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RAPIDITY DISTRIBUTION OF THE e-p ELASTIC FORM FACTOR*

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ABSTRACT

The deviation from the dipole fit of e-p elastic form factor has been observed experimentally. Based on a proposed parton-like hydrodynamical model of e-p elastic and deep inelastic scattering, a rapidity variable

$$y_i = \ln \left[\sqrt{\frac{Q^2}{4M_i^2} + 1} + \sqrt{\frac{Q^2}{4M_i^2}} \right]$$

is introduced. Experimental data up to $Q^2 = 25$ GeV^2 show that the form factor is gaussian in y_i.

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The slow falloff[1] of inclusive invariant cross sections for $pp \rightarrow \pi^0 X$ for high momentum pions at 90° in the c.m. frame has been viewed as a consequence of the gaussian distribution of this cross section in the transverse rapidity variable of the pion[2]

$$y_{\perp} = \frac{1}{2} \ln \frac{E + P_{\perp}}{E - P_{\perp}} \tag{1}$$

Similarly, for $P_{\perp} = 0$, the invariant cross section of the pions can be represented as a gaussian in the longitudinal rapidity[3]

$$y_{\parallel} = \frac{1}{2} \ln \frac{E + P_{\parallel}}{E - P_{\parallel}} \tag{2}$$

In solving the relativistic hydrodynamic equations during the expansion stage, a streamline is followed in space and time, using the variable

$$y_{\perp,\parallel} = \frac{1}{2} \ln \frac{t + x_{\perp,\parallel}}{t - x_{\perp,\parallel}} \tag{3}$$

Near the end of the expansion stage, we assume the velocity of the fluid constituents in question is constant, therefore,

$$y_{\perp,\parallel} = \frac{1}{2} \ln \frac{t + x_{\perp,\parallel}}{t - x_{\perp,\parallel}} = \frac{1}{2} \ln \frac{E + P_{\perp,\parallel}}{E - P_{\perp,\parallel}} \tag{4}$$

In solving the expansion equation, one finds that the distribution of the secondaries produced is approximately gaussian in $y_{\perp,\parallel}$.

In this Letter we introduce a new rapidity variable,

$$y_i = \ln \left[\sqrt{\frac{Q^2}{4M_i^2} + 1} + \sqrt{\frac{Q^2}{4M_i^2}} \right] \tag{5}$$

where Q^2 is the momentum transfer and M_i the parton mass, as explained in the next section. The elastic cross section can be written $\left(\text{assuming } G_E = \frac{G_M}{\mu}\right)$

$$\frac{d\sigma}{d\Omega} \Big/ \frac{d\sigma_{Mott}}{d\Omega} = \left(\frac{G_M}{\mu}\right)^2 \left[1 - 2\,\frac{\mu^2 Q^2}{4M^2}\,\tan^2\frac{\theta}{2}\right] \tag{6}$$

Then the form factor is

$$G_E^2 = \left(\frac{G_M}{\mu}\right)^2 = \frac{(d\sigma/d\Omega)/(d\sigma_{Mott}/d\Omega)}{1 - 2\,\frac{\mu^2 Q^2}{4M^2}\,\tan^2\frac{\theta}{2}} \tag{7}$$

It is traditionally assumed that the form factor can be parametrized as $\left(1 + \frac{Q^2}{0.71}\right)^{-2}$. Most experimental plots[4,5] are in terms of

$$R = \left[\left(\frac{d\sigma}{d\Omega} \Big/ \frac{d\sigma_{Mott}}{d\Omega}\right) \Big/ \left(1 - 2\,\frac{\mu^2 Q^2}{4M^2}\,\tan^2\frac{\theta}{2}\right)\right] \Big/ \left(1 + \frac{Q^2}{0.71}\right)^{-4} \tag{8}$$

If we assume that the form factor is $e^{-by_i^2}$, we define

$$R' = e^{-2by_i^2} \Big/ \left(1 + \frac{Q^2}{0.71}\right)^{-4} \tag{9}$$

Figure 1 is a plot of R' versus Q^2 for b = 0.88, M_i = 0.26 GeV. If $G_E(y_i)$ is plotted as a function of y_i, with the same coefficient b and M_i, we get an almost perfect gaussian in y_i, as shown in Fig. 2. This gaussian behavior of the form factor is a direct consequence of a parton-like model of deep inelastic and elastic e-p scattering.

In the Breit frame of e-p system, due to the Lorentz contraction, the proton shrinks to a disk of highly excited fluid, while constantly emitting and absorbing virtual states (partons), which are assumed to be responsible for the absorption of the photons. As in Feynman's picture, the parton longitudinal momentum $-k_i$ equals half of the momentum of the virtual photon. Its rapidity is

$$y_i = \ln \frac{\sqrt{k_i^2 + M_i^2} + k_i}{M_i} = \ln \left[\sqrt{\frac{Q^2}{4M_i^2} + 1} + \sqrt{\frac{Q^2}{4M_i^2}} \right] \tag{10}$$

It is assumed further that the expanded fluid has a certain probability of recombining to become just a proton, a proton and n mesons, etc. In elastic scattering, we limit ourselves to the first case. Pictorially, the elastic process can be represented as in Fig. 3.

In a collision, the probability that a certain set of final states are produced cannot be predicted from this simple model. However, certain simple features can be obtained. In Fig. 3, one sees three mechanisms: The emission of the virtual photon, the absorption of the virtual photon by the parton (proton fluid) and the parton distribution in the excited proton. The first mechanism is known. The second mechanism is borrowed from the parton model, i.e., in the Breit frame, it is most probable that only the parton which has a momentum of magnitude equal to half of the momentum of the virtual photon would absorb a photon. The third mechanism is obtained from Landau's hydrodynamic model: In the Breit frame, the virtual photon has no energy. The contracted proton is now composed of highly excited hadronic fluid which cools off by adiabatic expansion. Virtual states are continually emitted and absorbed in the rest frame of the fluid centers. We call these states partons. We can follow these centers in space x and time t using

$$y = \frac{1}{2} \ln \frac{t + x}{t - x}$$

Assuming the momentum of the parton is the same as the momentum of the fluid center and the velocity of the center is constant near the edge of the expansion stage, we obtain Eq. (4). From these considerations, the form factor should be gaussian in y_i if we assume that the fluid expands the same way as in p-p scattering where the rapidity distribution of the outgoing π's is gaussian in y_π, and that the fluid centers in question will become pions at the end of the expansion stage. Experimentally, $M_i \approx 2m_\pi$, suggesting that the elastic scattering is dominated by the absorption of the virtual photon very close to the end of the expansion stage where $M_i \approx m_\pi$.

It is interesting that several attempts were made in the past to understand the form factor at high Q^2. Our new variable, for high Q^2, is identical to Mack's variable which was derived from a perturbation-theoretic approach.[7] It is also interesting that from solving the differential equation for the form factor, Mack also obtained a gaussian distribution of the form factor in y_i' where $y_i' = 2 \ln \frac{Q}{a}$ where $a = \text{constant}$. The fact that our model works well at low Q^2 deserves further investigation.

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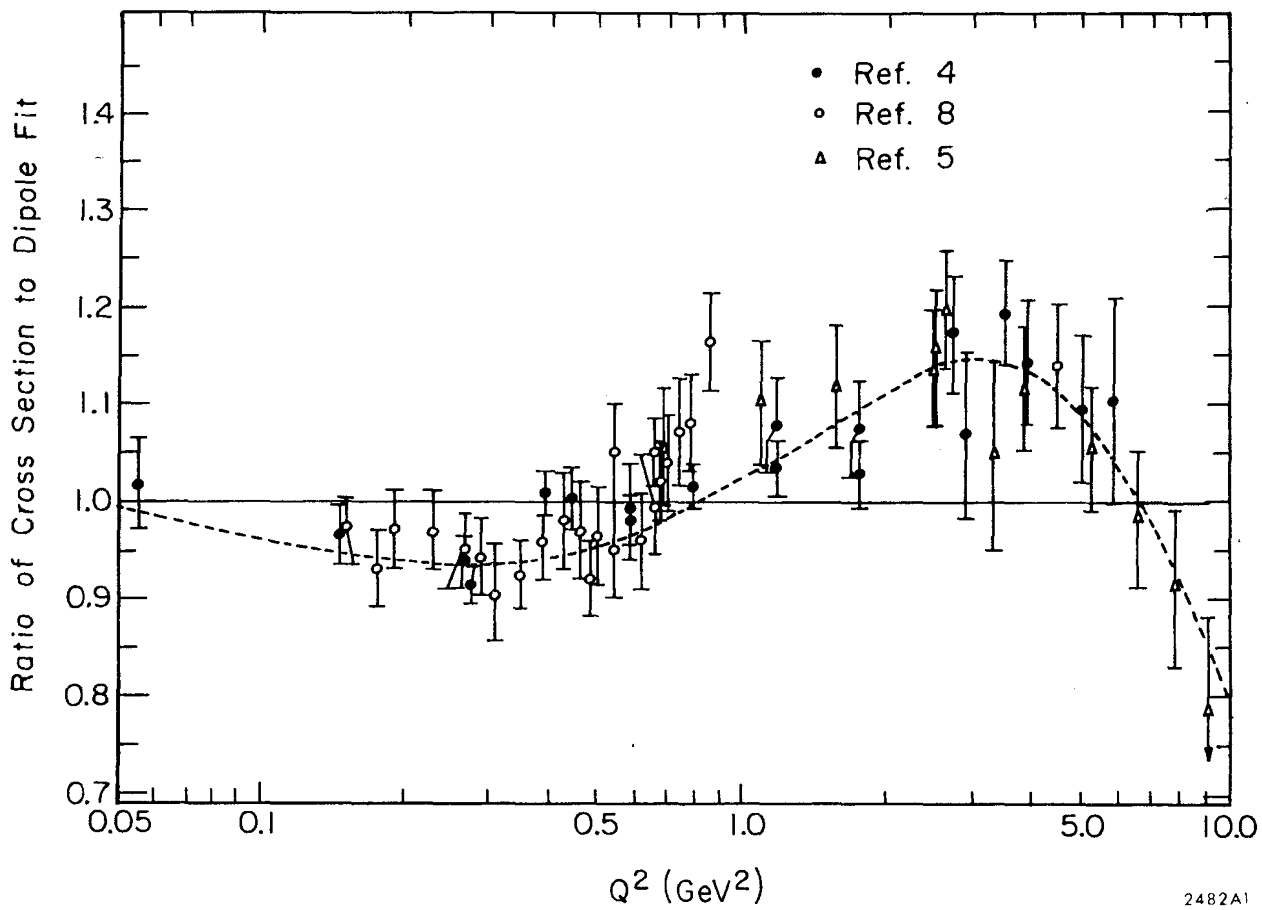
FIGURE CAPTIONS

1. The ratio of cross section to dipole fit as a function of Q^2. The data are taken from Refs. 4, 5 and 8. The dotted curve is the value of R' in Eq. (9) with b = 0.88 and M_i = 0.26 GeV.
2. Plot of G_E as a function of y_i^2. The data are taken from Table I of Ref. 5. The dotted curve is the assumed gaussian with b = 0.88 and M_i = 0.26 GeV.
3. The diagram for e-p elastic scattering.



Ref. 4
Ref. 8
Ref. 5
Ratio of Cross Section to Dipole Fit
1.4
1.3
1.2
1.1
1.0
0.9
0.8
0.7
0.05
0.1
0.5
1.0
5.0
10.0
Q2 (GeV2)
2482A1

Fig. 1

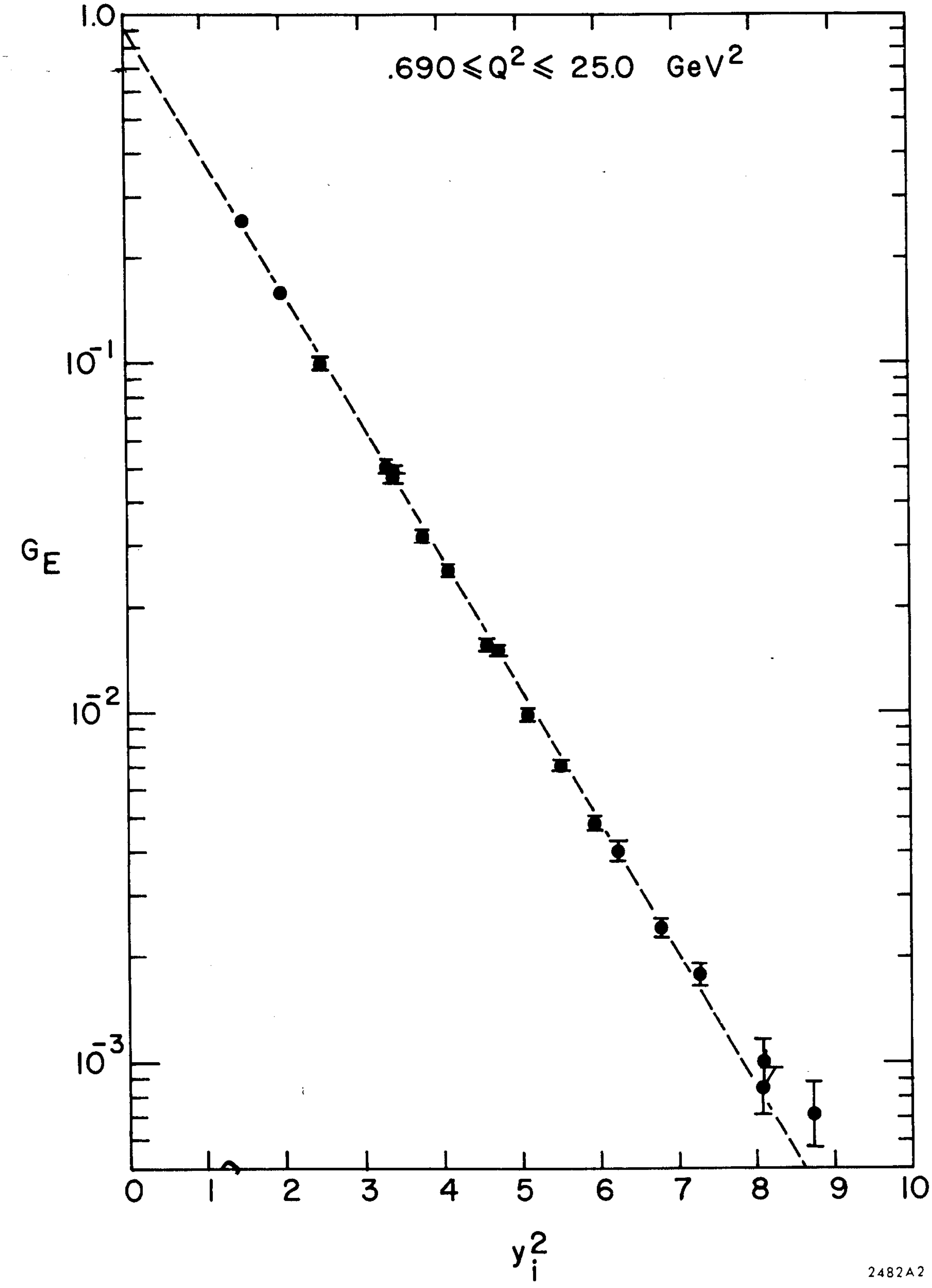


Fig. 2

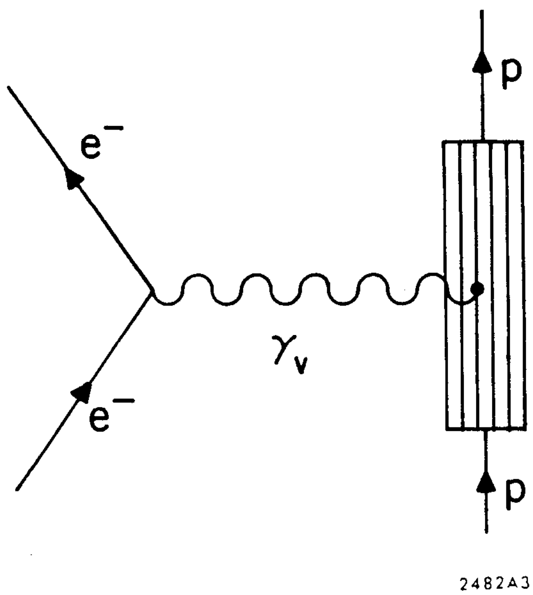


Fig. 3