

It would have to have L=S because of its zero spin. But then by (V.4) its C should be positive.

The mesons in (V.8) and (V.9) have not been observed. They may be called "C-exotic".

6. Individual quarks cannot be seen. Any theory must explain this one.* (See, e.g., Lipkin, 1973b.) So far, quarks are just a convenient figment of our imagination. The more abstract approach of the next section thus may be preferable until we understand why the model works.

B. Symmetry Approach

The assumptions just listed can be replaced by algebraic ones which are more economical. The group-theoretic statements that follow could be valid in a large variety of dynamical theories.

The quarks with three SU(3) and two SU(2) (spin) degrees of freedom, belong to a six-dimensional multiplet of SU(6). (For extensive reviews, see Lee, 1965, and Pais, 1966. SU(6) was first introduced for classifying particles by Gürsey and Radicati, 1964a, and Sakita, 1964.)

The states of three quarks are then

$$qqq = \underline{6} \times \underline{6} \times \underline{6} = \begin{cases} \underline{56} & \text{symmetric} \\ \underline{70} & \text{mixed} \quad \text{(twice)} \\ \underline{20} & \text{antisymmetric} \end{cases} \tag{V.10}$$

where the symmetry refers to what happens to SU(3) × quark spin when two quarks are interchanged.

The antiquarks are assumed to be distinguished from the quarks and to form a second SU(6) multiplet. Mesons are thus

$$q\bar{q} = (\underline{6}, \underline{\bar{6}}) \tag{V.11}$$

*Avoidance of such embarrassing questions has been termed the "Broom and Rug" model. See (Lipkin, 1968a).

of this $SU(6)_{quarks} \times SU(6)_{antiquarks}$. Baryons, lacking antiquarks, belong to $(\underline{56}, \underline{1})$, $(\underline{70}, \underline{1})$, and $(\underline{20}, \underline{1})$ of $SU(6) \times SU(6)$, but we shall usually omit the second SU(6) singlet index.

Finally, the orbital angular momentum forms an 0(3) group. Particle states at rest may thus be classified by

$$\text{"The rest symmetry"} \equiv SU(6)_q \times SU(6)_{\bar{q}} \times 0(3) , \qquad \text{(V. 12)}$$

for which we shall now discuss the evidence.*

C. Evidence for Multiplets of the "Rest Symmetry"

A possible assignment of the states in Figs. 17 and 18 is shown in Figs. 25 and 26. The reader is invited to trace these figures and superpose them on the earlier pair.

In each case, the levels fall into three major multiplets. (For the baryons, other multiplets must exist as well, which are far from filled.)

If we assigned as many states as possible to the three major multiplets, how many would be missing? Figures 27 and 28 give the answer. These "box scores" look rather good for the lowest L=0 and L=1 levels, especially when we recall that the completion of SU(3) multiplets is not easy (see section III, above). Gaps in the L=1 states are discussed in subsection J.

For the L=2 levels, the assignments of the baryon states are actually quite speculative, as there is also evidence (in the same mass range) for states belonging to other representations of the "rest symmetry". (These representations are indicated in Fig. 27.) The sorting out of the positive-parity baryons below ~2 GeV is a central unfinished task of low-energy resonance physics. The L=2 mesons are gradually falling into shape, and (except for the 1^{--} levels) are subject to fewer ambiguities.

*Strictly speaking, the symmetry of interest is $U(6) \times U(6) \times 0(3)$: see Eq. (VI. 12).

D. "Extra" States at Low Masses

Figures 17 and 18 contain states which cannot be assigned to the three major baryon or meson multiplets. These are indicated by letters in Figs. 25 and 26. They can be assigned tentatively to additional multiplets. These multiplets are in fact expected.

Let us assume that all baryon states below ~2 GeV and meson states below ~1.8 GeV have $L \leq 2$. Then the positive-parity baryons coupling to the lowest states must belong to

$$\underline{56} \text{ or } \underline{70}\ , \qquad L = 2 \text{ or } 0 \quad . \tag{V.13}$$

The value L=1 is excluded by the parity rule (V.2). Similarly, the negative-parity mesons must belong to

$$(\underline{6},\bar{\underline{6}})\ , \qquad L = 2 \text{ or } 0 \quad . \tag{V.14}$$

Experimentally there is evidence for all of the multiplets in (V.13) and (V.14), in addition to the lowest $\underline{56}$, L=0 and $(6,\bar{6})$, L=0 containing the nucleon and pion, respectively. This evidence is shown in Tables XI and XII for the baryons and mesons.

Aside from the major multiplets, the additional ones in Tables XI and XII do not have many candidates. This situation is due largely (in the baryons) to the assumption that as many of the observed states as possible fall into $\underline{56}$, L=2. In fact, however, only the assignments of the highest-spin states are unique (Faiman, 1970). Other states can mix, and probably do. If we assume the existence of all four baryon multiplets listed in (V.13), the progress toward filling these multiplets is shown in Table XIII.

As mentioned before, the N and Δ states are in best shape. Even here, however, we are missing things. (One expects more hyperons and they are harder to study.) In section IX we shall return to a coherent program for studying all the positive-parity baryons below ~ 2 GeV. Above 1.8 GeV, there is room for further experimental work if the multiplet structure is to be believed. We shall give some predictions that might be of use in this respect in sections VI and VIII. The $\underline{70}$, 2^+ states which have not yet been discovered will be rather hard to see in elastic πN phase shift analyses, for example (Faiman, 1973a).

For the mesons, only the 1^{--} states are ambiguous between L=2 and L=0 assignments. A discussion of couplings (section VI) is needed to resolve this ambiguity.

The existence of (possibly) three 0^{++} I=Y=0 states (II.A.5, above) requires an extra "no-quantum-numbers" state to mix with the $^3P_0$ $q\bar{q}$ states. Such a state can be postulated, e.g., in the theory of broken scale invariance. We shall see some consequences of this assumption in section VIII. An "extra" state with vacuum quantum numbers is certainly less embarrassing to the quark model than, say, an extra A_2 (see II.A.1, above).

E. Higher Levels

The Regge recurrences discussed earlier provide strong evidence for the baryon multiplets

$$\underline{70},\ L^P = 3^- \tag{V.15}$$

and

$$\underline{56},\ L^P = 4^+ \tag{V.16}$$

No concrete evidence exists for higher levels of mesons (the absence of reliable J^P analyses is the reason), though $\bar{p}p$ elastic scattering and annihilations into meson pairs are beginning to show some such evidence. The present evidence for the multiplets (V.15) and (V.16) is shown in Table XIV.

F. Other Suggested Baryon Multiplets

At various times one sees theoretical suggestions of the existence of such multiplets as $\underline{56}$, $L^P = 1^+$ or $\underline{56}$, $L^P = 1^-$.* The former violates the parity rule (V.2), which will be discussed below in this section. Moreover, there is no independent evidence for the states of such a multiplet. The latter would be characterized by a $\Delta(5/2^-)$, which seems to be totally absent from the latest elastic phase shift analyses (Almehed, 1972; Ayed, 1972). One solution (II) in $\pi N \to K\Sigma$ (Langbein, 1973) shows this state, the other (I) does not. Solution I is preferred on many other grounds (see section IX).

G. Oscillator Spectrum

If we take symmetric three-quark baryonic wave functions and roughly harmonic forces between quarks (Greenberg, 1964) the observed baryon levels fall neatly into the oscillator pattern shown in Fig. 29a. This pattern can be used in any case to count the levels of qqq systems (Karl, 1968; Walker, 1969). The $q\bar{q}$ states also suggest an oscillator spectrum (Fig. 29b).

The parentheses in Fig. 29a denote states which cannot couple to the lowest $\underline{56}$, L=0 when a single quark is disturbed in meson emission. No $\underline{20}$'s can couple in such a case since they are totally antisymmetric in space while the $\underline{56}$ is totally symmetric.** Moreover, it turns out (D. Faiman, private communication) that one can prove (using the scheme of Karl, 1968, for example) that such a mismatch occurs in this single-quark-transition picture whenever the parity rule (V.2) is violated. Hence the decoupling of $\underline{70}$, $L^P = 2^-$ (N=3) in Fig. 29a from the ground state baryons in $\underline{56}$, $L^P = 0^+$ is a symptom of a general theorem.

*See, for example, (Drell, 1972).

**Preliminary indications of a $\underline{20}$, $L^P = 2^+$ member have been mentioned by (Yaffe, 1973).

H. Mass Splittings

The fact that the SU(3) mass splittings are of the same order of magnitude in the $1/2^+$ octet and $3/2^+$ decimet can be interpreted in terms of the quark model (see, e.g., Dalitz, 1966b) or more abstractly in SU(6) language (Gürsey, 1964a). These methods can be extended to higher-lying multiplets. (See, e.g., Greenberg, 1967; Divgi, 1968; Feynman, 1971; Jones, 1973a; Horgan, 1973.) By choosing sets of operators responsible for the splitting and mixing of levels within multiplet of the rest symmetry, one can hope to gain a reasonable set of predictions for the masses of observed and, more important, unobserved states.

The difficulty with this approach is that it relies on the choice of operators which correspond essentially to quark masses, quark spin-spin and spin-orbit forces, and so on. By contrast, the mixing of levels probably occurs primarily via shared intermediate states, whose effects will be different in every specific case. (In the symmetry limit, the effects of all such intermediate states will cancel one another.)

It is thus not surprising that predictions of various schemes differ from one another and that they yield level mixings which are not the same (Jones, 1973a) as those obtained purely from analyses of decays (Faiman, 1972). Moreover, masses differ; compare (Horgan, 1973) and (Jones, 1973a). Our view, which may not be shared by others, is that mixing (and the consequent prediction of masses within a rest symmetry multiplet) is still very much an unsolved problem, and possibly one to which methods based on analyticity and unitarity are quite applicable. Nonetheless, the predictions of (Horgan, 1973) do seem to be borne out by the observed spectrum, and provide our best guess to date for the masses of unobserved states.

I. Models for Exotic States

While a basic precept of the quark model would seem to be the limitation of mesons to $\underline{1}$ and $\underline{8}$ and baryons to $\underline{1}$, $\underline{8}$, and $\underline{10}$ of SU(3), one can construct models of exotic resonances also based on $q\bar{q}$ or qqq systems. Such models (e.g., Greenberg, 1969) rely on excitations of additional degrees of freedom in the three-quark system and predict a host of new resonances. These models along with that of (Goebel, 1966), predict the lowest-lying exotic baryons to have positive parity. This is what seems to be observed (see section II), if these states do exist. Models based on S-wave $qqqq\bar{q}$ states would predict the lowest exotic baryons to have negative parity. So would the bootstrap approach of (Aaron, 1971).

J. Experimental Discussion of Gaps in the Lowest Multiplets

1. $\underline{70}$, L=1 baryons.

The states listed in Fig. 27 are generally based on phase shift analyses (except for Ξ^*'s, of which many are missing, and Ω^*'s, of which none has been detected). In the $\underline{70}$, L=1 baryons the other gaps refer to $J^P = 1/2^-$ and $3/2^-$ Λ's and Σ's.

Some indication of the properties of these missing states has been given by (Faiman, 1972). The discussion of these authors is based on mixing scheme that reproduce observed partial widths, using a version of $SU(6)_W$ to be discussed in the next section. No predictions are given for missing Σ's, but the authors' best guess for the masses, total widths, and dominant decay modes of the missing Λ's is:

$\Lambda(1830,\ 1/2^-)$

$$\begin{aligned} \Gamma_{tot} &= 426\ \text{MeV} \\ \Gamma_{N\bar{K}} &\simeq 0 \\ \Gamma_{\Sigma\pi} &\simeq 390\ \text{MeV} \\ \Gamma_{\Sigma^*\pi} &= 30\ \text{MeV} \end{aligned} \tag{V.17}$$

$\Lambda(1830,\ 3/2^-)$

$$\begin{aligned} \Gamma_{tot} &\cong 460\ \text{MeV} \\ \Gamma_{N\bar{K}} &\simeq 10\ \text{MeV} \\ \Gamma_{\Sigma\pi} &\simeq 55\ \text{MeV} \\ \Gamma_{\Xi K} &\simeq 30\ \text{MeV} \\ \Gamma_{\Sigma^*\pi} &\simeq 350\ \text{MeV} \end{aligned} \tag{V.18}$$

The first is dominantly an s-wave $\Sigma\pi$ state, and the second an s-wave $\Sigma^*\pi$ state. The first may be accessible in studies of $\Sigma\pi$ scattering, to be discussed in section X.B.3. Equations (V.17) and (V.18) illustrate the fact that gaps in low-lying SU(6) × 0(3) multiplets may well be due to experimental difficulties.

2. $(6,\bar{6})$, L=1 mesons.

Some of the states listed in Fig. 28 are in need of confirmation: for example, the A_1, the doubled Q (axial-vector K*'s), and the 0^+ mesons.

The A_1 is best looked for in nondiffractive processes (see, e.g., Garelick,1970 for examples of these). A partial-wave analysis of the three-pion state is still necessary, however, since Deck-type mechanisms may occur, e.g., in $\pi p \to pA_1$ (backward A_1 production). (See the discussion by Berger after the talk by Rosner, 1971c.) These kinematic reflections are not necessarily expected to lead to an S-wave $\rho\pi$ state, while the "true" A_1 should consist mainly of S-wave $\rho\pi$ with a small amount of D-wave (section VI. D). In this respect a recent result (Atherton, 1973) based on $\bar{p}p$ annihilations, claiming dominant D wave, is puzzling.

The Q has been observed in backward production (Firestone, 1972), but it cannot be resolved into the two expected peaks. Some predictions for the properties of these peaks, as well as a review of the literature on mixing between the expected 1^{++} and 1^{+-} states, are contained in (Colglazier, 1971a), and a recent brief experimental review is given by (Diebold, 1972). One very large analysis of $K^+p \to Q^+p$ has been carried out by (Bingham, 1972). We expect the Q situation to improve dramatically as a result of counter experiments at SLAC (D. Leith, private communication). One prediction (Colglazier, 1971a) is that the upper state should be dominantly a $K^*\pi$ and the lower one a $K\rho$ S-wave resonance. This conclusion is borne out to some extent by (Bingham, 1972).

The 0^+ mesons are discussed in various places throughout this review. Forthcoming studies of $K\pi$ scattering should improve the KN situation, but more work on the δ is needed. Again, the evidence from $\bar{p}p$ annihilations is confusing. (See Atherton, 1973.)

With regard to the gaps in the $(6, \bar{6})$, L=1 mesons, we should mention the possibility that the E(1420) could be mis-classified and really could have

$J^P = 1^+$. While this is considered less likely than the 0^- assignment (Baillon, 1967), the analysis rests on details of $\bar{p}p$ annihilations at rest, and is not straightforward. If the E(1420) had $J^P = 1^+$, it could be an SU(3) partner of the A_1, the Q_A, and the D. *

Some recent arguments have been advanced in favor of an E(1420) which is mainly a unitary singlet (Capps, 1973b). These rely on SU(3) properties and the observed narrow width of the E into $K^*\bar{K}$. A pure unitary singlet would be forbidden altogether from this decay mode, and the E is thus assumed to be dominantly a singlet with a small octet admixture.

An alternative suggestion for a $J^P = 1^+$ candidate (Rosner, 1971a) is the "M(953)" seen in $K^- p \to K^- p\, M(953)$ (Aguilar, 1970). This effect has not been confirmed, however.

The SU(3) partners of the B(1235) have been discussed in section III.B.2. If the I=0 partner "H" of the B were degenerate with it and "ideally" mixed (no strange quarks), it would have three times the B width:

$$\begin{aligned} &\Gamma[\text{"H(1235)"} \to \rho\pi] \\ &\quad = 3\Gamma[B(1235) \to \omega\pi] \\ &\quad \simeq 400 \text{ MeV} \end{aligned} \tag{V.19}$$

and thus could not be picked out except in three-body partial-wave analyses. Such analyses are expected in the near future based on spectrometer data, for example from Omega at CERN (section X).

*One scheme which proposes $J^P(E) = 1^+$ is advanced by (Carruthers, 1971b).

A "strange-quark" ($s\bar{s}$) $J^{PC} = 1^{+-}$ state would have a $K\bar{K}\pi$ decay mode, but not much else. It should have a mass of 1400-1500 MeV, and might be accessible in three-body partial-wave analyses. Since the E(1420) and f'(1514) also have $K\bar{K}\pi$ modes, mere "bump-hunting" probably will not be enough.

K. Conclusions

The general pattern of the "rest symmetry" indeed is confirmed for the low-lying mesons and baryons (Tables XI and XII, Figs. 27 and 28). The gaps in this pattern almost certainly may be traced to experimental complications, which gradually are being overcome. The pattern roughly resembles that of an oscillator spectrum, though we don't yet know why.

The sketchiness of information about the lower-spin, high mass levels invites further experimental work to see if the pattern really is what we think it is. Couplings, as described in the next section, can serve as a guide to these studies. Moreover, couplings provide a striking confirmation of the correctness of the assignments of existing resonances.

VI. DECAYS

Couplings of hadrons may be obtained from resonance partial widths, from magnitudes and signs of resonant amplitudes in inelastic processes, and from poles in dispersion relations (e.g., $G_{\pi NN}$). In principle one then has a wealth of data with which to compare various symmetry schemes. In practice stringent test of symmetries higher than SU(3) are just now becoming possible. These symmetries are the subject of the present section.

The hadrons seem to belong to families of simple quark-model states. Their couplings to one another may have some quark-ish features as well. All the symmetries to be discussed in what follows are motivated by the quark model but not dependent on it. They could arise from bootstraps, for example. Nobody has shown this yet, however. By learning which symmetry (if any) is valid, we learn what features of the quark model should be expected in a true theory with real dynamics.

The central question of this section is what to do with quark spins in decays. Quark spin is basically a nonrelativistic concept. In decays, however, particles (especially pions) are moving at a fair fraction of the velocity of light. One thus has to break the "rest symmetry" in describing decays; a <u>weaker</u> symmetry must hold. Here our theoretical intuition fails us; we do not know by <u>how much</u> the "rest symmetry" must be weakened. The resulting schemes form a well-defined hierarchy, from $SU(6)_W$ (the strongest) to SU(3) (the weakest). We shall see that most features of $SU(6)_W$ seem to be borne out by the study of hadron decays (and photoproduction of resonances). The recent developments concern a new set of selection rules for pion emission and electromagnetic transitions, indicating that "naive" $SU(6)_W$ (in its original form) is too strong a symmetry.

A. $SU(6)_W$ and its Limitations

1. Classification

As mentioned, the "rest symmetry" cannot hold for decays. Otherwise, for example, the decay

$$\Delta(\underline{56},\ \underline{1}) \rightarrow N(\underline{56},\ \underline{1}) + \pi(6, \underline{\bar{6}}) \qquad \text{(VI. 1)}$$

would be forbidden. Decays must be described by a symmetry generated by <u>fewer</u> operators than those that generate the rest symmetry. This symmetry will be correspondingly weaker.

The symmetry $SU(6)_W$ is one candidate for a scheme to describe resonance decays.*,** It is motivated in the following way (for a recent discussion, see Hey, 1973b).

---

*$SU(6)_W$ has been considered in various forms by a number of authors. In the absence of quark orbital angular momentum, it was treated by (Lipkin, 1965, 1966a; Barnes, 1965; Sakita, 1965; Dashen, 1965; Delbourgo, 1965a; Beg, 1965; Freund, 1965; Bardakci, 1965; Johnson, 1965; Oehme, 1965; and many others). The clearest statement of the algebraic structure may be found in the first two references, and a good review of the early literature is given by (Lee, 1965). "Kinetic" breaking of symmetries based on quarks, used in some of the above approaches, was proposed quite early by (Gell-Mann, 1965), whose discussion was also applicable to states with nonzero orbital angular momentum L.

**The $L \neq 0$ case has been treated explicitly by, for example (Delbourgo, 1965b; Costa, 1965; Volkov, 1966; Freund, 1967; Lipkin, 1967a, 1968b; Carter, 1968; Shafi, 1969; Horn, 1970; Carlitz, 1970b).

Consider the states of quarks or antiquarks at rest. The projection operator for a quark is then the Dirac matrix $(1+\beta)/2$. The "rest symmetry" is generated by

$$\begin{array}{ll} \lambda_i \ , & \lambda_i \beta \\ \lambda_i \vec{\sigma} \ , & \lambda_i \beta \vec{\sigma} \ , \qquad i=0,\ldots,9 \ , \end{array} \tag{VI. 2}$$

where the λ_i are the usual 3×3 matrices of SU(3) (Gell-Mann, 1961) and $\lambda_0 = (2/3)^{1/2}\, 1$. These matrices just measure spins and species of quarks and antiquarks at rest when sandwiched between quark fields.

Now consider quarks moving along the z-axis. The operator which takes a free quark at rest into one moving along the z-axis is

$$U(\chi) = \exp\left\{ \int d^3x \, q^+(x) \frac{\alpha_z \chi}{2} q(x) \right\} \ . \tag{VI. 3}$$

This just contains the Dirac matrix α_z, which we may think of as the <u>generator of Lorentz boosts in the z-direction</u>.

Those components of (VI. 2) which commute with α_z are then suitable for defining a relativistic "spin" that makes sense for a quark moving along the z-axis. This relativistic "spin" is thus generated by

$$\lambda_i \ , \quad \lambda_i \sigma_z , \quad \lambda_i \beta \sigma_x \ , \quad \lambda_i \beta \sigma_y \ . \tag{VI. 4}$$

It was dubbed "$SU(6)_W$" (Lipkin, 1965).

The crucial feature of (VI. 4) is that its SU(2) subgroup, $SU(2)_W$, coincides with quark spin for quarks but <u>reverses</u> the sign of the x and y components of quark spin for antiquarks. This becomes important when one comes to classify systems such as mesons, which involve both, under $SU(6)_W$.

So far no physical assumptions have been made. At this point, in order to guess at a plausible theory, one may assume that hadrons moving in the z direction are classified as if their quarks were stuck to them like raisins in a fruitcake. Then the breakdown of the rest symmetry to (VI.4) would continue to hold even for hadrons (multi-quark systems), though the generators of the $SU(6)_W$ applicable to particle classification would no longer be the quantities (VI.4) themselves in any interacting theory.

States of baryons are easily classified under $SU(6)_W$ since they contain only quarks. Hence, for example,

$$\begin{array}{ccc} \underline{\text{"rest symmetry"}} & & \underline{SU(6)}_W \\ (\underline{56},\underline{1}) & \longrightarrow & \underline{56} \\ (\underline{70},\underline{1}) & \longrightarrow & \underline{70} \end{array} \tag{VI.5}$$

For states of mesons, since

$$\begin{aligned} \vec{W}(q) &= \vec{S}(q) \\ W_z(\bar{q}) &= S_z(\bar{q}) \end{aligned} \tag{VI.6}$$

but

$$W_{x,y}(\bar{q}) = -S_{x,y}(\bar{q}) \quad ,$$

the lowering operators of W-spin for antiquarks are

$$\begin{aligned} W_-(\bar{q}) &= W_x(\bar{q}) - i\,W_y(\bar{q}) \\ &= -S_x(\bar{q}) + i\,S_y(\bar{q}) \\ &= -S_-(\bar{q}) \quad . \end{aligned} \tag{VI.7}$$

The $q\bar{q}$ states of W-spin and quark spin are then related by

$$
\begin{aligned}
|W=1,\ W_z=1\rangle &= |S=1,\ S_z=1\rangle \\
|W=1,\ W_z=0\rangle &= -|S=0,\ S_z=0\rangle \\
|W=1,\ W_z=-1\rangle &= -|S=1,\ S_z=-1\rangle
\end{aligned}
\tag{VI. 8}
$$

and

$$
|W=0,\ W_z=0\rangle = -|S=1,\ S_z=0\rangle \quad . \tag{VI. 9}
$$

Note that the W-spin triplet contains both S=0 and S=1, while the W-spin singlet contains S=0. This is known as "W-S flip" (Harari, 1966a). The quark model states are thus shuffled among each other in $SU(6)_W$. For example, the $\rho(\lambda = \pm 1)$ and the π form a W-spin triplet, while the $\rho(\lambda=0)$ forms a W-spin singlet.

Since the ρ and π belong to a $\underline{35}$ dimensional representation of $SU(6)_W$, while the N and Δ belong to a $\underline{56}$, it is clear that the masses of particles do not respect this symmetry very well. Splittings in m^2 of the order of 1/2 GeV^2 are quite to be expected within a multiplet. Nonetheless, it seems that the period of the parity alternation described above in section IV is somewhat greater than this. Hence, at least the lowest L=0 and L=1 multiplets for both mesons and baryons seem to be easily identified despite the splitting among their various members.

2. Interactions

Do any traces of the "classification symmetry" (for particles moving along the z-axis) describe their interactions as well? The original applications of $SU(6)_W$ assumed so. These applications included resonance decays and forward scattering — both collinear processes in a suitable frame.

The application of $SU(6)_W$ to forward scattering processes has led to the Johnson-Treiman relations (Johnson, 1965), which say

$$\Delta_{\pi p} = \Delta_{Kn} = \frac{1}{2} \Delta_{Kp} \qquad \text{(VI. 10)}$$

where

$$\begin{aligned} \Delta_{\pi p} &= \sigma_T(\pi^- p) - \sigma_T(\pi^+ p) \\ \Delta_{Kn} &= \sigma_T(K^- n) - \sigma_T(K^+ n) \\ \Delta_{Kp} &= \sigma_T(K^- p) - \sigma_T(K^+ n) \end{aligned} \qquad \text{(VI. 11)}$$

Experimentally in the region $\sim 6 - 30$ GeV, it appears that

$$\Delta_{\pi p} \simeq \frac{1}{2} \Delta_{Kn} \qquad \text{(VI. 12)}$$

and

$$\Delta_{\pi p} \simeq \frac{1}{3} \Delta_{Kp}$$

Both (VI. 10) and (VI. 12) satisfy a weaker relation (Barger, 1965)

$$\Delta_{\pi p} = \Delta_{Kp} - \Delta_{Kn} \qquad \text{(VI. 13)}$$

which can be derived purely on the basis of octet dominance in the t channel. The Johnson-Treiman relations follow if one assumes that this octet couples via pure F-type coupling to the baryons (Sawyer, 1965). This is not the case experimentally . However, one can expect the relations to provide some <u>idealized</u> picture of the hadrons which is not <u>too</u> <u>far</u> from reality. The physical origin of deviations from the Johnson-Treiman relations has been discussed by various authors (see, e.g., Lipkin, 1966b; Rosner, 1970a).

There are a number of notoriously <u>bad</u> $SU(6)_W$ predictions for collinear processes (Jackson, 1965). Some of these compare processes where pion exchange is possible (e.g., $\pi N \rightarrow \rho\Delta$) with ones where it is not (e.g., $\pi N \rightarrow \pi\Delta$ or $\pi N \rightarrow \pi N$). Others involve processes where <u>exotic</u> exchange must occur. In both cases the severe breaking of $SU(6)_W$ in the masses of the exchanged particles leads to a severe breaking in scattering amplitudes. The Johnson-Treiman relations involve exchanges very similar to one another, and are not expected to be affected by such considerations.

This situation is familiar from the case of SU(3), which works quite badly for four point functions (Meshkov, 1964) until one allows for breaking due to different masses of the exchanged particles. On the other hand, as we have seen, SU(3) for <u>couplings</u> works quite well, as the corrections for mass splittings are more straightforward. It thus makes sense to ask whether $SU(6)_W$ could <u>also</u> be better for couplings than for forward scattering.

The answer is that there are indeed some predictions of $SU(6)_W$ for couplings "worth saving", but that not all the predictions are good. Briefly, the "good" ones all involve relations between decays with the <u>same</u> final orbital angular momentum ℓ. The "bad" ones are relations between decays involving <u>different</u> ℓ. If the "good" predictions are to be preserved but the "bad" ones discarded, some well-defined breaking of $SU(6)_W$ must be adopted. Much recent work has dealt with this question (Colglazier, 1971a, b; Petersen, 1972; Petersen, 1973a, b; Faiman, 1972; Melosh, 1973; Gilman, 1973b, e; Hey, 1973b).

The "good" predictions of $SU(6)_W$ include the values of f/d for decays of octet resonances indicated in section II, Table IV. Most data on hyperon resonances can be checked with respect to $SU(6)_W$ by seeing if SU(3) holds with the predicted f/d value. As one sees, these values are in qualitative accord with experiment. (See section II. B. 3c for a discussion.)

A number of "good" predictions refer to decays of nonstrange baryons. For example, $SU(6)_W$ predicts

$$(\Delta(1236) \to N\pi) = \frac{12}{25} \frac{G^2_{\pi NN}}{4\pi} \frac{P^{*3}}{m^2_\Delta} \simeq 56 \text{ MeV} \tag{VI. 14}$$

(based on Gürsey, 1964b,

but with our interpretation

of kinematic factors)

while the present experimental value is about 115 MeV. There are also a number of predictions for partial decay widths of $\underline{35}$, L=1 mesons (Colglazier, 1971a,b; Gilman, 1973b,e), $\underline{70}$, L=1 baryons (Rosner, 1972c; Faiman, 1972; Gilman, 1973b,e) and $\underline{56}$, L=2 baryons (Faiman, 1973a; Gilman, 1973b,e), whose numerical values depend on dynamical barrier factors (to be discussed shortly). These are obeyed at least as well as Eq. (VI. 14).

The "bad" predictions of $SU(6)_W$ are considerably worse than Eq. (VI. 14). They entail, for example, a purely longitudinal ω in $B \to \omega\pi$ whereas transversely polarized ω's actually dominate (see section II). Another "bad" prediction involves the $\Delta\pi$ decay of the resonance $N(1520, 3/2^-)$. $SU(6)_W$

predicts the Δ to have helicity $\pm 1/2$; experimentally (Cashmore, 1973a, b) helicity $\pm 3/2$ seems to dominate.

Why does one set of predictions work (approximately) and not the other? The answer has been given in various languages. Basically they all amount to the same thing: $SU(6)_W$ involves the conservation of S_z (the component of quark spin along the decay axis) and thus of L_z. It thus entails both $\Delta S_z=0$ and $\Delta L_z=0$, and for this reason one sometimes speaks of the $SU(6)_W$ decay symmetry for L-excited hadrons as $SU(6)_W \times 0(2)_{L_z}$. The $0(2)_{L_z}$ is of course redundant. In any model entailing quark recoil, such a picture is no longer true. The transverse momentum of quarks in a hadron then can give rise to $\Delta L_z = \pm 1$ transitions as well as the $SU(6)_W$-invariant $\Delta L_z=0$ ones.

In a decay from (say) L=2 to a pair (L=1) + (L=0), $SU(6)_W$ does allow transitions from the $L_z=\pm 1$ states. Hence transverse momentum cannot be neglected consistently in $SU(6)_W$. The assumption is really that decays cannot change L_z (i.e., grossly speaking, transverse momentum). Such a rule is a direct consequence of assuming decays are collinear, even though the quarks participating in them may not be.

The $\Delta S_z=0$ rule of $SU(6)_W$ is a quick way to see how its "bad" predictions follow. By the rules for classifying the B meson, its quark spin must be zero, so that $SU(6)_W$ entails a final state in $B \to \omega\pi$ with $S_z=0$. The ω must then be longitudinally polarized, in contradiction to experiment. Similarly, the $N(1520, 3/2^-)$ is usually described as a quark-spin 1/2 state. If $\Delta S_z = 0$, this state cannot decay to $\Delta(\lambda = \pm 3/2)\pi$.

In the past few years, it has gradually become clear that the $\Delta L_z = \Delta S_z = 0$ rule of $SU(6)_W$ is not a fundamental limitation. An essentially unique algebraic structure can be preserved in which this rule is extended to allow pionic decays of resonances with $\Delta L_z = -\Delta S_z = \pm 1$ <u>and</u> 0. Various approaches differ as to kinematical details; the selection rules themselves also differ somewhat for resonance photoproduction. A brief outline of these approaches is in order before we deal with some details.

B. <u>Higher-Symmetry Models</u>

1. <u>The "random-breaking" approach</u>

The symmetry $SU(6)_W$ is equivalent to a single quark making a transition in a hadron. It thus entails $\Delta L = 1$.* For example, when a hadron with quark orbital angular momentum L decays to two others each with $L = 0$, their relative ("external") orbital angular momentum ℓ always satisfies the rule

$$\ell = \ell_{\pm} \equiv L \pm 1 \qquad \text{(VI. 15)}$$

Thus, in the decays

$$\begin{array}{ll} N(1670;5/2^-) & \rightarrow \Delta\pi \\ (L=1) & (\ell = 2, 4) \end{array} \qquad \text{(VI. 16)}$$

and

$$\begin{array}{ll} \Delta(1950;7/2^+) & \rightarrow \Delta\pi \\ (L=2) & (\ell = 3, 5) \end{array} \qquad \text{(VI. 17)}$$

only the lower value of ℓ is expected in each case in $SU(6)_W$. This is the value that corresponds to $\ell = L+1$.

*A good discussion of such selection rules may be found in (Gilman, 1973e).

Now, $SU(6)_W$ entails very specific relations between the amplitudes for $\ell_+ \equiv L+1$ and $\ell \equiv L-1$. In the $B \to \omega\pi$ decay, for example, S waves and D waves cooperate just so as to suppress the transverse coupling.

It can be argued that this "link" forged by $SU(6)_W$ between two partial waves is more disastrous for the lower one(ℓ_-). This is because in $SU(6)_W$ (or quark model calculations related to it, as in Faiman, 1971) the amplitudes for ℓ_- and ℓ_+ must have a <u>common</u> barrier factor, which will be p^{ℓ_+}*:

$$
\begin{aligned}
a\left[\ell_-,\ SU(6)_W\right] &\sim p^{\ell_+} \\
a\left[\ell_+,\ SU(6)_W\right] &\sim p^{\ell_+}
\end{aligned}
\qquad \text{(VI. 18)}
$$

Any "random" breaking can be expected to lead to amplitudes which have the "normal" barrier factors:

$$
\begin{aligned}
a\left[\ell_-,\ \text{breaking}\right] &\sim p^{\ell_-} \\
a\left[\ell_+,\ \text{breaking}\right] &\sim p^{\ell_+}
\end{aligned}
\qquad \text{(VI. 19)}
$$

This, of course, is also expected to be true in <u>specific</u> models of breaking, as it is (Colglazier, 1971a,b; Feynman, 1971). However, suppose the breaking does not obey rules suggested by our pet theory but is indeed "random" (or obeys the rules of <u>someone else's</u> pet theory). One can still expect that the terms in (VI. 19) will affect those in (VI. 18) much more for $a(\ell_-)$ than for $a(\ell_+)$. The "normally behaved" ℓ_- amplitude overwhelms the $SU(6)_W$ one, which is suppressed by an "anomalous" barrier factor.

Most of the "good" predictions of $SU(6)_W$ indeed involve decays with final orbital angular momentum $\ell = \ell_+ \equiv L+1$. The "random breaking" approach thus allows one to understand the most compelling of the successes of $SU(6)_W$.

---

*The unsatisfactory nature of this fact has been stressed by (Meshkov, 1970).

2. "ℓ-broken $SU(6)_W$"

The decays involving $\ell_- \equiv L-1$ are not wholly without systematics. For the $(6,\bar{6})$, L=1 mesons, the (very scanty) data on 0^+ decays to 0^-0^- (S waves) are roughly consistent with the S wave decay of the B into $\omega\pi$ (Colglazier, 1971a,b; Gilman, 1973b,e). For the 70, L=1 baryons, a host of S-wave decays may be related to one another in a manner which does not strain the data appreciably (Petersen, 1972; Faiman, 1972; Gilman, 1973b,e). The conclusions do depend to some extent on mixing, which is more of a problem for the low-spin resonances giving rise to S-wave decays. Finally, the 56, L=2 baryons have a few detectable P-wave decays (like $N(1690, 5/2^+) \to \Delta\pi$ and $N(1860, 3/2^+) \to N\pi$) which could be related to one another as in $SU(6)_W$ (Petersen, 1972; Faiman, 1973a).

One is thus tempted to ask (Capps, 1967, 1968a) whether the $SU(6)_W$ "link" between ℓ_+ and ℓ_- is the only thing grossly wrong with $SU(6)_W$. The resulting scheme which breaks this link can be called "ℓ-broken $SU(6)_W$" (Faiman, 1972

Calculations in "ℓ-broken $SU(6)_W$" are trivial given those in $SU(6)_W$. Any decay helicity amplitude which involves a single value of ℓ is left as it is and expressed in terms of a universal constant for that value of ℓ. Any helicity amp tude involving two or more values of ℓ may first be decomposed into its contributions from different ℓ, which are then assigned independent couplings in such a way as to be consistent with the $SU(6)_W$ limit.

The crucial tests of the need for "ℓ-breaking" in $SU(6)_W$ are those that measure the interference of the two waves which $SU(6)_W$ relates to each other. Thus, in the decays of the L=1 mesons (Colglazier, 1971a,b) it is the transvers nature of $B \rightarrow \omega\pi$ that demands the S and D wave amplitudes to have a relative phase opposite to that of $SU(6)_W$. Fits to partial widths alone, which depend only on squares of these amplitudes, cannot determine this relative sign. In fact, for both the L=1 mesons (Colglazier, 1971a, b; Gilman, 1973b, e) and the L=1 baryons (Faiman, 1972; Petersen, 1972; Gilman, 1973b, e) fits to partial widths alone allow for two solutions. In the first, the S/D wave ratio is roughly that of $SU(6)_W$. One may call this the "$SU(6)_W$-like" solution. In the second, the sign of S/D is reversed. This is the "anti-$SU(6)_W$" solution.

To decide between the two solutions in the case of the baryons, one may study the decays of resonances into $\pi\Delta$ or $\pi\Sigma(1385)$ (Rosner, 1971c; Petersen, 1972). The helicity of the isobar in the decay of a given resonance signals the way in which two given ℓ-values are interfering. We shall discuss the results of such tests, as applied to $\pi N \rightarrow \pi\Delta$ (Faiman, 1973b; Gilman, 1973b, e), in subsection D.

3. The quark-pair-creation (QPC) model

From the SU(3) structure of decays, one sees a big difference between graphs of the type shown in Fig. 30a and those of Fig. 30b. The latter seems to be absent since $\phi \rightarrow \rho\pi$ is weak and f' does not decay to $\pi\pi$ (Okubo, 1963). The former seems to be all one needs (Zweig, 1964).

What happens if we try to ascribe a spin structure to the graphs of Fig. 30a? An answer was first provided by (Micu, 1969). A decay via Fig. 30a involves the creation of an additional $q\bar{q}$ pair, shared among the two outgoing hadrons. This pair is assumed to have the quantum numbers of the vacuum. It should thus be an SU(3) singlet with $J^{PC} = 0^{++}$, i.e., a $^3P_0$ state. The calculation of decays then becomes a problem in the recoupling of angular momenta. A $q\bar{q}$ "spurion" has also been used by other authors to describe decays (Carter, 1968; Horn, 1970; Carlitz, 1970b).

In this "$^3P_0$" picture, some general spatial features of quark model wavefunctions have crept in through the back door. A $^3P_0$ $q\bar{q}$ pair should have definite weights for $L_z = 0, 1,$ and -1 (just given by the SU(2) Clebsch-Gordan coefficients for $S=1 \times L=1 \rightarrow J=0$). But the decays with $L_z=0$ and $L_z = \pm 1$ are assumed to be independent of one another. This can be ascribed to different overlaps of quark model wavefunctions (Le Yaouanc, 1973) or it can be left as a free parameter. In the latter case the "$^3P_0$" state is really like a combination of $^3P_0$ and $^3P_2$; the only non-trivial assumption is thus that it has L=1. The spins of the "spectator" quarks are also assumed not to change in the decay.

There is a particular limit in which the QPC model is just $SU(6)_W$. This is the case in which the $q\bar{q}$ pair has $L_z=0$ only (Carlitz, 1970b). In that case, one may check that the pair belongs to a singlet of $SU(6)_W$ and hence cannot lead to breaking of this symmetry. Allowing for the pair to have $L_z = \pm 1$ as well as $L_z=0$ breaks $SU(6)_W$ in a very specific way. It is clear that the L_z of the pair is the same as what we have called ΔL_z in section (VI. A), above.

For most cases of interest, "ℓ-broken $SU(6)_W$" and the QPC model turn out to be identical. They are the same whenever both final states have L=0 (Petersen, 1972, 1973a,b). In that case the initial L and that of the $q\bar{q}$ pair (L=1) just couple together to give $\ell = \ell_+ \equiv L+1$ and $\ell = \ell_- \equiv L-1$. If $\Delta L_z = 0$, the amplitudes for ℓ_+ and ℓ_- are related to one another. If $\Delta L_z = 0, \pm 1$, the ℓ_+ and ℓ_- amplitudes are free with respect to each other but all other relations of $SU(6)_W$ (i.e., among ℓ_+ and among ℓ_-) continue to hold.

The QPC model and "ℓ-broken $SU(6)_W$" also turn out to be identical for decays

$$\underline{A}(L) \rightarrow \underline{A}(L) + \pi \quad . \tag{VI. 20}$$

Here $\underline{A}(L)$ denotes a multiplet of the rest symmetry. The argument depends on crossing symmetry for the vertex (VI. 20) and is given by (Hey, 1973b).

In general it turns out the QPC model is slightly weaker than "ℓ-broken $SU(6)_W$". The first place where the difference could be tested would be in the decays (Hey, 1973b)

$$\underline{A}(L=2) \rightarrow \underline{B}(L=1) + \pi \quad , \tag{VI. 21}$$

but no results are available at present, and clean tests are hard to devise.

We would expect dual models for hadrons based on relativistic strings to give predictions similar to those of the QPC model. The picture of resonance decays in such models often resembles Fig. 30a.

4. Quark models

There is a vast body of work on "realistic" quark models for decays, gradually leading up to models whose algebraic structure is identical to that of the more abstract ones considered here.

The basic approach is to evaluate matrix elements of appropriate transition operators between quark model wavefunctions of the initial and final state.*, **

*The nonrelativistic form of this model has been considered by (Becchi, 1966; Dalitz, 1966a; Moorhouse, 1966; Mitra, 1967a,b; Lipkin, 1967b; Van Royen, 1967; Katyal, 1968; Faiman, 1968, 1969, 1971; Copley, 1969; Walker, 1969).

**The effects of relativistic or "recoil" corrections to quark model predictions for transitions have been discussed, for example, by (Katyal, 1970; Choudhury, 1970; Fujimura, 1970; Feynman, 1971; Ravndal, 1971; Copley, 1971; Close, 1972a,b; Abdullah, 1972; LeYaouanc, 1973; Böhm, 1973).

In the picture without quark recoil, the operator describing pion emission is usually taken to be σ_z, entailing the selection rule $\Delta S_z = \Delta L_z = 0$ and a consequent $SU(6)_W$ structure. The addition of a quark "recoil" term allows for $\Delta L_z = \pm 1$ transitions as well. The resulting algebraic structure is generally similar to that of ℓ-broken $SU(6)_W$ and the quark-pair-creation model, but additional parameters are specified. For example, decays involving different $SU(6)_W$ multiplets are related to one another. The ratios of $\Delta L_z = \pm 1$ to $\Delta L_z = 0$ amplitudes are also specified. We shall see one possibility where such a relation fails in subsection D, but most such relations seem to work fairly well as in $B \to \omega\pi$ (e.g., Choudhury, 1970; Feynman, 1971).

5. The Melosh approach

We have already mentioned in section II that the transformation between "current" quarks and "constituent" quarks (Melosh, 1973) has definite consequences for the decays and photoproduction of resonances. Many of these consequences turn out to be remarkably similar to ones obtained in the schemes mentioned above.

To recapitulate briefly:

The "constituent quarks" are the building blocks used in section V. A proton is made of three of them. The "current quarks" are what are measured for example, in deep inelastic lepton scattering. While currents may be expressed in terms of them simply, hadrons are very complicated states of current quarks, perhaps with additional $q\bar{q}$ pairs and so on (Bjorken, 1969; Kuti, 1971).*

*A suggestion that different kinds of quarks might exist and could be related to one another by a unitary transformation was made by (Ohnuki, 1965) before the successful applications of current algebra.

Each type of quark is associated with an algebra: a chiral $SU(3) \times SU(3)$, an $SU(6)_W$, and so on. One may classify particles according to representations of the group generated by this algebra. If the group is a symmetry of the one-particle Hamiltonian, the particles will fall into irreducible representations of the group.

The physical particles seem to belong to irreducible representations of $SU(6)_{W,\,\text{hadrons}}$ (generated by "constituent" quarks). But it is particularly easy to calculate their pionic transitions by evaluating matrix elements of the axial charge Q_5 using PCAC (see, e.g., Horn, 1966; Weinberg, 1970). These matrix elements are only expected to be simple between irreducible representations of $SU(6)_{W,\,\text{currents}}$. That is, we need

$$< B_{\text{hadrons}} | Q_5 | A_{\text{hadrons}} >$$

but it is easier to calculate

$$< B_{\text{currents}} | Q_5 | A_{\text{currents}} > \quad .$$

Here A and B stand for members of irreducible representations.

One approach is to construct a transformation V such that

$$| A_{\text{hadrons}} > = V | A_{\text{currents}} > \quad .$$

This transformation need not be written down explicitly; it is sufficient to specify in some self-consistent manner each hadron state as a sum of irreducible representations of the "currents" group. For practical purposes, this sum must be truncated (Altarelli, 1966; Harari, 1966b; Gerstein, 1966; Horn, 1966; Gilman, 1968a; Buccella, 1970, 1972; Casher, 1973a, b).

Another approach is to cast all the complexity of V onto the charges and currents themselves: we thus evaluate

$$
\begin{aligned}
&\langle B_{hadrons} | Q_5 | A_{hadrons} \rangle \\
&\quad = \langle B_{currents} | V^{-1} Q_5 V | A_{currents} \rangle \\
&\quad \equiv \langle B_{currents} | \widetilde{Q}_5 | A_{currents} \rangle ,
\end{aligned}
$$

where the states A and B belong to pure representations of $SU(6)_W \times 0(3)$ whose form we adopt from the quark model.

In the new representation, the transformed axial charges $\widetilde{Q}_5$ maintain their former $\Delta L_z = 0$ pieces and acquire new $\Delta L_z = \pm 1$ contributions (see Table XV). One thus has a few extra matrix elements to evaluate as compared with the old $SU(6)_W$ (ΔL_z=0) calculations. We shall sometimes speak of the new symmetry, when applied to pion emission, as $SU(6)_W$ (ΔL_z=0, ±1). *

The theoretical advantage of the Melosh method over those discussed in subsections 2 - 4 above is that it allows one to relate hadronic vertices to schemes for saturating sum rules (Adler, 1965b; Weisberger, 1965) based on current algebra (Gell-Mann, 1962b). The need for mixing is very clear from these sum rules. For example, saturation of the sum rules for πN and $\pi\Delta$ scattering by N and Δ alone would be justified if these states formed an approximately pure representation of $SU(3) \times SU(3)_{currents}$ or $SU(6)_{W, currents}$ However, this saturation leads to the prediction $|G_A/G_V| = 5/3$, and predicts no further resonances in πN scattering above the $\Delta(1236)$! Hence, one concludes that the N and Δ must contain mixtures of higher representations of these groups.

*A recent review of the applications of this method to hadronic and electromagnetic transitions has been given by (Weyers, 1973). More abstract properties have been reviewed by (Carlitz, 1973).

The new $\Delta L_z = \pm 1$ pieces (in Table XV) turn out to behave exactly as in the QPC model for pion emission. The pion combined with the $q\bar{q}$ pair certainly transforms as $\underline{35}$ (8, 3), $L_z = 0, \pm 1$ of quark spin. It also behaves this way under $SU(6)_W$, however. For baryonic decays the two pictures are manifestly identical: the quark spin of a baryon is the same as its W-spin. For mesonic decays the matrix elements in the two approaches simply turn out to be identical as well (Hey, 1973b). The QPC model thus turns out to be equivalent to one (the Melosh picture) in which we at least do not explicitly violate relativistic invariance. It is presumably for this reason that a covariant formulation of the QPC model can indeed be given (Colglazier, 1971a,b).*

6. Some comparisons

While the algebraic structure of the Melosh approach for pionic decays and that of the quark pair creation model are the same, the former makes an unambiguous prediction of how matrix elements should be compared with experiment: namely, via Eq. (II. 7). The covariant form of the quark pair creation model (Colglazier, 1971a,b) is essentially a picture of elementary Feynman diagram couplings, and thus suggests that the partial width for a given final orbital angular momentum ℓ be proportional to

$$\Gamma_\ell \sim P^{*\,2\ell+1}/M_A^2 \quad . \tag{VI. 22}$$

However, the factor (VI. 22) is actually somewhat arbitrary.

Some predictions fare better with Eq. (II. 7), while some favor (VI. 22) (see Rosner, 1973c; Kugler, 1973). We shall discuss these in the next subsection.

*The fact that the Melosh approach also makes statements about couplings of currents leads to such successful predictions as (a) $\mu(p)/\mu(n) = -3/2$, and (b) $f = 2/5$ for the axial current in the (Cabibbo, 1963) theory. This is in good agreement with recent fits. See (Ebenhoh, 1971; Roos, 1971).

The emphasis of the two approaches is also somewhat different. The "current-quark" method based on Melosh's work treats the pion as a source of the nonzero divergence of the axial current, while in the quark-pair-creation model the pion is treated as just another hadron. The algebraic identity of the two methods suggests that one may be able to believe both points of view simultaneously! However, the difference between Eq. (II. 7) and (VI. 22) cannot be ignored.

The fact that Eq. (II. 7) is assumed to hold for <u>any</u> ℓ may cause some concern, since the conventional centrifugal barrier factor is lacking. Since the axial charge is evaluated between infinite-momentum states, there is <u>no reason</u> to expect such a factor in this approach.

There are some predictions of the quark-pair-creation picture not obtained in the Melosh approach. These relate to decays involving vector mesons (see Petersen, 1973a,b). They are both a blessing and a curse. They lead to the interesting result that a $(\rho\epsilon)_{\ell=0}$ 1^- resonance must be an L=0 $q\bar{q}$ state, while a $(\rho\epsilon)_{\ell=2}$ 1^- resonance must be a $q\bar{q}$ state with L=2. The $\rho'(\sim 1500)$, seen decaying to $\rho\epsilon$ by $\ell=0$, must then be a radially excited $q\bar{q}$ state. These conclusions may be obtained by calculating the $\rho\epsilon$ helicity amplitudes for an L=0, 2 ρ' in "naive" $SU(6)_W$ with $\Delta L_z=0$. One then notes that these helicity structures are characteristic of $\ell(\epsilon\rho)=0,2$, respectively, a conclusion which will not be altered by "ℓ-breaking." The Melosh approach gives no predictions for longitudinal ρ's and hence can say nothing about $\rho' \to \epsilon\rho$ at present.

A difficulty with the quark-pair-creation picture for $\rho\rho$ decays of mesons has been noted by (Freund, 1973). In such a case the helicity constraints of the model cannot be satisfied without introducing Carlitz-Kislinger cuts (Carlitz, 1970b).

C. Partial-Width Predictions

Some specific predictions for the $\underline{35}$, L=1 mesons have been discussed recently by (Colglazier, 1971a; Gilman, 1973b, e; and Rosner, 1973c). Predictions for $\underline{35}$, L=2 mesons are mentioned by (Hey, 1973b; Gilman, 1973e).

At present no overall comparison has been made between the kinematic factors (II. 7) and (VI. 22), but it appears that (II. 7) is somewhat better for the mesons. This conclusion follows from the comparisons shown in Tables XVI and XVII.

The potentially significant numbers which may allow one to distinguish between the PCAC kinematic factor (II. 7) and the factor (VI. 22) are the ratios $A_2 \to \rho\pi / f_0 \to \pi\pi$, $K^{**} \to K^*\pi / K^{**} \to K\pi$, $\delta \to \eta\,\pi / B \to (\omega\pi)_{\ell=0}$, and $g \to \pi\pi / g \to \omega\pi$. These numbers all tend to favor the PCAC factor; a possible exception is the decay $A_2 \to \eta'\pi$. (See also section VII. D. 1.) We have not included predictions for decays in which no pions are emitted, such as $A_2 \to K\bar{K}$. These are related by SU(3) to ones shown in Tables XVI and XVII. We have seen in sections II and III that the kinematic factor (VI. 22) describes such decays well. They are not predicted reliably by the PCAC approach since $m_K^2 >> m_\pi^2$.

For the baryons, the most spectacular success of the PCAC factor is its modification of Eq. (VI. 14). In that relation, the kinematic factor is somewhat ambiguous, and modifications are possible. (See, e.g., Gürsey, 1964b; Schmid, 1972b.) On the other hand, the corresponding PCAC relation is

$$\Gamma(\Delta \to N\pi) = \frac{12}{25} \frac{G^2_{\pi NN}}{4\pi} \frac{P^*}{M^2_\Delta} \left\{ \frac{M^2_\Delta - M^2_N}{2M_N} \right\}^2 , \qquad \text{(VI. 23)}$$

$$\simeq 125 \text{ MeV} \quad (\Gamma_{expt} \approx 115 \text{ MeV})$$

where the factor in brackets arises from applying PCAC to $\Delta \to N\pi$ and using the Goldberger-Treiman relation (Goldberger, 1958) to eliminate the resulting pion decay constant in favor of $G^2_{\pi NN}$. Equation (VI. 23) cannot be more valid than the Goldberger-Treiman relation (about 20% in squares of couplings).

For other baryonic decays, we shall concentrate on cases where mixing effects are firmly under control. This means we shall restrict ourselves to the $\underline{70}$, L=1 multiplet, for which results are shown in Table XVIII.

Table XVIII does not permit one to choose between the two kinematic factors. One of the worst predictions of the PCAC factor is the $\Delta\pi/N\pi$ branching ratio of N(1670, $5/2^-$), while one of the worst of the $P^{*2\ell+1}$ is the prediction of an extremely wide N(1700, $3/2^-$). This state _is_ seen by (Herndon, 1972) but appears to be considerably narrower in their present solution. Overall, the agreement is reasonable, with several predictions for $\Delta\pi$ and $\Sigma^*(1385)\pi$ decays which we would hope to see confirmed in the next year or two.

The $\underline{56}$, L=2 multiplet has been discussed by (Petersen, 1972, 1973b; Rosner, 1973c; Gilman, 1973b, e). All these discussions omit the possibility of mixing with a $\underline{70}$, L=2 multiplet, which is treated in detail (for N and Δ states) by (Faiman, 1973a). One prediction which does not depend on these considerations is the $\Delta\pi/N\pi$ branching ratio of

$\Delta(1950, 7/2^+)$:

$$\frac{\Gamma(\Delta(1950) \to \Delta\pi)}{\Gamma(\Delta(1950) \to N\pi)} \begin{cases} = .88 & \text{(PCAC factor)} \\ = .29 & (P^{*2\ell+1} \text{ factor}) \\ \cong .4 \pm .2 & \text{(Herndon, 1972)} \\ \approx 1 & \text{(Mehtani, 1972)} \end{cases}$$

Both experimental analyses confirm the prediction (common to all the above theories), that $\Delta(1950) \not\to (\Delta\pi)_{\ell=5}$; (Herndon, 1972) see no H-wave, while (Mehtani, 1972) see very little.

Tables XVI - XVIII display a wealth of predictions only a few of which can be obtained via SU(3). Within the rather large experimental errors, these predictions do seem to provide a qualitative guide to observed partial widths. We have not quoted a number of predictions which follow from SU(3) or which relate to unobserved states; these may be found in the original references, or worked out as an exercise! The methods are straightforward and are given, for example, by (Rosner, 1972c; Hey, 1973b; Gilman, 1973e; or Weyers, 1973). They make use of standard tables of SU(6) unitary singlet factors (Cook, 1965), isoscalar factors (Lasinski, 1973), and angular momentum Clebsch-Gordan coefficients.

D. Phases in $\pi N \rightarrow \pi\Delta$

Since the N and Δ are in the same multiplet of SU(6), the resonant contributions in $\pi N \rightarrow \pi\Delta$ are related in a known way to those in elastic πN scattering. But the latter must have positive imaginary parts, by the optical theorem. Hence all the phases of resonant amplitudes in $\pi N \rightarrow \pi\Delta$ are specified with respect to one another, in a way which depends on the ΔL_z selection rules for pion emission.

An early analysis of $\pi N \rightarrow \pi\Delta$ by (Brody, 1971) showed prominent contributions from N(1670, $5/2^-$) and N(1688, $5/2^+$), but two solutions were found. In one ("A"), these two amplitudes had opposite relative phase, while in the other ("B") they had the same relative phase. Hence no conclusion was possible at the time. More recently, the analysis of (Herndon, 1972) leads to a preference for solution "A".

Some Argand circles resulting from the analysis of (Herndon, 1972) are shown in Fig. 31, along with magnitudes of resonant amplitudes. In a combination like "PP11", the first letter refers to the incident (πN) orbital angular momentum, the second to the $\pi\Delta$ orbital angular momentum, the first number to 2I, and the second to 2J.

One notices that the imaginary parts of resonant amplitudes seem to have well-defined phases: positive or negative. One also notes (on the right-hand side of the figures) rather well-defined bumps, except in the region of a gap between 1540 and 1650 MeV, mentioned in section II.

(Herndon, 1972) bridges the gap by demanding continuity of the PP11 wave (the first in Fig. 31). Two resonances appear in this wave: one above and one below the gap. The relative phase of amplitudes below and above the gap thus hangs on this rather slender thread.

Figure 32a shows the phases of the (Herndon, 1972) solution along with symmetry predictions. The arrows denote predicted resonant phases in the Argand diagram, referred to a baryon-first isospin convention. The phases in Fig. 31 refer to the isospin convention $\pi N \rightarrow \Delta\pi$, and thus have reversed relative I=1/2 - I=3/2 phase. The crosses are the experimental phases in the baryon-first convention.

A double-headed arrow in Fig. 32a indicates a phase which is sensitive to which value of ΔL_z dominates: $\Delta L_z=0$ (the "$SU(6)_W$" solution), or $\Delta L_z = \pm 1$ (the "anti-$SU(6)_W$" solution). For definiteness, we have shown the "anti-$SU(6)_W$" solution. The names stem from the relative phases of D/S and F/P waves: those of $SU(6)_W$ when $\Delta L_z=0$ dominates, and opposite to those of $SU(6)_W$ when $\Delta L_z = \pm 1$ dominates (Petersen, 1972; Rosner, 1972c).

The figure is cut in two at the gap.

The phases are defined with respect to the prominent FF37 resonance (Herndon, 1972; Kernan, 1973). One then sees that, above the gap, all of the "first-class predictions" hold that would be expected if $\Delta L_z = \pm 1$ dominated for $\underline{70}$, L=1 decays. (A "first-class prediction" is one that cannot be affected by mixing. A second-class prediction is one for which mixing can occur but is throught to be understood and does not change the predictions for unmixed states (Faiman, 1972, 1973a). A third-class prediction is one where the assignment is based on an educated guess.)

In fact, $\Delta L_z = \pm 1$ is expected to dominate in certain "realistic-quark" models based on harmonic oscillator wavefunctions (Feynman, 1971; Moorhouse, 1973b) both for 70, L=1 and for 56, L=2 decays. In the case of the latter, however, we see that $\Delta L_z = 0$ seems to dominate. A model has indeed been constructed (Buccella, 1972) in which $\Delta L_z = \pm 1$ dominates for odd L and $\Delta L_z = 0$ dominates for even L.*

The disagreements above the gap in PP11 and PP31 are both for states to whose assignments we are not committed firmly at present. Apparently our estimate of the experimental DS13 situation (Faiman, 1973b) was incorrect and the data actually agree with our prediction (Cashmore, 1973a). (This is one case in which the phase is not too well defined, since there are two overlapping DS13 resonances.) Hence one can be rather pleased with the overall pattern above the gap.

Below the gap, however, the disagreement is complete, leading one to suspect the continuation. At the urging of D. Faiman after the Purdue Baryon Conference in May, a new continuation was sought, and seems to have been obtained, in which the relative phase across the gap has changed and is now in accord with theory (Cashmore, 1973b). The situation has changed often enough that a little patience is probably in order till things settle down. (At one point, Faiman and I had our isospin conventions wrong!) Nonetheless, the situation looks very encouraging at present. Similar predictions for a number of multiplets (Faiman, 1973b) are shown in Fig. 32b.

Analyses of $\pi N \rightarrow \rho N$ and $\pi N \rightarrow \epsilon N$ are also performed by (Herndon, 1972). There may be some disagreement with the quark model in $\pi N \rightarrow \rho N$ (Moorhouse, 1973b); this should also apply, in principle, to the approach of (Petersen, 1973a),

*Recently it has been shown by (Eguchi, 1973b) that exact duality for baryons (see section VII. C) requires universal dominance of either $\Delta L_Z=0$ or $\Delta L_Z=\pm 1$ in $\pi N \rightarrow \pi\Delta$.

though the phase predictions still have not been worked out in full. The Melosh approach makes no predictions for this reaction without additional assumptions. The (Kernan, 1973) analysis of $\pi^+ p \to \pi\pi N$ sees very little evidence for resonant ρ production,* in contrast to (Herndon, 1972), and ascribes the large ρ signal to one-pion exchange. Hence results of this channel should be treated with some caution.

One straightforward prediction is obtained in any of the approaches discussed in the present section, <u>including the current-quark picture</u>. These are the relations (Petersen, 1973a; Moorhouse, 1973b):

$$\Gamma\left[\Delta(1950) \to (N\rho)_{\ell=5}\right] = 0 \quad , \tag{VI. 24}$$

$$\Gamma\left[\Delta(1950) \to (N\rho)_{\ell=3,\, S=3/2}\right]$$
$$= 3\Gamma\left[\Delta(1950) \to (N\rho)_{\ell=3,\, S=1/2}\right] \tag{VI. 25}$$

Experimental, the $N\rho$ decay of $\Delta(1950)$ <u>is</u> dominated by $\ell=3$, $S=3/2$.

The couplings of resonances to $\pi\Delta$ also are of interest in the discussion of duality (section VII).

C. <u>Resonances in $\gamma N \to \pi N$</u>

A large-scale analysis of single-pion photoproduction in the resonance region has recently been carried out (Moorhouse, 1973c, d). This analysis leads to resonant phases (and approximate magnitudes) which agree with quark model predictions (Feynman, 1971; Moorhouse, 1972, 1973c; Knies, 1973a). Another recent analysis has been made by (Devenish, 1973).

---

*See (Williamson, 1972).

A less predictive and more general discussion of resonant phases in $\gamma N \rightarrow \pi N$ may be founded on the Melosh transformation (Gilman, 1973d; Hey, 1973c). Here one needs the transformation properties of the dipole operator D_+ which induces electromagnetic transitions.

The analysis of (Gilman, 1973d) assumes the dipole operator to transform as a sum of

$$\begin{aligned} &\underline{35},\ (8,3)_{W_z=\pm1}\,, && \Delta L_z = 0 \\ +\ &\underline{35},\ (8,1)_{W_z=0}\,, && \Delta L_z = \pm1 \\ +\ &\underline{35},\ (8,3)_{W_z=\mp1}\,, && \Delta L_z = \pm2 \end{aligned} \tag{VI. 26}$$

A term also seems to be present (Hey, 1973c) which transforms as

$$\underline{35},\ (8,3)_{W_z=0}\,, \qquad \Delta L_z = \pm1\ . \tag{VI. 27}$$

This term also is required in the model of (Petersen, 1973a).

By neglecting the term (VI. 27) and the last term in (VI. 26), one obtains vertices for electromagnetic transitions which have the same algebraic structure as the quark model. There seems to be no compelling phenomenological need for the other terms at present. Based on the first two terms in Eq. (VI. 26) one can predict the signs of the resonant amplitudes in $\gamma N \rightarrow \pi N$. The results are shown in Fig. 33.

From Fig. 33 one sees that all the significant signs are in agreement with the theoretical expectations of (Gilman, 1973d). Moreover, the sign of the contribution of the S-wave πN resonance $\Delta(1610,\ 1/2^-)$ is that expected if

$\Delta L_z = \pm 1$ dominates in 70, L=1 pionic decays, as suggested by the $\pi N \to \pi\Delta$ case. Occasional discrepancies can almost certainly be traced to oversimplified (unmixed) assignments. Now, one will have to await quantitative comparisons, which are forthcoming. Since the algebraic structure is the same as the quark model, which does not fare too badly, one can expect reasonable agreement; the question is whether the agreement will be significantly better.

One very general quantitative relation predicted by all approaches is the M1 nature of $\Delta(1236)$ excitation:

$$\Delta(1236):\ A_{3/2} = \sqrt{3}\, A_{1/2} \quad . \tag{VI.28}$$

A similar relation holds for $\Delta(1950,\ 7/2^+)$, expressing M3 dominance:

$$\Delta(1950,\ 7/2^+):\ A_{3/2} = \sqrt{5/3}\, A_{1/2} \quad . \tag{VI.29}$$

This is borne out qualitatively by Fig. 32, where the quoted numbers are based on real parts coming from the tail of the resonance. The present upper limit of $E_{c.m.} = 1780$ MeV in the analysis of (Moorhouse, 1973c) is to be extended to 2 GeV in the near future, allowing a direct check of Eq. (VI.29). It is interesting that this relation holds even in the presence of the $\Delta L_z = \pm 2$ term in Eq. (VI.26).

Another benefit of the extended analysis will be the possibility of observing the photoproduction of P-wave πN resonances presumably belonging to 56, L=2, such as $\Delta(1910,\ 1/2^+)$. This will allow one to check the signs of P-wave πN couplings relative to F-waves. Are they the same as in $SU(6)_W$, as suggested by the $\pi N \to \pi\Delta$ analysis, or are they the opposite, as suggested by the quark model?

We have not discussed electroproduction or neutrino production here. The results of the Melosh transformation are unknown as yet. It is not yet known whether the very specific predictions of the quark model for lepto-production (see, e.g., Fujimura, 1970; Ravndal, 1971; Close, 1972a,b; Abdullah, 1972) are confirmed by the data.

F. "Lower Symmetries"

Up to now we have been discussing $SU(6)_W$ assignments of resonance multiplets, with the selection rules of Table XV characterizing pion emission (i.e., $\Delta L_z = 0, \pm 1$) and those of Eqs. (VI.26) and (VI.27) characterizing electromagnetic transitions. It may happen that these symmetries can be ruled out by future data in the same striking, qualitative way that "naive" $SU(6)_W$ (with $\Delta L_z = 0$ for pion emission) has already been disproven. (As we saw in subsection D, the $\pi N \rightarrow \pi \Delta$ data were very close to doing just that, until the new continuation across the gap was found.) In that case, one would like a "lower symmetry" than $SU(6)_W$ to which to retreat before being driven back all the way down to SU(3) by the data.

There have been two weaker symmetries discussed in the literature that fill the gap between $SU(6)_W$ (with the new selection rules) and SU(3). These are two forms of $SU(3) \times SU(3)$, known as coplanar and chiral.

1. Coplanar $SU(3) \times SU(3)$ (Dashen, 1965) was first applied to resonance decays (Rosner, 1972a) with the aim of allowing for transverse momentum of quarks inside a hadron.*

*The applicability of this symmetry to resonance decays was suggested by (Freund, 1971).

It assumes that $\Delta L_z = 0$ pion emission is governed by $SU(6)_W$ (as do all the symmetries described above). For the $\Delta L_z = \pm 1$ decays, however, the rules are more lax. In contrast to the quark-pair-creation picture, no assumption is made about the spins of "spectator quarks" remaining unchanged. As a result, the coplanar symmetry leads to additional $\Delta L_z = \pm 1$ matrix elements which may be termed "spin-orbit" effects. For the decays of L=1 mesons the general covariant formalism of (Colglazier, 1971a,b) turns out to be equivalent to this coplanar symmetry, until such "spin-orbit" processes are expressly forbidden. For higher-L decays, $\Delta L_z = \pm 2, \pm 3, \ldots$ transitions are not explicitly forbidden, though they may be absent in certain cases.

Some simple $SU(6)_W$ results which also follow from coplanar symmetry are listed in Table XIX. As in the quark-pair-creation model, the barrier factors in coplanar symmetry are arbitrary. (See the discussion by Rosner, 1972a.)

As in the case of $SU(6)_W$, one would really like a gross discrepancy if coplanar symmetry for decays were to be discarded. The experimental deviation of $(f/d)_{5/2^- \to 1/2^+ 0^-}$ from -1/3 cannot yet be ranked in this category, as mentioned above. More serious would be the phases of the lower PP11 contribution to $\pi N \to \pi\Delta$ relative to that of DD15. If the Roper resonance N(1470) really belongs to $\underline{56}$, L=0 (we need to see its SU(3) partners to verify this), and if the phases are really as in Fig. 32 (Herndon, 1972), then coplanar symmetry is rejected unambiguously. We have seen, however, that the existence of a gap in the data prevents this conclusion from being drawn at present.

Recently coplanar symmetry has been applied to the decays of the $\underline{70}$, L=1 baryons (Haut, 1973). In addition to the one $SU(6)_W$ ($\Delta L_z=0$) amplitude, these authors find <u>three</u> $\Delta L_z = \pm 1$ amplitudes (not four as claimed by Rosner, 1972a). One of their results is that the decay

$$N(1670,\ 5/2^-) \rightarrow \Delta\pi \qquad \text{(VI. 30)}$$

is pure D wave; the ℓ=4 amplitude must vanish. In fact, one would only obtain an ℓ=4 (G-wave) contribution to (VI. 30) if all the quarks in the initial state were to flip their spins in the transition. Coplanar symmetry apparently forbids this from happening. Another interesting relation is the $\Delta\pi/N\pi$ branching ratio of N(1670, 5/2$^-$). By SU(3), since coplanar symmetry implies f=-0.5 for the $5/2^- \rightarrow 1/2^+\ 0^-$ decays, this implies

$$\frac{\Gamma\left[\Lambda(1830,\ 5/2^-) \rightarrow \Sigma^*\pi\right]}{\Gamma\left[\Lambda(1830,\ 5/2^-) \rightarrow \Sigma\pi\right]} = \begin{cases} 1.4 & \text{PCAC} \\ .7 & p^{*2\ell+1} \end{cases} \qquad \text{(VI. 31)}$$

or $\widetilde{\Gamma}(\Sigma^*\pi)/\widetilde{\Gamma}(\Sigma\pi) = 7/2$, as quoted in Table XVIII. When analyses of $K^-p \rightarrow \pi\pi\Lambda$ such as that of (Prevost, 1971) and (Prevost, 1973) begin to show some stability and are confirmed by others, tests of Eq. (VI. 31) should be quite straightforward.

2. <u>Chiral SU(3) × SU(3)</u> (Gell-Mann, 1962) also can be applied to decays in the context of the Melosh transformation (Gilman, 1973a; Hey, 1973a,b). The transformed axial charge $\widetilde{Q}_5$ is assumed to behave as it does in the free quark model (Table XV), i.e., with (8, 1) - (1, 8), $\Delta L_z = 0$ and $(3, \bar{3})$ or $(\bar{3}, 3)$, $\Delta L_z = \pm 1$ pieces.

The hadrons are assumed to belong to pure representations of $SU(3) \times SU(3)$, in accord with their quark-model assignments. As in the case of decomposition of quark-model states to $SU(6)_W$, some vestige of the rest symmetry remains. Otherwise, for example, we would not know how much an A_2 with helicity 1 contains of $L_z=0$, $S_z=1$ and how much of $L_z=1$, $S_z=0$. What differs from $SU(6)_W$, however, is that in each helicity the quark spin multiplets are assumed to differ from one another. Hence the chiral predictions hold separately for each helicity.

For the most part, the chiral predictions follow from those of $SU(6)_W$ ($\Delta L_z=0, \pm 1$) by simply breaking the link implied by the latter between different $-\lambda$ processes. There are some relations of $SU(6)_W$ ($\Delta L_z=0, \pm 1$), however, in a given helicity, which do not follow in chiral $SU(3) \times SU(3)$ (Hey, 1973b).

An example of how the simplest $SU(6)_W$ predictions are relaxed when one retreats to chiral $SU(3) \times SU(3)$ is the relation (based on Hey, 1973a):

$$\Gamma(\Delta \rightarrow N\pi) = \frac{4}{3}(1-f)^2 \frac{G^2_{\pi NN}}{4\pi} \frac{P^*}{m^2_\Delta} \left\{\frac{m^2_\Delta - M^2_N}{2M_N}\right\}^2 , \qquad \text{(VI. 32)}$$

which replaces Eq. (VI. 23) and reduces to it when f equals its $SU(6)_W$ value of 2/5. Within the bounds allowed by Table IV for the experimental range of f, it is precisely the $SU(6)_W$ value of f which leads to the "best" prediction for $\Gamma(\Delta \rightarrow N\pi)$. In view of the uncertainties in the use of PCAC itself, one cannot use Eq. (VI. 32) to demonstrate that chiral $SU(3) \times SU(3)$ is "better" than $SU(6)_W$.

Other data are too imprecise at present to indicate that predictions of chiral $SU(3) \times SU(3)$ are any better than those of $SU(6)_W$. One can obtain additional relations by assumptions about the ΔL_z values which contribute to decays (Buccella, 1970; Gilman, 1973a), but these are outside the scope of the symmetry itself, and seem to hold only approximately.

The chiral approach also leads to predictions for matrix elements involving currents (Gilman, 1973a; Love, 1973). These are consistent with experiment. The original motivation of Melosh was to explain the SU(6) structure of the hadrons without obtaining those "bad" predictions that follow from assigning hadrons to pure representations of $SU(6)_{W,\,\text{currents}}$. Such predictions $\left[G_A/G_V = -5/3;\ \mu_A(p) = \mu_A(n) = 0\right]$ can be avoided in this framework. The $\rho\pi\pi$ coupling and G_A/G_V are both reduced by a factor $\eta \simeq 0.7$.

The relation (III. 15) also follows in this picture. The transverse ρ and ω (λ=1) belong to $(3,\bar{3})$, the dipole operator D_+ (see section VI. E) has only an effective $(3,\bar{3})$, ΔL_z=0 piece, and the pion belongs to $(8,1)-(1,8)$. Although there are two reduced matrix elements, they enter in the same combination in $\omega \to \pi^o\gamma$ and $\rho \to \pi^o\gamma$, leading to a unique relation.

In terms of generators, the coplanar and chiral symmetries are complementary to one another. Of those listed in Eq. (VI. 4),

$$\lambda_i, \quad \lambda_i\beta\sigma_y \quad (\text{or } \lambda_i\beta\sigma_x)$$

$$\to \text{coplanar } U(3) \otimes U(3) \qquad \text{(VI. 33)}$$

and

$$\lambda_i, \quad \lambda_i \sigma_z$$

$$\rightarrow \text{chiral } U(3) \otimes U(3) \quad . \qquad \text{(VI. 34)}$$

(This last holds at $p_z = \infty$ where $\sigma_z \sim \gamma_5$.) When commuted with one another, the generators (VI. 33) and (VI. 34) give those of $SU(6)_W$, i.e., Eq. (VI. 4).

Both the coplanar and the chiral symmetries apply to $\Delta L_z \neq 0$ transitions. The chiral symmetry, applied using Table XV, has the selection rules that would be expected of single-quark transitions, i.e., $\Delta L_z = 0, \pm 1$. The combination of the two symmetries just gives $SU(6)_W$ ($\Delta L_z = 0, \pm 1$). If $SU(6)_W$ ($\Delta L_z = 0, \pm 1$) is found to fail one can then take it apart into its component $SU(3) \times SU(3)$ subgroups and see where the trouble lies. In $\pi N \rightarrow \pi \Delta$, Fig. 32 (Herndon, 1972) would rule out at least the coplanar $SU(3) \times SU(3)$, but, as mentioned, a new continuation across the gap in data has been found in which this symmetry survives.

The way in which the two $SU(3) \times SU(3)$ symmetries are related is illustrated very nicely for the decays of the L=1 mesons. Let us consider the following processes listed in Table XX, from which all others can be obtained via SU(3) and the nonet coupling ansatz. The coplanar symmetry entails relations among <u>different</u> helicities in general, while the chiral symmetry relates only amplitudes in a <u>given</u> helicity. Their combination leads to the results of ℓ-broken $SU(6)_W$ or $SU(6)_W$ ($\Delta L_z = 0, \pm 1$), which is equivalent to the former in this case.

By the use of tables such as Table XX, one can pinpoint violations of $SU(6)_W$ (as extended in this section to include $\Delta L_z = 0, \pm 1$ pion emission). With sufficiently precise data, we may be able to determine whether coplanar symmetry, chiral symmetry, or both are at fault. At present both seem to be reasonably well obeyed.

G. Conclusions on Higher Symmetries for Decays

The question of what symmetry higher than SU(3) could apply (approximately) to resonance decays remains an open one. We have seen that $SU(6)_W$ (with the new selection rules) is not a quantitatively perfect symmetry, but it may describe the pattern of hadronic vertices well enough to let us understand many gross features of low-energy resonance physics. Tests of this idea are available at present, the most fruitful involving resonance decays in which two partial waves interfere with one another. The study of $\pi N \to \pi\Delta$, γN - and $\bar{K}N \to \pi\Sigma(1385)$ will provide much useful information in this respect.

Since $m_\rho \neq m_\pi$, we might not be too surprised if in the end violations of the newly interpreted $SU(6)_W$ turned out to be rather large — perhaps even as large as the invariant amplitudes themselves. We have seen such an effect in the case of "naive" $SU(6)_W$ ($\Delta L_z=0$), where the $\Delta L_z = \pm 1$ transitions seem to be at least as important as those with $\Delta L_z = 0$, and even dominant in some cases.

The successful description of pionic and electromagnetic transitions in algebraic terms serves as a useful complement to the more intuitive but less rigorous approach of the quark model. We are on the verge of seeing whether this model is compatible with the data mainly because of its algebraic features, which could be true in a wide class of theories, or whether its specific dynamical content is also correct.

VII. WHERE DUALITY FITS IN

The "matching" of the high-energy description of scattering amplitudes (via Regge pole exchange) and the low-energy description (via direct-channel resonances) leads to constraints on both descriptions. The idea that the two ways of writing scattering amplitudes are complementary, and should not be added to one another, has been called "duality" (Dolen, 1968).

The analytic basis of duality lies in finite-energy sum rules (FESR)*. These sum rules express an integral over the imaginary part of the amplitude up to some finite energy in terms of the contribution of the imaginary parts of the Regge exchange amplitudes at that energy. The crucial assumptions that convert FESR from tautologies to powerful sources of constraints are:

- Dominance of a few Regge poles at moderate energies, and
- Resonance saturation of the imaginary parts at low energies.

Such (clearly approximate) assumptions are what usually constitute "duality".

The concept of an "exotic channel" plays an important role in duality. Certain channels (such as K^+n) seem to have few or no resonances; these are the ones which cannot be made of $q\bar{q}$ or qqq (see sections III, V). On the other hand, Regge pole exchanges are certainly possible in $K^+n \rightarrow K^0p$. If the direct-channel and Regge pole descriptions are to be equivalent, the imaginary parts contributed by the Regge pole exchanges must cancel exactly.

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* These have been considered by many authors, for example: (Igi, 1967; Logunov, 1967; Horn, 1967; Gatto, 1967; Dolen, 1968).

Thus, for the leading (A_2 and ρ) trajectories, both the residues (in $K^+n \rightarrow K^op$) and the trajectories themselves (in general) must be degenerate. This circumstance is known as "exchange degeneracy" (Arnold, 1965; see also Ahmadzadeh, 1964) since it arises from the absence of an "exchange" force (in this case, in the $K^+n \rightarrow K^op$ channel). The Regge poles in the t channel ($K^+ \bar{K}^o \rightarrow \bar{n}p$) are "built" entirely out of the resonant contributions in the u channel ($K^+\bar{p} \rightarrow K^o\bar{n}$). This is the situation familiar from potential scattering, in which the Regge recurrences are spaced by a single unit.

The consequences of exchange degeneracy for resonances are far-reaching. They have been reviewed to a great extent by (Mandula, 1970). Here we shall recall them very briefly, starting with mesons (subsection A). Difficulties associated with elastic baryon-antibaryon scattering are noted in subsection B. For baryons (subsection C), a pattern of resonances is described which is consistent both with exact duality and with the symmetric quark model. Fianlly, some specific schemes, involving more local forms of duality, are mentioned in subsection D.

A. Exchange Degeneracy for Mesons

Any scattering process without baryon number exchange provides information on meson exchange degeneracy.

In order to separate out diffractive scattering (Pomeranchuk trajectory exchange) from the exchange of lower-lying trajectories, one must of course understand the nature of the Pomeranchuk trajectory (Pomeron) itself. The behavior of total cross sections up to ~ 30 GeV/c suggested that the Pomeron contribution was essentially constant. Higher-energy data, showing a rise in $\sigma_T(K^+p)$, $\sigma_T(K^+n)$, and $\sigma_T(pp)$ (Gorin, 1971; Amaldi, 1973)* indicate that our picture of the Pomeron is not so simple. Nonetheless, the following conjecture (Freund, 1968a; Harari, 1968a) seems borne out by the patterns to which it leads:

Low energy		High energy	
Resonances	$\longleftrightarrow$	Non-Pomeron Regge poles	(VII. 1)
Background	$\longleftrightarrow$	The Pomeron	

The double arrow $\longleftrightarrow$ in (VII. 1) indicates a relation between imaginary parts via FESR. With this conjecture, one can understand immediately why those total cross sections which lack prominent low-energy resonances (K^+p, K^+n, NN) are essentially flat (up to ~ 30 GeV/c) while those which do have prominent low-energy resonances have sizeable non-Pomeron contributions at higher energies. The identification of the Pomeron with background and non-Pomeron trajectories with resonances also is borne out by studies of pion-nucleon scattering (Gilman, 1968b; Harari, 1969b; Zarmi, 1971).

Applying the Freund-Harari conjecture to the scattering of pseudoscalar mesons off one another, one can establish the pattern of degeneracy of the leading non-Pomeron trajectories shown in Table XXI (without the Freund-Harari conjecture, we would not have been able to separate the f_0 from the

*See also (Amendolia, 1973).

Pomeron, which has identical quantum numbers).

The pattern in Table XXI refers only to natural-parity trajectories ($J^P = 1^-, 2^+, \ldots$), since these are the only ones that couple to pairs of pseudoscalar mesons.

Strictly speaking, the test of Table XXI via resonances alone has not yet been made. One would need more recurrences along the trajectories (see section IV). For example, the 3^- recurrence of the ρ (the g) has been seen. If the f_0, A_2, ω and ρ trajectories are really degenerate, the combined trajectory is remarkably straight. If extrapolated to J = 4, it would predict

$$m(f_0^*,\ J^{PC} = 4^{++}) \simeq m(A_2^*,\ J^{PC} = 4^{++}) = 1950 \text{ MeV} \quad \text{(VII.2)}$$

On the other hand, the intercept point $\alpha_\rho(0) \simeq 1/2$, the $\rho \approx \omega$, the $f \approx A_2$, and the $g \approx \omega_3$ do seem to lie on an approximately straight line.

Powerful constraints on couplings also follow from duality. For example, one obtains many features of SU(3) and the nonet ansatz (Okubo, 1963) merely from considering the processes of Table XXI (Chiu, 1968; Rosner, 1968).

The octet-vs. nonet structure of meson exchanges is of interest in the more general processes which involve external vector mesons as well as pseudoscalars. An elegant treatment in the SU(3) limit may be found in (Mandula, 1970). Suppose the external particles 1 and 2 coupling to a single Reggeon have charge parities b_1 and b_2 whose product is positive. This is so for $PP \rightarrow PP$ and $VV \rightarrow VV$; $P = 0^-$; $V = 1^-$. Then C = + trajectories couple symmetrically (d-type octets and singlets) while C = - trajectories couple antisymmetrically (f-type octets). For PV → VP and PP → VV, where C_1C_2 = -, the situation is reversed.

The types of predictions following from duality in these various cases are listed in Table XXII (from Mandula, 1970). The symmetrically coupled singlet is essential to the coupling pattern; its omission gives no solution.

Table XXII shows that the processes PP → PP, PV → VP, and PP → VV all give reasonable constraints, but that VV → VV does not. Specifically, when we apply exact duality to such processes as $\rho\rho \to \rho\rho$ and $K^*K^* \to K^*K^*$, we find that imaginary parts can be eliminated in all exotic channels only if the π and η (or η') are degenerate and obey nonet-type coupling patterns. This may indicate that the idea of duality is somewhat limited. Mandula, et. al. (Mandula, 1969b) have suggested that this is because of the high threshold (and consequent inelasticity) in VV → VV, which may make resonance saturation a poor approximation in this case. The ordering of duality predictions according to threshold has been termed "broken duality" and the VV → VV difficulties are evidence for it. Difficulties in $B\bar{B} \to B\bar{B}$ (section B) may also be avoided if duality is broken in the manner suggested by Mandula, et. al. The channel $B\bar{B} \to PP$ also was thought to present problems (Mandula, 1969a, b), but in fact does not (subsection C).

So far we have been considering constraints on meson trajectories that have followed from the absence of "SU(3) exotic" states. It is interesting that certain of these constraints also follow from the absence of "C-exotic" states, i.e., those which are forbidden as $q\bar{q}$ systems by virtue of their J, P, and C (see section V). An example is the case of $\pi - \eta$ scattering considered by (Schwimmer, 1969).

We may now turn briefly to constraints for meson trajectories obtained in meson-baryon scattering processes. No new exchange degeneracies occur, but a pattern of couplings emerges which is consistent with experiment (Rosner, 1969c). For example, if we impose duality constraints on meson-meson and meson-baryon scattering amplitudes, require factorizability of residues, and demand that the ϕ and f' decouple from nucleons (as seems to be true experimentally*) we obtain vanishing (net) contributions from the leading non-Pomeron trajectories to all baryon-baryon total cross sections. Moreover, in the SU(3) limit, one obtains equal F/D values for couplings of exchange degenerate trajectories (e.g. 2^+ and 1^-) to baryons, again in accord with experiment (Michael, 1972). Tests for exchange degeneracies in meson-baryon scattering are very simple. One may test for the degeneracy of trajectories alone (without requiring that of residues) by noting that for pairs of line-reversed reactions dominated by exchange-degenerate trajectories, the values of $d\sigma/dt$ should be equal at all t (Gilman, 1969). These tests do not fare spectacularly well (see, e.g., Michael, 1972), indicating either a breakdown of exchange degeneracy or - more likely - important Regge cut contributions. If residues are degenerate as well, polarizations should vanish. Again, as in $\pi^- p \rightarrow K^0 \Lambda$, where duality leads us to expect exchange degenerate

*D. Cline, unpublished compilation of data at 2.24 GeV/c (BNL - Syracuse), 3.0 GeV/c (Saclay), 3.5 GeV/c (RHEL), 4.1 and 5.5 GeV/c (Argonne-Northwestern). For 2.24 GeV/c see (London, 1966). The compilation indicates $\{g^2_{\phi N\bar{N}}/g^2_{\omega N\bar{N}}\} < 12\%$ at a two standard deviation limit (Rosner, 1971e).

K* and K** contributions, the Λ polarization is appreciable. A similar situation will occur in baryon exchange processes: whereas the data in the resonance region are consistent with exchange degeneracy, the processes which actually involve the exchange of supposedly degenerate trajectories do not have the features expected (Rosner, 1973a ; Minkowski, 1972). It is thus quite likely that the approximate duality first noted in πN charge exchange may only be a symptom of some deeper regularity which is not expressed in an equivalent way for all processes. Nonetheless, the approximate pattern suggested by duality may be an important guide to a correct description; we shall see whether this could be so in the next subsections.

B. The Baryon-Antibaryon Problem

The most stringent test of duality occurs in elastic baryon-antibaryon scattering (Rosner, 1968; Lipkin, 1969a). In this case, exact duality turns out to require exotic resonances coupled to baryon-antibaryon pairs*. Such resonances have not yet been observed.

The cleanest example which shows the difficulty is the charge-exchange process

$$\Delta^{++}\bar{\Delta}^{o} \rightarrow \Delta^{+}\bar{\Delta}^{+} \qquad \text{(VII.3)}$$

(Lipkin, 1970, 1973c). This process is assumed to be dominated by isospin one exchange in the t channel: $I_t = 1$. On the other hand, the s channel is

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* The algebraic features of this difficulty were noted early by (Capps, 1968b). Predictions of exotic $B\bar{B}$ resonances follow quite strongly from attempts to construct explicit dual models (Frampton, 1970). The first suggestion of the need for exotic $B\bar{B}$ resonances in the context of duality was by (Rosner, 1968). Exotic resonances also appear necessary in saturating current algebra (Young, 1972).

exotic: $I_s \geq 2$. If we demand that the imaginary part vanish, we will also find that the imaginary parts vanish in all the non-exotic charge exchange processes

$$\Delta^+ \bar{\Delta}^o \to \Delta^o \bar{\Delta}^+ , \qquad (VII.4)$$

$$\Delta^o \bar{\Delta}^o \to \Delta^- \bar{\Delta}^+ \qquad (VII.5)$$

$$\Delta^{++} \bar{\Delta}^- \to \Delta^+ \bar{\Delta}^o \qquad (VII.6)$$

etc., since these are all related to (VII.3) by Clebsch-Gordan coefficients if $I_t = 1$ is the only exchange allowed.

Similar difficulties involving external octet baryons also arise. They are more subtle since (in the SU(3) limit) the allowed t-channel octet exchanges may couple to external octet baryons via d-type or f-type coupling. Thus, for example (Roy, 1969; Kugler, 1970) there exist solutions for $B\bar{B} \to B\bar{B}$ ($B = \frac{1}{2}^+$ octet baryon) in which the baryons behave like mesons, for which a solution without exotics is clearly possible. This solution, however, has the undesirable feature that the leading non-Pomeron contribution to $\sigma_T(\bar{p}n)$ vanishes. Certainly this goes against the grain of most successful fits to this total cross section (see, e.g., Barger, 1971). Moreover, if factorizability is assumed, $\sigma_T(\pi^+ p)$ also becomes flat. If duality were to hold, this would mean the $\Delta(1236)$ is absent. Hence it is quite likely that octet baryons are subject to the same problems as those for Δ's. In the limit of factorizable Regge residues obeying SU(3) and the decoupling of ϕ from $N\bar{N}$, one has

$$\tilde{\sigma}_T(\bar{\Sigma}^+ p) = 1/2\, \tilde{\sigma}_T(\bar{p}n) \quad , \tag{VII. 7}$$

where $\tilde{\sigma}$ refers to the non-Pomeron contribution. The $\bar{\Sigma}^+ p$ channel is exotic.

The simplest way to see that total cross sections should have non-Pomeron contributions in exotic channels comes from a quark model mnemonic (Lipkin, 1966c). Suppose hadron A has an antiquark $\bar{q}_i$ corresponding to a quark q_i in hadron B. Then $\sigma_T(AB)$ will have a non-Pomeron contribution, as in the case of $K^- = \bar{u}s$ and $p = uud$. If the antiquarks $\bar{q}_j$ in A are all different from the quarks q_i in B (as for $K^+ = \bar{s}u$ and $p = uud$, or for any baryon-baryon system) then the leading non-Pomeron contribution to $\sigma_T(AB)$ should vanish. Processes of the first kind are shown in Figs. 34a, c, e, and those of the second in Figs. 34b, d, f.

Since Lipkin's rule follows from SU(3), exchange degeneracy, and the nonet coupling ansatz (as well as holding for all processes observed to date) we might well be tempted to believe it. However, Lipkin's rule implies that all $B\bar{B}$ channels which can form the graphs shown in Fig. 34e should have leading non-Pomeron contributions. These channels are all the ones which can be formed of two quarks and two antiquarks: the $\underline{1}$, $\underline{8}$, $\underline{10}$, $\underline{10}^*$, and $\underline{27}$ of SU(3).

A direct test of Lipkin's rule in exotic $\bar{B}B$ total cross sections may not be possible for some time. Measurements of $\sigma_T(\bar{\Lambda}N)$, $\sigma_T(\bar{\Sigma}N)$, and $\sigma_T(\bar{\Xi}N)$, which could test the whole pattern of Reggeon couplings in $\bar{B}B$ processes, would certainly be welcome, but they would require antihyperon beams comparable in intensity to present-day beams of pions. On the other hand, the systematics of the approach to hadronic scaling (Chan, 1971) in such reactions as

$$\bar{p}\,n \rightarrow \underset{\text{(fragment of } \bar{p})}{\pi^+} + \ldots \tag{VII. 8}$$

allows one to study the effective energy dependence of cross sections that would otherwise be inaccessible. The $\bar{p}$ fragments into a π^+ and a "$\bar{\Delta}^{--}$", whose cross section on n may then be studied as a function of energy. A slow approach to scaling would mean the $\bar{\Delta}^{--}$n cross section has the expected non-Pomeron contribution.

Even if Lipkin's rule should hold in exotic $\bar{B}B$ channels, we do not necessarily expect the non-Pomeron imaginary parts to arise from resonances. Experimentally, resonances in the $B\bar{B}$ channel do not seem to provide the whole difference $\sigma_T(\bar{p}p) - \sigma_T(pp)$, nor do they seem dominant in $B\bar{B} \rightarrow$ mesons (see, e.g., Fields, 1971). They also seem to be hard to detect in $\bar{p}p \rightarrow \bar{n}n$. * Duality says that such resonances <u>do</u> exist but have been missed up to now. Putting two graphs like Fig. 34 together end-to-end to obtain a duality graph (Imachi, 1968; Harari, 1969a; Rosner, 1969a) (Fig. 35a) we would expect the exotic s-channel, dual to the non-exotic t-channel, to be associated with resonances if duality holds. <u>This resonance dominance assumption is crucial to duality</u>.

There have been suggestions that the graphs of Fig. 35b are responsible for the major contribution to the difference between $\sigma_T(\bar{p}p)$ and $\sigma_T(pp)$. (See, e.g., Weiner, 1971; Chiu, 1971). Thus, these authors suggest that direct-

*See (Cutts, 1972; Grannis, 1972; Storer, 1972).

channel non-exotic resonances suffice to saturate $B\bar{B}$ non-Pomeron FESR. There are several indications weighing against this point of view.

First, the SU(3) exchange-degenerate description of total $\bar{B}B$ and BB cross sections appears to be reasonably correct. In addition to reasonably flat $\sigma_T(pp)$ and $\sigma_T(pn)$, one sees flat $\sigma_T(\Lambda p)$ in recent hyperon beam experiments*. In such a picture, we have seen that substantial non-Pomeron contributions to σ_T are expected in exotic direct channels.(See, e.g eq. (VII. 7)). One can only avoid such a prediction by giving up factorizabilit and SU(3) for the exchanges.

Secondly, there are no known exceptions (yet) to Lipkin's rule based on Fig. 34. It would be amusing if strong energy dependence in $\sigma_T(B\bar{B})$ in the range 6-30 GeV/c suddenly required the simultaneous occurrence of two suc $q\bar{q}$ annihilation processes (as in Fig. 35b).

Thirdly, the analysis of backward meson production in inclusive reactio (Hoyer, 1973) indicates that the "ordinary" ($q\bar{q}$) mesons in the s channel are "dual" to t-channel trajectories of very low intercept, which are presumabl exotic.

The most likely place where $B\bar{B}$ duality is to break down, therefore, is the resonance-saturation assumption. This then makes the search for exotic mesons of some interest. We shall call the $qq\bar{q}\bar{q}$ objects "gallons"***.

Any channel consisting of a baryon-antibaryon pair, whether one or bo are virtual, is suitable for looking for the predicted "gallons". There are

*(Gjesdal, 1972). $\sigma_T(\Sigma^- N)$ has been measured at 19 GeV/c (Badier, 1972).

**Proofs of factorizability, valid strictly only for potential scattering, are giv by (Gell-Mann, 1962a; Gribov, 1962a). The most likely cause of breakdown factorizability would be Regge cuts.

***I am indebted to P. G. O. Freund for this name. The "quark" as a unit of liq volume was first proposed by (Joyce, 1959). Its adoption as a part of a hadı is comparatively recent (Gell-Mann, 1964).

three main classes of such processes: direct-channel formation, baryon-antibaryon annihilation, and backward meson production.

1. Direct-channel formation

Whether or not exotic in the I, Y sense, the resonances formed in any baryon-antibaryon interactions are of considerable interest. At present we can study $\bar{p}p$ and $\bar{p}d$ interactions, and (as mentioned in section IX) the use of $\bar{n}$ beams will provide useful additional information in the near future, particularly at the lowest energies. One could expect - in principle - beams of $\bar{\Lambda}$ and even $\bar{\Sigma}$. In the latter case one could confirm the prediction (VII. 7) that $\sigma_T(\bar{\Sigma}^+p)$ should have a substantial energy-dependent part even though $\bar{\Sigma}^+p$ is an exotic channel. This would lend support to the usual SU(3) picture of non-Pomeron Regge pole exchanges which has been borne out by other experiments.

The $N\bar{N}$ system does not appear at present to have substantial resonances. One cannot be certain of this, however, until polarized-target studies are made at some length, preferably at the lowest possible energies. If one does find resonances in $N\bar{N}$, duality would predict ones in exotic channels like $\bar{\Sigma}^+p$ as well. Perhaps such effects are within the scope of future experiments in intense hyperon beams at the high-energy ($\geq$ 300 GeV) accelerators.

The baryon-baryon system is also of interest since duality predicts it not to have appreciable resonant structure. This system is much more easily accessible in the direct channel than $B\bar{B}$. While no statistically significant resonant behavior has been observed (see, however, Shabazian, 1973), systems like pp, pn, Λp, $\Sigma^{\pm}$p, and Ξp provide useful "calibrations". If one cannot demonstrate that there are many more resonances in $\bar{B}B \rightarrow \bar{B}B$ than in $BB \rightarrow BB$, duality will be proven wrong. The further measurement of $\sigma_T(\Sigma^- p)$, $\sigma_T(\Lambda p)$, etc., is also essential in checking exchange degeneracy for BB processes: these cross sections should be relatively flat in the 6-30 GeV/c range.

2. Baryon-antibaryon annihilation

When a baryon and an antibaryon annihilate into many mesons, one is assured of baryon exchange. One may then look for "gallons" coupling to baryon-antibaryon pairs via such reactions as

$$\bar{p}p \rightarrow \pi^{\pm} + \pi^{\pm} + (MM)^{\mp\mp} \qquad \text{(VII. 9)}$$

Selection rules dealing with the allowed decays of gallons and other exotic resonances have been proposed by (Freund, 1969a). These rules allow vertices of the type shown in Figs. 34a, c, e and Fig. 36, but forbid such vertices as those in Figs. 34b, d, f in which not every pair of hadrons is connected by quark lines. These selection rules ensure the self-consistency of the dual approaches discussed above. In the case of the process (VII. 9), for example, they allow any gallon $(MM)^{\mp\mp}$ to decay to another gallon plus an ordinary meson (Fig. 36) or to $B\bar{B}$ (Fig. 34e, read from right to left). If a gallon lies below threshold for either of these two processes, however, it must decay by a violation of the selection rules - for example, into many mesons, or even electromagnetically (if the selection rules are very good). The problem when the gallon in (VII. 9) decays into a $B\bar{B}$ pair is that one no longer is assured of baryon exchange, unless suitable kinematic selections are made.

3. Backward meson production

As stressed by (Freund, 1969a), (Jacob, 1970), and (Lipkin, 1973c), one way of looking for gallons is in reactions of the type

$$\begin{aligned} \pi^+ p &\rightarrow \text{(fast forward n)} + (X)^{++} \\ &\rightarrow \text{(fast forward } \Lambda) + (X)^{++} \end{aligned} \qquad \text{(VII. 10)}$$

and so on. The resonance production here (at least for MM $\gtrsim B\bar{B}$ threshold) should be comparable to that in the corresponding non-exotic reactions

$$\begin{aligned} \pi^+ p &\rightarrow \text{(fast forward n)} + (X)^{0} \\ &\rightarrow \text{(fast forward } \Lambda) + (X)^{0} \end{aligned} \qquad \text{(VII. 11)}$$

if duality holds. At present no convincing structure has shown up for $M \gtrsim 1.5$ GeV even in the non-exotic reactions (VII. 11). Here the decay channel $(X) \rightarrow B\bar{B}$ would be of considerable help in sorting out any possible resonances from background and determining their J^P properties (Faiman, 1973d).

Estimates of the mass of the lowest "gallons" can be made by noting tha any two hadrons allowed to resonate by the graphs of Figs. 34a, c do so at least once before $p_{CM} \approx (350$ MeV/c, 250 MeV/c), respectively. If similar behavior is guessed for Fig. 34e, one can estimate that there exists at least one exotic $\Delta^{++}\bar{n}$ resonance above, but no more than several tens of MeV abc threshold (Rosner, 1972b; see section IV. B). Such predictions also follow fr the results of (Schmid, 1969a): the graphs of Fig. 34e and 35a should be associated with a rotating phase $e^{-i\pi\alpha(t)}$, whose partial-wave projection would be expected to yield resonances not far above threshold.

C. Exchange Degeneracy for Baryons

The study of duality constraints on the baryon spectrum has been a topi of continued theoretical interest*,**.

*Early results on baryon exchange processes were obtained by (Capps, 1969a 1970a,b; Barger, 1969a,b; Mandula, 1969a,b; Chavda, 1969; Schmid, 1969b Mandelstam, 1970; and many others). A review of some of the early results is given by (Rosner, 1969b). That review is incomplete since the exact duali solution of (Mandelstam, 1970) was unknown at the time. Some specific predictions of this solution are noted by (Rosner, 1973a).

**Recent work has dealt with specific realizations of "non-null" duality conditions, whereby a specific pattern of s-channel resonances is used to "build" both t-channel meson exchange and u-channel baryon exchange. This approa is taken, for example, by (Capps, 1970a, 1973c), (Eguchi, 1973a,b), (Fukug 1973), and by authors attempting to construct dual scattering amplitudes in t manner of (Veneziano, 1968) for $0^- \frac{1}{2}^+$ scattering. One recent model of this sort has been proposed by (Igi, 1973). Specific constructions of algebraic models for baryon duality, based on duality graphs, have been undertaken by the Kyushu and Nagoya groups (see Ghoroku, 1973a,b for a review of son these).

The processes involving baryon exchange to which duality can be applied include

$$M_1 + B_1 \rightarrow B_2 + M_2 \tag{VII. 12}$$

(backward MB scattering)

and

$$B_1 + \overline{B}_2 \rightarrow M_1 + M_2 \tag{VII. 13}$$

(annihilation into meson pairs)

One demands that imaginary parts cancel among various exchanged baryon trajectories for all exotic direct channels in both types of process.

The experimentally established baryon multiplets include

$$\begin{array}{l} \underline{56}, \text{ even L} \\ \underline{70}, \text{ odd L} \end{array} \tag{VII. 14}$$

With this sequence, it was found impossible to satisfy constraints based on (VII. 12) and (VII. 13) simultaneously. This, together with difficulties in $VV \rightarrow VV$ and $B\overline{B} \rightarrow B\overline{B}$, led(Mandula, 1969b) to suggest the hypothesis of "broken duality", whereby channels with high threshold and great inelasticity - such as the two just mentioned and (VII. 13) - were not considered in deriving constraints. One then could find an acceptable solution to the remaining constraints.

It was shown by (Mandelstam, 1970) in a particular model that the spectrum

f

$$\begin{array}{l} \underline{56},\ L = 0 \\ \underline{70},\ L = 1 \\ \underline{56} \text{ and } \underline{70},\ L \geq 2 \end{array} \qquad (VII.15)$$

could indeed satisfy duality for both (VII.12) and (VII.13), as conjectured by (Freund, 1969b). The spectrum (VII.15) is in fact what one expects from a three-quark picture of baryons (see Fig. 37a). The three-particle system has two internal degrees of freedom, leading to an infinitely degenerate leading trajectory if one takes a harmonic-oscillator picture seriously (see Greenberg, 1964; Karl, 1968; Freund, 1969b). Properties of states of the crucial multiplets distinguishing (VII.15) from (VII.14) are summarized in Table XXIII.

1. 70, L = 2

This multiplet contains the characteristic states $N(7/2^+)$ and $\Lambda(7/2^+)$, for which some evidence indeed exists between 2 and 2.1 GeV (Lovelace, 1972; Barbaro-Galtieri, 1970b; see Rosner, 1973a).* All other states are not unique; they may belong to other multiplets expected nearby in mass, such as 56, L = 2; 56, L = 0; or 70, L = 0. Some evidence for a 70, L = 2 admixture in the $\Delta(1890,\ 5/2^+)$ (usually assigned to 56, L = 2) may exist (Faiman, 1973a).

The $N(7/2^+)$ should have a small elasticity (as observed). It is also expected to have a substantial $\pi\Delta$ coupling. The analysis of (Herndon, 1972) ends below the mass range of interest, but should see the effect if extended slightly. The $\Lambda(7/2^+)$ is expected to couple very weakly to $\bar{K}N$ (in the exact

*The $N(7/2^+)$ actually was included among the 1970 list of "established" particles (Barbaro-Galtieri, 1970a), but was dropped temporarily for lack of confirmation.

$SU(6)_W$ limit this coupling should vanish) and to have strong $\Sigma\pi$ and $Y_1^*\pi$ couplings (Faiman, 1971; Rosner, 1973a).

2. 56, L = 3

The most clearly defined state in this multiplet should be $\Delta(\sim 2200,\ 9/2^-)$. This state should be much more elastic than $N(\sim 2000,\ 7/2^+)$, at least with respect to its $\pi N/\pi\Delta$ coupling ratio. It could be responsible for the deep dip in backward $\pi^+ p$ scattering as a function of energy (Baker, 1972).

These crucial multiplets and the states identifying them are summarized in Fig. 37b.

If the sequence (VII. 15) is confirmed for the leading baryon trajectories, some of the motivation for breaking duality (Mandula, 1969b) will be lost. The outstanding problem for duality would then remain elastic $B\bar{B}$ scattering. As stressed, the study of the quantum numbers of all mesons up to a short distance above $B\bar{B}$ threshold is needed to resolve this question.

"Bootstrap" (non-null) conditions based on duality (see second footnote to this subsection) seem to have difficulty accommodating the spectrum (VII. 15), in which the ratios of residues change along the trajectory.* A solution proposed recently by (Capps, 1973c) encounters its first significant departure from (VII. 15) by the predicted absence of a 56, L = 3 multiplet. The question of whether a $\Delta(\sim 2200,\ 9/2^-)$ exists is then crucial in distinguishing the two schemes.

We have not dealt at all with important questions of parity doubling

*Recent results by (Eguchi, 1973b) are much more hopeful, however.

(Mac Dowell, 1959; Carlitz, 1970a,b; Schmid, 1972a; Minkowski, 1972), because we do not know how to. No spectroscopic evidence for parity doublets exist, but straight-line Regge trajectories require them (Gribov, 1962b). Until this problem is solved, explicit dual models for meson-baryon scattering cannot be constructed.

D. Other Duality Predictions

Some dual models, while devoted primarily to dynamical aspects of scattering amplitudes, also make statements about resonance couplings which are sufficiently precise that they can be compared with the algebraic features we have described in previous sections. While not discussing these models in any detail, we would like to mention some of their symmetry aspects.

1. Linear-zero (LZ) model

Odorico (1970-1973) has examined the consequences of assuming that scattering amplitudes possess straight-line zeroes in the Mandelstam s-t-u plane. The data on which this assertion is based are reviewed by (Odorico, 1973b).

The linear-zero suggestion is borrowed from the beta-function formula of (Veneziano, 1968). For $\pi\pi$ scattering, for example (Lovelace, 1968; Shapiro, 1969), this model assumes the amplitude to be

$$T(s,t) \sim \frac{\Gamma\left[1-\alpha(s)\right]\,\Gamma\left[1-\alpha(t)\right]}{\Gamma\left[1-\alpha(s)-\alpha(t)\right]} \qquad \text{(VII. 16)}$$

The denominator suppresses double poles at $\alpha(s) = J_1$, $\alpha(s) = J_2$, J_1 and J_2 integers ≥ 1, by acquiring poles of its own at these points. The denominator has poles, however, whenever $\alpha(s) + \alpha(t) =$ integer ≥ 1, implying zeroes of the amplitude at constant u. These linear zeroes are then assumed to be more universal than the specific formula (VII. 16). The allowed patterns for such zeroes have been listed by (Odorico, 1970-1973).

When these zeroes propagate into the physical scattering region, one can expect dips in the differential cross section. One example of this which has received widespread attention occurs in $\pi\pi$ scattering. We recall that the s-wave $\pi\pi$ phase shift passes rapidly through $180°$ just below $K\bar{K}$ threshold. (Fig. 3a). This is correlated with a very rapid decrease of the $< Y_1^0 >$ moment (see, e.g., Protopopescu, 1973; Hyams, 1973), and of the forward $\pi^-\pi^+$ cross section. The phenomenon is usually explained merely in terms of the effects of the S*(997), coupling strongly to kaons (see sections II, VIII). However, this behavior is also what is expected when a zero of the amplitude linear in u, passing through the double pole position

$$s = t = m_\rho^2 \quad (u = 4m_\pi^2 - 2m_\rho^2) \qquad \text{(VII. 17)}$$

enters the physical region at $t = 0$. In that case the zero would be expected at $s = 2m_\rho^2$ or

$$E_{zero} \cong 1040 \text{ MeV} \quad , \qquad \text{(VII. 18)}$$

which is not far from what is observed.

The existence of $\pi\pi$ phase shifts allows us to follow these zeroes into the physical region. They occur, in general, at different points for the real and imaginary parts of the amplitude. The position of the zeroes of the real part in one such solution (Estabrooks, 1973) is shown in Fig. 38.

One sees that the straight-line behavior is not exact, but neither are the phase shift solutions. Other processes, such as $K\pi$ scattering, are also expected to have the linear-zero structure. The entry into the physical region of such zeroes does seem to occur roughly as expected, leading to rapid changes in $< Y_1^0 >$ without corresponding threshold effects.

The linear-zero behavior is most compelling in channels such as $\pi^+\pi^- \rightarrow \pi^+\pi^-$ and $K^-p \rightarrow \bar{K}^0 n$, for which one crossed process is exotic. In more general cases, the expected pattern is more complicated, and it does not always work. Fig. 39 shows an example for the process $\bar{p}p \rightarrow \pi^-\pi^+$ and the crossed reactions of $\pi^{\pm}p$ elastic scattering.

Algebraic predictions of the LZ model generally stem from inconsistencies that are encountered in the model unless certain couplings vanish (Capps, 1973a; Odorico, 1973b). Some of these follow from the quark model. Others are inconsistent with it and require SU(3) - invariant couplings other than those suggested by the (Okubo, 1963; Zweig, 1964) rule. (In this context see also Kotlewski, 1973, and Finkler, 1973).

Two of the most-discussed non-quark model predictions are the vanishing of the $K^{**}_{(1420)}K\eta$ and $A_2\eta\pi$ couplings. In the case of $K^{**}_{(1420)} \rightarrow K\eta$, the SU(3) prediction is extremely small (see Table XVI). This comes about because of the destructive interference between the nonstrange and strange quarks in the η when one considers d-type coupling. By admixing more nonstrange quarks than one would have in a pure octet member, while still preserving Zweig's rule, one can suppress this coupling even further*. This mixing goes in the right direction to enhance the $\eta \rightarrow \gamma\gamma$ decay rate (see section III). The fact that K**(1420) exchange seems so much smaller than K*(890) exchange in $K^-p \rightarrow \eta \Lambda$ (Odorico, 1972) is actually evidence for SU(3) and exchange degeneracy. From SU(3) with the η an octet member, one predicts

$$\left[\frac{g(K^{**}\,K\eta)}{g(K^{**}\,K\pi)}\right]^2 \Bigg/ \left[\frac{g(K^{*}\,K\eta)}{g(K^{*}\,K\pi)}\right]^2 = 1/9 \qquad \text{(VII. 19)}$$

The K^{**} and K^* Reggeon couplings to $K\pi$ should be equal at the same t value, as required by exchange degeneracy in (say) $\pi^+K^+ \rightarrow K^+\pi^+$ (see section VII. A). One then expects the K^{**} contribution to $K^-p \rightarrow \eta\Lambda$ to be much smaller than that of K^*. As Table XVI shows, estimates of the $K^{**}K\eta$ coupling from decays of the K^{**}(1420) cannot rule out the SU(3) value at present.**

More interesting is the experimental suppression of the $A_2\eta'\pi$ coupling, a feature predicted in Odorico's model by the failure to obtain linear zeroes in $\pi\eta \rightarrow \pi\eta'$. Experimental bounds (Eisenstein, 1973) on $\Gamma(A_2 \rightarrow \eta'\pi)$ (see Table XVI) lie much lower than the quark model (or $SU(6)_W$) prediction when the PCAC kinematic factor is used. Mixing of the η' can improve the $SU(6)_W$ value. It is not known at present what effect kinematic factors have on this ratio.

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*The assignment $\eta = (n\bar{n} + d\bar{d} - s\bar{s})/\sqrt{3}$ would suppress this coupling entirely, but would lead to difficulties in Table XVI with $A_2 \rightarrow \eta\pi$.

**See also (Michael, 1972).

The linear-zero pattern also has been used to indicate possible qualitati violations of SU(3) in meson-baryon scattering (Odorico, 1973b). The algebr structure of such predictions is not stated explicitly, partly because spin has not been dealt with thoroughly. For example, only the invariant amplitude $A(s,t)$ (not the amplitude $B(s,t)$) is assumed to satisfy the linear-zero pattern in $0^- - 1/2^+$ scattering.

The linear-zero pattern thus is useful as a qualitative guide to the form of scattering amplitudes*. Its algebraic predictions for couplings are also borne out in a qualitative sense, and differ from quark model predictions in a couple of cases which deserve more experimental attention.

2. Structure of $\pi\Delta$ scattering

It has been observed that the helicity structure of resonance decays into $\pi\Delta$ is constrained very strongly by duality (Gell, 1971). In $\pi\Delta$ charge-exchan one seeks a pattern of s-channel helicity amplitudes that reproduces the desired features of ρ exchange. One would like the s-channel resonances to "build" a ρ trajectory passing through $\alpha_\rho(t) = 0$ at $t \simeq -0.6\ (\mathrm{GeV}/c)^2$, leadin to a vanishing of helicity-flip contributions to the differential cross section** This is what is responsible for the dip in πN charge-exchange, which had bee described previously with some success in terms of an s-channel pattern (see, e.g., Dolen, 1968).

The amplitudes for ρ exchange with s-channel helicity flip $\Delta\lambda$ are assum to behave as $J_{\Delta\lambda}(R\sqrt{-t})$. (see Harari, 1971). Here R is some effective "radiu

*It has been used as such in phase shift analyses: see, for example, (Langbei 1973).

**This comes as a consequence of the point $\alpha_\rho(t) = 0$ being a wrong-signature nonsense zero of the helicity-flip amplitude.

in $\pi\Delta$ scattering. The zeroes of $J_{\Delta\lambda}(R\sqrt{-t})$ must line up with one another to give the appropriate zero at a given value of t.

A solution was found in which the $\pi\Delta$ system coupled only via $\lambda = 3/2$ to most dominant resonances. The only amplitudes then refer to $\Delta\lambda = 0, 3$. The first zero of $J_3(X)$ at $X \simeq 6.4$ approximately corresponds to the second zero of $J_0(X)$ at $X \simeq 5.5$, leading to a rather large effective radius of $R \simeq 1.5$ f.

The evidence for $\lambda = 3/2$ dominance in resonance decays to $\pi\Delta$ is shown in Table XXIV. These resonances are the ones from Fig. 32a which appear prominently in $\pi N \rightarrow \pi\Delta$ (section VI).

For nearly every resonance which is to decay to $\pi\Delta$ via $\lambda = 3/2$, this helicity is at least as important as $\lambda = 1/2$. An important exception is $N(1690, 5/2^+)$. If its F-wave $\Delta\pi$ decay has really been observed, as suggested in a new solution mentioned by (Cashmore, 1973b), it turns out to interfere with the observed P-wave so as to reduce even further the $\Gamma(3/2) / \Gamma(1/2)$ ratio. This newly observed F-wave amplitude would have the predicted phase in $\pi N \rightarrow \pi\Delta$ (Fig. 32b, 56, L = 2 multiplet, FF15 amplitude).

One's amazement at the predictive power of duality is also somewhat diminished if we note that the whole unnatural-parity sequence of resonances tends to decay to $\Delta\pi$ via $\lambda = 3/2$ just because the lowest partial wave dominates. For an unnatural-parity resonance of spin J, when $\ell(\Delta\pi) = J - 1/2$, we have

$$\frac{\Gamma(\lambda=3/2)}{\Gamma(\lambda=1/2)} = 3\,\frac{J + 3/2}{J - 1/2} \tag{VII.20}$$

The two largest ratios in Table XXIV are examples of this result.

The one truly remarkable example of duality constraints seems to be the decay of $\Delta(1890,\ 5/2^+)$ via F-wave. However, if some P-wave were allowed, with the proper sign, the $\Gamma(3/2) / \Gamma(1/2)$ ratio could be enhanced even further.

We must conclude that duality suggests an approximate pattern in $\pi\Delta$ couplings of resonances, but one which still is in need of confirmation. The observation of more resonance decays involving both allowed $\pi\Delta$ partial waves, particularly that of $N(1690,\ 5/2^+)$, will be very important in this respect.

3. Trajectory inequalities

By summing s-channel exchanges and demanding that they reproduce a t-channel singularity, Coon and Geffen (Coon, 1971) have obtained a powerful set of inequalities regarding Regge trajectories. These predict, for example, that the leading trajectory in elastic processes must have $C = P = (-)^J$, and that magnetic couplings of vector mesons to baryons must be dominated by electric couplings. Some other results are mentioned by (Gasiorowicz, 1972).

4. Local and semi-local schemes

One can demand that the resonances which "build" a Regge trajectory do so within a small energy interval. This seems to be the case only if the interval is taken large enough. For example, in $\bar{K}N \rightarrow \pi\Lambda$, duality graphs predict the amplitude to be real on the average. This is the case in the energy range $1.54 \leq E_{CM} \leq 2.15$ GeV. (See Kernan, 1972; Shen, 1972.) On the other hand, if the $\Sigma(2030)$ is left out, the averaging in the range $1.54 \leq E_{CM} \leq 1.94$ GeV is not nearly as good (see Ferro-Luzzi, 1971b). The "local" pattern, suggested (for example) by (King, 1971), seems to be too strong an assumption.* This same "local" duality would have predicted a ρ' under the f_0 (Lovelace, 1968; Shapiro, 1969), a state for which there is no evidence whatever. **

Some conditions of duality can be satisfied on a very local basis (Schmid, 1972b). These lead to SU(6) - like relations for the lowest multiplets, but their general algebraic structure is still unclear.

Models of s-channel baryon resonances which lead to both t-channel meson exchange and u-channel baryon exchange have been mentioned at the beginning of subsection C. These models are also "local" in some sense; moreover, they assume the leading exchanged trajectories are "built" by the leading s-channel trajectories. This may not always be the case.

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*This "local" duality has been discussed by (Mandula, 1969c) in considerable generality.

**Such a $\rho'(1250)$ would be predicted to have a $\pi\pi$ width of about 100 MeV (Shapiro, 1969) and negligible $\omega\pi$ coupling (Veneziano, 1968). Some claims for a 1^- state around 1250 MeV do appear in $\bar{p}p \rightarrow \omega\pi^+\pi^-$ at rest and in $e^+e^- \rightarrow \pi^+\pi^-$ ($\geq 2\pi^0$) (M. Greco, private communication). Even if confirmed, such effects clearly would not be much help to "local" duality.

VIII. THE 0^+ MESONS

Recently it has been suggested that there are three I=Y=0 scalar mesons, all inferred from $\pi\pi$ scattering (Estabrooks, 1973). They are:

$$\epsilon(700) \tag{VIII. 1}$$

$\Gamma \approx$ several hundred MeV

couples to $\pi\pi$; $K\bar{K}$ coupling unknown

$$S^*(997) \tag{VIII. 2}$$

$\Gamma \simeq$ a few MeV

couples to $K\bar{K}$; much more weakly to $\pi\pi$

$$\epsilon'(1240) \tag{VIII. 3}$$

$\Gamma \simeq$ at least 100 MeV

couples to $\pi\pi$; $\pi\pi$ amplitude nearly elastic at resonance

These constitute too many states for the quark model: only two such resonances can be formed from $q\bar{q}$ with L=1. The third state must come from somewhere else.

An additional $q\bar{q}$; L=1 multiplet would be embarrassing since none of its other members has been seen. Fortunately there exists a plausible source of an additional I=Y=0 scalar meson which does not open such a Pandora's box. This source is the theory of spontaneously broken dilatation (or conformal) invariance.

Usually one would associate conformal invariance with purely massless fields. Another way of satisfying conformal invariance, however, is to surround a massive particle with a cloud of massless scalar particles which can "soak up" the effects of a conformal transformation. It is most economical to invent a single such (Nambu-Goldstone) boson, assuming it to behave as a unitary singlet. This boson is a natural candidate for the additional 0^+ isoscalar meson. (Kastrup, 1970; Chang, 1970; Crewther, 1971). Similar arguments can also account for an additional 0^-. See the review by (Carruthers, 1971a).

The masses and couplings in (VIII. 1 to VIII. 3) indicate the importance of mixing effects, as we shall see.

The remaining 0^+ states are the following:

$$\begin{array}{l} \delta(970) \\ \Gamma \simeq 30 \text{ to } 60 \text{ MeV} \\ \text{Couples to } \eta\pi \end{array} \tag{VIII. 4}$$

and

$$\begin{array}{l} \text{KN } (1100 \text{ to } 1400) \\ \Gamma \simeq \text{couple of hundred MeV} \\ \text{Couples to } K\pi \end{array} \tag{VIII. 5}$$

Before the discovery of the ϵ', it was very hard to understand the structure of the remaining 0^+ nonet. The two observed I=Y=0 members both have masses less than the strange member, a situation incompatible with the Gell-Mann-Okubo mass formula in view of $m_{\pi_N} < m_{K_N}$. Moreover, though the S* is roughly degenerate in mass with $\delta(970)$, it behaves (in its suppressed $\pi\pi$ coupling) very much like an $s\bar{s}$ state (e.g., the f').

The presence of an additional SU(3) singlet which can mix with the nine $^3P_0$ quark-antiquark states improves the situation. First, it can give the additional state which seems to be required by the data. Second, it can alter the couplings of the I=Y=0 states so as to agree with the pattern (VIII. 3), at least in principle.

It is intriguing that — in addition to $m_{S^*} \simeq m_\delta$ — the ϵ' (mass)^2 lies approximately the same distance above m_δ^2 as would be expected in an "ideal" nonet, like the 2^+ multiplet. This suggests that the ϵ might be taken as primarily the dilaton state,* with small $q\bar{q}$ admixtures, and the S* and ϵ' be viewed as "ideal" nonet states $(u\bar{u} + d\bar{d})/\sqrt{2}$ and $s\bar{s}$ with just enough dilaton admixed to reproduce the observed couplings (see Fig. 40). ** The small mass difference between S* and δ suggests that this mixing may be very small. If this is the case, however, one requires a large dilaton -$\pi\pi$ coupling to cancel the S* $\rightarrow$ $\pi\pi$ coupling of the $q\bar{q}$ system. The scale of this latter coupling is set by the decay $\delta \rightarrow \eta\pi$, where the dilaton has no effect, and in higher symmetries (SU(6)-like; see section VI) by $B \rightarrow \omega\pi$ as well.

The details of this exercise are presented in Appendix B. Our best guess as to scales of the decays of $^3P_0$ mesons (based on $\Gamma(\delta \rightarrow \eta\pi) = 60$ MeV) leads to a dilaton width far in excess of that compatible with the Adler-Weisberger relation for $\pi\pi$ scattering (Adler, 1965b; Weisberger, 1965) or with broken scale-invariance estimates (see, e.g., Ellis, 1971). With modest

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*(Nagels, 1973, et al.) argue that the ϵ should be dominantly a unitary singlet on the basis of its effects when exchanged in nucleon-nucleon and hyperon-nucleon scattering.

**A different suggestion has been made by (Lipkin, 1973f), who views the ϵ' as $(u\bar{u} + d\bar{d})/\sqrt{2}$ in analogy with the f_0. This suggestion fails to explain the low mass of S*(presumed $s\bar{s}$) or the existence of the $\epsilon(700)$.

improvements in data, one may already see the simple model of Appendix B ruled out.*

In order to guide the way toward a full understanding of mixing in the 0^+ mesons, one needs more information from both experiment and theory:

1) Can we live with an extremely broad $\epsilon(700)$? I am not aware that this possibility has been explored fully.

2) How elastic is $\epsilon'(1240)$? We noted that the amplitude at resonance in Fig. 3b is elastic, but the size of the resonant circle indicates a resonance which has substantial couplings other than $\pi\pi$. For example, if the resonance has a large $K\bar{K}$ coupling which interferes destructively with background, one can imagine such behavior. The behavior of the moments in $\pi^- p \rightarrow K^0 K^0 n$ (Beusch, 1970) in this region is suggestive of a rapidly varying S-wave $\pi\pi \rightarrow K\bar{K}$ amplitude around $m_{\epsilon'}$, but with present statistics one cannot say more.

If the $\epsilon'(1240) \rightarrow \pi\pi$ coupling is really not very large the dilaton coupling to $\pi\pi$ inferred from the model mentioned above can be reduced somewhat (though, as mentioned, the scale can be set in other ways, and one probably expects $\Gamma(\epsilon' \rightarrow \pi\pi)$ of at least a hundred MeV).

3) What is the exact mass and width of $\delta(970)$ and $K_N(1100-1400)$? The widths (particularly for $\delta \rightarrow \eta\pi$) set the scale for decays of ${}^3P_0$ $q\bar{q}$ states. The mass of the K_N should not exceed that of the ϵ' ($\sim$1240 MeV) if SU(3)-breaking occurs via the usual octet dominance. SU(3) predictions for $K_N \rightarrow K\pi$ widths are noted in section VI. C.

*Difficulties in identifying the $\epsilon(700)$ as the Goldstone boson of broken scale invariance have been pointed out by (Renner, 1972).

4) What is the theoretical significance of mixing between the dilaton and $q\bar{q}$ states? The parameter describing this mixing may appear elsewhere, since it essentially involves an amplitude for quark pair creation in a $^3P_0$ state. This same process appears in certain pictures of resonance decays (section VI). Hence it is not unreasonable to expect that the same process which causes resonances to decay also causes the dilaton to mix with the remaining isoscalar numbers of a 0^+ nonet, whatever the specific model involved.

To conclude, the 0^+ mesons in many ways are "special". There are probably ten, not nine of them, and their masses and decays are very likely to be more complex (but also more interesting) than in the case of the nonet states.

IX. FORMATION EXPERIMENTS

The direct channel has traditionally been the major source of information regarding baryon resonances. With some exceptions to be discussed in section X, we expect this situation to continue.

The πN system has continued to yield interesting information up to about $E_{c.m.} \simeq 2$ GeV; phase shift analysts do not all agree that unique analyses at much higher c.m. energies are possible with present data. In subsection A we update the situation regarding elastic πN measurements and show where our knowledge is wearing thin.

Inelastic πN channels are valuable sources of future information on the N^* and Δ resonances. These channels and the information they are expected to provide are treated in subsection B. The status of photoproduction phase shifts is described in subsection C.

The hyperons are discussed in subsection D, where both elastic and inelastic $\bar{K}N$ channels are treated. Subsection E deals with the KN system, and subsection F with baryon-antibaryon and baryon-baryon channels. Subsection G contains a brief summary.

Although some mesonic resonances can be studied in the "direct channel" in e^+e^- collisions, they are discussed in section X.

A. Elastic πN Phase Shifts

A good review of the data a couple of years ago was given by (Manning, 1971). More recent data are quoted by (Almehed, 1972). There have been two significant gaps which are in the process of being filled. Until recently, no charge-exchange polarization data have existed; measurements at 1030, 1245, 1440, 1590, and 1790 MeV/c now have been made (Shannon, 1973). Charge-exchange differential cross sections have recently been measured by (Nelson, 1973) and (Yamamoto, 1972). These measurements have been mentioned in section II.

For πN scattering (other cases will be mentioned) a tape of all available data in the resonance region is available from the Particle Data Group at Berkeley. A brief guide to these tapes is given by (Kelly, 1973).

There is some room for further measurements of the πN CEX differential cross section, particularly for 1.7 GeV $\leq E_{c.m.} \leq$ 1.8 GeV where there is a gap in the data and at higher energies to resolve the discrepancy between (Nelson, 1973) and (Yamamoto, 1972). (See Fig. 11 of Lovelace, 1972.)

Measurements of spin-rotation parameters still have not been made in elastic πN scattering in the resonance region. They would be of great help in confirming the stability of present solutions, even though they may not be much help in distinguishing between these solutions (Wagner, 1972b).

From the standpoint of symmetry physics, one would like to know several things more about elastic πN resonances. The low partial waves in πN scattering are not in satisfactory shape above ~1800 MeV, especially if one bears in mind the quark model prediction that they should be rather complicated and full of resonances. Above 2 GeV, one can probably hope only for information on the high partial waves, which in any event provide the cleanest tests of the symmetries in question.

Let us summarize some of these tests.

1. P-wave πN states. The positive-parity baryons expected as N=2 excitations in a harmonic-oscillator quark model consists of $\underline{56}$ and $\underline{70}$, $L^P = 0^+$ and 2^+. Candidates for members of all these multiplets have been observed, but there are many gaps, as shown in Table XXV. (See also Faiman, 1973c.)

2. Stray P = - states below 2 GeV. All the expected nonstrange members of $\underline{70}$, $L^P = 1^-$ have been found. Any more P = - states in the same mass range would have to belong to a new SU(6) multiplet, for which there is no evidence at present.

3. $\underline{70}$, $L^P = 2^+$ and $\underline{56}$, $L^P = 3^-$. These multiplets are of interest in dual schemes (section VII) and as tests of the degrees of freedom in the quark model (section V). They would exist in addition to the established $\underline{56}$, $L^P = 2^+$ and the very likely $\underline{70}$, $L^P = 3^-$.

The nonstrange states of $\underline{70}$, $L^P = 2^+$ have been studied by (Faiman, 1973a). It is interesting that most of them have small expected πN couplings. The only states with appreciable πN couplings are $N(\sim 2025, 7/2^+)$ (probably seen), $\Delta(5/2^+)$ (which probably mixes with $\Delta(1890, 5/2^+)$), $N([8,2], 3/2^+)$ (which may mix with $N(1850, 3/2^+)$), and $N([8,2], 5/2^+)$ (which may have been seen, around 2100 MeV: see Lovelace, 1972).*

*The numbers in the square brackets are SU(3) and quark-spin dimensions, respectively.

4. Second P = - group (above 2 GeV). We recall (Fig. 29) that the quark model predicts a rich structure of negative-parity N* and Δ states just above 2 GeV. These will correspond to S-wave, D-wave and G-wave πN resonances. Possible candidates for some of these may be found in Table V; the Regge recurrences of $\underline{70}$, L=1 states would naturally fall into $\underline{70}$, L=3. Evidence for the $\underline{70}$, L=3 multiplet is discussed by (Moorhouse, 1973d). In general, since the multiplet assignments will not be unique, the most one will be able to demonstrate will be consistency with the quark model. Other (less specific) theoretical descriptions are clearly needed.

5. Parity alternation. As mentioned in sections IV and V, parity alternation is crucial in using the quark model to classify states. Without it, multiplets would overlap one another too strongly.

Measurements of backward cross sections, as in $\pi^+ p \rightarrow p\pi^+$ (Fig. 8), indicate that prominent groups of resonances indeed alternate in parity: the dip-bump structure is more pronounced than in the forward direction, where resonances of all parities add up. This structure persists to rather high energies, indicating that some conclusions about resonances can probably be drawn as high as $E_{c.m.} = 3$ GeV (Baker, 1972).

Another interesting area of high-mass resonance physics has been indicated by results of (Schmidt, 1973) on πN scattering at 5 GeV/c. This work indicates possible evidence for Ericson fluctuations: levels whose prominence and narrowness is of a statistical nature, arising from the presence of many levels at that energy. (Carlson , 1973).

B. Inelastic Two-Body πN Channels

We shall discuss $\pi N \to N\eta$, $K\Lambda$, $K\Sigma$, $\pi\Delta$, ρN, ωN, and ϕN.

1. $\pi N \to N\eta$

The present experimental situation is shown in Fig. 41a. New differential cross sections are available (Lemoigne, 1973; Chaffee, 1973). Polarization measurements are forthcoming from the Rutherford Laboratory (J. Thresher, private communication). Until then, analyses (Deans, 1971; Lemoigne, 1973) can only guess at the resonant structure. Signs of resonant amplitudes are interesting. (See section III. C. 2e, and Fig. 15.) This channel is of considerable use in sorting out f/d values for octet assignments, which in turn help one to classify states according to pure or mixed representations of the rest symmetry. For example, the large $N\eta$ coupling of $P_{11}(1780)$ has led to the suggestion that it be assigned to $\underline{70}$, L=0 rather than $\underline{56}$, L=0 (Faiman, 1968; Heusch, 1970; Feynman, 1971). (This assignment conflicts with present data on $\pi N \to \pi\Delta$; see section VI. D and Fig. 32a.)

In general one expects stronger $N\eta$ couplings from $\underline{70}$ members than from $\underline{56}$ members. The $N\eta$ channel is especially useful in looking for $\underline{70}$ (8, 4) states. In Appendix D we have listed products of SU(6) factors and SU(3) (isoscalar) factors which enable one to see what decay modes of a given resonance are dominant. As an example of the use of Table D. 1, the ratio $\tilde{\Gamma}(N\eta)/\tilde{\Gamma}(N\pi)$ is 1/25 for a $\underline{56}$(8, 2) N*, but 1 for a $\underline{70}$(8, 4) member.

Some specific questions regarding $N\eta$ couplings of resonances are the following.

a. $\underline{N(1520,\ 3/2^-) \rightarrow N\eta}$. Some determinations are too large for SU(3) (Petersen, 1972).

b. $\underline{P_{11}\text{ states near 1500 MeV}}$. A recent claim for a "second Roper" resonance coupling to $N\eta$ exists (Lemoigne, 1973). This would affect (a), above.

c. States around 1.7 GeV. An upper limit on the $N\eta$ coupling of the second S_{11} resonance, $\tilde{N}(1700,\ 1/2^-)$, would help specify mixing properties of this state. One would also like bounds on the D_{15} [$N(1670,\ 5/2^-)$] and F_{15} [$N(1688,\ 5/2^+)$] couplings to $N\eta$, to test SU(3). Table D. 1, incidentally, indicates that structure in high $N\eta$ partial waves around this mass (Chaffee, 1973) is much more likely to come from the former resonance than from the latter.

d. P-waves, 1.7 - 1.9 GeV. $N\eta$ couplings will help sort out the classification of P_{11} and P_{13} πN states in this mass range.

e. Daughter states. The $N\eta$ channel, having a high threshold, tends to be sensitive to lower-spin states at a given mass than $N\pi$. This is also true of ΛK and ΣK. All these channels thus provide useful information on "daughter" states.

f. $\underline{N(\sim 2000,\ 7/2^+) \rightarrow N\eta}$. This decay mode should be visible since such a resonance would belong to $\underline{70}(8,4)$. [See Table D. 1.]

2. $\underline{\pi N \to \Lambda K}$

The data used in the analysis by (Wagner, 1971a) are shown in Fig. 41b.* This work raises several interesting questions. Most resonances seem to contribute with the same phase, as expected from Fig. 15.

The $\pi N \to \Lambda K$ reaction is sensitive to quark-spin-1/2 resonances: $\underline{56}$ and $\underline{70}$ (8,2). (See Table D.1.) It thus serves as a useful complement to $\pi N \to N\eta$.

Polarization measurements in $\pi N \to \Lambda K$ are generally made by analyzing the decay of the Λ. Measurements on polarized targets, contemplated at the Rutherford Laboratory, should provide worthwhile additional information. Some gaps in the data are mentioned by (Wagner, 1971a).

3. $\underline{\pi N \to \Sigma K}$

A recent analysis by (Langbein, 1973) quotes previous analyses and experiments. The only data not included are polarizations in $\pi^+ n \to K^o \Sigma^+$ recently published by (Davies, 1973). These have helped to fill an important gap. In the isospin-reflected reaction $\pi^- p \to K^+ \Sigma^-$, the only real way to measure polarization is to use a polarized target, since the $\Sigma^- \to n\pi^-$ decay does not analyze the Σ^- polarization effectively. Some of these difficult measurements have in fact been made at lower energies. They are of great importance over the whole energy range, as are measurements of $\pi^- p \uparrow \to K^o \Sigma^o$. Measurements of $\pi^+ p \uparrow \to K^+ \Sigma^+$ provide new constraints when the Σ^+ polarization is also analyzed, and are highly desirable.

Gaps in present $\pi N \to \Sigma K$ data are shown in Fig. 41c.

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*For a recent measurement near ΣK threshold, see (Nelson, 1973b).

Many interesting resonance questions can be studied in $\pi N \rightarrow \Sigma K$. As implied by Table D. 1, this reaction is particularly sensitive to decimets and to members of $\underline{70}$ (8, 4).

a. Resonant signs. (Section III. C. 2a.) All $\underline{10}$'s in $\pi^+ p \rightarrow K^+ \Sigma^+$ should have the same sign. A $\underline{27}$ would have opposite sign. In the fits of (Kalmus, 1971) all roughly the same phase in the mass range covered except for P3 ($J^P = 3/2^+$), which is wildly out of line. Could this be due to a direct-channel $\underline{27}$ effect (not necessarily resonant)? This $\underline{27}$ would also show up as a Z_1^* in K^+p scattering (possibly seen) and as $3/2^+$ effects in $\Sigma^\pm \pi^\pm$, $\Xi^o \pi^+$ or $\Xi^- \pi^-$, $\bar{K}^o \Sigma^+$ or $K^- \Sigma^-$, and $\Omega^- \pi^\pm$ channels. The possibility of studying hyperon-pion scattering (see section X. B) makes these questions less remote. In any case the channel $\pi^+ p \rightarrow K^+ \Sigma^+$ offers a rare opportunity to study exotic direct-channel effects in $0^- \ 1/2^+ \rightarrow 0^- \ 1/2^+$ scattering in the absence of diffractive background.

4. $\pi N \rightarrow \pi\Delta$, ρN, ωN, ϕN.

These channels are treated together because they test for similar kinds of resonance physics. In each of the above final states, a resonance can decay into more than one partial wave. The interference between these partial waves can serve as a test of higher symmetry schemes such as those discussed in section VI.

Much progress has been made in the analysis of $\pi N \rightarrow \pi\Delta$ and $\pi N \rightarrow \rho N$ (Cashmore, 1973a,b and Kernan, 1973). (See section VI. D.) However, some questions exists as to whether one should use an isobar analysis or isolate the final state by cuts on the Dalitz plot. In the first case, one runs the risk of neglecting contributions from higher isobars,* while in the second one must avoid regions of the Dalitz plot in which resonances overlap, thereby losing valuable information about the helicity structure of decays. Because of this ambiguity, it is of considerable interest to study related reactions in which the resonance in the final state sits above as little background as possible. This should be true in the case

$$\begin{array}{l} \pi^- + p \rightarrow \omega + n \\ \qquad\qquad\; \lfloor\!\rightarrow \pi^0 + \gamma \end{array} \tag{IX. 1}$$

for example. Predictions for ωN resonances have been given by (Petersen, 1973a) in the context of symmetries beyond SU(3) and are implied by the work of (Moorhouse, 1973b). No resonances are expected in the ϕN system on the basis of the quark model and its associated couplings (sections V, and VII). Nonetheless, the cross section for $\pi^- p \rightarrow \phi n$ is appreciable near threshold (see Bracci, 1972 for references) and falls off rapidly with increasing energy. This behavior is worth investigating in a partial-wave analysis.

Returning to the best-studied reaction $\pi N \rightarrow \pi\Delta$, we recall the discussion of section VI regarding the present experimental situation. The most pressing problem is the existence of a gap in the data available to (Herndon, 1972), between $E_{c.m.} = 1540$ and 1650 MeV. Above 2 GeV, data are more scarce and harder to analyze. One might hope to see the signal of $N(\sim 2000, 7/2^+) \rightarrow \Delta\pi$ in such analyses. (See Table XXIII.)

*Some questions of procedure in the isobar analysis of (Herndon, 1972) also have been raised by (Aaron, 1973).

C. Photoproduction and Electroproduction of N* and Δ

Many reactions are available in which a real photon γ or a virtual one γ^* excites a nonstrange baryon resonance, which then decays into an easily-studied two-body final state:

$$\gamma\ (\text{or}\ \gamma^*) + N \rightarrow (N^+\ \text{or}\ \Delta) \rightarrow N\pi,\ \eta N,\ K\Lambda,\ K\Sigma\ ,\ \ldots \tag{IX.2}$$

By exciting resonances via photons rather than pions, one utilizes all these final states as "SU(3)-inelastic" reactions.

Resonant phases in $\gamma N \rightarrow \pi N$ are not constrained by the optical theorem, in contrast to the elastic πN case. These phases provide useful tests of higher symmetries, (Moorhouse, 1972; Gilman, 1973d, Hey, 1973c) as we have seen in section VI. E. Moreover, the predicted magnitudes of the amplitudes are also in rough agreement with experiment.

A recent analysis of $\gamma N \rightarrow \pi N$ has been performed by (Moorhouse, 1973c) in the range 1160 MeV $\leq E_{c.m.} \leq$ 1780 MeV.* Figure 42 shows the data used and its quality. (I am indebted to R. G. Moorhouse for help in preparing this figure.) The differential cross sections σ in the case of π^o consist of some experiments with rather striking discrepancies from one to the other, and the situation could stand improvement. However, the largest gaps are in the quality (and sheer existence) of experiments measuring asymmetry (A) using polarized photons, recoil nucleon polarization (P), or using polarized targets (T). Simultaneous measurements using linearly polarized photons on a target polarized perpendicular to the scattering plane can provide all the necessary asymmetry information and are now in progress at

*Another recent analysis is that of (Devenish, 1973).

Daresbury (R. G. Moorhouse, private communication). Bubble chamber experiments with linearly polarized photons (Ballam, 1971) have also been performed at SLAC, roughly in the $E_{c.m.}$ region of 1.5 to 1.7 GeV. Their results will be incorporated into future analyses. Data above $E_{c.m.} = 1780$ MeV, and recent Daresbury measurements of $T(\pi^0)$ (Booth, 1973) have been included in a newer analysis by (Knies, 1973a) whose final results are not available at the time of writing. Questions one might pose for this and subsequent analyses include these:

a. Sign of $\Delta(1910; 1/2^+)$ contribution. This provides a handle on the P-wave couplings of πN to $\underline{56}$, L=2 states, assuming that to be the correct assignment for this resonance (section VI.E). In $\pi N \rightarrow \pi\Delta$ (section VI.D), we have seen that this sign disagrees with the quark model prediction.

b. Classification of $N(1860, 3/2^+)$. Table XXV shows that this is quite ambiguous.

c. Classification of $\Delta(1890, 5/2^+)$. Does this state contain a substantial mixture of $\underline{70}$, L=2 (Faiman, 1973a) in addition to the usual $\underline{56}$, L=2?

d. M3 dominance of $\Delta(1950, 7/2^+)$. (Section VI.E.) One will be able to test this prediction quantitatively.

e. Existence of $N(\sim 2000, 7/2^+)$. According to the selection rule of (Moorhouse, 1966), this resonance should not be photoproduced off protons. Its photoproduction off neutrons should be dominated by M3.

Resonance production by $q^2 \neq 0$ leptonic currents is still in its infancy, but has allowed one to study the helicity structure of resonances such as the $\Delta(3/2^+, 1236)$

and $N(3/2^-, 1520)$ out to $-q^2 \simeq 0.6\ \mathrm{GeV}^2$ (Close, 1972a,b). In contrast to at leas one explicit quark model (Ravndal, 1971), the data do not show much variation of the ratios of $\lambda = 1/2$ to $\lambda = 3/2$ excitations. Such data are a good test of specific quark model details, and also should provide insight into the manner in which resonance excitation ($q^2 = 0$) merges into the deep inelastic region (large negative q^2). These data also are welcome tests of other dynamical models (Adler, 1968; Walecka, 1967; Zucker, 1971). Recent experimental aspects are reviewed by (Clegg, 1973; Fischer, 1973).

The photoproduction of η and K^+ can provide information on the SU(3) properties of resonances. The $N(1780, 1/2^+)$ has been suggested as a $\underline{70}$, L=0 member on the basis of its behavior in $\gamma p \to \eta p$, for example (Heusch, 1970). A $\underline{56}$, L=0 member (see, e.g., Harari, 1968b) would couple too weakly to ηp.

In practice, the data on $\gamma N \to K\Lambda$, $\gamma N \to \eta N$ and $\gamma N \to K\Sigma$ do not permit the detailed analysis possible in $\gamma N \to \pi N$. Reviews are given by (Fischer, 1971 1973; Donnachie, 1971); samples of the quality of analysis possible are the work of (Donnachie, 1972), (Renard, 1972), and (Hicks, 1973).