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N* RESONANCE PARAMETERS AND K-MATRIX FITS TO THE REACTIONS $\pi N \rightarrow \Delta\pi + \rho N + \epsilon N^*$

R. S. Longacre,† A. H. Rosenfeld, T. Lasinski, G. Smadja†
Department of Physics and Lawrence Berkeley Laboratory
University of California, Berkeley, California 94720

R. J. Cashmore†† and D. W. G. S. Leith
Stanford Linear Accelerator Center
Stanford University, Stanford, California 94305

ABSTRACT

From a partial wave analysis of the reaction $\pi N \rightarrow \pi\pi N$ we extract 50 couplings and partial widths for N* resonances decaying into $\Delta\pi$, ρN and ϵN. Three different methods of determining the resonance parameters are compared. The signs of the $\pi\Delta$ couplings are found to be consistent with the predictions of ℓ-broken $SU(6)_W$, if one assigns $P'_{11}(1415)$, $P''_{11}(1730)$ and $P_{33}(1700)$ to $[56, L = 0^+]$ supermultiplets. The signs of the ρN couplings are inconsistent with $SU(6)_W$ symmetry in its ℓ-broken form, but in good agreement with the observed photon couplings.

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† Present address: D. Ph. P.E., CEN, Saclay, B. P. No. 2, 91 Gif-Sur-Yvette, France.
†† Present address: University of Oxford, Keble Road, Oxford, England.

We have completed a partial wave analysis of the reaction $\pi N \rightarrow \pi\pi N$ for energies up to 2000 MeV, using a generalized isobar model in which the $N\pi\pi$ final state is treated as a coherent sum of $\Delta\pi + \rho N + \epsilon N$ sub-states.[1] A unique set of amplitudes has been found describing the inelastic πN scattering throughout this entire energy range.[2] In Ref. (1) we presented 28 Argand plots of these energy independent $N\pi\pi$ amplitudes. We have now made K-matrix fits to these Argand amplitudes and have extracted the resonance couplings and partial widths. These parameters should be expressed in terms of T-matrix poles, but are more usually described as parameters in various approximations to Breit-Wigner amplitudes. Since these methods cannot agree in general, we have estimated the resonance parameters in three different ways described below. The signs of the resonance couplings are accessible only from the study of inelastic scattering, and are very powerful tools in the classification of resonant states and in testing models.[3]

We now summarize the three methods used in finding the resonance parameters; further details may be found in Ref. (4). The first two methods rely on approximating the T-matrix $T_{if}(LJ^P)$ between an initial state, i, and a final state, f, as a Breit-Wigner amplitude:

$$T_{if} = \frac{\pm\sqrt{x_i x_f}\ \Gamma/2}{E_R - E - i\Gamma/2} + (\text{constant background})$$

where for any channel $x_i = \Gamma_i/\Gamma$. T_{if} then describes a circle of diameter $\sqrt{x_i x_f}$. We determine the sign of the coupling from the direction of the amplitude at resonance; if the circle lies above the origin we say that the final state has a positive coupling[5] and on Table I we attribute a positive sign to the resonant amplitude $\sqrt{x_i x_f}$.

Method I: "Eyeball Fits to Existing Argand Plots".

This is the most direct approach. We consider only the initial state $i = 1 = \pi N$, and rely on the elastic partial wave analysis of Ayed and Bareyre[6] for the parameters E_R, Γ and x_1. We then draw by eye and compass a best circle through the $\pi\Delta$ amplitude on the Argand plot, measure its diameter and call it $\sqrt{x_1 x_\Delta}$. The $\pi\Delta$ width Γ_Δ is then $x_\Delta \Gamma$ (Ayed). We independently repeat the procedure for the ρN and ϵN channels, and do not ask whether the whole T-matrix would satisfy unitarity. The results are displayed in Table I for each resonance, as the first row labelled "Argand (T_{1j})". The ± sign in Table I indicates that the resonant contribution to those amplitudes is not well determined.

Method II: "Eyeball Fits to the Whole Unitarized T-Matrix".

Here we still think in terms of drawing Breit-Wigner circles of radius $\sqrt{x_i x_f}$ but we invoke the constraint of unitarity. For each of the 11 different partial waves (IJ^P), we perform a K-matrix fit to the Argand amplitudes, using the amplitudes of Ayed for the πN channel[6] and our own amplitudes for the $\Delta\pi$, ρN, ϵN channels[1] adding when needed a $N\eta$ or "dummy" channel. From the K-matrix we calculate a T-matrix, $\underset{\sim}{T}(\underset{\sim}{K})$, which satisfies unitarity. Now, even if we looked only at the Argand curves for the top row, T_{1f}, of the matrix we could make better estimates because the curves have been smoothed and modified to satisfy unitarity. But we actually go further and take into account the Argand plots of all the elements of the T-matrix. Thus, we can look at T_{22}, which describes the reaction $\pi\Delta \rightarrow N^* \rightarrow \pi\Delta$. Eyeball circles through the Argand curves T_{11}, T_{22}, T_{33} — should then provide better-constrained estimates of Γ_1, Γ_2, Γ_3 — . In fact we not only draw Breit-Wigner circles through all the Argand plots, but even slightly modify the Ayed values of E_R and Γ in an attempt to find an overall best fit. The parameters estimated from this whole $\underset{\sim}{T}$-matrix method are shown on the second row for each resonance in Table I.

Method III: "Search for Poles in the Unitary $\underset{\sim}{T}(\underset{\sim}{K})$-Matrix of Method II.

This time we abandon Breit-Wigner approximations and use a computer to hunt for the poles and the residues of the T-matrix. The definition for each Γ_f is not entirely unique; they are, of course, determined by the complex residues γ_f of T_{ff} at the pole using a formula

$$\frac{\Gamma}{2} = \gamma^2 \text{ (kinematics)} \quad .$$

However we have a choice of evaluating the kinematics at the pole or on the real axis at a point $E_R = \mathrm{Re}E_{pole}$. We choose to evaluate the kinematic quantity on the real axis. These values of Γ_f are listed in Table I as the third row, labelled "Pole".

Method IV: (Not presented because it gave ridiculous values).

For nearly-elastic resonances the poles in the T-matrix are closely related to K-matrix parameters. Thus K-matrix pole energies have sometimes been used to describe E_R. But the resonances considered here are inelastic and have large backgrounds, and we found that the K-matrix parameters yielded meaningless values for the masses and partial widths of the resonances (i.e., if interpreted literally they correspond to resonances which are shifted by $\sim$100 MeV from their nominal value and have much greater widths than given by Methods I, II, or III). This is not surprising since the K-matrix is merely a good way to parameterize the T-matrix in terms of real numbers, and the K-matrix pole positions and residues even change along with the number of channels considered.

Discussion

Of the three methods described above, obviously (III) is the most stable and attractive method of obtaining resonance parameters: (1) we expect pole positions and residues in the T-matrix to be independent of our parameterisation of the T-matrix providing that it is good. This expectation stems from work on the $P_{33}(1236)$ resonance[7] and investigations of our own;[4] (2) we expect the pole

position and residue to be closely related but not equal to the Breit-Wigner parameters. For example, consider the elastic background-free, text-book resonance "$P_{33}(1236)$" (which we have omitted from Table I because it has no $N\pi\pi$ channels). Ayed and Bareyre[6] quote $E_R = 1231$, $\Gamma = 109$, but we find the pole to be at $(1212 - i\ 101/2)$ MeV and its complex width on Table I would be $|\Gamma| = 97$ MeV at an angle of 24^o. This large angle is also found by all other pole searches.[7] We expect, and find, that pole and Breit-Wigner parameters will diverge even more when we have either large backgrounds, or wider resonances.

Results.

1. Agreement between the three methods:

(a) The spread in masses is typically 10-20 MeV, although for $F_{37}(1930)$ and $P_{33}(1700)$ the agreement is rather poor;

(b) in general, the total widths agree to $\sim \pm 20\%$ with $P_{11}(1700)$ and $D_{13}(1700)$ showing bad discrepancies;

(c) one can actually predict just by looking at the Argand plots how well the three methods will agree. For example, $D_{13}(1700)$ has mis-shapen $N\pi\pi$ Argand plots with a very small circle in $\Delta\pi DS_{13}$ ($\sqrt{x_1 x_\Delta} \sim 0.1$), and one expects discrepancies, since for such a shape a pole does not simulate a Breit-Wigner. On the other hand, D_{15} and F_{15} show smooth circles with little background, and the three methods agree to within $\pm 30\%$.

To be more objective we have guessed that the diameter of the resonance "circle" is a measure of the agreement to be expected between the different methods. Figure 1 shows the ratio of the resonance widths as estimated by the three methods, as a function of the diameter of the resonance circle, $(\sqrt{x_1 x_f})$.

It is clear that for circles of diameter > 0.3 the estimates for Γ_f agree to within about $\pm 20\%$, but this spread increases roughly inversely as the coupling.

It will be necessary for future theories to present resonance parameters in terms of T-matrix poles. When we bear in mind the 50% errors possible in going from Breit-Wigner to pole parameters the factors of 1.5 to 2 discrepancy in present tests of SU(3) and quark model predictions seem quite reasonable.

2. Quark Model Assignments from Signs in Table I.

It has recently been emphasized that the determination of coupling signs and branching ratios is important for the classification of resonant states, and testing our understanding of underlying constituent dynamics.[3,8,9] The signs of the resonant amplitudes and couplings of the N* to the $\pi\Delta$, ρN, and ϵN channels using the three methods described above, all agree. Further, the couplings imply: (a) Signs of $\Delta\pi$ decays are consistent with L-broken $SU(6)_W$ or Melosh transformations. For the $[70, 1^-]$ supermultiplet the signs are those of "Anti-$SU(6)_W$", while for the $[56, 2^+]$, our FP_{15} sign selects "Normal-$SU(6)_W$" (see references 3,8,9). (b) that the new $D_{13}(1700)$ should be classified as a $[70, 1^-]$ member while the $P_{11}(1700)$ and $P_{33}(1700)$ should be classified as members of a $[56, 0^+]$. (c) that there are indications of radial excitation structure now emerging in the baryon spectrum:

$[56, 0^+]$	State	
n = 0	Proton	$P_{33}(1236)$
n = 2	$P_{11}(1420)$	$P_{33}(1700)$
n = 4	$P_{11}(1700)$	

(d) that the signs of the ρN couplings agree with the related photoproduction data.[10]

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FIGURE CAPTION

1. Ratios of Resonance Partial Widths as estimated by our methods I, II, and III. "P" (for pole) means Γ^{III}/Γ^{I} where Γ^{I} comes from the eyeball fit to a single row of energy-independent Argand plots. "T" (for "T-matrix) means Γ^{II}/Γ^{I}. We plot only cases with partial widths $\Gamma > 10$ MeV. The ordinate $\sqrt{x_1 x_f}$ is the "diameter" of the smaller of the two resonance circles (Method I or Method II).

Table I. Resonance Parameters evaluated three different ways (see text). If there is more than one resonance per Argand plot, we add a prime to the first, two primes to the second, e.g. $P'_{33}(1232)$, $P''_{33}(1900)$. Notation such as $\rho_1 N\ SS_{11}$ is explained in the previous letter, ref. 1. The partial widths via method III (Pole) are calculated by $\Gamma/2$ = (residue)^2 × Kinematics evaluated at E = Re(E_{pole}). We then tabulate $|\Gamma|$ and phase. (8 signs for the $\rho_1 N$, ϵN amplitudes corrected July 1974)

T-matrix property used in estimate		Resonance J^P	Mass (MeV)	Γ (MeV)	$N\pi$ x_1	Γ_1 (MeV)	$\Delta\pi$ $\sqrt{x_1 x_\Delta}$	Γ_Δ (MeV)	$\sqrt{x_1 x'_\Delta}$	Γ'_Δ (MeV)	$N\rho$ $\sqrt{x_1 x_\rho}$	Γ_ρ (MeV)	$N\epsilon$ $\sqrt{x_1 x_\epsilon}$	Γ_ϵ (MeV)	Check Σx_i
					S_{11}		SD_{11}				$N\rho_1, SS_{11}$		SP_{11}		
I.	Argand(T_{1j})	S'_{11}	1520	75	0.34	26	0.00	0			+0.12	3	±0.1	2	0.41 +Nη
II.	$T_{ij}(K)$		1510	100	0.20	20	-0.06	2			+0.09	4	−0.09	4	0.30 +Nη
III.	Pole		1496	103		|38| ∠-43°	+*	|1| ∠0°			+*	|5| ∠58°	−*	|4| ∠34°	|0.47|+Nη
I.	Argand(T_{1j})	S''_{11}	1675	150	0.54	81	-0.16	7			±0.23	15	−0.23	15	0.79 +Nη
II.	$T_{ij}(K)$		1660	130	0.45	58	-0.15	6			+0.16	8	−0.25	18	0.69 +Nη
III.	Pole		1648	117		|61| ∠-58°	-*	|3| ∠27°			+*	|5| ∠12°	−*	|16| ∠-11°	|0.72|+Nη
					P_{11}		PP_{11}				$N\rho_1, PP_{11}$		PS_{11}		
I.	Argand(T_{1j})	P'_{11}	1415	180	0.54	97	-0.30	30			0.00	0	±0.18	11	0.77
II.	$T_{ij}(K)$		1390	200	0.54	110	-0.37	50			−0.23	20	+0.23	20	1.00
III.	Pole		1381	209		|136| ∠-68°	-*	|14| ∠-32°			−*	|15| ∠+37°	+*	|10| ∠+108°	|0.85|
I.	Argand(T_{1j})	P''_{11}	1730	165	0.16	28	±0.13	16			+0.32	99	+0.18	31	1.05
II.	$T_{ij}(K)$		1710	75	0.20	15	-0.20	15			+0.20	15	+0.28	30	1.00
III.	Pole		1708	17		|3| ∠-146°	-*	|5| ∠-37°			+*	|3| ∠+18°	+*	|7| ∠+130°	|1.05|
					P_{13}						$N\rho_1, PP_{13}$				
I.	Argand(T_{1j})	P_{13}	1695	115	0.14	16					−0.35	101			1.02
II.	$T_{ij}(K)$		1720	150	0.20	30					−0.40	120			1.00
III.	Pole		1716	124		|16| ∠-83°					−*	|168| ∠-46°			|0.68|
					D_{13}		DS_{13}		DD_{13}		$N\rho_3, DS_{13}$		DP_{13}		
I.	Argand(T_{1j})	D'_{13}	1524	120	0.56	67	+0.27	16	+0.24	12	+0.32	22	0.0	0	0.98
II.	$T_{ij}(K)$		1520	150	0.60	90	+0.24	15	+0.30	23	+0.24	15	+0.17	7	1.00
III.	Pole		1514	146		|88| ∠+2°	+*	|17| ∠54°	+*	|23| ∠-22°	+*	|22| ∠-90°	+*	|2| ∠0°	|1.03|
I.	Argand(T_{1j})	D''_{13}	1710	100	0.09	9	+0.15	25	-0.10	11	0	0	−0.2	44	0.89
II.	$T_{ij}(K)$		1710	300	0.1	30	+0.16	75	-0.14	60	-0.07	15	−0.2	120	1.00
III.	Pole		1710	607		|14| ∠-138°	+*	|77| ∠-49°	-*	|54| ∠-67°	-*	|6| ∠+162°	−*	|357| ∠+72°	|0.84|
					D_{15}		DD_{15}								
I.	Argand(T_{1j})	D_{15}	1660	145	0.41	59	-0.45	72							0.90
II.	$T_{ij}(K)$		1660	150	0.45	67	-0.50	83							1.00
III.	Pole		1663	146		|62| ∠-17°	-*	|93| ∠+5°							|1.06|
					F_{15}		FP_{15}		FF_{15}		$N\rho_3, FP_{15}$		FD_{15}		
I.	Argand(T_{1j})	F_{15}	1680	125	0.59	74	+0.26	14	0	0	+0.27	15	−0.28	16	0.95
II.	$T_{ij}(K)$		1670	130	0.59	78	+0.25	13	-0.08	1	+0.30	19	−0.30	19	1.00
III.	Pole		1668	132		|75| ∠-15°	+*	|15| ∠0°	+*	|1| ∠-90°	+*	|13| ∠-39°	−*	|28| ∠-32°	|1.00|
					S_{31}		SD_{31}				$N\rho_1, SS_{31}$				
I.	Argand(T_{1j})	S_{31}	1625	160	0.32	51	+0.40	80			±0.18	16			0.92
II.	$T_{ij}(K)$		1600	150	0.40	60	+0.40	60			-0.28	30			1.00
III.	Pole		1583	143		|37| ∠-136°	+*	|17| ∠-22°			-*	|26| ∠-152°			|0.98|
					P_{33}		PP_{33}								
I.	Argand(T_{1j})	P''_{33}	1900	205	0.19	39	-0.36	140							0.87
II.	$T_{ij}(K)$		1640	300	0.10	30	-0.30	270							1.00
III.	Pole		1609	323		|17| ∠-77°	-*	|157| ∠-94°							|0.72|
					D_{33}		DS_{33}		DD_{33}		$N\rho_3, DS_{33}$				
I.	Argand(T_{1j})	D_{33}	1725	190	0.17	32	-0.25	70	0	0	±0.20	45			0.77
II.	$T_{ij}(K)$		1680	240	0.20	48	-0.24	72	-0.10	12	+0.30	108			1.00
III.	Pole		1681	245		|136| ∠+16°	-*	|113| ∠+29°	+*	|28| ∠-145°	+*	|84| ∠+104°			|1.06|
					F_{35}				FF_{35}		$N\rho_3, FP_{35}$				
I.	Argand(T_{1j})	F_{35}	1870	255	0.14	36			-0.12	26	-0.28	143			0.80
II.	$T_{ij}(K)$		1830	220	0.18	40			-0.20	48	-0.33	132			1.00
III.	Pole		1813	193		|32| ∠-46°			-*	|140| ∠+19°	-*	|196| ∠-59°			|0.87|
					F_{37}		FF_{37}				$N\rho_3, FF_{37}$				
I.	Argand(T_{1j})	F_{37}	1930	235	0.41	96	-0.25	36			±0.18	19			0.64+?
II.	$T_{ij}(K)$		1925	240	0.40	96	-0.32	60			-0.24	36			0.80+?
III.	Pole		1924	258		|185| ∠-17°	-*	|177| ∠-9°			-*	|139| ∠+52°			|1.00|+?

*In Method III we get the widths directly from the T-matrix residues, and are unconcerned with $\sqrt{x_1 x_f}$, except for its sign.

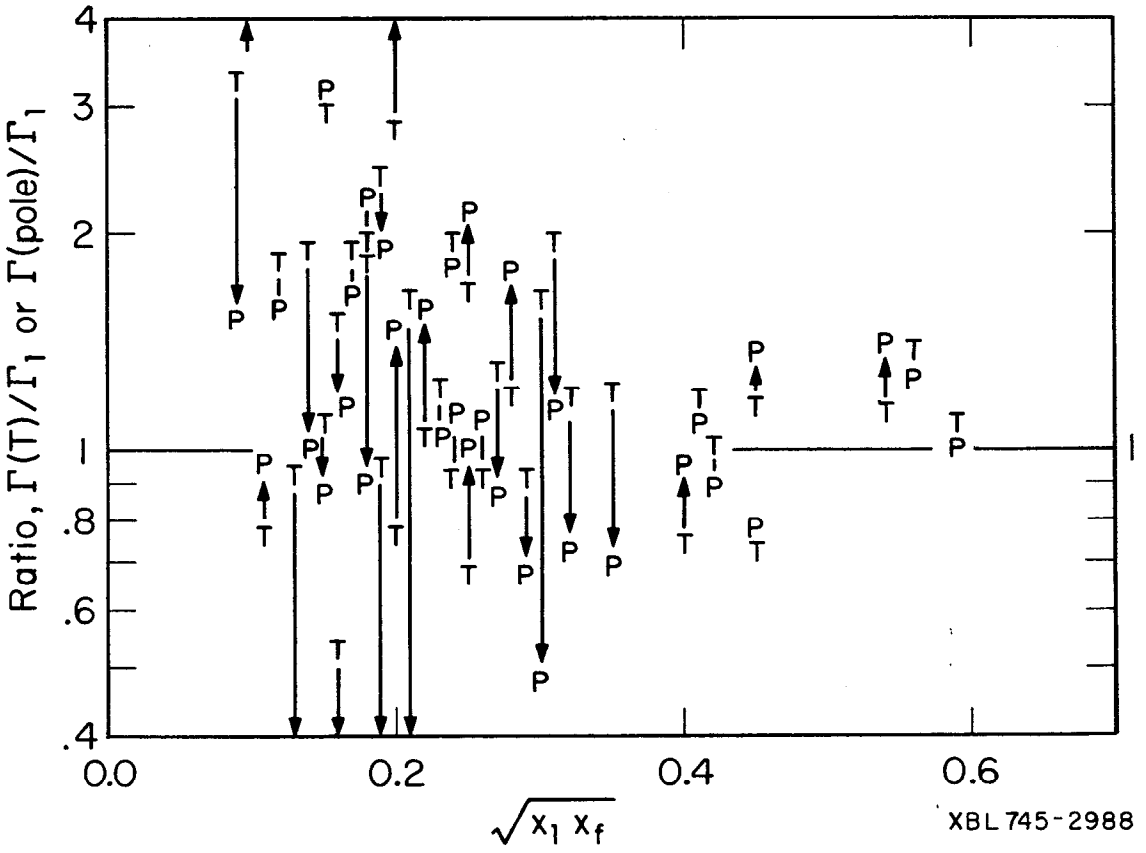

Fig. 1. Ratios of Resonance Partial Widths as estimated by our methods I, II, and III. "P" (for "pole") means Γ^{III}/Γ^{I} where Γ^{I} comes from the eyeball fit to a single row of energy-independent Argand plots. "T" (for "T-matrix") means Γ^{II}/Γ^{I}. We plot only cases with partial widths $\Gamma > 10$ MeV. The ordinate $\sqrt{x_1 x_f}$ is the "diameter" of the smaller of the two resonance circles (Method I or Method II).