$K_{L}^{o} p \rightarrow K_{S}^{o} p$ SCATTERING FROM 1 TO $10 \mathrm{GeV} / \mathrm{c}^{*}$

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ABSTRACT
The differential cross sections for $K_{L}^{o} p \rightarrow K_{S}^{o} p$ scattering are presented in several momentum intervals between 1 and $10 \mathrm{GeV} / \mathrm{c}$. The data are strongly peaked in the forward direction, characteristic of a large $s$ channel helicity nonflip scattering amplitude in this reaction, and a distinct break in the differential cross section occurs at $|t|=0.3 \mathrm{GeV}^{2}$. The phase of the forward scattering amplitude, $\phi$, is consistent with being independent of momentum. The average value of the phase, $\phi=-133.9 \pm 4.0^{\circ}$, corresponds to a Regge trajectory $\alpha(0)=0.49 \pm 0.05$ in agreement with the canonical $\rho, \omega^{\circ}$ Regge intercept, $\alpha(0) \sim 0.5$. However, this result disagrees with the Regge trajectory determined from the energy dependence of the forward cross section, $\alpha(0)=0.30 \pm 0.03$, indicating a breaking of the Regge phase-energy relation. Comparisons of $K_{L}^{o} p \rightarrow K_{S}^{0} p$ and $\pi^{-} p \rightarrow \pi^{0} n$ scattering data reveal substantial differences in the energy dependence of the differential cross sections. Comparisons to KN charge-exchange data then suggest that direct channel (absorption) effects may explain the differences in $\pi \mathrm{N}$ and KN channels.

[^0]
## I. INTRODUCTION

The simple structure of $K_{L}^{o} p \rightarrow K_{S}^{o} p$ scattering makes this a particularly relevant reaction in the study of two body interactions. Of the known hyperons only $\Sigma$ resonances can be formed in the low energy or s channel scattering region. Similarly in the u channel the reaction $\mathrm{K}^{0} \mathrm{p} \rightarrow \mathrm{pK}^{\circ}$ allows only $\Sigma$ exchange whereas the channel $\overline{\mathrm{K}}^{0} \mathrm{p} \rightarrow \mathrm{p} \overline{\mathrm{K}}^{\mathbf{0}}$ is exotic. In the t channel only mesons with natural spin-parity and odd charge conjugation can be exchanged; of the possible candidates $\rho$ and $\omega$ contributions are thought to dominate. Thus, the reaction $K_{L}^{o} p \rightarrow K_{S^{\prime}}^{o} p$ can be considered the $K p$ scattering analogue of $\pi^{-} p \rightarrow \pi^{o} n$ scattering. This similarity will be exploited in our analysis.

Previous results on the reaction $K_{L}^{0} p \rightarrow K_{S}^{0} p$ have come primarily from $K_{S}^{0}$ coherent regeneration experiments, ${ }^{1-3}$ however these analyses determine the scattering amplitude only at $\mathrm{t}=0$. In contrast the present experimental results provide complete angular distributions for $K_{L}^{o} p \rightarrow K_{S}^{0} p$ scattering in the momentum interval 1 to $10 \mathrm{GeV} / \mathrm{c}$. ${ }^{4}$

Details of the experiment are described in Section II. Qualitative features of the data and the differential and total cross sections for $K_{L}^{0} p \rightarrow K_{S}^{0} p$ scattering are discussed in Section III. The forward $K_{L}^{0} p \rightarrow K_{S}^{0} p$ differential cross sections are then discussed in detail in Section IV. Finally, in Section V the tabular results and a brief summary of our large angle analyses ${ }^{5}$ are presented. Section VI is the summary and conclusions.

## II. EXPERIMENTAL DETAILS

The $K_{L}^{o} p \rightarrow K_{S}^{o} p$ data come from an approximately one million picture exposure of the SLAC 40 -inch hydrogen bubble chamber to a $K_{L}^{0}$ beam. The typical $\mathrm{K}_{\mathrm{L}}^{\mathrm{o}}$ flux was between 20 and $40 \mathrm{~K}_{\mathrm{L}}^{\mathrm{O}}$ /pulse over the momentum interval $\sim 600 \mathrm{MeV} / \mathrm{c}$ to $\sim 12 \mathrm{GeV} / \mathrm{c}$. The beam spectrum peaked at $\sim 4 \mathrm{GeV} / \mathrm{c}$,
decreasing to $<10 \%$ of maximum intensity at momenta of $\sim 1$ and $11 \mathrm{GeV} / \mathrm{c}$. Details on the construction of the beam and the $K_{L}^{0}$ momentum spectrum are discussed elsewhere. ${ }^{6}$

The film was scanned once with approximately $10 \%$ of the film rescanned a second time. In addition a special scan for backward-vee events (events with the laboratory scattering angle of the $\mathrm{K}_{\mathrm{S}}^{0},{ }^{\circ}{ }_{\mathrm{LAB}}>45^{\circ}$ ) was made on $\sim 15 \%$ of the film. Scan efficiencies, as well as corrections for losses of events with steeply dipping (primarily short) protons were determined as a function of momentum transfer. These two corrections were consistent with being uncorrelated and with being independent of beam momentum. The scan efficiencies and azimuthal loss corrections are recorded in Table I for events with momentum transfers $|t|>0.025 \mathrm{GeV}^{2}$. For events with $|t|<0.025 \mathrm{GeV}^{2}$ (proton ranges in the bubble chamber $\lesssim 1.5 \mathrm{~cm}$ ) the statistics were too sparse to determine the scanning efficiencies; therefore the data in this interval has been omitted.

The fiducial regions chosen for the primary vertex and the $K_{S}^{0}$ decay vertex allowed at least 15 cm for the $\mathrm{K}_{\mathrm{S}}^{0}$ decay region, and at least 20 cm for measurement of the two pions from the $\mathrm{K}_{\mathrm{S}}^{\mathrm{O}}$ decay. With this interaction region, scanning efficiencies were found to be independent of the $K_{S}^{O}$ decay lengths, $\ell_{K_{S}}^{0}$, for $0.3 \leq \ell_{\mathrm{K}_{\mathrm{S}}^{0}} \leq 20 \mathrm{~cm}$. For $\mathrm{K}_{\mathrm{S}}^{\mathrm{o}}$ decay lengths $\ell_{\mathrm{K}_{\mathrm{S}}^{0}}>20 \mathrm{~cm}$ corrections were made for a slow decrease in the scanning efficiency up to $\ell_{K_{S}^{0}}=40 \mathrm{~cm}$, the $K_{S}^{o}$ decay length cutoff used in the analysis. Accepted events were weighted by the reciprocal of the detection probability:

$$
W^{-1}=\exp \left(-\ell_{\min / \lambda}\right)-\exp \left(-\ell_{\max / \lambda}\right)
$$

where $\ell_{\min }$ is $0.3 \mathrm{~cm}, \ell_{\text {max }}$ is the potential decay length to the boundary of the decay volume ( $\ell_{\max } \leq 40 \mathrm{~cm}$ ), and $\lambda$ is the mean decay length for a $K_{S}^{0}$ of the given momentum.

The events were measured both on conventional film plane machines and on the SLAC Spiral Reader, and processed with the computer programs TVGP-SQUAW. Events were accepted with kinematic reconstruction probabilities $\geq 1 \%$. Contamination from the reactions $\overline{\mathrm{K}}^{0} \mathrm{p} \rightarrow \pi^{+} \Lambda^{0}, \overline{\mathrm{~K}}^{0} \mathrm{p} \rightarrow \pi^{+} \Sigma^{0}$, $K_{L}^{o} p \rightarrow K_{L}^{o} p$ and $K_{S}^{o} p \rightarrow K_{S}^{o} p$ was determined to be $<1 \%$. The inclusion of $K_{L}^{o} p \rightarrow K_{S}^{o} \pi^{+} n$ or $K_{L}^{o} p \rightarrow K_{S}^{o} \pi^{o} p$ final states into the $K_{L}^{o} p \rightarrow K_{S}^{o} p$ data was estimated to be $\lesssim 1 \%$ for forward scattering events, and $\lesssim 5 \%$ for events in the backward direction, $\cos \theta_{\mathrm{cm}}<-0.5$.

The final sample of $K_{L}^{o} p \rightarrow K_{S}^{o} p$ data used in the analysis consisted of 1929 events in the momentum interval 1 to $10 \mathrm{GeV} / \mathrm{c}$.
III. DIFFERENTIAL AND TOTAL CROSS SECTIONS

The differential cross sections for our $K_{L}^{o} p \rightarrow K_{S}^{o} p$ data are shown in Fig. 1 and recorded in Table II for six momentum intervals between 1 and $7.5 \mathrm{GeV} / \mathrm{c}$. The cross sections are corrected ${ }^{7}$ for the unseen decay mode $\mathrm{K}_{\mathrm{S}}^{\mathrm{o}} \rightarrow \pi^{\mathrm{o}} \pi^{\mathrm{o}}$. The uncertainties in the data include the statistical errors as well as the uncertainties in the shape of the $K_{L}^{o}$ momentum spectrum, the scanning efficiency and the azimuthal loss correction (see Section II). An overall normalization uncertainty of $\sim 10 \%$ has not been included, however.

To compensate for the loss of events with momentum transfers $|t|<0.025 \mathrm{GeV}^{2}$ (see Section II), data in the interval $0.025 \leq|t| \leq 0.25 \mathrm{GeV}^{2}$ were parameterized with the form ${ }^{8}$ :

$$
\begin{equation*}
\frac{\mathrm{d} \sigma}{\mathrm{dt}}(\mathrm{~s}, \mathrm{t})=\left(\frac{\mathrm{As}}{\mathrm{p}_{\mathrm{LAB}}^{2}}\right) \exp \left\{\left(\mathrm{b}_{0}+2 \mathrm{~b}^{\prime} \ln \mathrm{s}\right) \mathrm{t}\right\} \tag{1}
\end{equation*}
$$

between 1 and $10 \mathrm{GeV} / \mathrm{c}$. The $K_{I}^{0} p \rightarrow K_{S}^{0} p$ cross section in the interval $0.0 \leq|t| \leq 0.025 \mathrm{GeV}^{2}$ was then determined from this parametrization, and included in the quoted differential cross sections for $\cos \theta>0.9$ (see Table II) and in the $K_{L}^{o} p \rightarrow K_{S}^{o} p$ total cross sections.

The $K_{L}^{o} p \rightarrow K_{S}^{0} p$ differential cross sections are characterized by distinct peaks in the forward and backward regions (t and $u$ channel Regge regions) that are observed at all energies. Some structure occurs through the entire angular distribution in the $s$ channel resonance region, but disappears above 3.5 GeV/c. The cross sections at backward and middle angles decrease with energy much more rapidly than in the forward direction.

The general energy dependence of the data can be seen in Fig. 2 where the cross section for $K_{L}^{O} p \rightarrow K_{S}^{0} p$ is plotted as a function of beam momentum. Cross sections from previous measurements ${ }^{9-11}$ are also included in this figure. Numerical values for the present data are recorded in Table III. For momenta above $\sim 1 \mathrm{GeV} / \mathrm{c}$ the $\mathrm{K}_{\mathrm{L}}^{0} \mathrm{p} \rightarrow \mathrm{K}_{\mathrm{S}}^{0} \mathrm{p}$ cross sections can be parametrized with the power law form:

$$
\begin{equation*}
\sigma_{K_{L}^{o}} p \rightarrow K_{S}^{o} p=A p_{L A B}^{-n} \tag{2}
\end{equation*}
$$

giving $\mathbf{n} \sim 2.1$ as shown in Table IV. This energy dependence is substantially greater than for the $\pi$ induced reactions $\pi^{-} p \rightarrow \pi^{0} n$ and $\pi^{-} p \rightarrow \eta^{0} n$ where $\mathrm{n}=1.09 \pm 0.03,{ }^{12}$ and $\mathrm{n}=1.37 \pm 0.04,{ }^{13}$ for data in the interval $\sim 5$ to $50 \mathrm{GeV} / \mathrm{c}$. Somewhat better agreement is obtained from comparisons with the $K$ induced reactions $\mathrm{K}^{-} \mathrm{p} \rightarrow \overline{\mathrm{K}}^{\circ} \mathrm{n}$ and $\mathrm{K}^{+}{ }_{\mathrm{n}} \rightarrow \mathrm{K}^{\circ} \mathrm{p}$ where $\mathrm{n}=1.5 \pm 0.1$, ${ }^{14}$ and $\mathrm{n}=2.10 \pm 0.05,15$ respectively.

Little structure is apparent in the $K_{L}^{0} p \rightarrow K_{S}^{0} p$ cross section in Fig. 2 where the data are plotted with a log scale. The low energy data do reveal resonance enhancements, however, as shown in Fig. 3 where the cross section is replotted
as a function of center-of-mass energy. These data are also recorded in Table V. Although the $\mathrm{K}^{\mathrm{O}} \mathrm{p} \rightarrow \mathrm{K}^{0} \mathrm{p}$ and $\overline{\mathrm{K}}^{0} \mathrm{p} \rightarrow \overline{\mathrm{K}}^{\mathrm{O}} \mathrm{p}$ channels are mixed in the data, if the exotic $K^{0} p \rightarrow K^{0} p$ cross section is smoothly varying with energy only $\Sigma$ resonances should appear in the $K_{L}^{0} p \rightarrow K_{S}^{o} p$ cross section. The enhancements can in fact be explained by the known resonances, the $\Sigma(1765)$, $\Sigma(2030)$ and $\Sigma(2250)$ as indicated in Fig. 3.

## IV. FORWARD CROSS SECTIONS

## A. Phenomenology

For $K_{L}^{o} p \rightarrow K_{S}^{0} p$ scattering the known $t$ channel exchanges are the $\rho, \omega^{0}$ and $\phi$ mesons. Neglecting the $\phi$ contribution, since the $\phi N \bar{N}$ coupling is thought to be small, ${ }^{16}$ the $K_{L}^{o} p \rightarrow K_{S}^{o} p$ reaction can be parametrized in terms of $\rho$ and $\omega^{\circ}$ exchanges. ${ }^{17}$

The reactions $K_{L}^{o} p \rightarrow K_{S}^{0} p$ and $\pi^{-} p \rightarrow \pi^{0} n$ can then be related by adopting a particular model for the coupling constants. For example assuming $\operatorname{SU}(3)$ we obtain:

$$
\begin{equation*}
\mathrm{A}_{\pi-\mathrm{p} \rightarrow \pi_{\mathrm{n}}}=\sqrt{2} \rho \equiv \sqrt{2} \mathrm{~V} \tag{3}
\end{equation*}
$$

and

$$
A_{K_{L}^{o} p \rightarrow K_{S}^{o}}=\frac{1}{2}\left\{(4 \mathrm{~F}-1) \omega^{o}-\rho\right\} \equiv(2 \mathrm{~F}-1) \mathrm{V}
$$

where the exchange degeneracy of $\rho$ and $\omega^{\circ}$ Regge trajectories is used to equate these amplitudes to a "universal" vector exchange amplitude $\mathrm{V}(\mathrm{s}, \mathrm{t})$. In Eq. (3) the symmetric and antisymmetric $\operatorname{SU}(3)$ octet couplings are defined such that $\mathrm{F}+\mathrm{D}=1$. Similar relations also follow for the differences of $\pi \mathrm{N}$ and KN total cross sections. These can be obtained from the optical theorem results:

$$
\begin{equation*}
\Delta \sigma_{\pi^{ \pm} \mathrm{p}}^{\mathrm{TOT}}=\sigma_{\pi^{-} \mathrm{p}}^{\mathrm{TOT}}-\sigma_{\pi^{+} \mathrm{p}}^{\mathrm{TOT}} \equiv-\frac{4 \sqrt{2} \pi}{\mathrm{k}} \operatorname{Im} \mathrm{~A}_{\pi^{-} \mathrm{p} \rightarrow \pi^{\mathrm{o}_{\mathrm{n}}}} \tag{4a}
\end{equation*}
$$

and

$$
\begin{equation*}
\Delta \sigma_{K \pm n}^{\mathrm{TOT}}=\sigma_{K_{n}^{-}}^{\mathrm{TOT}}-\sigma_{K_{n}^{+}}^{\mathrm{TOT}} \equiv-\frac{8 \pi}{k} \operatorname{Im} A_{K_{L}^{o} p \rightarrow K_{S}^{o} p}^{o} \tag{4b}
\end{equation*}
$$

For the present analysis the scattering will be discussed in terms of $s$ channel helicity amplitudes, $\mathrm{f}_{\Delta \lambda}$, where the net helicity flip in the reaction is specified by $\Delta \lambda$. From hyper charge exchange reactions estimates of the $\operatorname{SU}(3)$ factor F (see Eq. (3)) have been obtained for both helicity flip and nonflip amplitudes ${ }^{18}$ :

$$
\begin{equation*}
F_{\Delta \lambda=0} \sim 1.25 \tag{5}
\end{equation*}
$$

and

$$
\mathrm{F}_{\Delta \lambda=1} \sim 0.25
$$

Using these values, we observe (cf. Eq. (3)) that the $\omega^{\circ}$ contribution should dominate the $\rho$ contribution to the $K_{L}^{o} p \rightarrow K_{S}^{o} p$ helicity nonflip amplitude. However, the $\omega^{0}$ approximately decouples irom the helicity flip amplitude such that

$$
\begin{equation*}
{ }^{f_{\Delta \lambda=1}}\left(\mathrm{~K}_{\mathrm{L}}^{o} \mathrm{p} \rightarrow \mathrm{~K}_{\mathrm{S}}^{0} \mathrm{p}\right) \approx-\frac{1}{2} \rho=\frac{1}{2 \sqrt{2}} \mathrm{f} \Delta \lambda=1^{\left(\pi^{-} p \rightarrow \pi^{0} \mathrm{n}\right)} \tag{6}
\end{equation*}
$$

This yields an estimate for the helicity flip contribution to the $K_{L}^{o} p \rightarrow K_{S}^{o} p$ cross section:

$$
\begin{equation*}
\left(\frac{d \sigma}{d t}\right)_{\Delta \lambda=1}\left(K_{L}^{o} p \rightarrow K_{S^{o}}^{o} p\right) \approx \frac{1}{8}\left(\frac{d \sigma}{d t}\right)\left(\pi^{-} p \rightarrow \pi^{o} n\right) \tag{7}
\end{equation*}
$$

where the $\pi^{-} p \rightarrow \pi^{\circ} \mathrm{n}$ cross section is taken to be dominantly helicity flip.

## B. Differential Cross Sections

The forward differential cross sections for our data are shown in Fig. 4 and recorded in Table VI for five momentum intervals between 1.5 and 10 $\mathrm{GeV} / \mathrm{c}$. The data are strongly forward peaked, with a distinct break in the
cross section slope at $|t| \sim 0.3 \mathrm{GeV}^{2}$. No other fixed $t$ structure is observed in the differential cross section for $|t| \lesssim 2 \mathrm{GeV}^{2}$.

The forward peaking in the $K_{L}^{0} p \rightarrow K_{S}^{0} p$ cross section indicates that the s channel helicity nonflip amplitude is large for this reaction. By comparison, in the dominantly helicity flip reaction $\pi^{-} p \rightarrow \pi^{\circ} \mathrm{n}$ the cross section actually decreases in the very forward direction, and has a significant minimum near $|t| \sim 0.6 \mathrm{GeV}^{2}$. This latter feature has been offered as confirmation for the Regge signature factor in scattering amplitudes:

$$
\begin{equation*}
\mathrm{f}_{\Delta \lambda}(\mathrm{t})=\widetilde{\mathrm{f}}_{\Delta \lambda}(\mathrm{t})\left(1-\mathrm{e}^{-\mathrm{i} \pi \alpha_{\rho}(\mathrm{t})}\right) \tag{8}
\end{equation*}
$$

Since a linear trajectory $\alpha_{\rho}(\mathrm{t})$ crosses zero near $|\mathrm{t}| \sim 0.6 \mathrm{GeV}^{2}, \rho$ exchange dominated cross sections are predicted to have a minimum near this value of momentum transfer, in agreement with the $\pi^{-} p \rightarrow \pi^{0} \mathrm{n}$ data. Although $\alpha_{\omega}(\mathrm{t})$ is thought to be essentially equal to $\alpha_{\rho}(\mathrm{t})$ no similar minimum is observed in the $K_{L}^{o} p \rightarrow K_{S}^{o} p$ cross section (see Fig. 4). Further comparisons of the $K_{L}^{o} p \rightarrow K_{S}^{o} p$ and $\pi^{-} p \rightarrow \pi^{0} n$ differential cross sections are presented in Section IV.E.

## C. Phase of the $K_{L}^{0} p \rightarrow K_{S}^{0}$ Forward Amplitude

In the very forward direction the s channel helicity flip amplitude must vanish to conserve angular momentum. Thus at $\mathrm{t}=0$ the helicity nonflip amplitude provides the only contribution to the differential cross section. Since the imaginary part of this amplitude is determined by $\Delta_{\mathrm{K}^{ \pm} \mathrm{n}}^{\mathrm{TOT}}$ (see Eq. (4)), it is possible to evaluate the phase of the forward amplitude:

$$
\begin{equation*}
\phi=\tan ^{-1}\left(\operatorname{Im} \mathrm{f}_{\Delta \lambda=0} / \operatorname{Ref}_{\Delta \lambda=0}\right) \tag{9}
\end{equation*}
$$

We first consider the imaginary part of the $K_{L}^{o} p \rightarrow K_{S}^{o} p$ amplitude. The optical theorem (Eq. (4)) indicates that the sign of the imaginary part is
negative, that is $\sigma_{\mathrm{K}^{-} \mathrm{n}}^{\mathrm{TOT}}>\sigma_{\mathrm{K}^{+} \mathrm{n}}^{\text {TOT }}{ }^{22-24}$ Evaluating the imaginary contribution to the forward $K_{L}^{o} p \rightarrow K_{S}^{o} p$ cross section we obtain:

$$
\begin{equation*}
\left(\frac{\mathrm{d} \sigma}{\mathrm{dt}}\right)_{\mathrm{opt}}=\frac{1}{64 \pi}\left(\frac{\Delta \sigma_{\mathrm{K}^{ \pm} \mathrm{n}}^{\mathrm{TOT}}}{\overline{\mathrm{c}}}\right)^{2} \tag{10}
\end{equation*}
$$

This then allows the ratio of real to imaginary parts of the forward amplitude to be determined from the data:

$$
\begin{equation*}
\frac{\operatorname{Ref}_{\Delta \lambda=0}}{\operatorname{Im} f_{\Delta \lambda=0}}= \pm \sqrt{\frac{\left(\frac{d \sigma}{d t}\right)_{0}}{\left(\frac{\mathrm{~d} \mathrm{\sigma}}{\mathrm{dt}}\right)_{\text {opt }}}-1} \tag{11}
\end{equation*}
$$

The quadratic sign ambiguity in Eq. (11) can be resolved by recourse to dispersion relations. ${ }^{1,25}$ An equivalent result is obtained using simple Regge theory where the signature factor gives the phase of the forward amplitude:

$$
\begin{equation*}
\left(\frac{\operatorname{Ref}}{\operatorname{Im} \mathrm{f}}\right) \sim \tan \left(\frac{\pi \alpha(0)}{2}\right) \tag{12}
\end{equation*}
$$

For reasonable $\omega^{\circ}, \rho$ Regge trajectories $\alpha(0)$ lies in the interval $0<\alpha(0)<1$ implying that Ref/Imf is positive. The phase, $\phi$, is thus defined to lie in the third quadrant.

The $K_{L}^{o} p \rightarrow K_{S}^{o} p$ cross sections are extrapolated to $t=0$ using the exponential parametrization:

$$
\begin{equation*}
\frac{\mathrm{d} \sigma}{\mathrm{dt}}=\left(\frac{\mathrm{d} \sigma}{\mathrm{dt}}\right)_{0} \mathrm{e}^{\mathrm{bt}} \tag{13}
\end{equation*}
$$

in the momentum transfer interval $0.025 \leq|t| \leq 0.25 \mathrm{GeV}^{2}$. The forward differential cross sections are determined in five momentum intervals between 1.5 and $10 \mathrm{GeV} / \mathrm{c}$. The resulting cross sections and slopes are shown in Figs. 5 and 6, and recorded in Table VII.

Previous experimental results ${ }^{1-3,9,11}$ on the forward $K_{L}^{o} p \rightarrow K_{S}^{0} p$ cross section are also included in Fig. 5. Of these, Darriulat et al. , ${ }^{1}$ Buchanan et al. , ${ }^{2}$ and Birulev et al. , ${ }^{3}$ are $K_{S}^{o}$ coherent regeneration experiments measuring the $K_{L}^{o} p \rightarrow K_{S}^{0} p$ amplitude only at $t=0$. This technique exploits the observed interference in the proper time distribution of $\pi^{+} \pi^{-}$decays of both $K_{S}^{0}$ and $K_{L}^{o}$ mesons to determine the magnitude and the phase of the forward $K_{L}^{o} p \rightarrow K_{S}^{o} p$ scattering amplitude. Good agreement is found between the present extrapolations of the $K_{L}^{o} p \rightarrow K_{S}^{o} p$ cross section and the results of the coherent regeneration experiments.

We note that the forward cross section data are well described by the power law form:

$$
\begin{equation*}
\left(\frac{\mathrm{d} \sigma}{\mathrm{dt}}\right)_{0}=\mathrm{Ap}_{\mathrm{LAB}}^{-\mathrm{n}_{0}} \tag{14}
\end{equation*}
$$

Values of these parameters determined in several different momentum intervals are recorded in Table VIII. The curve in Fig. 5 results from the parametrization ( $\mathrm{n}_{0}=1.40$ ) of the $\mathrm{K}_{\mathrm{L}}^{\mathrm{o}} \mathrm{p} \rightarrow \mathrm{K}_{\mathrm{S}}^{\mathrm{o}} \mathrm{p}$ data above $2.5 \mathrm{GeV} / \mathrm{c} .{ }^{26}$

Values for ( $\mathrm{d} \sigma / \mathrm{dt})_{\text {opt }}$, obtained from $\Delta \sigma_{\mathrm{K}^{ \pm} \mathrm{n}}^{\mathrm{TOT}}$ (see Eq. (10)), are shown in Fig. 7. The uncertainties in the cross section differences are calculated summing the $\mathrm{K}^{ \pm} \mathrm{n}$ cross section errors in quadrature. For the data of Ref. 22 a $2 \%$ systematic normalization uncertainty is included in both the $\mathrm{K}^{+} \mathrm{n}$ and $\mathrm{K}^{-} \mathrm{n}$ data. Systematic uncertainties are not included for the Galbraith et al., , ${ }^{23}$ or the Denisov et al. , ${ }^{24}$ data where the statistical errors are already large. ${ }^{27}$

Data on both $\mathrm{K}^{ \pm} \mathrm{n}$ total cross sections exist below $3.3 \mathrm{GeV} / \mathrm{c}$ and above $6 \mathrm{GeV} / \mathrm{c}$. To obtain estimates of the $\mathrm{K}^{ \pm} \mathrm{n}$ cross section differences in the momentum interval 1.5 to $10 \mathrm{GeV} / \mathrm{c}$ two procedures are used. Between 1.5 and $2.5 \mathrm{GeV} / \mathrm{c}$ the ( $\mathrm{d} \sigma / \mathrm{dt})_{\text {opt }}$ data are directly averaged for comparison with
the $K_{L}^{0} p \rightarrow K_{S}^{0}$ forward data. Above $2.5 \mathrm{GeV} / \mathrm{c}$ the ( $\mathrm{d} \sigma / \mathrm{dt}$ ) opt data are fitted to the power law form:

$$
\begin{equation*}
\left(\frac{\mathrm{d} \sigma}{\mathrm{dt}}\right)_{\mathrm{opt}}=\mathrm{Bp}_{\mathrm{LAB}}^{-\mathrm{n}_{\mathrm{opt}}} \tag{15}
\end{equation*}
$$

giving $B=1965 \pm 202 \mu \mathrm{~b} / \mathrm{GeV}^{2}$ and $\mathrm{n}_{\text {opt }}=1.32 \pm 0.06$, as shown by the curve in Fig. 7. This result is then used to determine the average ( $\mathrm{d} \sigma / \mathrm{dt}$ ) opt cross sections in the momentum interval recorded in Table VII . Finally, the quoted errors in (d $\sigma / \mathrm{dt}$ ) ${ }_{\text {opt }}$ are scaled by a factor of two to allow for possible uncertainties coming from our choice of averaging procedures.

The results for the ratio of real to imaginary parts of the forward amplitude, the phase, and the intercept of the effective Regge trajectory then follow from Eqs. (9)-(12) and are recorded in Table VII. The phase of the $K_{L}^{o} p \rightarrow K_{S}^{o} p$ scattering amplitude is plotted together with previous measurements ${ }^{1-3}$ in Fig. 8. The data indicate that the phase, $\phi$, has little or no energy dependence in the interval 1.5 to $50 \mathrm{GeV} / \mathrm{c}$. The average values of $\phi$ from the present experiment, $\phi=-133.9 \pm 4.0^{\circ}$, and from the Serpukhov results, $\phi=-132.3 \pm 5.7^{\circ}, 3$ are in good agreement and are consistent with the constant phase $\phi=-133.4 \pm 3.3^{\circ}$ for the combined data.

Recently the possibility has been suggested ${ }^{28}$ that the value of the $\mathrm{K}_{\mathrm{L}}^{\mathrm{o}} \rightarrow \pi^{+} \pi^{-}$ decay parameter $\eta_{+-}$is different from the previously accepted value. ${ }^{7}$ A change in $\eta_{+-}$could potentially alter the results of the $K_{S}^{o}$ coherent regeneration experiments, and in particular the highest energy (Serpukhov) data. However, the agreement of the present experiment with the coherent regeneration results in the same momentum interval (see Figs. 5 and 8) suggests that the existing $\mathrm{K}_{\mathrm{S}}^{0}$ regeneration analyses are correct or insensitive to the possible change in $\eta_{+-}$. A further check is provided by the consistancy of the phase determined
directly from the $K_{S}^{0}$ coherent regeneration experiments with the phase determined using their values of ( $\mathrm{d} \sigma / \mathrm{dt})_{0}$ 'gether with Eqs. (9)-(11). To illustrate this comparison the solid curve in Fig. 8 shows the phase resulung from the $\mathrm{p}_{\mathrm{LAB}}^{-\mathrm{n}}$ parametrizations of $(\mathrm{d} \sigma / \mathrm{dt})_{0}$ and ( $\left.\mathrm{d} \sigma / \mathrm{dt}\right)_{\text {opi }}$ or $\mathrm{p}_{\mathrm{LAB}} \geq 2.5 \mathrm{GeV} / \mathrm{c}$ (see Eqs. (14) and (15)). Although the slight inequality of $n_{0}$ and $n_{\text {opt }}$ does predict a small change in $\phi$ with energy, it is clear that the values of $\phi$ obtained in this manner are consistent with all the $\mathrm{K}_{\mathrm{S}}^{0}$ coherent iegeneration data.

Having determined that the phase of the forward $K_{L}^{0} p \rightarrow K_{S}^{0} p$ amplitude is approximately constant, the average phase from the present experiment, $\phi_{\mathrm{AVG}}=-133.9 \pm 4.0^{\circ}$ is used to evaluate the contribution of the imaginary part of the forward cross section for our data. Thus our ( $\mathrm{d} \sigma / \mathrm{dt})_{0}$ data have been scaled by the factor $\sin ^{2} \phi_{\text {AVG }}$ and plotted with dashed error bars in Fig. 7 to illustrate the common energy dependence of the $(\mathrm{d} \sigma / \mathrm{dt})_{0}$ and $(\mathrm{d} \sigma / \mathrm{dt})_{\mathrm{opt}}$ data.

The intercepts of the $\omega^{\circ}, \rho$ Regge trajectory calculated from the present data (see Eq. (12)) are given in Table VII and yield the average value $\alpha(0)=0.49 \pm 0.05$. The forward differential cross section for $\pi^{-} p \rightarrow \pi^{0}{ }^{0},{ }^{29,32}$ and the total cross section differences, $\Delta \sigma_{\pi^{ \pm} \mathrm{p}}^{\mathrm{TOT}},{ }^{24}$ can be used in an analogous manner to determine the phase of the forward amplitude and therefore the $\rho$ Regge trajectory, $\alpha_{\rho}(0)$. These latter data indicate that $\alpha_{\rho}(0)$ is almost independent of momentum, and has a value $\alpha_{\rho}(0) \sim 0.56,{ }^{30}$ in the momentum interval spanned by our $K_{L}^{o} p \rightarrow K_{S}^{o} p$ data. The phases of the forward $K_{L}^{o} p \rightarrow K_{S}^{o} p$ and $\pi^{-} p \rightarrow \pi^{o} n$ scattering amplitudes are similar therefore, and in agreement with the canonical value for the $\omega^{\circ}, \rho$ Regge intercept, $\alpha(0) \sim 0.5$.

## D. Energy Dependence of the $K_{L}^{0} p \rightarrow K_{S}^{0} p$ Differential Cross Section

Simple Regge theory predicts that for reactions with only one (or two exchange degenerate) $t$ channel Regge exchange(s) the cross section is proportional to:

$$
\begin{equation*}
\frac{\mathrm{d} \sigma}{\mathrm{dt}}(\mathrm{~s}, \mathrm{t}) \propto \frac{\mathrm{s}^{2 \alpha(\mathrm{t})}}{\mathrm{p}_{\mathrm{LAB}}^{2}} \tag{16}
\end{equation*}
$$

where $\alpha(t)$ is the appropriate Regge trajectory. For $\omega^{\circ}$ or $\rho$ exchange $\alpha(t)$ is approximately of the form

$$
\begin{equation*}
\alpha(\mathrm{t})=0.5+\alpha^{\prime} \mathrm{t} \tag{17}
\end{equation*}
$$

with $\alpha^{\prime} \sim 1 \mathrm{GeV}^{2}$.
Thus, if the forward differential cross sections are approximately exponential in momentum transfer, Eq. (16) predicts an increase in the forward slopes with energy:

$$
\begin{equation*}
\frac{\mathrm{d} v}{\mathrm{dt}}=\left(\frac{\mathrm{d} \tau}{\mathrm{dt}}\right)_{0} \exp \left\{\left(\mathrm{~b}_{0}+2 \alpha^{\prime} \ln \mathrm{s}\right) \mathrm{t}\right\} \tag{18}
\end{equation*}
$$

The forward slopes for $\mathrm{K}_{\mathrm{L}}^{0} \mathrm{p} \rightarrow \mathrm{K}_{\mathrm{S}}^{0} \mathrm{p}$ plotted in Fig. 6 are consistent with an increase in the slopes with increasing momentum. Parametrizing the energy dependence of the slopes as in Eq. (18) we obtain $b_{0}=3.1 \pm 4.5 \mathrm{GeV}^{-2}$ and $\alpha^{\prime}=1.4 \pm 1.1 \mathrm{GeV}^{-2}$, where the large uncertainties result in part from the parameters $b_{0}$ and $\alpha^{\prime}$ being highly correlated.

Alternatively the data can be parametrized with the form of Eq. (16); $\alpha(t)$ can then be directly determined as a function of momentum transfer. Tabular results from fits of our $K_{L}^{0} p \rightarrow K_{S}^{0} p$ data to Eq. (16), and the alternate forms:

$$
\begin{equation*}
\frac{\mathrm{d} \sigma}{\mathrm{dt}} \propto \mathrm{p}_{\mathrm{LAB}}^{2 \alpha(\mathrm{t})-2} \tag{19a}
\end{equation*}
$$

and

$$
\begin{equation*}
\frac{\mathrm{d} \sigma}{\mathrm{dt}} \propto \frac{(-\mathrm{u})^{2 \alpha(\mathrm{t})}}{\mathrm{p}_{\mathrm{LAB}}^{2}} \tag{19b}
\end{equation*}
$$

are recorded in Table IX. The values of $\alpha(\mathrm{t})$ are observed to be only slightly different in the three parametrizations.

For uniformity of comparison with other analyses, the $\alpha(t)$ determined from Eq. (19a) are plotted in Fig. 9. The points for $t<0$ are from the present experiment, however the solid point at $\mathrm{t}=0$ is obtained using all the data above $2.5 \mathrm{GeV} / \mathrm{c}$ (see Table VIII). In addition, the energy dependence of the contribution of the imaginary part of the forward cross section (from the $\Delta \sigma_{\mathrm{K}^{ \pm} \mathrm{n}}^{\mathrm{TOT}}$ data) has been plotted as the open point in Fig. 9. For comparison the canonical $\omega^{\circ}, \rho$ Regge trajectory, $\alpha(\mathrm{t})=0.5+\mathrm{t}$, is shown as the solid line in Fig. 9. Approximate agreement is observed between the $K_{L}^{0} p \rightarrow K_{S}^{0} p$ data and the linear Regge trajectory, except near $|t| \approx 0$.

Interestingly, the value for $\alpha(0)$ determined from the energy dependence of the forward cross sections, $\alpha(0)=0.30 \pm 0.03$, is in substantial disagreement with the value determined from the phase of the forward amplitude, $\alpha(0)=0.49 \pm 0.05$. This result indicates a failure of the Regge phase-energy relation. Similar disagreements with the Regge predictions for helicity nonflip amplitudes at momentum transfers, $\mathrm{t}<0$, have been observed in the $\pi \mathrm{N}$ amplitude analyses at $6 \mathrm{GeV} / \mathrm{c}^{19-21}$

For comparison with the present results, the $\rho$ Regge trajectory derived from $\pi^{-} p \rightarrow \pi^{\circ}$ n data ${ }^{31}$ is shown shaded in Fig. 9. Since $\omega^{\circ}$ and $\rho$ Regge trajectories are essentially exchange degenerate for $t>0$, the trajectories are expected to be nearly equal in the physical scattering region, $t \leq 0$. This is seen to be true for $|t| \gtrsim 0.4 \mathrm{GeV}^{2}$, however the $K_{L}^{o} p \rightarrow K_{S}^{o} p$ data are
systematically lower in the very forward direction. In particular, at $t=0$,


This result has a direct bearing on the SU(3) relations in Eq. (3). For $t$ channel exchange models satisfying factorization and having exchange degenerate $\rho$ and $\omega^{o}$ trajectories, the $K_{L}^{o} p \rightarrow K_{S}^{0} p$ and $\pi^{-} p \rightarrow \pi^{\circ} n$ cross sections are predicted to have the same energy dependence, and to be simply related by the constant, F, in Eq. (3). A direct comparison of the $\pi^{-} p \rightarrow \pi^{\circ} n,{ }^{29,32}$ and $K_{L}^{0} p \rightarrow K_{S}^{0} p$ forward cross sections is shown in Fig. 10 where the dashed curve represents the $K_{L}^{o} p \rightarrow K_{S}^{o} p$ data (see Fig. 5). Clearly a fixed ratio between these cross sections does not exist, implying that the simple picture given by Eq. (3) is incorrect, or at least incomplete.

Since $\omega^{0}$ and $\rho$ exchanges dominate $K_{L}^{O} p \rightarrow K_{S}^{0} p$ and $\pi^{-} p \rightarrow \pi^{0} n$ scattering respectively, one explanation for the dissimilarity in the energy behavior of the forward cross sections is simply that $\omega^{0}$ and $\rho$ exchange amplitudes have intrinsically different energy dependences. However, as an additional check the forward cross sections for $K_{L}^{o} p \rightarrow K_{S}^{o} p$ and $K N$ charge exchange scattering ${ }^{33}$ (having $\rho$ and $\mathrm{A}_{2} \mathrm{t}$ channel quantum numbers) are compared in Fig. 11. In contrast to the result in Fig. 10, the $K_{L}^{O} p \rightarrow K_{S}^{0} p$ and $K N$ charge exchange forward cross sections in Fig. 11 are observed to agree in magnitude (a mere coincidence ?) as well as in energy dependence. These results then suggest that $\omega^{\circ}, \rho$ and $\mathrm{A}_{2}$ exchange amplitudes are consistent with exchange degeneracy (equal energy dependence), but that $t$ channel factorization is broken for reactions with different particles in the s channel. Absorption or direct channel effects are thought to be important for helicity nonflip amplitudes for $t<0 ;{ }^{34}, 35$ the present result suggests that direct channel effects are also important at $\mathrm{t}=0$.

The approximate energy independence of the phases of the forward scattering amplitudes for $K_{L}^{o} p \rightarrow K_{S}^{o} p$ and $\pi^{-} p \rightarrow \pi^{o} n$, combined with the observed inequality of the energy dependence of their forward cross sections, then implies that the energy dependences of the total cross section differences, $\Delta \sigma_{\mathrm{K}^{ \pm} \mathrm{n}}^{\mathrm{TOT}}$ and $\Delta \sigma_{\pi^{ \pm} \mathrm{p}}^{\mathrm{TOT}}$ (see Eq. (4)), must also be unequal This is experimentally the case ${ }^{24} ; \Delta \sigma_{\pi^{ \pm} \mathrm{p}}^{\mathrm{TOT}}$ decreases with energy substantially more slowly than $\Delta \sigma_{\mathrm{K}^{ \pm} \mathrm{n}}^{\mathrm{TOT}}$. Similar discrepancies between the energy dependence of forward cross sections are also observed in other channels. ${ }^{36}$
E. Amplitude Structure of $K_{L}^{o} p \rightarrow K^{o}{ }^{p}$ Forward Cross Sections

Amplitude analyses of $\pi \mathrm{N} \rightarrow \pi \mathrm{N}$ scattering data have recently been completed at $6 \mathrm{GeV} / \mathrm{c}^{19-21}$ One result is the essentially model independent determination of the $t$ channel isospin one " $\rho$ " exchange amplitude. Assuming that $\rho$ and $\omega^{\circ}$ vector exchange amplitudes have similar structures (as a function of momentum transfer), the $\rho$ amplitudes from $\pi \mathrm{N}$ scattering should then be simply related by a multiplicative factor, a $\Delta \lambda$, to the $\rho+\omega^{\circ}$ amplitude in $K_{L}^{o} p \rightarrow K_{S}^{o} p$ scattering:

$$
\begin{equation*}
\mathrm{f}_{\Delta \lambda}\left(\mathrm{K}_{\mathrm{L}}^{o} \mathrm{p} \rightarrow \mathrm{~K}_{\mathrm{S}}^{\mathrm{o}} \mathrm{p}\right)=\mathrm{a}_{\Delta \lambda} \mathrm{V}_{\Delta \lambda} \tag{20}
\end{equation*}
$$

where $\Delta \lambda$ is the net helicity flip in the reaction.
In the following analysis the $V_{\Delta \lambda}$ are chosen to be the Saclay $I_{t}=1$ amplitudes ${ }^{20}$ (see Eq. (3)) and the coefficients in Eq. (20) are obtained by fitting the sum of the $\pi N$ amplitudes to the $K_{L}^{o} p \rightarrow K_{S}^{0} p$ differential cross section in the momentum transfer interval $0.025 \leq|t| \leq 0.30 \mathrm{GeV}^{2}$. Fits to the $K_{L}^{0} p \rightarrow K_{S}^{0} p$ data in larger momentum transfer intervals disagree substantially with the data near $\mathrm{t}=0$ and are not considered. The analysis of the 5 to $7.5 \mathrm{GeV} / \mathrm{c}$ $K_{L}^{o} p \rightarrow K_{S}^{0} p$ data is shown in Fig. 12. The solid curve in Fig. 12a represents
the best fit of the SACLAY amplitudes ${ }^{20}$ to the $\mathrm{K}_{\mathrm{L}}^{\mathrm{O}} \mathrm{p} \rightarrow \mathrm{K}_{\mathrm{S}}^{\mathrm{O}} \mathrm{p}$ data; the shaded region in Fig. 12a displays the uncertainties in the structure of the SACLAY amplitudes. For completeness, the polarization prediction for this solution is shown in Fig. 13; again the shaded region reflects the uncertainties on the on the $\pi \mathrm{N}$ amplitudes.

The coefficients corresponding to the fit in Fig. 12a are: $a_{\Delta \lambda=0}=1.85 \pm 0.77$ and $\mathrm{a}_{\Delta \lambda=1}=-0.48 \pm 0.09$. This result is in approximate agreement with the $\mathrm{SU}(3)$ predictions, $\mathrm{a}_{\Delta \lambda}=(2 \mathrm{~F}-1)$ of Eqs. (3) and (5). We note that the sign of the a ${ }_{\Delta \lambda=1}$ coefficient has been chosen to agree with the $\operatorname{SU}(3)$ prediction. Experimentally this sign could be determined from the polarization in $K_{L}^{O} p \rightarrow K_{S}^{o} p$ scattering.

The comparison of the SACLAY amplitudes to the $K_{L}^{o} p \rightarrow K_{S}^{o} p$ data (Fig. 12a) indicates that the $\pi \mathrm{N}$ amplitudes predict too small a cross section for $|t| \gtrsim 0.3 \mathrm{GeV}^{2}$. The recent Argonne $\pi \mathrm{N}$ amplitude analysis, ${ }^{21}$ which has a smaller $I_{t}=1$ helicity nonflip amplitude than the SACLAY solution in this momentum transfer region, is in even greater disagreement with the $K_{L}^{o} p \rightarrow K_{S}^{o} p$ differential cross section.

To investigate the discrepancy between the $K_{L}^{o} p \rightarrow K_{S}^{o} p$ differential cross section and the $\pi \mathrm{N}$ amplitude results, the contribution of the separate helicity nonflip and flip amplitudes to the cross section are shown in Fig. 12b, c respectively. In these figures the SACLAY amplitudes are simply scaled by the appropriate ${ }_{\Delta \lambda}$ coefficient, Eq. (20). The SU(3) prediction, Eq. (7), for the helicity flip contribution to the $K_{L}^{0} p \rightarrow K_{S}^{0} p$ cross section is shown as the shaded curve in Fig. 12c. Thus the helicity nonflip amplitude is predicted to dominate the differential cross section in the very forward direction, $|t|<0.05 \mathrm{GeV}^{2}$, and for $|t| \gtrsim 0.4 \mathrm{GeV}^{2}$; whereas at intermediate values of
momentum transfer, $0.05<|\mathrm{t}|<0.4 \mathrm{GeV}^{2}$ the helicity flip amplitude is the most important. The predicted composition of the differential cross section is therefore quite complex.

Interestingly, in the momentum transfer interval where the largest discrepancy exists between the $K_{L}^{0} p \rightarrow K_{S}^{0} p$ data and the $\pi N$ amplitudes, the cross section is dominated by the helicity nonflip amplitude. Since $\omega^{0}$ exchange is thought to dominate the helicity nonflip amplitude (see Eqs. (3) and (5)), an intrinsic difference in the structure of $\rho$ and $\omega^{\circ}$ exchange amplitudes could account for the discrepancy observed in Fig. 12a. Alternatively, helicity nonflip amplitudes are thought to be substantially influenced by direct channel or absorptive effects. Thus while the $\rho$ and $\omega^{\circ}$ amplitudes may have a similar structure in a given reaction, these amplitudes, and in particular the $\rho$ exchange amplitude, may be significantly different in $\pi \mathrm{N}$ and KN channels.

Assuming that it is the helicity nonflip amplitude that differs substantially between $K N$ and $\pi N$ reactions, ${ }^{37}$ a good description of the $K_{L}^{o} p \rightarrow K_{S}^{o} p$ data can in fact be obtained using a parametrization that allows the real part of the helicity nonflip amplitude to differ significantly between $K_{L}^{o} p \rightarrow K_{S}^{o}{ }^{0}$ and $\pi^{-} \mathrm{p} \rightarrow \pi^{\circ} \mathrm{n}$ reactions. The amplitudes have the form:
$\operatorname{Im} f_{\Delta \lambda=0}(\mathrm{~s}, \mathrm{t})=\mathrm{g}_{0}\left(\frac{\mathrm{~s}}{\mathrm{~s}_{0}}\right)^{\alpha(\mathrm{t})+\Delta \alpha} \mathrm{e}^{A t} \mathrm{~J}_{0}(\mathrm{r} \sqrt{-\mathrm{t}})$
$\operatorname{Ref}{ }_{\Delta \lambda=0}(\mathrm{~s}, \mathrm{t})=\mathrm{g}_{0}\left(\frac{\mathrm{~s}}{\mathrm{~s}_{0}}\right)^{\alpha(\mathrm{t})+\Delta \alpha} \mathrm{e}^{\mathrm{At}}\left\{\left(1+\mathrm{at}+\mathrm{bt}{ }^{2}\right) \mathrm{e}^{\mathrm{Bt}}\right\} \tan \frac{\pi \alpha(0)}{2}$
and

$$
\begin{equation*}
\mathrm{f}_{\Delta \lambda=1}=\mathrm{g}_{1}\left(\frac{\mathrm{~s}}{\mathrm{~s}_{0}}\right)^{\alpha(\mathrm{t})+\Delta \alpha} \mathrm{e}^{\mathrm{At}} \mathrm{~J}_{1}(\mathrm{r} \sqrt{-\mathrm{t}})\left\{\tan \frac{\pi \alpha(\mathrm{t})}{2}+\mathrm{i}\right\} \tag{21}
\end{equation*}
$$

as suggested by the dual absorptive model (DAM) of Harari, ${ }^{34,38}$ and previously discussed in Ref. 39. In Eq. (21) $\alpha(\mathrm{t})$ is the $\omega^{\circ}, \rho$ Regge trajectory (see Eq. (7)) and $s_{0}$ is set to $1 \mathrm{GeV}^{2}$. The parameters obtained by fitting the $K_{L}^{0} p \rightarrow K_{S}^{0} p$ data are given in Table $X$; the curves in Fig. 4 show the comparison of the DAM to the $K_{L}^{o} p \rightarrow K_{S}^{0} p$ data. The contributions of the helicity nonflip and flip DAM amplitudes to the 5-7.5 GeV/c data are shown in Figs. 12b, c, and the polarization prediction is shown in Fig. 13.

We note that the DAM, the $\pi \mathrm{N}$ amplitudes, and the $\mathrm{SU}(3)$ prediction (Eq. (7)) are in approximate agreement for the helicity flip contribution to the cross section, Fig. 12c. In contrast, the DAM helicity nonflip amplitude is much larger than the $\pi N$ amplitude result for $|t| \gtrsim 0.1 \mathrm{GeV}^{2}$. In particular for $0.1 \leq|t| \leq 0.6 \mathrm{GeV}^{2}$ the magnitude of the imaginary part of the DAM nonflip amplitude, shown dashed in Fig. 12b, is sufficient to duplicate the $\pi N$ amplitude result which itself is predominantly imaginary in this momentum transfer interval. ${ }^{19-21}$ Thus the difference in the $K_{L}^{o} p \rightarrow K_{S}^{o} p$ and $\pi N$ amplitudes is attributed, in the DAM parametrization, to the much larger real part in the helicity nonflip amplitude in the $K_{L}^{0} p \rightarrow K_{S}^{0} p$ reaction.

## V. CROSS SECTIONS AT LARGE ANGLES

The analyses of our $K_{L}^{0} p \rightarrow K_{S}^{0} p$ backward scattering data and $90^{\circ}$ scattering data ${ }^{5}$ have been published previously. However for completeness a summary is provided here, and the data presented in tabular form for reference purposes.

Typically, scattering in the backward direction is discussed in terms of the possible $u$ channel exchanges. Reducing the $K_{L}^{o} p \rightarrow \mathrm{pK}_{S}^{\mathrm{o}}$ scattering
amplitudes into the states of well-defined strangeness:

$$
\mathrm{A}_{\mathrm{K}_{\mathrm{L}}^{\mathrm{o}} \mathrm{p} \rightarrow \mathrm{pK}_{\mathrm{S}}^{\mathrm{o}}}=\frac{1}{2}\left\{\mathrm{~A}_{\mathrm{K}^{\mathrm{o}} \mathrm{p} \rightarrow \mathrm{pK}^{\mathrm{o}}}-\mathrm{A}_{\mathrm{K}_{\mathrm{p}}^{\mathrm{o}} \rightarrow \mathrm{p}^{\mathrm{o}}}\right\}
$$

we observe that the reaction $\mathrm{K}^{\circ} \mathrm{p} \rightarrow \mathrm{pK}^{\circ}$ allows only $u$ channel $\Sigma$ exchanges, whereas the channel $\overline{\mathrm{K}}^{\mathrm{O}} \mathrm{p} \rightarrow \mathrm{p} \mathrm{K}^{0}$ is exotic. Thus at sufficiently high energies the $\mathrm{K}^{\mathrm{o}} \mathrm{p} \rightarrow \mathrm{pK}{ }^{\mathrm{o}}$ amplitude should dominate the $\overline{\mathrm{K}}^{\mathrm{o}} \mathrm{p} \rightarrow \mathrm{pK}^{\mathrm{o}}$ amplitude, isolating the $\Sigma$ exchange contribution to the $K_{L}^{0} p \rightarrow \mathrm{pK}_{S}^{0}$ differential cross section:

$$
\frac{\mathrm{d} \sigma}{\mathrm{dt}}\left(\mathrm{~K}_{\mathrm{L}}^{\mathrm{o}} \mathrm{p} \rightarrow \mathrm{pK}_{\mathrm{S}}^{\mathrm{o}}\right) \xrightarrow[\text { energy }]{\text { large }} \frac{1}{4} \frac{\mathrm{~d} \sigma}{\mathrm{dt}}\left(\mathrm{~K}^{\mathrm{o}} \mathrm{p} \rightarrow \mathrm{pK}^{\mathrm{o}}\right)
$$

Comparisons of the backward $\mathrm{K}_{\mathrm{L}}^{0} \mathrm{p} \rightarrow \mathrm{pK}_{\mathrm{S}}^{0}$ cross sections to backward $\mathrm{K}^{+} \mathrm{p} \rightarrow \mathrm{pK}^{+}$ scattering (where $\Lambda$ and $\Sigma$ exchanges occur in the $u$ channel) and to the reaction $\pi^{-} \mathrm{p} \rightarrow \Lambda^{\circ} \mathrm{K}^{0}$, which also isolates u channel $\Sigma$ exchange, are presented in Ref. 5.

To determine the cross sections at $180^{\circ}\left(u^{\prime}=0\right)$, the $K_{L}^{o} p \rightarrow p K_{S}^{o}$ data, shown in Fig. 14 and presented in Table XI, are parametrized with the form:

$$
\frac{\mathrm{d} \sigma}{\mathrm{~d} \mathrm{u}^{\prime}}=\left(\frac{\mathrm{d} \sigma}{\mathrm{du} u^{\prime}}\right)_{0} e^{\mathrm{b} \mathrm{u}^{\prime}}
$$

Momentum transfer intervals are chosen consistent with the backward cross section being described by a single exponential. The $u^{\prime}$ intervals, the resulting slopes and cross sections are recorded in Table XII. The $K_{L}^{o} p \rightarrow p K_{S}^{o}$ slopes do vary substantially through the resonance region; however, the average slope below $5 \mathrm{GeV} / \mathrm{c},\langle\mathrm{b}\rangle \sim 6 \mathrm{GeV}^{-2}$, is in qualitative agreement with the slope for the reaction $\pi^{-} p \rightarrow \Lambda^{\circ} \mathrm{K}^{\circ},\langle b\rangle=5.5 \pm 0.5 \mathrm{GeV}^{-2} .^{40}$ This slope is chosen therefore to determine the $K_{L}^{o} p \rightarrow K_{S}^{o} p$ backward cross section in the highest momentum interval (see Table XII) where there are insufficient data to determine a slope.

The energy dependence of the backward cross sections and the cross sections at $90^{\circ}$ are presented in Tables XII and XIII respectively. Both cross sections decrease by over three orders of magnitude between 1 and $7.5 \mathrm{GeV} / \mathrm{c}$. In contrast the forward cross sections, Table VI, decrease by less than a factor of ten in the same momentum interval.

The energy dependence of the $90^{\circ}$ scattering cross sections is of particular interest since large transverse momentum reactions are potential probes of the hadronic structure at small distances. ${ }^{41}$ Comparisons of the $90^{\circ}$ data to parton model predictions and to the predictions of single particle inclusive cross sections are discussed in Ref. 5.

## VI. SUMMARY AND CONCLUSIONS

Differential cross sections are presented for $K_{L}^{0} p \rightarrow K_{S}^{0} p$ scattering in several momentum intervals between 1 and $10 \mathrm{GeV} / \mathrm{c}$. The general features of the data are the following.
(a) The differential cross sections show forward and backward peaks in all momentum intervals. In particular the strong forward peak indicates the importance of the s-channel helicity nonflip scattering amplitude in $K_{L}^{o} p \rightarrow K_{S}^{o}{ }^{0}$.
(b) The energy dependence of the $K_{L}^{0} p \rightarrow K_{S}^{0}$ p cross section is approximately $\sigma \propto \mathrm{p}_{\mathrm{LAB}}^{-2.1}$ in the interval 1 to $10 \mathrm{GeV} / \mathrm{c}$. The energy dependence of the cross section at $\mathrm{t}=0$ is more gradual however with: $\left(\frac{\mathrm{d} \sigma}{\mathrm{dt}}\right)_{0} \propto \mathrm{p}_{\mathrm{LAB}}^{-1.33 \pm 0.24}$ for the present data, or $\left(\frac{d \sigma}{d t}\right)_{0} \propto p_{\mathrm{LAB}}^{-1.40 \pm 0.05}$ for all data in the momentum interval 2.5 to $50 \mathrm{GeV} / \mathrm{c}$.
(c) The phase of the $K_{L}^{o} p \rightarrow K_{S}^{o} p$ scattering amplitude at $t=0$ is consistent with being independent of energy: $\phi=-133.9 \pm 4.0^{\circ}$ for the present data, and $\phi=-133.4 \pm 3.3^{\circ}$ between 1.5 and $50 \mathrm{GeV} / \mathrm{c}$.
(d) Expressing the energy dependence of the data in the form $\frac{\mathrm{d} \sigma}{\mathrm{dt}}(\mathrm{s}, \mathrm{t}) \propto \mathrm{p}_{\mathrm{LAB}}^{2 \alpha(\mathrm{t})-2}$, the Regge trajectory for the $\mathrm{K}_{\mathrm{L}}^{0} \mathrm{p} \rightarrow \mathrm{K}_{\mathrm{S}}^{0} \mathrm{p}$ data is in approximate agreement with the canonical $\omega^{\circ}$, $\rho$ trajectory, $\alpha(\mathrm{t})=0.5+\mathrm{t}$, for $|t| \geq 0.4 \mathrm{GeV}^{2}$, but falls significantly below the prediction for $|t|<0.4 \mathrm{GeV}^{2}$. In addition the value at $t=0, \alpha(0)=0.30 \pm 0.03$, is substantially different from the Regge trajectory intercept determined from the phase of the forward amplitude, $\alpha(0)=0.49 \pm 0.05$. This indicates a breaking of the Regge phase-energy relation at $\mathrm{t}=0$.
(e) The $K_{L}^{0} p \rightarrow K_{S}^{o} p$ differential cross section has a distinct break at $|t| \sim 0.3 \mathrm{GeV}^{2}$, but does not have a minimum at $|t| \sim 0.6 \mathrm{GeV}^{2}$ in contrast to the pronounced dip observed in the related reaction $\pi^{-} p \rightarrow \pi^{\circ} n$.

Comparisons of the energy dependences of the forward $K_{L}^{o} p \rightarrow K_{S}^{0} p$ and $\begin{array}{ll}\pi^{-} p \rightarrow \pi^{0} n \text { cross sections find a substantial disagreement: } \quad \alpha(0) \\ 0.30 \pm 0.03 \text { while } \alpha(0) & =0.58 \pm 0.02 .29 \text { A similar } K_{L}^{o} p \rightarrow K_{S}^{o}{ }^{\mathrm{p}}=\end{array}$ $0.30 \pm 0.03$ while $\alpha(0)_{\pi-\mathrm{p} \rightarrow \pi^{\circ} \mathrm{n}}=0.58 \pm 0.02 .^{29}$ A similar result is obtained by comparing only the imaginary parts of these cross sections, $\Delta \sigma_{\mathrm{K}^{ \pm} \mathrm{n}}^{\mathrm{TOT}}$ and $\Delta \sigma_{\pi^{ \pm} \mathrm{p}}^{\mathrm{TOT}} .24$ In contrast the energy dependence of the forward cross sections for $K_{L}^{o} p \rightarrow K_{S}^{o} p$ and $K N$ charge-exchange are in good agreement. These comparisons suggest that direct channel or absorption effects are breaking $t$ channel factorization for those reactions with different particles in the $s$ channel.

The comparison of the $I_{t}=1 \pi N$ amplitude analysis results to the $K_{L}^{o} p \rightarrow K_{S}^{0} p$ differential cross section suggests that the helicity nonflip amplitude for vector exchange is different for $\pi^{-} p \rightarrow \pi^{\circ} n$ and $K_{L}^{o} p \rightarrow K_{S}^{o} p$ scattering. Since direct channel effects are thought to be important for helicity nonflip amplitudes, absorption may also explain the momentum transfer dependent differences between the $K_{L}^{o} p \rightarrow K_{S}^{o} p$ and $\pi^{-} p \rightarrow \pi^{o} n$ differential cross sections.

## Acknowledgements

We wish to acknowledge the assistance provided by R. Watt and the crew of the SLAC 40 -inch bubble chamber, by J. Brown and the scanning and measuring staff at SLAC, and by D. Johnson.

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Table I

Scanning Efficiencies Determined for $K_{L}^{O} p \rightarrow K_{S}^{O} p$

| Momentum Transfer <br> Interval <br> $(\Delta t)^{2}$ <br> $\left(\mathrm{Gev}^{2}\right)$ | Scan Efficiency <br> $(\%)$ | Azimuthal <br> Loss <br> Correction |
| :---: | :---: | :---: |
| $0.025-0.05$ | $50 \pm 20$ | $1.30 \pm 0.08$ |
| $0.05-0.1$ | $80 \pm 12$ | $1.14 \pm 0.05$ |
| $0.1-0.2$ |  |  |
| $0.2-0.4$ |  |  |
| $>0.4$ and $\theta_{\text {LAB }} \leq 45^{\circ}$ | $90 \pm 7$ |  |
| $\theta_{\text {LAB }}>45^{\circ}$ | $94 \pm 4$ | $1.10 \pm 0.04$ |

(a) $\theta_{I A B}$ is the laboratory scattering angle of the $K_{S}^{0}$. For initial $K_{L}^{O}$ momenta $\gtrsim 1 \mathrm{GeV} / \mathrm{c}$, a laboratory scattering angle $\theta_{\mathrm{LAB}} \geq 45^{\circ}$ corresponds to $\cos \theta_{\mathrm{cm}} \leq 0.0$.
Table II

| Differential Cross Sections: $\frac{d \sigma}{d \Omega}\left(\mathrm{~K}_{\mathrm{L}}^{\mathrm{p}} \rightarrow \mathrm{K}_{\mathrm{S}}^{\mathrm{p}}\right)$ |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $\mathrm{P}_{\mathrm{LAB}}(\mathrm{Gev} / \mathrm{c})$ | 1.0-1.5 | 1.5-2.0 | 2.0-2.5 | 2.5-3.5 | 3.5-5.0 | 5.0-7.5 |
| Cose Interval | $\frac{\mathrm{d} \sigma}{\mathrm{~d} \Omega}(\mu \mathrm{~b} / \mathrm{sr})$ |  |  |  |  |  |
| 1.0-0.9 | $319 \pm 87.5$ | $246 \pm 59.6$ | $167 \pm 34.2$ | $125 \pm 19.7$ | $95.3 \pm 14.7$ | $52.8 \pm 8.3$ |
| 0.9-0.8 | $128 \pm 31.7$ | $83.6 \pm 17.4$ | $49.8 \pm 10.3$ | $47.1 \pm 6.18$ | $30.7 \pm 3.8$ | $15.3 \pm 2.5$ |
| 0.8-0.7 | $103 \pm 27.4$ | $69.4 \pm 14.5$ | $44.5 \pm 9.0$ | $32.0 \pm 4.82$ | $19.9 \pm 2.9$ | $2.57 \pm 0.87$ |
| 0.7-0.6 | $111 \pm 27.3$ | $53.6 \pm 12.4$ | $57.2 \pm 10.3$ | $37.7 \pm 5.22$ | $6.9 \pm 1.7$ | $0.73 \pm 0.30$ |
| $0.6-0.5$ | $89.8 \pm 23.1$ | $52.1 \pm 11.9$ | $60.7 \pm 10.8$ | $15.7 \pm 3.30$ | $3.6 \pm 1.1$ |  |
| 0.5-0.4 | $40.0 \pm 14.3$ | $77.2 \pm 15.1$ | $44.5 \pm 9.2$ | $8.6 \pm 2.3$ | $\} 1.3 \pm 0.5$ | $1$ |
| $0.4-0.3$ | $137 \pm 30.9$ | $83.0 \pm 15.5$ | $21.2 \pm 6.2$ | $6.7 \pm 2.0$ | $\{1.3$ |  |
| 0.3-0.2 | $48.4 \pm 17.3$ | $60.0 \pm 13.1$ | $)^{16.2} \pm 5.7$ | $3.7 \pm 1.0$ | $1.3 \pm 0.5$ |  |
| 0.2-0.1 | $115 \pm 30.8$ | $41.5 \pm 10.6$ | $8.0 \pm 2.7$ | 3.7 | $\underline{0}$ | ( $00.0{ }^{\text {a }}$ |
| 0.1-0.0 | $137 \pm 40.1$ | $36.0 \pm 9.9$ | $\{8.0-2.7$ | $4.2 \pm 1.1$ |  | <<0.043 |
| 0.0--0.1 | $141 \pm 38.5$ | 46.9 $\pm 14.0$ | 12.0 $\pm 3.9$ | - | $\int 0.0$ | ) |
| -0.1--0.2 | $138 \pm 38.6$ | 17.8 $\pm 6.1$ | $\left\{\begin{array}{l}12.0-3.9\end{array}\right.$ | $3.9 \pm 1.3$ | $\} 0.10 \pm \begin{aligned} & 0.14 \\ & 0.08\end{aligned}$ | $1$ |
| -0.2--0.3 | $85.1 \pm 28.2$ | $\{17.8 \pm 6.1$ | 6.6 $\pm 2.9$ | 3.9 | 0 0.08 |  |
| -0.3--0.4 | $128 \pm 36.7$ | $\} 19.0 \pm 6.3$ | $\int_{03.6}+7.9$ | $4.0 \pm 1.4$ | $1$ | ) |
| -0.4--0.5 | $138 \pm 37.7$ | $19.0 \pm 6.3$ | $23.7 \pm 7.9$ | H.0 1.4 | I |  |
| -0.5--0.6 | $106 \pm 31.9$ | $28.4 \pm 10.8$ | $26.2 \pm 8.4$ | $1.2 \pm 1.2$ |  |  |
| -0.6--0.7 | $107 \pm 33.2$ | $36.7 \pm 13.3$ | $\left.10.7 \pm \begin{array}{l}7.9 \\ 4.9\end{array}\right]$ | 1.2 -0.7 | $0.20 \pm 0.27$ | $0.064 \pm 0.156$ |
| -0.7--0.8 | $142 \pm 37.5$ | $23.8 \pm 10.2$ | $\} 7.9 \pm 3.2$ | $2.4 \pm 1.0$ | $0.20 \pm 0.13$ | 0.052 |
| -0.8--0.9 | $124 \pm 37.4$ | $41.4 \pm 14.3$ | $\int 7.9 \pm 3.2$ | ) $2.4 \pm 1.0$ | ) |  |
| $-0.9--0.95$ $-0.95--1.0$ | $\} 188 \quad \pm 50.5$ | 81.1 $\pm 21.3$ | $\begin{aligned} & 18.4 \pm 15.7 \\ & 53.6 \pm 18.5 \end{aligned}$ | $3.9 \pm \begin{aligned} & 2.9 \\ & 1.8\end{aligned}$ | $\} \begin{aligned} & \left.1.8 \pm \begin{array}{l}1.5 \\ 0.9\end{array}\right\}\end{aligned}$ | $\} 0.30 \pm \begin{aligned} & 0.72 \\ & 0.24\end{aligned}$ |

Table III
Total Cross Section for $K_{L}^{0} p \rightarrow K_{S}^{O} p$

| $\begin{gathered} \mathrm{P}_{\mathrm{IAB}} \\ (\mathrm{GeV} / \mathrm{c}) \end{gathered}$ | EVENTS | $\begin{gathered} \sigma \\ (\mu \mathrm{b}) \end{gathered}$ |
| :---: | :---: | :---: |
| 0.6-0.8 | 44 | $2385 \pm 561$ |
| 0.8-1.0 | 138 | $3692 \pm 633$ |
| 1.0-1.2 | 151 | $2398 \pm 298$ |
| 1.2-1.4 | 101 | $1002 \pm 128$ |
| 1.4-1.6 | 132 | $952 \pm 117$ |
| 1.6-1.8 | 153 | $807 \pm 88.5$ |
| 1.8-2.0 | 116 | $501 \pm 61.3$ |
| $2.0-2.2$ | 130 | $491 \pm 44.2$ |
| $2.2-2.4$ | 104 | $329 \pm 32.2$ |
| $2.4-2.6$ | 104 | $290 \pm 28.1$ |
| $2.6-2.8$ | 91 | $241 \pm 24.8$ |
| 2.8-3.0 | 84 | $210 \pm 23.5$ |
| $3.0-3.2$ | 96 | $205 \pm 20.4$ |
| $3.2-3.4$ | 58 | $133 \pm 17.5$ |
| $3.4-3.6$ | 51 | $120 \pm 17.0$ |
| $3.6-3.8$ | 66 | $141 \pm 17.3$ |
| $3.8-4.0$ | 56 | $123 \pm 16.6$ |
| $4.0-4.4$ | 87 | $101 \pm 11.0$ |
| $4.4-4.8$ | 76 | $90.8 \pm 10.4$ |
| $4.8-5.2$ | 53 | $73.0 \pm 9.9$ |
| $5.2-6.0$ | 65 | $49.0 \pm 5.8$ |
| $6.0-7.0$ | 55 | $47.0 \pm 6.4$ |
| $7.0-8.0$ | 19 | $23.2 \pm 5.2$ |
| 8.0-10.0 | 22 | $25.2 \pm 5.1$ |
| 10.0-12.0 | 4 | $14.6 \pm 16.9$ |

(a) Totals for events with $K_{S}^{0}$ decay lengths $0.3 \leq \ell_{K_{S}^{0}}^{0} \leq 40$. cm, see sect. II.

Table IV

Energy Dependence of the $K_{L}^{0} p \rightarrow K_{S}^{0} p$ Cross Section,

$$
\left(\hat{K}_{\mathrm{L}}^{\circ} \mathrm{p} \rightarrow \mathrm{~K}_{\mathrm{S}}^{\circ} \mathrm{p}\right)=A p^{-n}
$$

| Momentum Interval <br> $(\mathrm{GeV} / \mathrm{c})$ | A <br> $(\mu \mathrm{b})$ | n |
| :---: | :---: | :---: |
| $1.0-12.0$ | $2255 \pm 114$ | $2.18 \pm 0.05$ |
| $2.5-12.0$ | $1812 \pm 241$ | $2.02 \pm 0.10$ |
| $4.0-12.0$ | $2000 \pm 767$ | $2.07 \pm 0.23$ |

Table V

Cross Section for $K_{\mathrm{L}}^{0} p \rightarrow K_{S}^{o} p$ in the Resonance Region

| $\mathrm{E}_{\mathrm{c} \cdot \mathrm{m}}$. <br> GeV | $\sigma$ <br> $(\mu \mathrm{b})$ |
| :---: | :---: |
| $1.60-1.70$ | $2465 \pm 564$ |
| $1.70-1.80$ | $3602 \pm 619$ |
| $1.80-1.85$ | $2888 \pm 419$ |
| $1.85-1.875$ | $1586 \pm 313$ |
| $1.875-1.90$ | $1597 \pm 292$ |
| $1.90-1.925$ | $1171 \pm 230$ |
| $1.925-1.95$ | $1009 \pm 212$ |
| $1.95-1.975$ | $756 \pm 162$ |
| $1.975-2.00$ | $777 \pm 177$ |
| $2.00-2.025$ | $1048 \pm 180$ |
| $2.025-2.05$ | $990 \pm 175$ |
| $2.05-2.075$ | $1038 \pm 194$ |
| $2.075-2.10$ | $775 \pm 142$ |
| $2.10-2.125$ | $966 \pm 145$ |
| $2.125-2.15$ | $605 \pm 107$ |
| $2.15-2.175$ | $478 \pm 95.6$ |
| $2.175-2.20$ | $548 \pm 106$ |
| $2.20-2.225$ | $559 \pm 97.4$ |
| $2.225-2.25$ | $364 \pm 64.7$ |
| $2.25-2.275$ | $603 \pm 87.6$ |
| $2.275-2.30$ | $498 \pm 78.3$ |
| $2.30-2.325$ | $290 \pm 58.6$ |
| $2.325-2.35$ | $427 \pm 64.0$ |
| $2.35-2.40$ | $302 \pm 37.5$ |
| $2.40-2.45$ | $296 \pm 35.1$ |
| $2.45-2.50$ | $287 \pm 33.9$ |
| $2.50-2.60$ | $211 \pm 19.9$ |
| $2.60-2.70$ | $188 \pm 16.4$ |
| $2.70-2.80$ | $136 \pm 15.2$ |
| $2.80-2.90$ | $123 \pm 13.0$ |
| $2.90-3.00$ | $115 \pm 13.1$ |

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| $\mathrm{P}_{\mathrm{LAB}}(\mathrm{GeV} / \mathrm{c})$ |  | $-2.5$ | 2.5-3.5 | 3.5-5.0 | 5-7.5 | 7.5-10 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $\begin{aligned} & \text {-t Interval } \\ & \left(\mathrm{ccv}^{2}\right) \end{aligned}$ | $\frac{d \sigma}{d t}\left(\mu \mathrm{~h} / \mathrm{gev}^{2}\right)$ |  |  |  |  |  |
| 0.025-0.05 | 175 | $\pm 795$ | $520 \pm 25.5$ | $359 \pm 172$ | $209 \pm 109$ | $75.1 \pm 37.6$ |
| 0.05-0.10 | 702 | $\pm 138$ | $344 \pm 79.8$ | $353 \pm 73.6$ | $141 \pm 35.4$ |  |
| 0.10-0.15 | 514 | $\pm 101$ | $308 \pm 60.3$ | $1 / 6 \pm 35.3$ | $90.7 \pm 21.8$ |  |
| 0.15-0.20 | 413 | $\pm 89.2$ | $213 \pm 51.7$ | $92.8 \pm 24.4$ | $61.9 \pm 18.1$ | $41.3 \pm 15.3$ |
| 0.20-0.25 | 207 | $\pm \quad 58.9$ | $111 \pm 31.6$ | $63.7 \pm 18.1$ | $18.9 \pm 8.6$ |  |
| 0.25-0.30 | 210 | $\pm \quad 59.6$ | $1.22 \pm 33.5$ | $96.0 \pm 24.0$ | $39.3 \pm 14.4$ |  |
| 0.30-0.35 | 27. | $\pm 67.0$ | $105 \pm 29.9$ | $46.1 \pm 15.6$ | $26.0 \pm 8.2$ |  |
| 0.35-0.40 | 302 | $\pm 73.7$ | $158 \pm 37.5$ | $48.5 \pm 15.6$ |  | $20.6 \pm 8.2$ |
| 0.40-0.45 | 187 | $\pm \quad 50.9$ | $136 \pm 34.7$ | $38.4 \pm 13.7$ | $19.2 \pm 6.5$ |  |
| 0.4; - 0.50 | 234 | $\pm 56.3$ | $88.2 \pm 27.0$ | $69.0 \pm 18.7$ |  |  |
| 0.50-0.55 | 263 | $\pm 63.4$ | $106 \pm 32.3$ | $98.3 \pm$ 2a. | $22.4 \pm 6.6$ | ) |
| 0.55-0.60 | 38. | $\pm 80.7$ | $155 \pm 36.3$ | $74.6 \pm 19.0$ |  |  |
| 0.60-0.65 | 299 | $\pm 66.8$ | $85.5 \pm 27.4$ | $34.4 \pm 9.0$ |  |  |
| 0.65-0.70 | 393 | $\pm 76.8$ | $96.5 \pm 28.2$ |  | $16.3 \pm 4.5$ | $\left.2.7 \pm \begin{array}{r}3.6 \\ 2.0\end{array}\right]$ |
| 0.70-0.80 | 275 | $\pm 47.2$ | $74.7 \pm 18.0$ | $40.3 \pm 10.5$ |  |  |
| 0.80-0.90 | 257 | $\pm 44.0$ | $93.9 \pm 19.7$ | $36.6 \pm 9.6$ | $13.2 \pm 4.0$ |  |
| 0.90-1.00 | 166 | $\pm 34.8$ | $80.9 \pm 18.5$ | $32.8 \pm 8.9$ |  |  |
| 1.00-1.10 | 164 | $\pm 38.6$ | 29.6 * 7.8 | $15.8 \pm 4.4$ | $11.5 \pm 3.9$ |  |
| 1.10-1.20 | 202 | $\pm 44.0$ |  |  |  |  |
| 1.20-1.30 |  | $\pm 27.7$ | $21.9+6.7$ | 11.7 $\pm 3.7$ |  |  |
| 1.30-1.40 |  | $\pm 30.1$ |  |  | $\pm \quad 1.4$ |  |
| 1.10-1.50 |  | $\pm 30.6$ | $7.3 \pm 6.0$ | $4.4 \pm 3.6$ |  | $0.98 \pm 1.32$ |
| 1.50-1.60 |  | $\pm 29.4$ | - 3.7 | - 2.2 |  | ( $0.98 \pm 0.72$ |
| 1.60-1.70 |  | $\pm 35.1$ | $25.4 \pm 5.5$ |  |  |  |
| 1.70-1.80 | 151 | $\pm 46.5$ |  | $2.6 \pm 1.8$ | $0.41 \pm 1.00$ |  |
| 1.80-1.90 | 11.2 | $\pm 40.5$ | $25.3+5.5$ |  | 0.70 .33 |  |
| 1.90-2.00 | 155 | $\pm 51.5$ |  |  |  |  |

Table VII
Determination of the Phase of the Forward Amplitude for $K_{I^{\prime}}^{O} p \rightarrow K_{S}^{O} p$

| $\begin{gathered} \mathrm{P}_{\mathrm{LAB}} \\ (\mathrm{GeV} / \mathrm{c}) \end{gathered}$ | $\begin{gathered} \text { SLOPE }^{(a)} \\ \left(\mathrm{GeV}^{-2}\right) \end{gathered}$ | $\begin{gathered} \left(\frac{d \sigma}{d t}\right)_{0} \\ \left(\mu \mathrm{~b} / \mathrm{Gev}^{2}\right) \end{gathered}$ | $\begin{aligned} & \left(\frac{d \sigma}{d t}\right)_{o p t}^{(b)} \\ & \left(\mu \mathrm{b} / \mathrm{GeV}^{2}\right) \end{aligned}$ | (ReA/ImA) | (degrees) | $\alpha(0)$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 1.5-2.5 | $8.1 \pm 1.6$ | $1425 \pm 282$ | $663 \pm 60.8$ | $1.07 \pm 0.23$ | $-137.0 \pm 6.9$ | $0.52 \pm 0.08$ |
| 2.5-3.5 | $6.9 \pm 1.6$ | $646 \pm 134$ | $465 \pm 55.3$ | $0.62 \pm 0.27$ | $-122.0 \pm 11.0$ | $0.36 \pm 0.12$ |
| $3.5-5.0$ | $10.7 \pm 1.8$ | $665 \pm 138$ | $294 \pm 31.0$ | $1.12 \pm 0.25$ | $-138.4 \pm 7.1$ | $0.54 \pm 0.08$ |
| 5.0-7.5 | $10.6 \pm 2.2$ | $305 \pm 78$ | $177 \pm 18.8$ | $0.85 \pm 0.28$ | $-130.4 \pm 9.3$ | $0.45 \pm 0.10$ |
| 7.5-10.0 | $7.8 \pm 4.8$ | $138 \pm 82$ | $112 \pm 13.6$ | $0.48 \pm 0.77$ | $-115.6 \pm 36.0$ | $0.29 \pm 0.40$ |
|  |  |  | Weighted Average | $.92 \pm 0.13$ | $-133.9 \pm 4.0$ | $0.49 \pm 0.05$ |
|  |  |  | Birulev et |  | $-132.3 \pm 5.7$ |  |
|  |  |  | Combined Da |  | $-133.4 \pm 3.3$ |  |
| (a) Slopes are determined in the interval $0.025 \leq\|t\| \leq 0.25 \mathrm{GeV}^{2}$. |  |  |  |  |  |  |
| (b) Optical points are determined by parameterizing the $K^{ \pm} \mathrm{n}$ total cross section differences |  |  |  |  |  |  |
| $\left(\frac{\mathrm{d} \sigma}{\mathrm{dt}}\right)_{\mathrm{opt}}=\left(\Delta \sigma_{\mathrm{K}^{ \pm} \mathrm{n}}^{\mathrm{TOT}} / \sqrt{ }\right.$ |  |  | and averag | his result | appropriat | ntum inte |

Table VIII
Energy Dependence of the $K_{L}^{O} p \rightarrow K_{S}^{O} p$ Forward Cross Section,

$$
\left(\frac{d \sigma}{d t}\right)_{0}\left(K_{L}^{o} p \rightarrow K_{S}^{o} p\right)=A p^{-n_{0}}
$$

| Momentum Interval $(\mathrm{GeV} / \mathrm{c})$ | $\begin{gathered} \mathrm{A} \\ \left(\mu \mathrm{~b} / \mathrm{GeV}^{2}\right) \end{gathered}$ | $\mathrm{n}_{0}$ | Reference |
| :---: | :---: | :---: | :---: |
| 1.5-10.0 | $3255 \pm 988$ | $1.33 \pm 0.24$ | Present Expt. |
| $14-50$ | $1118 \pm 996$ | $1.04 \pm 0.26$ | Birulev et al ${ }^{(3)}$ |
| $\begin{aligned} & 1.5-50 \\ & 2.5-50 \\ & 3.5-50 \end{aligned}$ | $\begin{aligned} & 3274 \pm 607^{(a)} \\ & 3082 \pm 749 \\ & 4013 \pm 1400 \end{aligned}$ | $\begin{aligned} & 1.36 \pm 0.06^{(\mathrm{a})} \\ & 1.34 \pm 0.08 \\ & 1.42 \pm 0.11 \end{aligned}$ | Present Expt. $\begin{gathered} + \\ \text { Birulev et al } \end{gathered}$ |
| 2.5-50 | $3834 \pm 476$ | $1.40 \pm 0.05$ | Present Expt. + Ref. (1-3) |

(a) For the combined data samples, the uncertainties in the forward cross sections have been augmented by a $10 \%$ systematic uncertainty combined in quadrature.

Table IX
Effective Regge Trajectory, $\alpha_{\text {eff }}$, for $K_{L}^{O} p \rightarrow K_{S}^{\circ} p$

| $\|t\|$ Interval <br> $(\mathrm{GeV})^{2}$ | Momentum <br> Interval <br> $(\mathrm{GeV} / \mathrm{c})$ | $\alpha_{\text {eff }}(\mathrm{a})$ | $\alpha_{\text {eff }}(b)$ | $\alpha_{\text {eff }}(\mathrm{c})$ |
| :---: | :---: | :---: | :---: | :---: |
| $0.025-0.10$ | $3-8$ | $0.26 \pm 0.18$ | $0.30 \pm 0.19$ | $0.23 \pm 0.15$ |
| $0.10-0.20$ | $3-8$ | $0.14 \pm 0.18$ | $0.16 \pm 0.20$ | $0.12 \pm 0.15$ |
| $0.20-0.40$ | $3-8$ | $-0.08 \pm 0.20$ | $-0.09 \pm 0.22$ | $-0.07 \pm 0.16$ |
| $0.40-0.60$ | $3-7$ | $-0.02 \pm 0.22$ | $-0.03 \pm 0.25$ | $-0.01 \pm 0.18$ |
| $0.60-0.80$ | $3-7$ | $-0.14 \pm 0.29$ | $-0.16 \pm 0.33$ | $-0.11 \pm 0.22$ |
| $0.80-1.20$ | $3-7$ | $-0.25 \pm 0.23$ | $-0.28 \pm 0.26$ | $-0.19 \pm 0.17$ |
| $1.20-2.00$ | $3-6$ | $-0.97 \pm 0.45$ | $-1.14 \pm 0.53$ | $-0.62 \pm 0.30$ |

(a), (b), (c) $\alpha_{\text {eff }}$ determined using Eqs. (19a), (16), and (19b) respectively. To determine u for the purpose of our maximum likelihood fits we use: $u=\Sigma m_{i}^{2}-s-<t>$, where $\langle t>$ is the average momentum transfer for the events in the given |t|interval.

Dual Absorptive Model Parameters for Fit to

$$
K_{I}^{0} p \rightarrow K_{S}^{0} p \text { Forward Cross Sections }
$$

| Reaction | $\mathrm{K}_{\mathrm{L}}^{\mathrm{O}} \mathrm{p} \rightarrow \mathrm{K}$ | $\pi^{-} p \rightarrow \pi^{0} n$ |
| :---: | :---: | :---: |
| $r\left(\mathrm{GeV}^{-1}\right)$ | 5.19 | 5.19 |
| A $\left(\mathrm{GeV}^{-2}\right)$ | -0.88 | -0.93 |
| $g_{0}$ | -19.0 | -15.0 |
| $\mathrm{E}_{1}$ | 1.7 .0 | -30.9 |
| $a\left(\mathrm{GeV}^{-2}\right)$ | 2.97 | 5.56 |
| b ( $\mathrm{GeV}^{-4}$ ) | 8.79 | 10.20 |
| c ( $\mathrm{GeV}^{-6}$ ) | ---- | 5.22 |
| $B\left(\mathrm{GeV}^{-2}\right)$ | 2.0 | 1.5 |
| $\alpha^{\prime}\left(\mathrm{GeV}^{-2}\right)$ | 0.88 | 0.9 |
| $\Delta \alpha$ | -0.14 | --- |

(a) Results from a similar comparison

$$
\text { to } \pi^{-} p \rightarrow \pi^{\circ} n \text {, ref. } 39
$$

IX $\partial T q e \amalg$

$K_{L}^{\circ} p \rightarrow \mathrm{pK}_{\mathrm{S}}^{\circ}$ Backward Differential Cross Sections at $180^{\circ}\left(\mathrm{u}=\mathrm{u}_{\mathrm{MAX}}\right)$

(a) Slope value taken from the ralated reaction $\pi^{-} p \rightarrow \Lambda^{0} K^{0}$

$$
\mathrm{K}_{\mathrm{L}}^{\circ} \mathrm{p} \rightarrow \mathrm{~K}_{\mathrm{S}}^{\circ} \mathrm{p} \text { Cross Section Near } 90^{\circ}
$$

| $P_{\mathrm{IAB}}$ <br> $(\mathrm{GeV} / \mathrm{c})$ | $\left(\frac{d \sigma}{d \Omega}\right)_{90^{\circ}}^{(\mathrm{a})}$ <br> $(\mu \mathrm{b} / \mathrm{sr})$ | $\left(\frac{\mathrm{d} \sigma}{\mathrm{dt}}\right)_{90^{\circ}}^{(\mathrm{a})}$ <br> $\left(\mu \mathrm{b} / \mathrm{GeV}^{2}\right)$ |
| :---: | :---: | :---: |
| $1.0-1.5$ | $133 \quad \pm 19.8$ | $1104 \pm 165$ |
| $1.5-2.0$ | $35.6 \pm 5.6$ | $190 \pm 29.9$ |
| $2.0-2.5$ | $10.0 \pm 2.3$ | $38.6 \pm 9.0$ |
| $2.5-3.5$ | $4.52 \pm 0.84$ | $12.3 \pm 2.3$ |
| $3.5-5.0$ | $0.43 \pm$0.30 <br> 0.19 | $0.79 \pm$0.55 <br> $5.0-7.5$ |

(a) Determined in the center of mass scattering interval $-0.2<\cos \theta<0.2$
(b) Cross section upper bounds correspond to $85 \%$ confidence limit (1.9 events when no events were observed)

## FIGURE CAPTIONS

1. Differential cross sections for $K_{L}^{0} p \rightarrow K_{S}^{0} p$ in six momentum intervals between 1.0 and $7.5 \mathrm{GeV} / \mathrm{c}$. The dashed error bar in the highest momentum data indicates the $85 \%$ confidence level upper limit on this cross section (this corresponds to 1.9 events when no events are observed).
2. Total cross section for the reaction $K_{L}^{O} p \rightarrow K_{S}^{O} p$. The solid points come from the present experiment; data from Refs. 9-11 are also included.
3. Cross section for the reaction $K_{L}^{0} p \rightarrow K_{S}^{O} p$ in the resonance region. The masses of known $\Sigma$ resonances are included for comparison with the data.
4. Forward differential cross sections for $K_{L}^{0} p \rightarrow K_{S}^{0} p$ in 5 momentum intervals between 1.5 and $10.0 \mathrm{GeV} / \mathrm{c}$. The solid curves are from a fit using the dual absorptive model described in Section IV. E.
5. Differential cross section at $t=0$ for $K_{L}^{o} p \rightarrow K_{S}^{o} p$. The solid points, determined by exponential extrapolation of the forward differential cross sections, are from the present experiment. Data from Refs. 1-3, 9, 11 are also included. The solid curve is a parametrization of the data above $2.5 \mathrm{GeV} / \mathrm{c}$ with the form $(\mathrm{d} \sigma / \mathrm{dt})_{0}=\mathrm{Ap}_{\mathrm{LAB}}^{-\mathrm{n}_{0}}$.
6. Exponential slopes of the forward $K_{L}^{o} p \rightarrow K_{S}^{o} p$ differential cross section (see Eq. (13)) determined in the momentum transfer interval $0.025 \leq|t| \leq 0.25 \mathrm{GeV}^{2}$. The curve is a simple Regge parametrization of the data: $b(s)=b_{0}+2 \alpha^{\prime} \ln s$.
7. The imaginary part of the $K_{L}^{O} p \rightarrow K_{S}^{O} p$ differential cross section at $t=0$ determined from the optical theorem. The $K^{ \pm} n$ total cross section data are from Refs. 22-24. The solid curve is a parametrization of the data above 2.5 GeV/c with the form $(\mathrm{d} v / \mathrm{dt})_{\mathrm{opt}}=\mathrm{Bp}_{\mathrm{LAB}}^{-\mathrm{n}}{ }_{\mathrm{opt}}$. The (*) points are
found by scaling the forward $K_{L}^{o} p \rightarrow K_{S^{p}}^{0} p$ cross sections by $\sin ^{2} \phi_{A V G}$, where $\phi_{A V G}$ is the average phase of the forward cross section (see Section IV. C).
8. Phase of the $K_{L}^{o} p \rightarrow K_{S}^{0} p$ scattering amplitude at $t=0$. The ( $\bullet$ ) data are from the present experiment. Results from $K_{S}^{0}$ coherent regeneration experiments, Refs. 1-3 are also included. The curve shows the phase at $t=0$ resulting from the power law fits displayed in Figs. 5 and 7.
9. Regge trajectory for $K_{L}^{O} p \rightarrow K_{S}^{O} p$ data resulting from a parametrization of the data with the form: $\frac{d \sigma}{d t}(s, t) \propto p_{L A B}^{2 \alpha(t)-2}$. The points at $t<0$ are from the present experiment; the solid point at $t=0$ includes all the data ${ }^{1-3}$ above $2.5 \mathrm{GeV} / \mathrm{c}$. The energy dependence of the imaginary part of the forward cross section (see Fig. 7) is shown by the open circle at $t=0$. The shaded region represents the results of a similar analysis of $\pi \bar{p} \rightarrow \pi^{\circ} \mathrm{n}$ scattering. ${ }^{31}$ The solid curve depicts the canonical $\omega^{\circ}, \rho$ Regge trajectory, $\alpha(\mathrm{t})=0.5+\mathrm{t}$.
10. The differential cross sections at $t=0$ for $\pi-p \rightarrow \pi^{0} n$. ${ }^{32}$ The solid curve is a parametrization of the data above $5 \mathrm{GeV} / \mathrm{c}$ with the form $(d \sigma / d t)_{0}=A p_{L A B}^{-n}{ }^{29}$ The dashed curve represents the $K_{L}^{o} p \rightarrow K_{S^{0}}^{0} p$ forward cross sections from Fig. 5.
11. The differential cross section at $\mathrm{t}=0$ for KN charge-exchange scattering. 33 The dashed curve represents the $K_{L}^{O} p \rightarrow K_{S}^{o} p$ forward cross sections from Fig. 5.
12. Comparison of $s$ channel helicity amplitudes to the $K_{L}^{O} p \rightarrow K_{S}^{0} p$ data in the interval 5 to $7.5 \mathrm{GeV} / \mathrm{c}$. The differential cross section is shown in (a); the helicity nonflip and flip contributions to the cross section are shown in (b) and (c) respectively. The curves are explained in Section IV.E.
13. Predicted polarization for $K_{L}^{O} p \rightarrow K_{S}^{0} p$ scattering at $6 \mathrm{GeV} / \mathrm{c}$. The curves are discussed in Section IV.E.
14. Backward differential cross section for $K_{L}^{o} p \rightarrow K_{S}^{o} p$ in 5 momentum intervals between 1 and $5 \mathrm{GeV} / \mathrm{c}$.


Fig. 1


Fig. 2


Fig. 3


Fig. 4


Fig. 5


Fig. 6


Fig. 7


Fig. 8


Fig. 9


Fig. 10


Fig. 11

Fig. 12

NOI $\perp \forall$ ZIV $\forall 7$ Od


Fig. 14


[^0]:    *Work supported by the U. S. Atomic Energy Commission.
    **Now at Duke University, Durham, North Carolina.
    ***Now at University of Toronto, Toronto, Canada.
    $\dagger$ Now at University of Washington, Seattle, Washington. $\dagger \dagger$ Now at National Accelerator Lab, Batavia, Illinois.
    $\dagger \dagger \dagger$ Now at Lawrence Berkeley Lab, Berkeley, California. $\ddagger$ Now at Purdue University, Lafayette, Indiana.

