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COMMENT ON MULTIPLICITIES IN CHENG-WU FIELD THEORY*

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ABSTRACT

It is noted that the model of Cheng and Wu used to explain high energy hadronic cross sections implies average multiplicities much smaller than those experimentally observed.

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Although many types of functional s-dependence [1] can be invoked to fit the observed rise of the pp total cross section through the ISR energy range [2,3], the most exciting possibility under current consideration is that the observed structure presages an indefinitely rising cross section with a $\ln^2 s$ energy behavior. The energy behavior of the total cross section would thus saturate in form, if not in magnitude, the Froissart bound [4]

$$\sigma_{tot}(s) \leq \frac{\pi}{m_\pi^2}\left(\ln(s/s_0)\right)^2 \qquad (1)$$

as was predicted by Cheng and Wu [5] based on a rather detailed analysis of Feynman graphs. The fulfillment of the prediction must certainly by considered partial support for the basic assumptions underlying the calculations of Cheng and Wu and it is therefore important to find out if their scheme makes other testable predictions.

In seeking possible experimental tests we must distinguish between those predictions which depend on the field theory substructure and those which depend on the general "impact picture" prescription abstracted from the field theory. Experimental tests of the latter kind such as the energy behavior of other cross sections, slope parameters and the real part of the forward scattering amplitudes have been proposed by Cheng, Walker and Wu [6,7]. These tests are currently not decisive. Also, since the same general features of the impact picture appear in quite distinct dynamical schemes such as Finkelstein and Zachariasen's absorbed multiperipheral model [8] it is inherently more interesting to test instead the underlying field theory.

In fact, there are several specific predictions of Cheng and Wu's massive QED [9]. The most easily testable is the behavior with energy of the average multiplicity. The purpose of this comment is to point out that this prediction is in considerable disagreement with experimental data. Surprisingly, the prediction of the field theory model of Cheng and Wu translates into an average multiplicity of produced hadrons

$$\langle n \rangle \cong 1.7 - 4.2 \tag{2}$$

at the highest ISR energy. The magnitude of the disagreement between this prediction and the experimentally observed copious production of hadrons [10] in high energy collisions makes the relevance of the other predictions of the field theory as presented in Ref. 9 highly questionable.

The basis for the prediction, Eq. (2), of the average multiplicity is Cheng and Wu's formulation of massive quantum electrodynamics [9]. The fundamental diagram in the production process is that with a multiperipheral chain as shown in Fig. 1a. The exchanges in this diagram consist of vector mesons and the sum over all single chain diagrams yields, at high energy, an amplitude in impact parameter space

$$A(s,b) \sim h(b)\, s^{a} (\ln s)^{-2} \tag{3}$$

The power, a, is given by

$$a = c \frac{\pi}{32} \alpha^{2} \tag{4}$$

where $c = 11$ for spin $-\frac{1}{2}$ particles,

$c = 5$ for spin -0 particles,

and α is the fundamental coupling constant in the theory. As in the multiperipheral model the average number of particles produced in such a chain is

$$\frac{<n>}{\text{chain}} \sim 2a \ln s \tag{5}$$

where the factor of 2 appears in converting the number of pairs in the chain to the number of particles. If this were the only set of diagrams considered Eq. 5 would give the average multiplicity. However, the amplitude (3) violates the Froissart bound and the prescription of Cheng and Wu is to recover s-channel unitarity by summing over diagrams such as shown in Fig. 1b where there are multiple towers and chains. The leading terms are shown to give the eikonal prescription for the S-matrix. The t-channel tower is then interpreted as the "potential".

If we define $\Omega(s,b)$ as the "opaqueness" or "blackness" in impact parameter space and assume that Ω is predominantly real, we have

$$\sigma_{tot}(s) = 2\int d^2b \left[1 - \exp\left(-\Omega(s,b)\right)\right] \tag{6}$$

$$\sigma_{el}(s) = \int d^2b \left[1 - \exp\left(-\Omega(s,b)\right)\right]^2 \quad . \tag{7}$$

The cross section for producing m distinct chains in the model of Cheng and Wu is then [9]

$$\sigma(m,s) = \int d^2b \exp\left[-\Omega(s,b)\right] \frac{\left(2\Omega(s,b)\right)^m}{m!} \tag{8}$$

and the average number of chains produced is

$$<m> = \sum_{m=0}^{\infty} m\, \sigma(m,s) \Big/ \sum_{m=0}^{\infty} \sigma(m,s) \tag{9}$$

$$= \sigma_{tot}^{-1}(s) \; 2\int d^2b\, \Omega(s,b) \quad .$$

The important aspect of this connection between the average number of multiperipheral chains and the opaqueness is that the opaqueness can be determined separately from the differential cross section. The analysis of Chao and Yang [11] has determined $\Omega(s,b)$ at 3 energies as shown in Fig. 2. From their analysis we can conclude that the average number of chains has remained approximately constant at 1.3 between $P_{LAB} = 30$ Gev/c and 1500 GeV/c. Using the parameterization of Ref. 6 we also reach the conclusion that the average number of multiperipheral chains is a slowly varying quantity whose magnitude is near one at ISR energies.

We combine this fact with Eq. (5) to get

$$\langle n \rangle \simeq (2a \ln s) \langle m \rangle \simeq 2.6\, a \ln s \tag{10}$$

which should be approximately valid at current energies. This is in strong conflict with data since self-consistency of the field theory approach requires that a, given by Eq. (4), be a small number. The explicit parameterization of Cheng, Walker and Wu in Ref. 7 gives two possible values of a depending on whether they take seriously the logarithmic factors in Eq. 3

$$\begin{aligned} a_0 &= 0.083 \Longrightarrow \langle n \rangle \cong 0.22 \ln s \\ a_1 &= 0.20 \Longrightarrow \langle n \rangle \cong 0.52 \ln s \quad . \end{aligned} \tag{11}$$

Fits to the observed hadronic multiplicities give numbers like [10]

$$\langle n \rangle \simeq 3.5 \ln s$$

so there is a factor of 7-16 contradiction.

The smallness of the parameter a is no fluke of the fitting procedure. It is a necessary consequence of the assumption that the observed pp cross section has something to do with an elementary vector exchange. In Ref. 9, Cheng and Wu demonstrate that the opaqueness at fixed impact parameter eventually has the asymptotic behavior

$$\Omega(s,b) \sim a^{2(1-2a)} s^{a/(1+2a)} (\ln s)^{-2/(1+2a)} [h(b)]^{1/(1+2a)} \quad . \tag{12}$$

The observation of only small energy dependence of the opaqueness implies that a is small. In fact, if a were large, the theory would predict that a black disc would develop at low energies. The fact that σ_{el}/σ_{tot} in the data is far from the black disc limit of one half,

$$\sigma_{el}^{pp}(s) / \sigma_{tot}^{pp}(s) \cong 0.18 \text{ at ISR} \quad , \tag{13}$$

is sufficient within the framework of Cheng and Wu to require that a be small.

We could generate the same type of impact picture with a large coupling constant and so a large multiplicity if, for example, instead of vector exchange we started with scalar exchange such as in the ϕ^3 model of Chang and Yan [12]. In such non-QED approaches however, we achieve saturation of the Froissart bound only if some coupling constant is large enough. One of Cheng and Wu's repeated arguments for their massive QED is that it leads to saturation of the Froissart bound for any value of the coupling constant. They therefore claim that their impact picture is not accidental but required.

An interesting interpretation of Cheng and Wu's field theory has been presented by Zachariasen [13]. Since observed hadrons have form factors, and are obviously not fundamental in the sense of a Lagrangian field theory, Zachariasen suggests that Cheng and Wu's calculations represent interaction of fundamental constituents, quarks or partons, mediated by the exchange of a massive vector gluon. As the hadronic cross sections are appropriate convolutions of quark cross sections, they would exhibit the same high energy behavior. The considerations presented here indicate that this interpretation can be made consistent with existing multiplicity data only if there are, on the average, many hadrons per parton [14].

In Ref. 9, Cheng and Wu point out that (9) combined with (12) leads to multiplicities which eventually grow as small as power of s. This is in conflict with currently popular ideas: If Mueller's generalized optical theorem [15] is valid and if we assume that the same j-plane singularities occur in the forward 3-3 amplitude as in 2-2 amplitudes then the average multiplicity can grow only logarithmically. However, the prediction of power behavior for the average multiplicity from a Cheng-Wu (s-channel iteration) type of calculation of diagrams for n-particle exclusive processes should not necessarily be taken seriously unless one is convinced that the (very complicated) constraints of s-channel unitarity on the production amplitudes are properly implemented. There is no guarantee that this is done in the calculation of Cheng and Wu. The 2-2 unitarity condition is implemented via the eikonal approximation but multiparticle unitarity is ignored. It may be that other corrections cancel the power of s growth of the single particle inclusive distribution in the central region just as multiple tower corrections removed the power behavior of the total cross section implied by the single tower amplitude [3]. If the power of s behavior persists despite such corrections, then the experimentally small variation of the opaqueness $\Omega(s,b)$ at present energies (and the experimentally large multiplicities) indicates that, at best, the asymptotic regime of applicability of Cheng-Wu field theory has not yet been reached.

We are grateful to J. D. Jackson for an important discussion concerning the average number of multiperipheral chains.

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FIGURE CAPTIONS

Fig. 1 Taken from Ref. 9. Diagram (a) represents one multiperipheral production chain. Diagram (b) is an example with arbitrary number of production towers and chains.

Fig. 2 The opaqueness in pp collisions as determined by the analysis of Ref. 11. The values of $\langle m \rangle$, Eq. 9, obtained are

PLAB = 29.7	$\langle m \rangle = 1.33 \pm .03$
= 245	$= 1.28 \pm .03$
= 1480	$= 1.35 \pm .04$

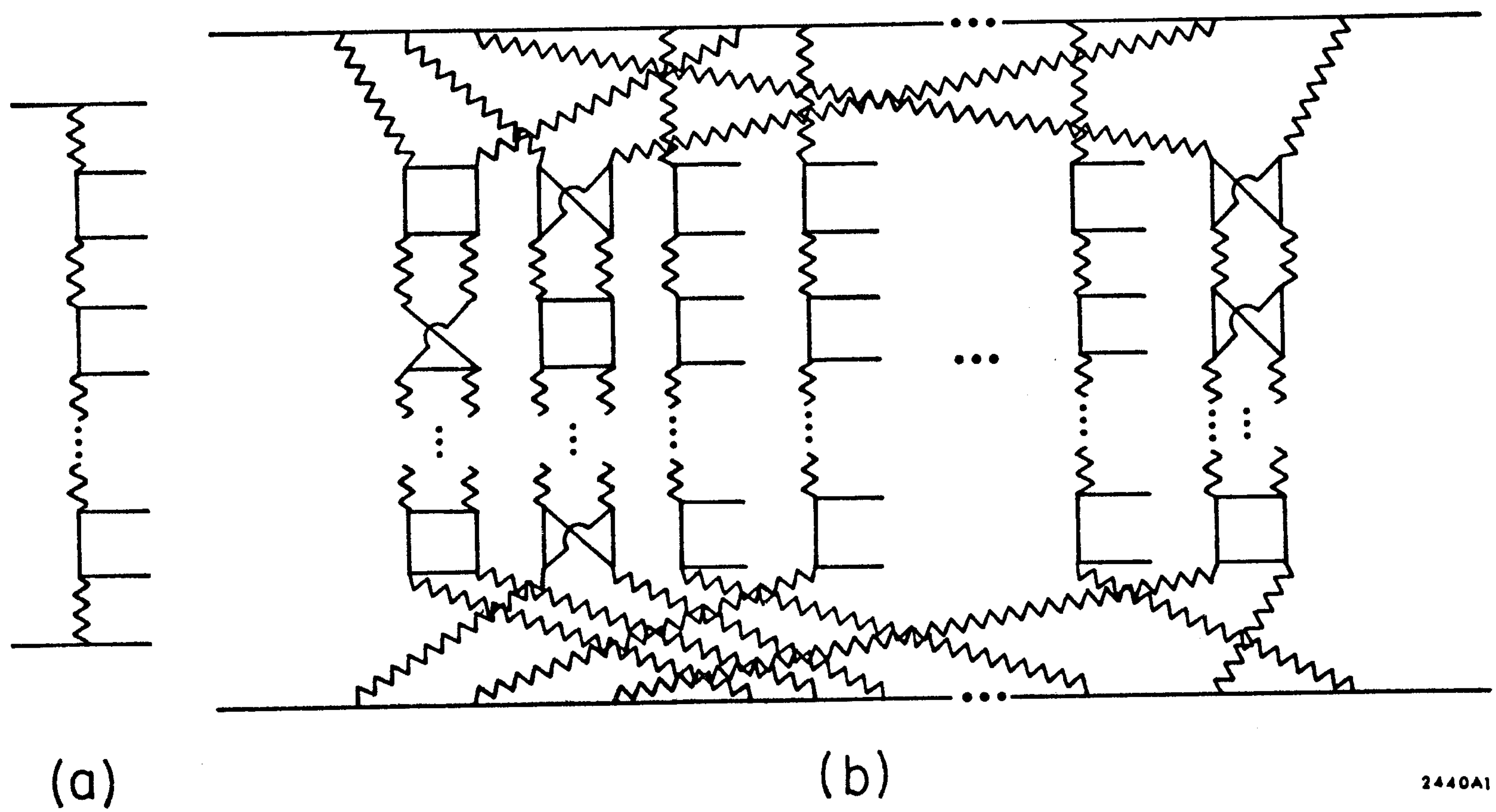
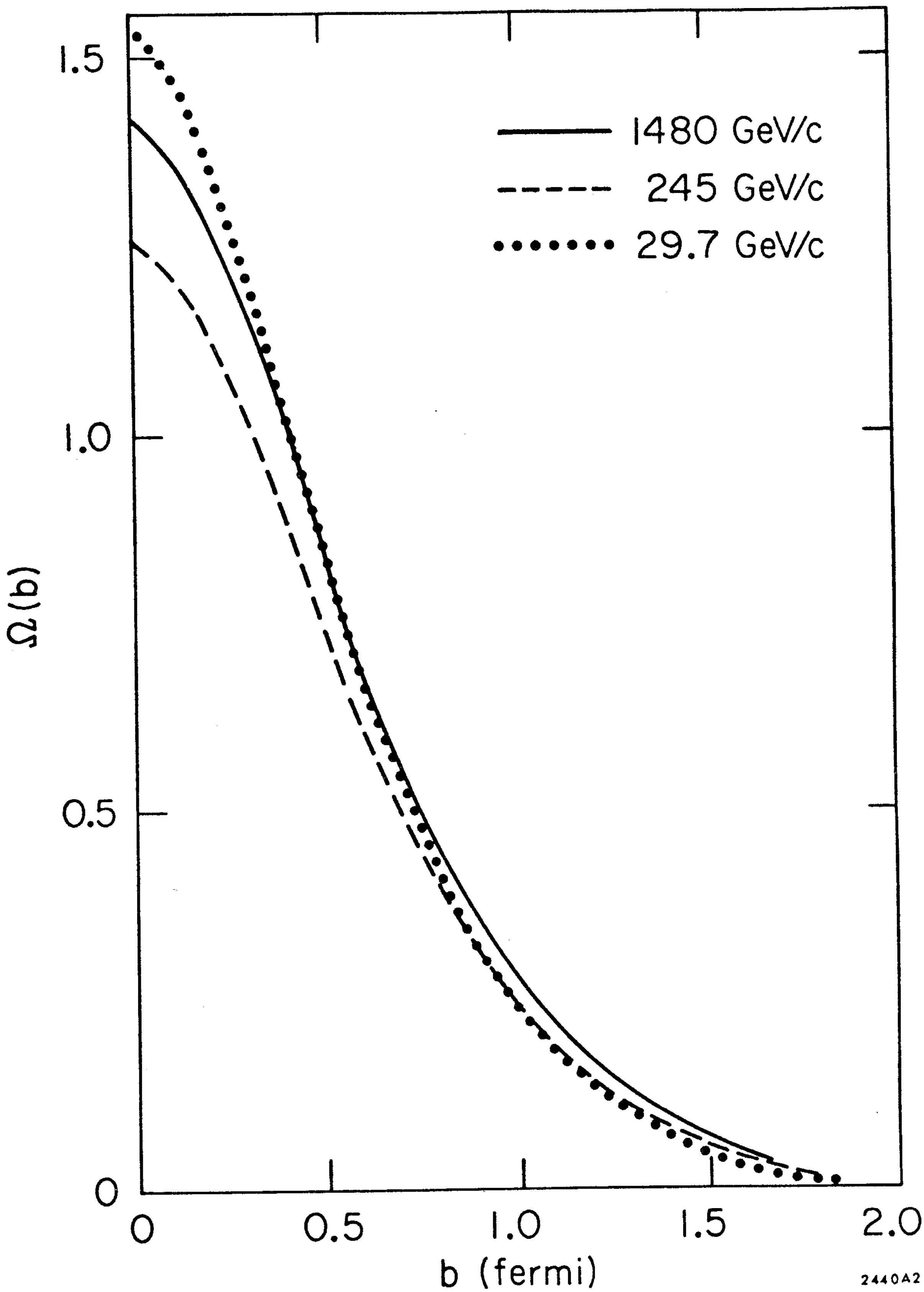


FIG. 1



1480 GeV/c
245 GeV/c
29.7 GeV/c
Ω(b)
b (fermi)
1.5
1.0
0.5
0
0.5
1.0
1.5
2.0
2440A2

FIG. 2