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SOME REMARKS ON e^+e^- INCLUSIVE SCALING*

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ABSTRACT

The nonscaling behavior manifested by preliminary data at SPEAR may be due to the fact that the scaling region in e^+e^- has not yet been reached and in fact, scaling will not set in until q^2 is an order of magnitude higher.

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To interpret the interesting behavior of the preliminary data of e^+e^- annihilation at SPEAR presented at the 1973 Irvine Conference,[1] the popular view is that inclusive and cross section scalings have already been reached and the deviation from scaling observed in the data manifests the scale breaking mechanism. We take a different point of view, namely that scaling has not been reached yet, and it will not be reached until q^2 is an order of magnitude larger than current SPEAR energies.

To postulate a priori that scaling should be observed for e^+e^- annihilation, we have assumed that the scaling of $e^- p \to e^- x$, can be analytically continued from the scattering to the annihilation region.

The kinematics notation is similar to that in the recent article by Sullivan[2] as shown in Fig. 1. For the scattering process:

$$e^-(\ell) + P(p) \to e^-(\ell') + X$$

$$q^2 = (\ell - \ell')^2 < 0$$

$$M\nu = p \circ q = p \cdot (\ell - \ell') > 0$$

$$\cos\theta_s = \hat{\ell}' \cdot \hat{\ell}$$

$$S = (p + q)^2 = M^2 + q^2 + 2m\nu \geq M^2$$

$$\omega \equiv \frac{2m\nu}{-q^2} = \frac{s - m^2}{-q^2} - 1$$

$$1 \leq \omega$$

For the annihilation process:

$$e^-(\ell^-) + e^+(\ell^+) \rightarrow \overline{P}(p) + X$$

$$q^2 = (\ell^- + \ell^+)^2 > 0$$

$$m\nu = -p\cdot q = -p\cdot(\ell^- + \ell^+) < 0$$

$$\cos\theta_a = \hat{\ell}\cdot\hat{p}$$

$$S = (q-p)^2 = m^2 + q^2 + 2m\nu \geq m^2$$

$$\omega = \frac{2m\nu}{-q^2} = \frac{s-m^2}{q^2} - 1$$

$$0 \leq \omega \leq 1$$

We define $x = \frac{2E}{\sqrt{S}}$ where E is the CM energy of the outgoing hadron. Since $\omega \equiv \frac{2m\nu}{-q^2}$ and $\nu \equiv \frac{-p\cdot q}{m} = \frac{-Eq_0}{m}$ we can see that $x = \omega$ for this case.

The structure function is

$$\frac{1}{\pi}\overline{W}^{\mu\nu}(\nu, q^2) = \sum_n <0|j^\mu(0)|\overline{P}(p), n>_c \; <P(p), n|j^\nu(0)|0>_c (2\pi)^3\delta^4(q-p-p_n)$$

and the cross section is

$$\frac{d^2\sigma}{dE\,d\Omega_p} = \frac{2\alpha^2}{q^4} m^2 \left(\frac{\nu^2}{q^2} - 1\right)^{1/2} \left[2\overline{W}_1 + \left(\frac{\nu^2}{q^2} - 1\right)\overline{W}_2 \sin^2\theta_a\right]$$

Integrating out $d\Omega_p$, making use of the definition for ω, we get:

$$\frac{Sd\sigma}{d\omega} = 2\pi\alpha^2 m\left(\omega^2 - \frac{4m^2}{q^2}\right)^{1/2}\left(2\overline{W}_1 + \frac{1}{3}\left(\omega^2 - \frac{4m^2}{q^2}\right)\frac{1}{\omega}\;\frac{\nu\overline{W}_2}{m}\right)$$

Neglecting $\frac{4m^2}{q^2}$, we should expect scaling for $\frac{Sd\sigma}{d\omega}$ if we believe that the scaling limit has been set and the analytic continuation is valid. (From kinematics and the relation $\omega \overline{F}_2 = 2\,\overline{F}_1$, we suggest that instead of plotting $\frac{S\,d\sigma}{d\omega}$, we should plot:

$$\frac{S\,d\sigma}{d\omega}\left[\omega^2 - \frac{4m^2}{q^2}\right]^{-1/2}\left[1+\frac{1}{3\omega^2}\left(\omega^2 - \frac{4m^2}{q^2}\right)\right]^{-1}$$

versus ω to check the scaling for the structure function.)

Preliminary e^+e^- results show that $\frac{S\,d\sigma}{d\omega}$ does not scale except in the large ω region. We will make use of the K. Wilson's scaling criterion[3] to see if the scaling region has been reached at SPEAR energies: In a collision, if p_1 and p_2 are the momenta of the target and the incident particle then if the produced hadron k is relativistic with respect to p_1 and p_2, the probability that a particle is produced with a longitudinal momentum and arbitrary transverse momentum would manifest factorization and scaling, i.e.,

$$\rho(k_{\parallel})\,dk_{\parallel} = \int \rho_{\perp}(k_{\perp}^2)\,d^2k_{\perp}\,\frac{dk_{\parallel}}{k_{\parallel}}$$

In other words, if $k_{\parallel}$ is relativistic, $\rho(k_{\parallel})$ is proportional to $\frac{1}{k_{\parallel}}$ and the proportionality constant does not depend on $k_{\parallel}$ or p_1 or p_2. Recent pp data at ISR (1973) has shown that, for relativistic k (large $x_{\parallel} \equiv 2k_{\parallel}/\sqrt{S}$) scaling has been observed. However, at 90° CM where the pion's transverse momentum is bounded, this criterion has not been satisfied, and we should not yet expect scaling even at ISR energies.

Using this criterion, in the annihilation process, p is now just q at rest in the CM system, "decaying" into hadrons of momentum k. Experimentally, it appears that $\langle k \rangle$ is bounded, hence the bulk of the produced hadrons (mostly pions) have a rather low momentum. In fact $\langle k \rangle \simeq \langle p_{\perp} \rangle$ for pp collisions at comparable energies. However only the relativistic particles, i.e., $x \approx 1$ should scale, a fact verified experimentally. In order to determine how relativistic k must be to manifest scaling, we look at the 90^{o} production data for $pp \rightarrow \pi^{\pm}x$. A striking similarity between the momentum distribution of pions in $e^{+}e^{-}$ and pp at large angles is revealed.[4,5] For pp collisions, at S = 13 and 24 GeV^2 the value for small $p_{\perp}$ at 90^{o} CM is $\frac{1}{\pi} \frac{d\sigma}{dp_{\perp}^{2}\, dy} = 16$ and 30 mb $(GeV/c)^{-2}$. A factor of 2 in deviation from scaling is observed from S = 13 to S = 24 GeV^2. (A factor of 4 is observed if we look at $\left.\frac{d\sigma}{dp_{\perp} dy}\right|_{90^{o}}$). Hence, if we postulate that the mechanism for producing a pion is similar for pp and $e^{+}e^{-}$ (e.g., thermodynamical for small $p_{\perp}$ or hydrodynamical for large $p_{\perp}$) then we should expect a substantial deviation in $e^{+}e^{-}$ scaling for S = 16 and S = 25 GeV^2. For large $p_{\perp}$ in pp collisions, deviation from Blankenbecler-Brodsky-Gunion[6] scaling can be roughly represented by $\Gamma(p_{\perp}, S) = (1 - \frac{2p_{\perp}}{\sqrt{S}})^{9}$ in the relation:

$$\frac{Ed^{3}\sigma}{dp^{3}} = p_{\perp}^{-8} \left(1 - \frac{2p_{\perp}}{\sqrt{S}}\right)^{9}$$

Again, if we assume the transverse momentum distribution for pions is similar for $e^{+}e^{-}$ and pp collisions, then $\Gamma(p_{\perp}, S)$ is far from being energy independent at SPEAR energies.

In the argument above, we have assumed that the production mechanism for $e^{+}e^{-}$ is similar to that of pp in the 90^{o} CM region. If one is interested in the particle type rather than the momentum distribution of the final state, one should instead compare $e^{+}e^{-}$ with 90^{o} data of $\bar{p}p$ annihilation since in both cases the total charge of the "prematter"[7] is zero. Recent data[8] of $\bar{p}p$ reveals an enhancement of π^{o}'s at rather low energy ($p_{inc} \approx 5$ GeV/c). It is interesting to notice that the ratio of neutral to charge pions is approximately equal to that of the recent

e^+e^- experiment.[1] If it is reasonable to assume that the pions should have equal shares of entropy when the initial temperature (energy) is asymptotically large, then an enhancement of π^0 may indicate that asymptopia is still orders of magnitude away.

A remark may be made on the scaling thresholds of the scattering and annihilation processes. In the $ep \to ex$ process, the Bjorken's scaling limit is conventionally quoted as $\nu \to \infty$, $q^2 \to \infty$ while instead this should be $p \cdot q \to \infty$, $q^2 \to \infty$. In this process, since the proton mass in GeV is roughly 1, one may use ν and $p \cdot q$ interchangeably. However for $e\pi \to ex$ process and for e^+e^- where the observed particle is a pion $p \cdot q << \nu$. At the lowest SPEAR energy the value of $p \cdot q \approx 1.35\ \mathrm{GeV}^2/c^2$ ($\nu \approx 10$ GeV) for $x = .3$ is below the scaling limit for ep process ($p \cdot q > 2.5$, $q^2 > 1.5$). However, the large deviation from scaling at $x = .3$ may not be accounted by this value of $p \cdot q$ alone. Two speculations may be made:

1. The scaling thresholds are different for $e\pi$ and ep.

2. The singularity structure of the scaling function in ω is not so simple. In probing the internal structure of the hadron (Bjorken's limit) we may have encountered anomalous singularities, one example is the better known triangle singularities.[2]

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FIGURE CAPTION

1. (a) Scattering process; (b) the annihilation process.

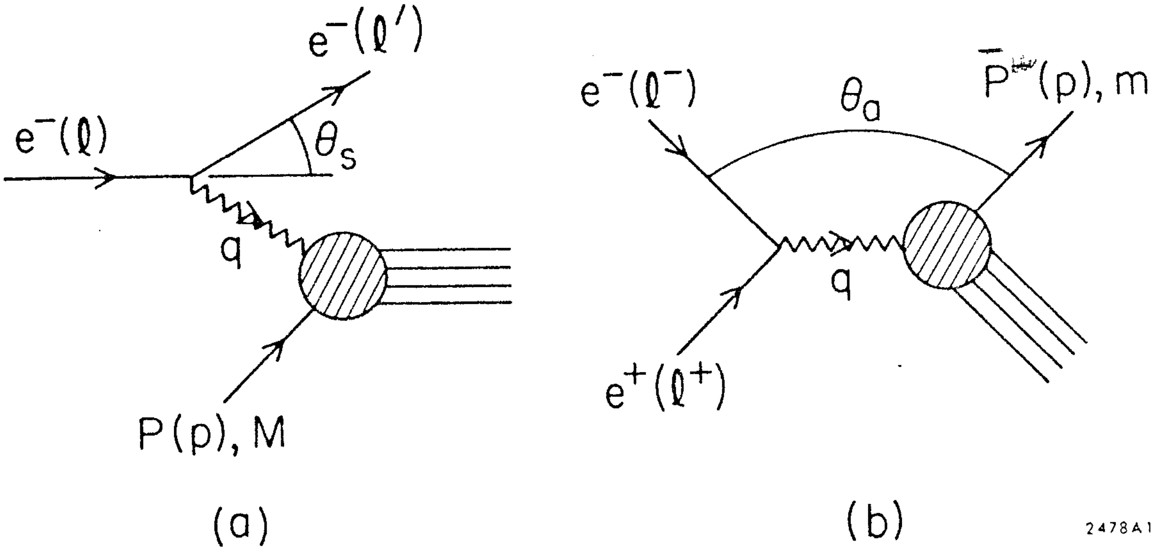

Fig. 1