SLAC-PUB-1347
(T/E)
November 1973

CHARGE FLUCTUATIONS IN e^+e^- ANNIHILATIONS*

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ABSTRACT

We consider what can be learned about the nature of the annihilation process $e^+e^- \to$ hadrons from the measurement of the mean square charge transfer between right and left C. M. hemispheres. Predictions of parton models where this quantity goes to a finite constant and fireball models where it grows with multiplicity are compared. We conclude that it should be possible to discriminate between these two classes of models at SPEAR energies. We also comment on difficulties associated with testing in e^+e^- annihilations Feynman's conjecture that quark quantum numbers remain in the parton fragmentation region.

(Submitted to Phys. Rev.)

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* Supported by the U. S. Atomic Energy Commission.

I. INTRODUCTION

There have been two basic approaches to the annihilation process

$$e^+e^- \to \gamma^*(q^2) \to \text{hadrons} \tag{1.1}$$

The first approach is that of the parton model in which one assumes that the photon couples to charged elementary constituents which then convert into hadrons[1-6]. The parton point of view has many attractive features but its interpretation of (1.1) has the drawback that many experimental observables in the annihilation process depend sensitively on the unknown nature of the parton-hadron dynamics. For this reason the parton model is unable to make many unambiguous predictions. For example, in various versions of the parton model the average hadronic multiplicity can be asymptotically finite[2], can grow as $\ln(q^2)$[3,4] or can grow as a power of (q^2)[5]. One basic prediction of the parton model is that a "jet" structure should develop due to the fact that the hadrons should maintain the direction of the elementary constituents[6]. This feature, unfortunately, may prove difficult to test at anything short of very high energies.

The second point of view is generally referred to as Generalized Vector Dominance (GVD)[7-10] in which the photon couples directly to a spectrum of vector mesons which then decay into stable hadrons. The GVD approach needs as input various assumptions concerning the mass spectrum of the vector mesons, their electromagnetic couplings, and their decay mechanisms. It therefore contains a great deal of flexibility and makes few predictions which would allow it to be distinguished from the parton model. Certain simplifying approximations must be made in order for the model to have predictive power. In a large class of GVD models, the decay products of the vector meson are treated statistically. The original Fermi statistical model[11], the Hagedorn thermodynamic model[12] and the Landau hydrodynamic model[13] can be cast in the language of GVD and included in this category.

Even within the GVD category there are variations in the predictions for as basic a dynamical feature as hadronic multiplicity. The thermodynamic model has the natural prediction that the dominant decay mode of a massive vector meson is characterized by a limited average energy, $\langle E_\pi \rangle$, so that[14]

$$\langle n \rangle \sim (q^2)^{\frac{1}{2}} / \langle E_\pi \rangle \ . \tag{1.2}$$

The other models give $\langle n \rangle$ proportional to smaller powers of q^2 [14,15], behaviors which cannot easily be distinguished from the options available in the parton model.

The problem, then, is that if we are going to attempt to distinguish experimentally between the general class of parton models and the class of statistically decaying GVD models we will have to measure quantities more sensitive than the average hadronic multiplicity. The fact that quantum numbers are treated differently in the two approaches suggests that measurements of fluctuations in the density of charged particles in local regions of phase space should enable one to test for these basic differences.

In this paper we concentrate on what can be learned from one such type of measurement, namely that of the mean-square charge transfer between C. M. hemispheres. In parton models there is an ordering principle in the production of charged particles which leads to semi-local neutrality and forbids large charge transfers regardless of multiplicity; in the statistical decay of a massive vector meson, however, the fluctuations in charge density are not limited but grow with the multiplicity. We will see that measurement of the mean square charge density is therefore capable of distinguishing between parton and GVD models in much the same way that it distinguishes between short-range-order and fragmentation models for purely hadronic interactions[16-18].

The plan of this paper is as follows. In Sec. II we discuss the experimental definition of mean-square charge transfer. Since the one-photon intermediate state does not necessarily preserve a special axis to use in dividing phase space into two separate regions we discuss a possible method of defining the appropriate C.M. hemispheres on an event-by-event basis. The method is based on the existence, within the parton model, of an underlying jet structure and simplifies our parton calculations without biasing any of the predictions of the statistical GVD models. This discussion is not crucial for what follows it since it should be possible (within limits) to use charge fluctuations defined with respect to fixed axes to distinguish models. In Sec. III we discuss a parton jet model and a cascade model containing a parton loop. Sec. IV is devoted to the Generalized Vector Dominance model and the assumption of statistical decay. Sec. V discusses the strong ordering approximation used in our parton model calculations and the consequences for our results of relaxing the assumption. Our conclusions are presented in Sec. VI, and an Appendix is included which illustrates the inappropriateness of using e^+e^- annihilations to test Feynman's hypothesis[19] that parton quantum numbers may be retained on the average in the fragmentation regions in parton models.

II. THE MEASUREMENT OF MEAN-SQUARE CHARGE TRANSFER

We want to describe the charge transfer, u, in the one-photon annihilation process $e^+e^- \to$ hadrons occurring in unpolarized colliding beams. In hadronic collisions the charge transfer in a given event is defined in terms of hemispheres of phase space (here referred to as left and right) in the C.M. frame. The hemispheres are centered around the beam and

$$u = \frac{1}{2}\left[(Q_R^F - Q_R^I) - (Q_L^F - Q_L^I)\right] \tag{2.1}$$

where Q_R^F is the total charge of the final state particles going in the rightward C.M. direction and Q_R^I is the initial (beam or target) charge going rightward.

If we let $\sigma(u)$ represent the cross section for a given value of u, then the inclusive average

$$\langle u^2\rangle = \sum_{-\infty}^{+\infty} u^2\,\sigma(u) \tag{2.2}$$

is an indicator whose measurement has been shown to be capable of distinguishing between short-range-order and fragmentation mechanisms[16-18] for hadronic production processes.

In the absence of one-photon - two-photon interference effects or of weak parity violation effects, either of which leads in some models[20] to slight overall charge asymmetries of the final state hadrons, information concerning the direction of the charge in the initial e^+e^- state is not transmitted to the final particles so the relevant quantity in e^+e^- annihilations is

$$u = \frac{1}{2}\left[Q_R^F - Q_L^F\right] \tag{2.1'}$$

where only the charge of the final state particles is considered. Inclusive expectation values of even moments of u are in general nonzero and will be shown to distinguish between models of the annihilation dynamics.

Since the photon in the intermediate state has spin one the most general inclusive distribution to produce a hadron of momentum p and angle θ from the e^+e^- direction is

$$p\, d^2\sigma/d\Omega dp = A(p) + B(p)\cos^2\theta \tag{2.3}$$

The beam direction in e^+e^- annihilations does not therefore play as important a role in determining the properties of the final state as does the direction of the beam in, for example, pp collisions where momenta of final state particles in directions transverse to the beam are extremely limited.

The absence of a preferred spatial direction complicates the problem of interpreting the experimentally determined charge transfer. A choice of an axis has to be made in order to define the left and right hemispheres in (2.1'). This choice can either be made once and for all (on the basis of experimental configuration) or can be made on an event-by-event basis.

The predictions of the parton model are particularly transparent if the axis can be chosen in each event to coincide with the jet axis. The possible existence of hadron jets in e^+e^- annihilations has been frequently discussed[6,etc.]. It is a clear prediction of parton models that hadrons should be emitted in clusters with limited relative transverse momenta. The hadrons are supposed to retain approximately the direction of the intermediate virtual partons. The axis can be defined in terms of a picture such as that of Fig. 1 where a parton and antiparton are produced and assumed not to interact after production. If the hadrons which are the decay products of the parton have small momenta transverse to the parton's direction we will, at high enough energy, have jet structure. The jet axis will be that describing the average motion of each group of hadrons. No experimental evidence for jet structure has yet been reported and no clear-cut evidence for jets is expected at currently available energies. The average particle momentum at CEA energies is approximately 1 GeV/c so that transverse momenta of .3-.4 GeV/c as seen in hadron collisions would lead only to a very diffuse jet structure[15].

In the absence of definite jet structure the optimum event-by-event axis is one which coincides with the direction of the most energetic hadron. As long as we restrict attention to $\langle u^2 \rangle$ or even powers of u it doesn't matter which direction along the axis is defined as left and which is defined as right. Naively, it might be thought possible to test Feynman's conjecture that, on the average, the charge of a parton is deposited in its fragmentation region[19] by choosing left to be along the direction of the overall positive charge. This turns out not to be the case, as we will show in the appendix.

In a situation where the detection apparatus has limited acceptance, there may be sound experimental reasons for not attempting to define the left-right axis differently for each event. We will attempt to discuss as we go along how the predictions of the parton model will change when we define the left and right hemispheres relative to a fixed axis. It certainly does not affect GVD predictions.

III. PARTON MODELS

In this Section we shall examine the predictions of two classes of parton models for the behavior of charge fluctuations.

A. The Parton Jet Model

The parton jet model is suitable to begin our considerations since its predictions and limitations are very straightforward. The basic situation is indicated in the diagram of Fig. 1 where a parton-antiparton pair is produced. The parton and antiparton are assumed not to interact after production and subsequently decay independently into hadrons. This picture is justified by its simplicity and by the fact that it emerges in virtually all field theory models.[2] It has the disadvantage that we are forced to abandon the identification of partons with quarks, since the independent decay of the produced partons rules out the possibility that they possess any bizarre quantum numbers. At first glance the parton jet model seems to imply asymptotically finite hadron multiplicities.[2] It is not clear how rigorous this prediction is but it should not be considered a drawback. The observed rise of multiplicities through current energies is consistent with the possibility that new parton-antiparton channels are being opened as we increase the energy.

If we can define the left-right axis along the parton direction, the measurement of charge transfer $u \equiv \pm Q_a$ can give a great deal of information. In the limit that there is no overlap between the fragmentation regions the charge in each hemisphere is simply that of the parton. Since the photon couples only to charged objects we can never have neutral jets

$$\lim_{q^2 \to \infty} |u|_{min} \neq 0 \quad . \tag{3.1}$$

The only way we can have a neutral cluster of particles would be if the decay products of the parton and the antiparton are mixed.

The behavior of $<u^2>$ in the model is strongly correlated with the behavior of

$$R = \sigma(e^+e^- \to \text{hadrons}) / \sigma(e^+e^- \to u^+u^-) \quad . \tag{3.2}$$

If we assume that the (pointlike) partons in Fig. 1 have either spin -$\frac{1}{2}$ or spin -0, we get at large q^2

$$R \sim \left(\sum_{s=\frac{1}{2}} Q_a^2 + 1/4 \sum_{s=0} Q_a^2 \right) \quad . \tag{3.3}$$

Experimentally, the value of R is rising through CEA energies and is substantially greater than 1. (See Fig. 2, taken from reference 21). The large value of R may give support to the idea of integrally charged partons, which is a fundamental requirement of the parton jet model if fractionally charged particles are not to appear in the final state. In the limit that there is no overlap between fragmentation regions the value of $<u^2>$ is given by

$$<u^2> \sim \left(\sum_{s=\frac{1}{2}} Q_a^4 + 1/4 \sum_{s=0} Q_a^4 \right) / R \quad . \tag{3.4}$$

Combining this knowledge with the values of

$$|u|_{max} \sim \sup_a |Q_a| \; , \tag{3.5a}$$

$$|u|_{min} \sim \min_a |Q_a| \; , \tag{3.5b}$$

it should be possible to reconstruct a good deal about the number of each type of charged parton.

The viability of the parton jet model depends, of course, crucially on the appearance at high enough energies of a distinct charged-jet structure. It is not clear whether this possibility can be either supported or rejected within the energy limits of machines projected in the next few years.

B. The Quark-Parton Loop Model : Meson Production

For the rest of this Section we will identify partons with quarks. This identification will be made because of the success the quark-parton model has had in quantifying electroproduction data[22]. The fact that the experimental value of R as defined in (3.3) is, at the highest energies currently available, quite different from the simple quark model prediction

$$R \sim 2/3 \tag{3.6}$$

must be considered an embarrassment[23]. However, we won't deal with this problem here.

A more immediate problem is the absence of observed quarks. If we want to take seriously the suggestion that partons possess quark quantum numbers we must seek a more complicated description of the annihilation process into hadrons than that of the parton jet model. The simplest diagram in which quarks are not emitted contains a "loop" as shown in Fig. 3. If the triality of each "link" exchanged in the loop remains constant the only particles emitted will be mesons, baryons, and antibaryons. Because of the kinematic restrictions on producing a massive baryon-antibaryon pair we believe it a good approximation to consider here only the production of mesons. We will return to baryon emission in subsection C.

The dynamical implications of the fact that quarks don't get out in spite of the large gap in momentum space between the original parton-antiparton pair must be considered severe. The authors in Ref. 24 , for example, have discussed the complicated space-time structure implied by this kind of loop. The overall picture is workable but cannot be considered attractive. We will treat it here as an alternative forced upon us by our prejudice that partons must be identified with quarks.

The diagram of Fig. 3 will be treated in the strong-ordering limit. In this limit each final state is determined by specifying the parton coupled to the photon as well as the position and type of each hadron. Thus there is no interference between final states; and since the original partons are produced incoherently we can deal with probabilities rather than amplitudes. The strong ordering principle is supported in a QED analogy where the preference of photons is to be emitted softly[25]. Soft emission of hadrons would suggest the presence, at sufficiently high q^2, of some jet-like structure in this model as well. The distinction between the structure of the final state in this model and that in the jet model of Sec. III. A is that in general it will not be possible here to separate unambiguously two groups of hadrons. There will be particles in the central or plateau region which cannot be associated with either the parton or the antiparton. We will return in Sec. V to the relaxation of the strong-ordering principle and will find that most of the results calculated here remain approximately valid.

If we choose the axis for defining our two hemispheres to coincide approximately with the parton-antiparton direction (as can be done by using the direction of the most energetic hadron in the absence of clear-out jet structure) then the diagram of Fig. 3 can be used directly to calculate the charge transfer. The left-going and right-going particles can be separated by a line through the diagram which intersects the propagator of the parton coupled to the photon as well as one other parton link. If we call these partons respectively a and a', where a and a' are u, d or s-type quarks, the expression for the mean square charge transfer is

$$\langle u^2 \rangle = \left(\sum_{a,a'} (Q_a - Q_{a'})^2 \; P(a \mid a') \; \sigma(a) \right) \Big/ \left(\sum_a \sigma(a) \right) \tag{3.7}$$

In this expression $P(a \mid a')$ is the conditional probability that parton a will yield parton a' averaged over all possible intermediate emission of hadrons. The cross section, $\sigma(a)$, for producing the parton-antiparton pair is

$$\sigma(a) \sim Q_a^2 \; \sigma_{e^+e^- \to \mu^+\mu^-} \tag{3.8}$$

for spin $-\frac{1}{2}$ quarks.

The structure of Eq. (3.7) is then contained in the expression for the conditional probability $P(a \mid a')$. The difference between (3.7) and the expression (3.4) in the parton jet model is that parton a' here carries some quantum numbers between the left and right hemispheres and the values the spin and charge of parton a are assumed known. Because we are dealing with quarks the values of $\langle u^2 \rangle$ are limited

$$|u| \equiv |(Q_a - Q_{a'})| = 0,1 \tag{3.9}$$

only. In the limit that the strong-ordering and the jet structure approximations are valid the value of $\langle u^2 \rangle$ in (3.7) is necessarily less than one. The quark-parton model carries a prediction of semilocal charge conservation connected with the absence of exotic quantum number exchanges. When the axis used to define the two hemispheres is not aligned with the initial parton-antiparton pair direction or when strong ordering is violated there will, of course, be some events with charge transfer two or greater. We will argue in Sec. V that these effects should be small so that the result

$$\langle u^2 \rangle \lesssim 1 \tag{3.10}$$

is expected in the quark-parton model.

We can go further in calculating the charge transfer in the quark-parton model if, at high energies, the average number of hadrons emitted in one hemisphere becomes large. If this is so it makes sense to assume that the frequency with which the different partons occur as links in the plateau region becomes insensitive to the identity of the parton coupled to the photon. In terms of the conditional probability $P(a|a')$ this can be expressed

$$P(a|a') \xrightarrow[\langle n_L \rangle \text{ large}]{} p(a') \quad . \tag{3.11}$$

Equation (3.11) is a slightly weakened formulation of Feynman's assumptions concerning the plateau region discussed in Ref. 1. If we enforce isospin invariance but allow for an SU_3-breaking distinction between strange and nonstrange quarks we get

$$p(u) = p(d) = \frac{1}{2}\,(1 - p(s)), \quad p(s) \leq 1/3 \tag{3.12}$$

Inserting this into (3.7) and using (3.8) we get

$$\langle u^2 \rangle = 1/2 + \frac{p(s)}{6} \quad . \tag{3.13}$$

Because of the large ratio of the mass of the kaon to the mass of the pion it should be expected that, in the central region, fluctuations in strangeness are more localized than charge fluctuations and that p(s) is small. In this case $\langle u^2 \rangle$ would be expected to be nearer 1/2 than the SU_3 symmetry value of 5/9.

If we define, in analogy to (2.1′)

$$v = \frac{1}{2}\left[(S_R^F - S_L^F)\right] \tag{3.14}$$

where S_R^F (S_L^F) is the total strangeness of the particles in the right (left) hemisphere we have an analogous expression to (3.7) for

$$\langle v^2 \rangle = \left(\sum_{a,a'}{}' (S_a - S_{a'})^2 \, P(a|a') \, \sigma(a)\right) \Big/ \left(\sum_a{}' \sigma(a)\right) \quad . \tag{3.15}$$

Using the assumptions leading to (3.12) we get

$$\langle v^2 \rangle = 1/6 + (2/3)\, p(s) \quad . \tag{3.16}$$

Eliminating p(s) between (3.13) and (3.16) we get the connection

$$\langle u^2 \rangle = 11/24 + 1/4 \langle v^2 \rangle \quad . \tag{3.17}$$

We can also calculate correlations between u and v. If we restrict attention to those events with $|v| = 1$, we must have the conditional probabilities

$$\begin{aligned} \widetilde{P}(u|s) &= \widetilde{P}(d|s) = 1 \\ \widetilde{P}(s|u) &= \widetilde{P}(s|d) = 1/2 \end{aligned} \tag{3.18}$$

Using (3.8) for $P(a|a')$ in (3.7), we get

$$\langle u^2 \rangle_{|v|=1} = 3/4 \tag{3.19}$$

A word of caution about the assumption (3.11) is in order: it does not follow trivially from assumptions of short range order in purely hadronic interactions. This distinction is important since the short range order hypothesis in hadron production processes is separately testable. The problem can be illustrated using a simple cascade model due to Cahn and Colglazier[4]. We assume for simplicity that the only partons are u-type and d-type quarks and the only emitted hadrons are pions. We then make the assumption the probabilities for emission at each step along the loop is given by the quark decomposition of the pions illustrated in Fig. 4

$$\begin{aligned} p(u \to u\pi^0) &= p(d \to d\pi^0) = 1/3 \\ p(u \to d\pi^+) &= p(d \to u\pi^-) = 2/3 \end{aligned} \tag{3.20}$$

In the matrix notation of Cahn and Colglazier we have the conversion matrix

$$A = \begin{pmatrix} 1/3 & 2/3 \\ 2/3 & 1/3 \end{pmatrix} \tag{3.21}$$

where the basis is chosen $u = (1,0)$ and $d = (0,1)$.

The conditional probability that parton a will yield parton a' after N intermediate emissions is given as a power of A

$$P^N(a|a') = (A^N)_{aa'} \tag{3.22}$$

For large N this rapidly converges toward the limit[4]

$$\lim_{N\to\infty} A^N = \begin{pmatrix} 1/2 & 1/2 \\ 1/2 & 1/2 \end{pmatrix} \tag{3.23}$$

which agrees with the assumption (3.11). The problem is we cannot use this particles cascade formulation for the whole loop: if we go all the way around the loop in a given final state,resulting from M emissions, quantum number conservation requires that we find

$$P^M(a\,|\,a') = \delta_{aa'}$$

and so all the emissions cannot be independent, contrary to Eq. (3.20). (At most M-1 can be independent, and there is no way of telling which ones these are.)

It can be argued that this formalism would be useful in treating "half the loop" at a time, i.e. restricting the final quark a' to be one in the central region. However, we prefer to take the point of view that, except for the parton-anti-parton pair link attached to the photon, the frequencies of appearance of u, d, or s-type links are determined in the large by the symmetry properties of the hadronic would, as in (3.12), and on the average their (random) ordering within the loop results in the (observed symmetry properties of the) various final states.

C. The Quark-Parton Loop Model: Baryon Production

The quark-loop diagram, Fig. 3, considered so far does not treat baryon emission on the same footing as meson emission. It is possible to have baryons in the final state only by letting the emitted mesons represent heavy states which can decay into a baryon-antibaryon pair. In the strong-ordering limit we are then neglecting the possibility that the baryon could appear in one hemisphere and the antibaryon in the other. To allow for separation of a baryon and an antibaryon we have to consider diagrams such as shown in Fig. 5 where they are separated by exchanged $\bar{q}\bar{q}$ links. Each $\bar{q}$ is an antiquark so the triality of the exchange links is still +1.

With the double links $\bar{u}\bar{u}$, $\bar{d}\bar{d}$, $\bar{s}\bar{s}$, $\bar{u}\bar{d}$, $\bar{u}\bar{s}$, and $\bar{d}\bar{s}$, if we continue to assume that the photon only couples to single quarks there is now the possibility of $|u| = 2$ events corresponding to a u-quark entering one hemisphere and $\bar{u}\bar{u}$ antiquarks leaving so that the quantum numbers of a Δ^{++} are deposited there.

We can estimate the charge fluctuations here if we assume that the emission of baryons is rare so that the average concentration of $\bar{q}\bar{q}$ links is also rare. We parameterize the probability that a given exchange is a double link as β,

$$\beta \approx \langle n_B \rangle / \langle n \rangle \qquad (3.24)$$

$$\beta \ll 1$$

and assume that the relative frequency of occurrence of each quark or antiquark behaves independently: e.g., $P_{\bar{u}\bar{u}} : P_{\bar{u}\bar{s}} : P_{\bar{s}\bar{s}} :: P_u P_u : 2\, P_u P_s : P_s P_s$, and we do not distinguish between $P_{\bar{u}\bar{s}}$ and $P_{\bar{s}\bar{u}}$ (hence the factor of two). The probabilities are then as given in Table 1. Inserting these into (3.7) we now have

$$\langle u^2 \rangle = 1/2 + \frac{p(s)}{6} + \beta\, \frac{(4 - 9\,p(s) + 3\,p^2(s))}{6} \qquad (3.25)$$

Because β is assumed to be small this is very close to (3.13).

In analogy to what was done with strangeness we can define the net baryon number transfer

$$b \equiv \frac{1}{2} (B_R - B_L) \tag{3.26}$$

The mean square expectation of this should be simply

$$<b^2> = \beta \quad . \tag{3.27}$$

We can also study correlations between charge transfer and baryon number transfer

$$<u^2>_{|b|=0} = \frac{7 - 8\,p(s) + 3\,p^2(s)}{6} \quad \epsilon\,(7/9,\ 7/6) \tag{3.28}$$

Of course $<u^2>_{|b|=0}$ is given by (3.13).

We now leave the strong ordering limit of the quark parton model to consider the behavior of charge fluctuations in models (like GVD) where there is no ordering at all. Following that we shall return to parton models to consider the consequences of weak ordering.

IV. MODELS WITHOUT CONSTRAINTS ON ORDERING OF CHARGED PARTICLES

As we have seen, the parton loop picture puts strong constraints on the ordering of charged particles. In the calculation of Sec.'s III. B and III. C two negative pions are prohibited from occupying adjacent phase space regions so that charge is conserved semi-locally and charge fluctuations are small. Many pictures of the annihilation process, in contrast, make no ordering assumptions at all. Particles are produced subject to overall conservation of charge and momentum but there are no additional constraints on where the particles of a given charge end up.

We will phrase our discussion of unordered models in the language of the Generalized Vector Dominance model[7,8]. In GVD a photon couples directly to a heavy vector meson which decays into stable hadrons. In the absence of a detailed description of the decay mechanisms of massive vector mesons which incorporates ordering constraints we must rely on a statistical description of the final state. The original Fermi statistical model[11] is probably the simplest example of this type of treatment. Other examples of models which treat charge in a statistical way include Landau's hydrodynamic model[13] and various versions of the Hagedorn thermodynamic bootstrap[12].

We do not want to claim that GVD cannot be augmented by an ordering principle if one wishes to make the charge fluctuations in its final states look like those in the parton model. Dual models for coupling currents to hadrons[26] constitute a specific example where this can be done. The ordering of charged particles in dual models is governed by Harari-Rosner[27] diagrams which have the same structure as the quark-parton diagrams in Figs. 3-5. The point is that in the absence of extra dynamic assumptions we expect a statistical description of the charged decay products of a neutral meson to be valid. This means the behavior of the mean square charge transfer, $<u^2>$, should be closely related to fluctuations in the number of particles. If this relation does not hold it means there is some underlying dynamics, such as partons, which we have omitted.

Consider a given final state which has n_L charged particles in the left hemisphere and n_R charged particles in the right hemisphere. Here we do not assume any underlying jet structure so it does not matter how the hemispheres are chosen. We will assume for convenience that all particles are singly charged. Since only overall charge conservation is in force and not semilocal charge conservation, we have large possible fluctuations

$$|u|_{max} = \min(n_L, n_R) \tag{4.1}$$

Let $P_{n+}(n_{R+}, n_{R-})$ be the probability that a final state with n+ positive (and by charge conservation $n- = n+$ negative) particles should send n_{R+} positive and n_{R-} negative particles into the right hemisphere. If we assume independence of positive and negative particle motions then

$$P_{n+}(n_{R+}, n_{R-}) = P_{n+}(n_{R+})\, P_{n+}(n_{R-}) \quad . \tag{4.2}$$

This assumption should be good at high energies where particles are produced copiously. Momentum and charge conservation may make some corrections when $n_{R+} \approx 0$ or n+, but we will assume these configurations are negligible. We then have

$$\langle u^2\rangle_{n+} = \sum P_{n+}(n_{R+}, n_{R-})(n_{R+}-n_{R-})^2$$

$$\equiv 2\left(\langle n_{R+}^2\rangle_{n+} - \langle n_{R+}\rangle_{n+}^2\right) . \qquad (4.3)$$

For example, if we let $P_{n+}(n_{R+})$ take the form of a binomial distribution,

$$P_{n+}(n_{R+}) = r_+^{n_{R+}} \ell_+^{n_{L+}} \binom{n+}{n_{R+}} \qquad (4.4)$$

where $r_+ = 1 - \ell_+$ is the probability for a positively charged particle to go to the right. When we work in C. M. and choose right and left without regard to charges then $r = \frac{1}{2}$ and we have

$$\langle u^2\rangle_{n+} = \tfrac{1}{2}\, n+ \qquad (4.5)$$

If we average over the number of produced particles

$$\langle u^2\rangle = \sigma_{tot}^{-1} \sum \sigma(n+)\ \langle u^2\rangle_{n+}$$

$$= 2\left(\langle n_{R+}^2\rangle - \langle n_{R+}\rangle^2\right) \qquad (4.6)$$

(4.6)

If we assume the binomial distribution (4.4) is approximately valid we then get

$$\langle u^2\rangle = \frac{1}{2}\langle n+\rangle = \frac{1}{4}\langle n_{ch}\rangle . \qquad (4.7)$$

An alternate formultion of this result can be given using simple generating functional techniques. Let $\sigma(n_{L+}, n_{L-}, n_{R+}, n_{R-})$ be the cross section for producing n_{L+} positively charged particles in the left hemisphere, etc.

We form the generating functional

$$Q(Z_{L+}, Z_{L-}, Z_{R+}, Z_{R-}) = \sum_{n's} \sigma(n_{L+}, n_{L-}, n_{R+}, n_{R-}) Z_{L+}^{n_{L+}} Z_{L-}^{n_{L-}} Z_{R+}^{n_{R+}} Z_{R-}^{n_{R-}} \tag{4.8}$$

Overall charge conservation gives

$$n_{L+} - n_{L-} + n_{R+} - n_{R-} = 0 \tag{4.9}$$

In the absence of any dynamical mechanism giving semilocal charge conservation we would have

$$\sum_{n_{L+}, n_{L-}} \sigma(n_{L+}, n_{L-}, n_{R+}, n_{R-}) = a(n_{R+})\, a(n_{R-}) \tag{4.10}$$

so that

$$Q(1,1\,;\, Z_{R+}, Z_{R-}) = \exp\left\{ f(Z_{R+}) + f(Z_{R-}) \right\} \tag{4.11}$$

where we have written $f(Z) \equiv \ln a(Z)$. If we introduce the variables

$$\begin{aligned} u &= n_{R+} - n_{R-} \\ n_R &= n_{R+} + n_{R-} \\ X &= (Z_{R+}/Z_{R-})^{\frac{1}{2}} \\ Z_R &= (Z_{R+} Z_{R-})^{\frac{1}{2}} \end{aligned} \tag{4.12}$$

we can write

$$Q(1,1\,;\, Z_R, X) = \exp\left\{ f(Z_R X) + f(Z_R/X) \right\} \tag{4.13}$$

and note that

$$\langle u^2 \rangle = \left(\frac{X\partial}{\partial X}\right)^2 \ln Q(1,1\,;\,Z_R,X)\Bigg|_{Z_R,X=1} = \left[\frac{\partial^2}{\partial Z_R^2} + \frac{\partial}{\partial Z_R}\right] \ln Q(1,1\,;\,Z_R,X)\Bigg|_{Z_R,X=1} \tag{4.14}$$

which agrees with (4.6).

The problem then boils down to determining what the (charged) multiplicity distribution is in the various forms of the statistical decay models. In the Hagedorn "bootstrap" model[12] we have a form of pulverization where the energy carried off by each hadron is limited to some $\langle E_\pi \rangle$ so that asymptotically[14]

$$\langle n_{ch} \rangle \sim \frac{2}{3} \frac{\left(q^2\right)^{\frac{1}{2}}}{\langle E_\pi \rangle} \quad .$$

The Fermi statistical model[11] gives asymptotically[14]

$$\langle n_{ch} \rangle \sim a\,(q^2)^{1/3}$$

while the Landau hydrodynamical model[13] gives [15]

$$\langle n_{ch} \rangle \sim a(q^2)^{3/8}$$

None of these alternatives is completely ruled out by existing multiplicity data especially since we do not necessarily expect the multiplicity distribution to reach its asymptotic form early. Projected high energy forms, based on empirical fits to average multiplicities, are given for these statistical models in Refs. 14 and 15. The $\langle u^2 \rangle \sim \frac{1}{4} \langle n_{ch} \rangle$ predictions of (4.7) for these statistical models are given as a function of energy in Fig. 6 , and compared there with an estimate of $\langle u^2 \rangle$ $\left(1/2 << u^2 > < 1 \text{ asymptotically} \right)$ for the quark-parton loop model, Eq. (3.25) or (3.13). (The cross-hatched area in the figure represents the estimated error expected from a possible breakdown of pure strong ordering in the parton loop model, as will be discussed in Sec. V.) We can see from the figure that it may be feasible to distinguish these alternatives at currently accessible energies, at least between the Hagedorn and parton-loop models. This possibility is important since average multiplicities have been shown not to provide a crucial test.

Raw $\langle u^2 \rangle$ data at current energies is probably not sufficient to discriminate between the parton-loop and all versions of the statistical model. However, data taken only from the higher multiplicity events may do the trick as long as the number of prongs is low enough to make it still reasonable to assume some sort of jet structure in the parton approach. Monte Carlo calculations performed by Gary Feldman and Gail Hanson of SLAC, using corrections for experimental acceptances, indicate that this may be the case[28]. Plots of $\langle u^2 \rangle$ vs. prong number can prove especially valuable.

V. CORRECTIONS TO STRONG-ORDERING IN THE PARTON MODEL

The calculations of Sec. III. B in the quark-parton model were made in the limit that the diagram in Fig. 3 determines the ordering in phase space of the produced mesons. In analogy with the situation in hadronic physics, it is important to consider the corrections to strong-ordering in order to evaluate the extent to which formulas such as (3.13) or (3.25) deserve confidence.

For example, one important breakdown of strong ordering occurs when a resonance is produced in one hemisphere and decays giving a charged particle in the opposite hemisphere. Since clustering or resonance effects are known to be important in hadronic production processes[29], we must obviously consider them here. We therefore supplement the formula (3.7) for charge transfer by including in it a probability $P(\Delta u)$ for an extra transfer of charge Δu due to the decay of a resonance. We then have

$$\langle u^2\rangle = \left(\sum_{a,a',u} (Q_a - Q'_a + \Delta u)^2 \; P(a\,|a')\, P(\Delta u)\, \sigma(a)\right) \Big/ \left(\sum_a \sigma(a)\right) \tag{5.1}$$

We have assumed, as seems natural, that the resonance decay contribution is independent of the quark involved. Eq. (5.1) yields

$$\langle u^2\rangle = \langle u^2\rangle_{P\bar{P}} + \langle (\Delta u)^2\rangle \tag{5.2}$$

where $\langle u^2\rangle_{P\bar{P}}$ is the parton-antiparton contribution given for example by (3.13) or (3.25) and

$$\langle (\Delta u)^2\rangle = \sum_{\Delta u} P(\Delta u)(\Delta u)^2 \; . \tag{5.3}$$

To find the behavior of $\langle (\Delta u)^2\rangle$ we can transcribe directly the results of a calculation by Quigg and Thomas[17]. They assume pions are produced in isoscalar clusters of $(\pi^+ \pi^- \pi^0)$. On the average there are

$$\langle N\rangle < \frac{1}{2} \langle n_{ch}\rangle \tag{5.4}$$

such clusters. They assume furthermore that phase space is approximately one-dimensional and that resonances produced within a rapidity interval $(-\Delta, \Delta)$

(the "active zone") of the total rapidity interval $(-Y/2, Y/2)$ can produce a charge transfer and find[17]

$$\langle (\Delta u)^2 \rangle = \frac{4}{3} \Delta \frac{\langle N \rangle}{Y} \qquad (5.5)$$

where $Y = \ln(q^2)$. Using (5.4) and the estimate that Δ in (5.5) is of the same order of magnitude as the transverse momentum cutoff we get

$$\langle (\Delta u)^2 \rangle \lesssim .3 - .4 \quad . \qquad (5.6)$$

The shaded region in Fig. 6 represents the allowance for this extra charge transfer due to a breakdown of strong ordering in the quark-parton model. Since clusters with approximately three pions are needed to explain short-range-order effects in multiperipheral cluster models of high energy hadron collisions[29] we believe that the calculation of Quigg and Thomas gives a valid estimate of resonance effects in annihilations as well.

Other contributions to charge fluctuations are associated with a particle produced in one fragmentation region ending up in the other C. M. hemisphere. These vanish rapidly with increasing energy in the parton model if jet structure is observed and the left-right axis is chosen along the direction of the leading hadron. Relative to a fixed axis, this effect should lead to an extra nonzero constant. The size of this constant depends on many dynamical quantities and it is best estimated in explicit Monte Carlo calculations.

VI. CONCLUSIONS

We have studied charge transfer fluctuations in various types of parton and generalized vector dominance models for $e^+ e^-$ annihilations into hadrons. Very general final-state ordering properties in these two classes of models - its presence (to some degree) in the parton models and absence in GVD models - lead to radically different predictions for the behavior of charge transfer $\langle u^2 \rangle$ with multiplicity when quantum number conservation laws are enforced. In parton jet or loop models, $\langle u^2 \rangle$ tends to a finite constant ($\approx \frac{1}{2}$ for quark partons) independent of multiplicity, whereas in GVD (statistical) models it rises with the multiplicity, $\langle u^2 \rangle \sim n_{ch}$. Energies and multiplicities soon to be available at SPEAR may be sufficient to decide which of these two general classes of models is the more appropriate for treating annihilation.

APPENDIX

Feynman's hypothesis that in a parton model the parton charges should be retained on the average in the fragmentation regions in electroproduction has stirred up considerable interest lately. Although the hypothesis in its strongest form - that the parton charges are on the average exactly retained - has been shown[4, 19] to be not generally true, the possibility that some average residuum of the parton quantum numbers may be present is one that should not be overlooked. In this Appendix we shall demonstrate how easily one might be led astray in testing this hypothesis in $e^+e^- \rightarrow$ hadrons.

The natural way to test the hypothesis for annihilation would be to define a suitable right-left axis, say along the fastest particle's direction, and measure $\langle |u| \rangle$ - which is the same as measuring $\langle u \rangle$ with "right" taken to be the direction of the net positive hemisphere. (In electroproduction these definitions are made without reference to the final state: "right" is the direction of the virtual photon in the photon-proton C. M.) In the strong form of Feynman's hypothesis, then, a parton jet model as in Fig. 1 predicts $\langle |u| \rangle = \frac{5}{9}$ - we have taken the partons to be quarks for convenience here.

But say that the true model of the annihilation process was the GVD, or statistical fireball, model of Sec. IV, and let us for simplicity consider the four-charged pions final state. Then, again choosing our right-left axis along the line of greatest rapidity difference, there are six possible orderings in rapidity space, each equally likely: $\pi^+\pi^+ / \pi^-\pi^-$, $\pi^+\pi^- / \pi^+\pi^-$, $\pi^+\pi^- / \pi^-\pi^+$, $\pi^-\pi^+ / \pi^+\pi^-$, $\pi^-\pi^+ / \pi^-\pi^+$, and $\pi^-\pi^- / \pi^+\pi^+$. (We have assumed that equal numbers of pions always go to right and left.) Thus we find $\langle |u| \rangle = \frac{2}{3} \approx \frac{5}{9}$ and could be deceived into thinking that we had found evidence that quark charge does tend to be retained in its hemisphere (or fragmentation region) on the average.

In general, when directions right and left cannot be meaningfully defined without reference to the final state (as they can be in electroproduction), we must look at higher moments of u in testing such a hypothesis based on averages over a number of events. Unless measurements of $\langle u^2 \rangle$ first convince us that a parton model is appropriate for annihilation, tests of Feynman's hypothesis in annihilation are pointless.

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Table 1

Occurrences and properties of various link types in the quark-parton loop model for meson and baryon emission.

Charge	Strangeness	Type and Probability
$-\frac{4}{3}$	0	$P_{\bar{u}\bar{u}} = \beta \left(\frac{1-p(s)}{2}\right)^2$
$-\frac{1}{3}$	0	$P_{\bar{u}\bar{d}} = 2\beta \left(\frac{1-p(s)}{2}\right)^2$
	-1	$P_{\bar{u}\bar{s}} = 2\beta\, p_s \left(\frac{1-p(s)}{2}\right)$
	0	$P_d = (1-\beta)\left(\frac{1-p(s)}{2}\right)$
	+1	$P_s = (1-\beta)\ p(s)$
$+\frac{2}{3}$	0	$P_{\bar{d}\bar{d}} = \frac{\beta}{4}\,(1 - p(s))^2$
	-1	$P_{\bar{d}\bar{s}} = \beta\, p(s)(1 - p(s))$
	-2	$P_{\bar{s}\bar{s}} = \beta\, p(s)^2$
	0	$P_u = (1-\beta)\, \frac{1-p(s)}{2}$

FIGURE CAPTIONS

1. The parton jet model for $e^+e^- \rightarrow$ hadrons in the one-photon approximation. The parton a and antiparton $\bar{a}$ fragment independently into hadrons, and their quantum numbers are found in the produced jets.

2. Data from Ref. 21 on the ratio R defined in Eq. (3.2). Figure taken from Ref. 21 where references to the original experimental papers can be found.

3. The parton (quark) loop model for meson production discussed in Sec. III. B. Parton quantum numbers are not left in the final state. The dashed line represents the C. M. separation into left and right hemispheres, and strong ordering of the final state is assumed.

4. Detail from Fig. 3 of how u and d-type links join to form pions. See also Eq. (3.20).

5. Same as Fig. 3 but including baryon emission as well as meson emission.

6. This plot compares $<u^2>$ predictions of three common statistical (GVD) models with that of a quark parton model as a function of energy $s = q^2$. By restricting measurements at each energy to the higher multiplicity events, the energy at which GVD and quark model predictions separate can be lowered slightly. The shaded region in the quark model predictions represents uncertainty due to failure of strong ordering discussed in Sec. V.

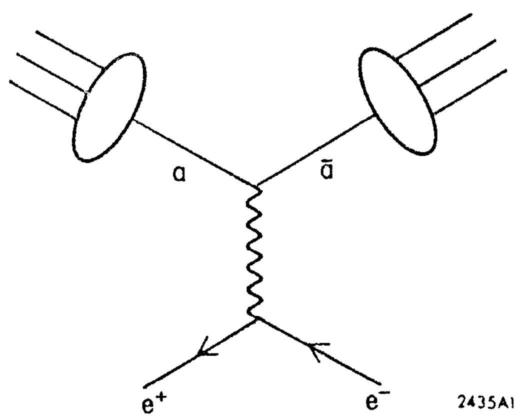

Fig. 1

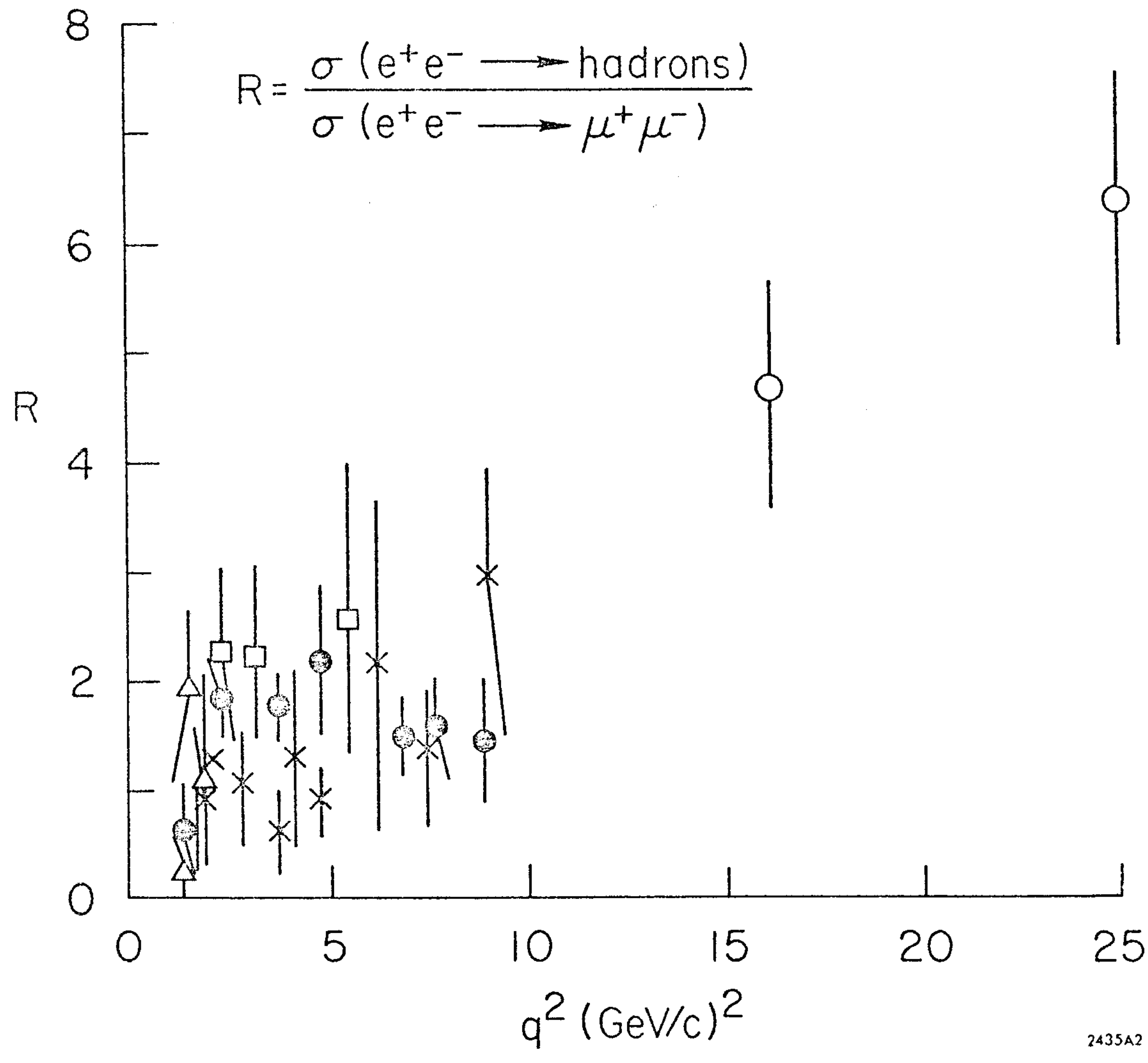
8
6
4
2
0
R
$R = \frac{\sigma(e^+e^- \longrightarrow \text{hadrons})}{\sigma(e^+e^- \longrightarrow \mu^+\mu^-)}$
0
5
10
15
20
25
q^2 (GeV/c)^2
2435A2

Fig. 2

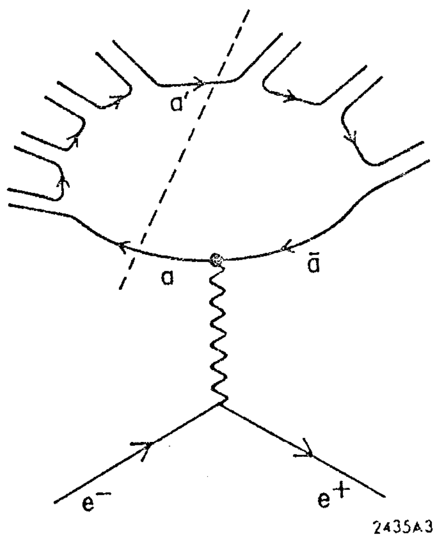

Fig. 3

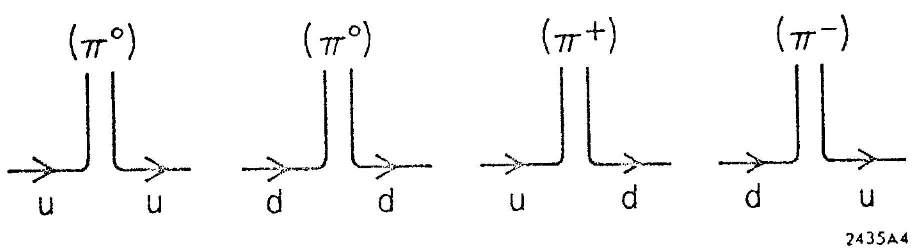

Fig. 4

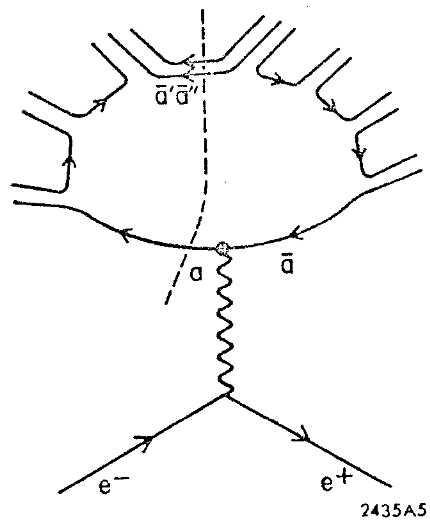

Fig. 5

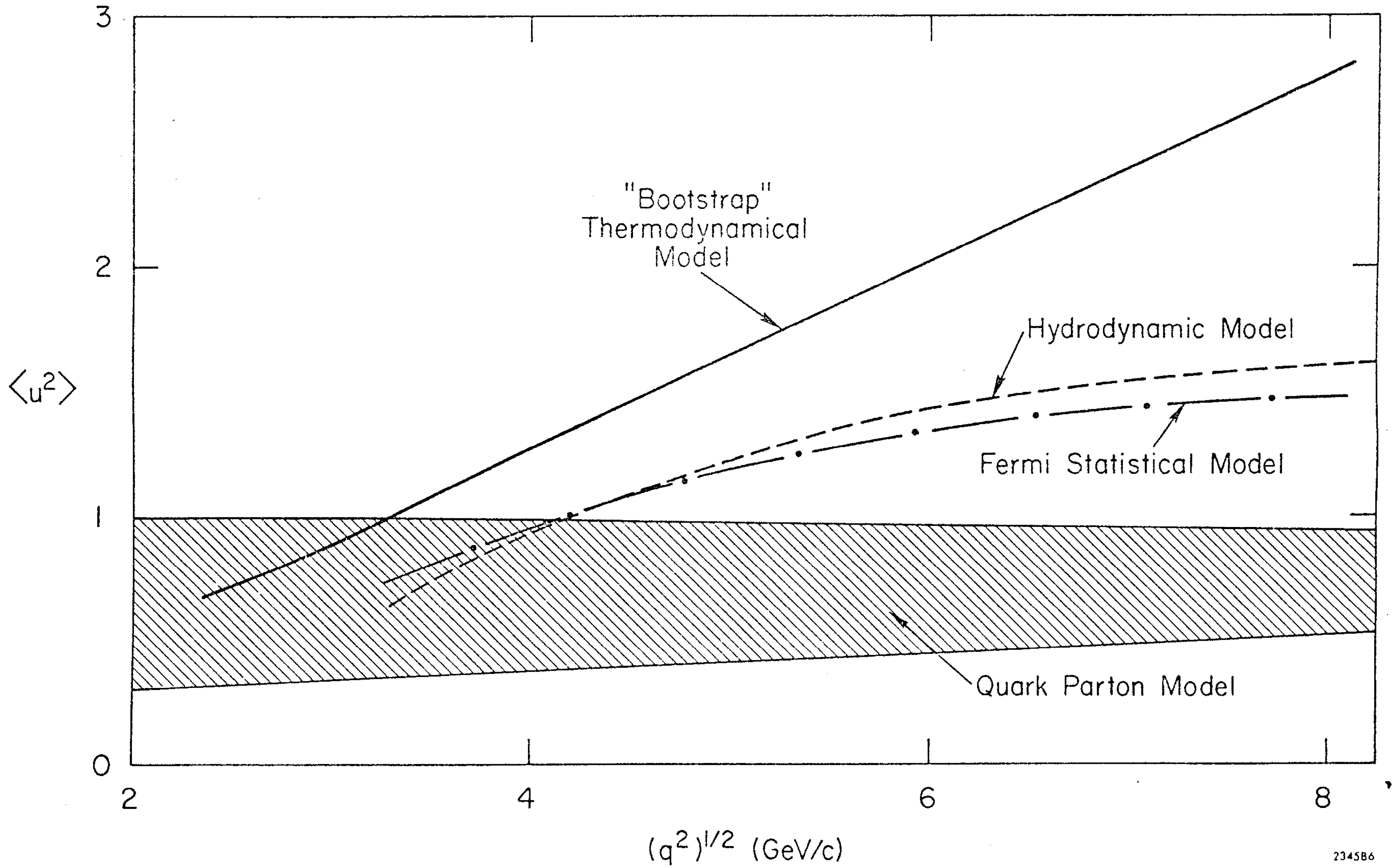
"Bootstrap" Thermodynamical Model
Hydrodynamic Model
Fermi Statistical Model
Quark Parton Model
$\langle u^2 \rangle$
3
2
1
0
2
4
6
8
$(q^2)^{1/2}$ (GeV/c)
2345B6

Fig. 6