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WEAK NEUTRAL CURRENTS IN ELECTRON AND MUON SCATTERING*

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ABSTRACT

Contributions to inelastic scattering arising from the interference of a weak neutral current with the electromagnetic current are calculated using the Weinberg model and the quark parton model. These interferences produce nonvanishing $(d\sigma^- - d\sigma^+)$, $(d\sigma^{-(L)} - d\sigma^{+(R)})$, $(d\sigma^{-(L)} - d\sigma^{-(R)})$ which give rise to sizeable affects when compared to the purely electromagnetic scattering.

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Preliminary measurements of inelastic neutrino and anti-neutrino reactions at CERN[1] and NAL[2] indicate the possibility of neutral neutrino currents which are in turn coupled to hadrons. These measurements of $\nu + N \rightarrow \nu + x$ and $\bar{\nu} + N \rightarrow \bar{\nu} + x$ are in rough agreement with the predictions of the Weinberg $SU(2) \times U(1)$ model gauge theory of weak interactions coupled with the model of nucleons composed of fractionally-charged quarks.[3] In particular the fact that the observed neutrino and anti-neutrino cross sections are not equal requires an interference between vector and axial vector currents and necessarily excludes the possibility that the effect is electromagnetic in origin (such as the charge radius of the neutrino).

It is the purpose of this modest note to point out that in the same model, quite sizeable weak neutral current effects are predicted in charged lepton deep inelastic scattering $\ell^{\pm} + N \rightarrow \ell^{\pm} + x$.[4,5] The estimates reported in this paper can be regarded as indicating typical magnitudes for neutral current effects, with the specific numbers dependent on the Weinberg model as well as on the parton model. Because the discovery of weak neutral currents coupled to leptons is fundamental to the connection of weak interactions with electromagnetism and because rough theoretical estimates indicate that the measurements are not beyond present experimental possibilities, moderately difficult experimental searches for such weak effects should be considered at present accelerators.

We consider in this paper three experimental measurements of weak neutral current contributions to deep inelastic lepton scattering — unpolarized lepton vs. antilepton scattering (typically a SLAC experiment):

$$R(\ell^- - \ell^+) \equiv \frac{d\sigma(\ell^- + N \rightarrow \ell^- + x) - d\sigma(\ell^+ + N \rightarrow \ell^+ + x)}{d\sigma(\ell^- + N \rightarrow \ell^- + x) + d\sigma(\ell^+ + N \rightarrow \ell^+ + x)} \quad , \qquad (1)$$

the same measurement, but with longitudinally polarized leptons (such as in the muon beams at NAL, with R for right and L for left handed):

$$R(\ell_L^- - \ell_R^+) \equiv \frac{d\sigma(\ell_L^- + N \to \ell^- + x) - d\sigma(\ell_R^+ + N \to \ell^+ + x)}{d\sigma(\ell^- + N \to \ell^- + x) + d\sigma(\ell^+ + N \to \ell^+ + x)} \tag{2}$$

and a similar measurement comparing the scattering of leptons of the same charge and opposite longitudinal polarizations (potentially feasible with muons at NAL and with polarized beams at SLAC):

$$R(\ell_L^- - \ell_R^-) \equiv \frac{d\sigma(\ell_L^- + N \to \ell^- + x) - d\sigma(\ell_R^- + N \to \ell^- + x)}{2\, d\sigma(\ell^- + N \to \ell^- + x)} \tag{3}$$

The denominator in each of the above equations is to a good approximation just twice the ordinary deep inelastic differential cross section for electroproduction, while the numerator results from the interference of the electromagnetic current with the weak neutral current.

To lowest order in electromagnetic and neutral weak current the various R's can be expressed as follows:

$$\begin{aligned} R(\ell^- - \ell^+) &= -\frac{1}{2} \frac{Q^2}{Q^2 + M_z^2} \frac{g_A R_3 Q^2 (E + E') M^{-1}}{2 W_1 Q^2 + W_2 (4EE' - Q^2)} \\ &\to \frac{Q^2}{M_z^2} \frac{xy(2-y)}{2 - 2y + y^2} (-g_A) \frac{\nu R_3(x)}{F(x)} \end{aligned} \tag{4}$$

$$R(\ell_L^- - \ell_R^+) = \frac{g_V + g_A}{g_A} R(\ell^- - \ell^+) \tag{5}$$

$$R(\ell_L^- - \ell_R^-) = \frac{Q^2}{Q^2 + M_z^2} \; \frac{g_A\left[2R_1Q^2 + R_2(4EE' - Q^2)\right] - g_V R_3 Q^2(E+E')M^{-1}}{2W_1Q^2 + W_2(4EE' - Q^2)}$$

$$\rightarrow \frac{Q^2}{M_z^2}\left[g_A \frac{\nu R_2(x)}{F(x)} - 2g_V \frac{\nu R_3(x)}{F(x)} \; \frac{xy(2-y)}{2-2y+y^2}\right] \tag{6}$$

The first of Eqs. (4) and (6) follow from the introduction of neutral vector and axial vector current and do not depend on the assumptions of the Weinberg or quark models. The expression following the arrow is obtained in the scaling limit under the assumption that $2M\,x\,W_1 = \nu W_2 \equiv F$. Here E and E' are the initial and final lepton energy in the laboratory frame, M is the nucleon mass, and we use the standard variables $x = Q^2/2M\nu$, $y = \nu/E = 1 - E'/E$, $s\,x\,y = Q^2$.

The functions R_1, R_2, R_3 are defined analogously to the usual W_1, W_2, W_3 (see Derman Ref. 4) from the interference tensor $R_{\mu\nu}$ of the electromagnetic and weak neutral hadron currents

$$R_{\mu\nu} = \frac{4\pi^2 P_0}{M} \int d^4x\, e^{iq.x} \left\langle P_{in} \left| \left[J_\mu^\gamma(x)\, J_\nu^W(0) + J_\mu^W(x)\, J_\nu^\gamma(0)\right] \right| P_{in} \right\rangle \tag{7}$$

The constants g_V and g_A arise in the analogous lepton tensor in the form

$$J^R_{\mu\nu} = \left[(g_V - \lambda g_A)(k_\mu k'_\nu + k'_\mu k_\mu - g_{\mu\nu} kk') - (\lambda g_V - g_A)\, i\,\epsilon_{\mu\nu\alpha\beta} k_\alpha k'_\beta\right]$$

and are given in the Weinberg model as

$$g_A = \left[-2\sin 2\theta_w\right]^{-1} \qquad \text{and} \qquad g_V = (1 - 2\cos 2\theta_w)/2\sin 2\theta_w \quad .$$

The polarization $\lambda = +1$ (-1) for right-handed (left-handed) leptons. To obtain the antilepton tensor, set $g_A \rightarrow -g_A$, $g_V \rightarrow g_V$. The cross section is then given by

$$\frac{d^2\sigma}{d\nu\, dQ^2} = \frac{\pi\, \alpha^2}{Q^4 E^2}\left[J^{\gamma}_{\mu\nu}\; W_{\mu\nu} + J^{R}_{\mu\nu}\; R_{\mu\nu}\right] \qquad (8)$$

where $J^{\gamma}_{\mu\nu}$ and $W_{\mu\nu}$ are the usual electromagnetic lepton and hadron tensors.

Applying the quark model to the Weinberg model, the weak neutral current is given by

$$J^{W}_{\mu} = \sum_{q} \bar{q}\, \gamma_{\mu} (G^{q}_{V} - G^{q}_{A}\, \gamma_5)\, q$$

so that

$$2\, M\, x\, R_1 = \nu\, R_2 = 2x \sum_{q} Q_q (G^{q}_{V} + G^{\bar{q}}_{V}) \quad ,$$

$$\nu\, R_3 = -2 \sum_{q} Q_q (G^{q}_{A} - G^{\bar{q}}_{A})$$

where the sums are over the three usual quarks p, n, λ, and the "charmed" quark p'. ("Charm" is introduced in order to avoid unobserved strangeness-changing neutral processes.[3,7]) For p and p', $G_V = (\frac{1}{2} - 2Q\sin^2\theta_W)/\sin 2\theta_W$, $G_A = (2\sin 2\theta_W)^{-1}$; for n and λ, $G_V = (-\frac{1}{2} - 2(Q-1)\sin^2\theta_W) / \sin 2\theta_W$, $G_A = (-2\sin 2\theta_W)^{-1}$. In this paper we assume the usual quark charge assignments ($Q = 2/3$).

In the Weinberg four-quark model[3] it is strenghtforward to derive sum rules and linear relations between the interference functions R_i and the familiar structure functions $\nu W_2 = F$ (the electromagnetic structure function) and $F_i^{\nu,\bar{\nu}}$ (the usual neutrino scattering structure functions[9]). Perhaps the most interesting case experimentally is the sum of neutron plus proton structure functions. Some relations for these sums in the approximation of zero Cabibbo angle are

$$\sin 2\theta_W \, \nu R_2^{(P+N)} = (1 - 4\sin^2\theta_W)\, F^{P+N} + \frac{2}{9}\left(F_1^{\nu\,(P+N)} + F_1^{\bar{\nu}\,(P+N)}\right) \quad (9)$$

$$\sin 2\theta_W \, \nu R_3^{(P+N)} = \frac{1}{4}\left(F_3^{\nu\,(P+N)} + F_3^{\bar{\nu}(P+N)}\right) + \frac{1}{6}\left(F_1^{\nu\,(P+N)} - F_1^{\bar{\nu}\,(P+N)}\right) \quad (10)$$

$$\int_0^1 \nu R_3^{(P+N)}(x)dx = \frac{1}{4\sin 2\theta_W}\int_0^1 \left(F_3^{\nu\,(P+N)} + F_3^{\bar{\nu}\,(P+N)}\right)dx \quad (11)$$

The two photon exchange interfering with the one photon exchange can also give contributions to $R(\ell^- - \ell^+)$ and $R(\ell_L^- - \ell_R^+)$. Its expected kinematic dependence is of the form[8] $\log Q^2$ rather than linear in Q^2 and thus in principle could be separated out. However, the function $R(\ell_L^- - \ell_R^-)$ does not get any contribution from the two photon interference term since it is purely parity violating.

Numerical estimates for the asymmetries (4), (5), (6) can be made without detailed application of the above sum rules under the approximation that only valence quarks contribute to the various processes. This approximation is in good agreement with the observed ratio ~ 3 for the total inelastic neutrino to anti-neutrino cross sections, and furthermore should be especially good near $x \approx 1$ where our effect is maximum.

Introducing the Fermiconstant $G \approx 10^{-5}/M^2$, Eqs. (4), (5), (6) become in this approximation

$$R(\ell^- - \ell^+) = -\left(\frac{G\,Q^2}{2\sqrt{2}\,\pi\alpha}\right) \frac{y(2-y)}{2 - 2y + y^2}\,\frac{9}{10} \quad (4')$$

$$R(\ell_L^- - \ell_R^+) = 2\cos 2\theta_W \, R(\ell^- - \ell^+) \quad (5')$$

$$R(\ell_L^- - \ell_R^-) = -\left(\frac{G\,Q^2}{2\sqrt{2}\,\pi\alpha}\right)\frac{9}{5}\left[\left(1 - \frac{20}{9}\sin^2\theta_W\right) - 2(1 - 2\cos 2\theta_W)\frac{y(2-y)}{2 - 2y + y^2}\right] \quad (6')$$

We note that $G/(2\sqrt{2}\,\pi\alpha) = 1.8 \times 10^{-4}\ \mathrm{GeV}^{-2}$. For Q^2 of order several hundred GeV^2, which is feasible with NAL muon beams, the size of these weak-electromagnetic interference terms could be as large as 5%. More precisely, for $\sin^2\theta_W = 0.3$, the value presently favored by experiment, and for $y = \nu/\nu_{max} = 1$, the above equations yield respectively

$$R(\ell^- - \ell^+) = -(1.8\%)\,(Q^2/100\ \mathrm{GeV}^2)$$

$$R(\ell^-_L - \ell^+_R) = -(1.4\%)\,(Q^2/100\ \mathrm{GeV}^2)$$

$$R(\ell^-_L - \ell^-_R) = -(0.2\%)\,(Q^2/100\ \mathrm{GeV}^2)$$

We note that in the Weinberg model the sign of these interference terms is determinate, and that (4') is independent of the angle θ_W. Furthermore, Eq. (6') is very sensitive to the value of θ_W; for example, changing $\sin^2\theta_W$ to a value of 0.4 increases the above estimate for $R(\ell^-_L - \ell^-_R)$ by a factor of 16.

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6. Eqs. (4), (5), and (6) apply as well to elastic scattering with the substitutions $R_1 = (\nu/M) G_M^\gamma G_M^z \delta(\nu - Q^2/2M)$

$$R_2 = 2\left[F_1^\gamma F_1^z + (Q^2/4M^2) F_2^\gamma F_2^z\right] \delta(\nu - Q^2/2M)$$

$$R_3 = -2 G_A^z G_M^\gamma \delta(\nu - Q^2/2M)$$

$$W_1 = (Q^2/4M^2)(G_M^\gamma)^2 \delta(\nu - Q^2/2M)$$

$$W_2 = \left[(F_1^\gamma)^2 + (Q^2/4M^2)(F_2^\gamma)^2\right]\delta(\nu - Q^2/2M)$$

where F_i^γ $\left[= F_i^\gamma(Q^2)\right]$ are the usual elastic nucleon electromagnetic form factors, with $F_2^\gamma(0)$ normalized to the magnetic moment and $G_M = F_1^\gamma + F_2^\gamma$. The functions F_i^Z are the analogous form factors of the neutral vector current coupled to the Z boson, and G_A^Z is the axial form factor (the coefficient of $-\gamma_\mu \gamma_5$) for the corresponding neutral axial vector current.

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