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K^o REGENERATION ON DEUTERIUM*

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ABSTRACT

A calculation of the process $K_L^o d \rightarrow K_S^o d$ is made using the phenomenological fits of Loos and Matthews to the meson-nucleon amplitudes in the Glauber multiple scattering formalism. The results of a numerical calculation for the scattering cross section are presented for representative values of incident kaon energy for $0 \leq t \lesssim 1.4\ \mathrm{GeV}^2$.

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I. Introduction

The process $K_L^o d \to K_S^o d$ is unique among inelastic meson-nucleon processes in that it allows the direct experimental separation of the properties of the ω^o trajectory from its closely-related relative the ρ. In anticipation of an upcoming experiment to measure this process at SLAC[1], a calculation has been done using the available information about the K-nucleon amplitudes. A detailed fit to inelastic meson-nucleon cross section and polarization data has been performed by Loos and Matthews[2] using the parametrization of the dual absorption model (DAM) of Harari[3]. Within the confines of the model this fit provides a complete description of the amplitudes involving non-vacuum t-channel quantum numbers; the vacuum or Pomeron (IP) amplitude can be crudely modeled from high energy elastic processes. The Glauber multiple scattering formula (GMSF)[4] provides a reasonably accurate description of meson-dueteron scattering if double scattering corrections are included.[5]

In Section II the kaon nucleon amplitudes will be defined. In Section III the calculation for the deuteron will be shown and numerical results presented.

II. The K-Nucleon Amplitudes

The most general amplitude for K-nucleon scattering may be written as:

$$\begin{aligned}\mathcal{M} = \bar{u}(p', s')\, \chi^{\dagger}_{\alpha}(N')\, \chi^{\dagger}_{\beta}(K') \Big[& a^0(s,t)\, \delta_{\alpha\alpha'}\, \delta_{\beta\beta'} + a^1(s,t)\, \tau_{\alpha\alpha'} \cdot \tau_{\beta\beta'} \\ & + \left(b^0(s,t)\, \delta_{\alpha',\alpha}\, \delta_{\beta',\beta} + b^1(s,t)\, \tau_{\alpha'\alpha} \cdot \tau_{\beta'\beta} \right) (q + q') \cdot \gamma \Big] \\ & \times \chi_{\alpha'}(N)\, \chi_{\beta'}(K)\, u(p, s)\end{aligned} \tag{2.1}$$

where $\mathcal{M}$ is normalized to:

$$\frac{d\sigma}{dt} = \frac{(\hbar c)^2}{64\pi s p_i^{*2}} |\mathcal{M}|^2 \equiv \frac{(\hbar c)^2}{16\pi \lambda^2} |\mathcal{M}|^2 \tag{2.2}$$

$$\lambda = \left((s - m_N^2 - m_K^2)^2 - 4 m_N^2 m_K^2\right)^{\frac{1}{2}} \tag{2.3}$$

and u(p, s) to:

$$\bar{u}(p, s)\, u(p, s') = 2m_N \delta_{s's} . \tag{2.4}$$

An analogous amplitude may be written for the $\bar{K}$-nucleon system:

$$\mathcal{M} = \bar{u}(p', s')\chi^\dagger(N')\chi^\dagger(\bar{K}')\left[\bar{a}^0(s,t) + \bar{a}^1(s,t)\,\tau\cdot\tau + \left(\bar{b}^0(s,t) + \bar{b}^1(s,t)\,\tau\cdot\tau\right)(q+q')\cdot\gamma\right]$$
$$\times\ \chi(N)\,\chi(\bar{K})\,u(p, s) . \tag{2.5}$$

From these amplitudes, linear combinations can be formed which have definite t-channel isospin and crossing properties. These will then be labeled with the leading trajectories which have those quantum numbers:

$$\begin{aligned}
A^{IP} &= \bar{u}\left(a^0 + \bar{a}^0 + (b^0 + \bar{b}^0)\,\not{Q}\right)u \\
A^{\omega^0} &= \bar{u}\left(a^0 - \bar{a}^0 + (b^0 - \bar{b}^0)\,\not{Q}\right)u \\
A^{\rho} &= \bar{u}\left(a^1 + \bar{a}^1 + (b^1 + \bar{b}^1)\,\not{Q}\right)u \\
A^{A_2} &= \bar{u}\left(a^1 - \bar{a}^1 + (b^1 - \bar{b}^1)\,\not{Q}\right)u
\end{aligned} \tag{2.6}$$

$$\not{Q} = (q + q')\cdot\gamma . \tag{2.7}$$

Evaluating the s-channel center of mass (CM) helicity amplitudes from an explicit representation for the spinors gives for a particular amplitude:

$$\begin{aligned} \mathcal{M}^{i}_{++} &= 2\cos\Theta^*/2\left(m_N a^i + (s - m_N^2 - m_K^2) b^i\right) \\ \mathcal{M}^{i}_{+-} &= 2\sin\Theta^*/2\left(E_N^* a^i + 2E_K^* m_N b^i\right) \end{aligned} \tag{2.8}$$

(* denotes CM) which invert to give the invariant amplitudes:

$$\begin{aligned} a^i(s,t) &= \frac{-1}{\lambda}\left[\frac{2m_N(\lambda^2/4s + m_K^2)^{\frac{1}{2}}\mathcal{M}^{i}_{++}}{(\lambda^2/s + t)^{\frac{1}{2}}} - \frac{(s - m_N^2 - m_K^2)\mathcal{M}^{i}_{+-}}{(-t)^{\frac{1}{2}}}\right] \\ b^i(s,t) &= \frac{1}{\lambda}\left[\frac{(\lambda^2/4s + m_N^2)^{\frac{1}{2}}\mathcal{M}^{i}_{++}}{(\lambda^2/s + t)^{\frac{1}{2}}} - \frac{m_N\mathcal{M}^{i}_{+-}}{(-t)^{\frac{1}{2}}}\right] . \end{aligned} \tag{2.9}$$

In the GMSF it will be necessary to know laboratory frame (LF) scattering amplitudes normalized to $d\sigma/d\Omega = (\hbar c)^2 \,|\, f \,|^2$ with the spin quantized along a fixed $\hat{z}$ axis which will be taken to be the direction of the incident K. The kinematics are depicted in Fig. 1. These amplitudes are given by:

$$\begin{aligned} f^{i}_{s',s(LF)} &= \frac{1}{8\pi\sqrt{s}}\left(\frac{P_K^f\,\xi}{E_N^f}\right)^{\frac{1}{2}}\left(2\,m_N(E_N^f + m_N)\right)^{\frac{1}{2}} \\ &\left\{\left[a^i + b^i\left(E_K^i + E_K^f + \frac{\underset{\sim}{q}\cdot(2\underset{\sim}{K} + \underset{\sim}{q})}{E_N^f + m_N}\right)\right]\delta_{s's} + \left[b^i\left(2i\frac{\chi^{\dagger}_{s'}\,(\underset{\sim}{q}\times\underset{\sim}{K})\cdot\underset{\sim}{\sigma}\,\chi_s}{E_N^f + m_N}\right)\right]\right\} \end{aligned} \tag{2.10}$$

where

$$\xi \equiv \frac{2}{|\,1 + dE_N^f(E_K^f)/dE_K^f\,|}$$

The DAM[3] gives as the general form of the s-channel helicity amplitudes:

$$\mathrm{Im}\ \mathcal{M}_{\Delta\lambda} \propto \text{"}J_{\Delta\lambda}(r\sqrt{-t})\text{"} \ . \tag{2.11}$$

The $\Delta\lambda = 1$ amplitudes are well described by simple Regge pole exchange, and take the conventional phases $\tan\left(\frac{\pi\alpha(t)}{2}\right) + i$ for vector exchange, and $\cot\left(\frac{\pi\alpha(t)}{2}\right) - i$ for tensor exchange. The $\Delta\lambda = 0$ amplitudes neet cut contributions to describe them and hence the DAM makes no statement about the real part of the $\Delta\lambda = 0$ amplitude. The $\mathbb{P}$ amplitude empirically is dominated by $\Delta\lambda = 0$ and is essentially pure imaginary for small t. Loos and Matthews[2] find in their fit that the energy dependence of the amplitude is well described by the conventional Regge form $(s/s_0)^{\alpha(t)}$, and the phases of the forward $\Delta\lambda = 0$ amplitude by the conventional Regge phase. The remainder of the $\Delta\lambda = 0$ amplitude is fit by them to an arbitrary function in t. The results of their fit are given by:

$$\begin{aligned}
\mathcal{M}^{\omega^o}_{++} &= g^{\omega^o}_{++}\left(\frac{s}{s_0}\right)^{\alpha(t)} e^{A_V t}\left(P_V(t)\tan\left(\frac{\pi\alpha(0)}{2}\right) + i\,J_0(r\sqrt{-t})\right) \\
&\left(P_V(t) \equiv e^{B_V t}\left(1 + a_V t + b_V t^2 + \cdots\right)\right) \\
\mathcal{M}^{\omega^o}_{+-} &= g^{\omega^o}_{+-}\left(\frac{s}{s_0}\right)^{\alpha(t)} e^{A_V t}\,\text{"}J_1^V(r\sqrt{-t})\text{"}\left(\tan\left(\frac{\pi\alpha(t)}{2}\right) + i\right) \\
\mathcal{M}^{\rho}_{++} &= \mathcal{M}^{\omega^o}_{++}\,(g^{\omega^o}_{++} \to g^{\rho}_{++}) \\
\mathcal{M}^{\rho}_{+-} &= \mathcal{M}^{\omega^o}_{+-}\,(g^{\omega^o}_{+-} \to g^{\rho}_{+-}) \\
\mathcal{M}^{A_2}_{++} &= g^{A_2}_{++}\left(\frac{s}{s_0}\right)^{\alpha(t)} e^{A_T t}\left(P_T(t)\cot\left(\frac{\pi\alpha(0)}{2}\right) - i\,J_0(r\sqrt{-t})\right)
\end{aligned} \tag{2.12}$$

$$\mathcal{M}^{A_2}_{+-} = g^{A_2}_{+-}\left(\frac{s}{s_0}\right)^{\alpha(t)} e^{A_T t} \text{"}J_1^T(r\sqrt{-t})\text{"} \left(\cot\left(\frac{\pi\alpha(t)}{2}\right) - i\right)$$

$$\text{"}J_1^V(r\sqrt{-t})\text{"} \equiv J_1(r\sqrt{-t})\left(\frac{\cos\left(\frac{\pi\alpha(t)}{2}\right)\cos\left(\frac{\pi\beta(0)}{2}\right)}{\cos\left(\frac{\pi\alpha(0)}{2}\right)\cos\left(\frac{\pi\beta(t)}{2}\right)}\right)$$

$$\text{"}J_1^T(s\sqrt{-t})\text{"} \equiv J_1(r\sqrt{-t})\left(\frac{\sin\left(\frac{\pi\alpha(t)}{2}\right)\sin\left(\frac{\pi\beta(0)}{2}\right)}{\sin\left(\frac{\pi\alpha(0)}{2}\right)\sin\left(\frac{\pi\beta(t)}{2}\right)}\right) \tag{2.13}$$

$$\beta(t) = \frac{x_1^2 + r^2 t}{(x_2^2 - x_1^2)} \tag{2.14}$$

where:

$$J_1(x_1) = J_1(x_2) = 0 \tag{2.15}$$

are the first and second zeros of J_1. The form of "J_1" is intended to remove an unphysical pole coming from the Regge phase at $\alpha(t) = 0$. It does not remove additional poles at $-t$ large enough so that $\alpha(t) = -n$ so the amplitudes cannot be expected to make sense for large negative t. The $\mathbb{P}$ amplitude will be given by the form:

$$\mathcal{M}^{\mathbb{P}}_{++} = i g_{\mathbb{P}}\, s\, e^{A_{\mathbb{P}} t}$$
$$\mathcal{M}^{\mathbb{P}}_{+-} = 0 \ . \tag{2.16}$$

The model parameters for the amplitudes are given in Table 1. In particular, the vector parameters are from a fit to $K^o_L p \to K^o_S p$, g^ω and g^ρ are from a comparison of $K^o_L p \to K^o_S p$ and π nucleon charge exchange assuming good SU(3), the A_2 parameters are from the fit to $K^+ n \to K^o p$. The $\mathbb{P}$

amplitude parameters are from an analysis of K^+p and K^-p elastic scattering. This completes the definition of the K-nucleon amplitudes which will now be used in the GMSF to determine the deuteron regeneration amplitude.

III. The Regeneration Amplitude

Following Ref. 5 a LF deuteron scattering amplitude may be expressed as:

$$F(p^{lab},\Delta) = \frac{1}{4\pi}\int d^3x_1\, d^3x_2\, \phi^{*f}(x_1,x_2)\,\phi^{i}(x_1,x_2)$$

$$< I=0, M' \mid \frac{ip^{lab}}{2\pi}\int d^2b\, e^{i\underset{\sim}{b}\cdot\underset{\sim}{\Delta}}\left[1-\prod_{j=1}^{2}\left(1-\frac{1}{2\pi i p^{lab}}\int d^2\delta\, e^{-i\delta\cdot(b-x_j)} f_{(j)}(p^{lab},\delta)\right)\right] \mid I=0, M >$$

$$\times \frac{1}{(2\pi)^3\delta^3(P_f-P_i)} \tag{3.1}$$

where:

$$\psi_d^{\,i}(x_1,x_2) = Y_{00}(\hat{r})\phi(r) \mid I=0;\ M >$$

$$\psi_d^{\,f}(x_1,x_2) = Y_{00}(\hat{r})\phi(r)\, e^{i\underset{\sim}{P}\cdot\underset{\sim}{X}} \mid I=0,\ M' > \tag{3.2}$$

$$r = \underset{\sim}{x}_1 - \underset{\sim}{x}_2 \qquad X = \frac{\underset{\sim}{x}_1 + \underset{\sim}{x}_2}{2}$$

defining:

$$g^{e\ell}(|q|) = \frac{1}{4\pi}\int d^3x\ |\phi(r)|^2\, e^{i\underset{\sim}{q}\cdot\underset{\sim}{x}} \tag{3.3}$$

some of the integrals may be performed by:

$$F(p^{lab},\Delta) = g^{e\ell}(|\Delta|/2) < I=0;\ M' \mid f_{(1)}(p^{lab},\Delta) + f_{(2)}(p^{lab},\Delta)$$

$$+ \frac{i}{2\pi p^{lab}} \int d^2x\, g^{e\ell}(|x|) \quad f_{(1)}(p^{lab}, \tfrac{\Delta}{2}+x) \quad f_{(2)}(p^{lab}, \tfrac{\Delta}{2}-x) \quad \mid I=0, M > . \tag{3.4}$$

The regeneration amplitude can be expressed as the difference of the two elastic amplitudes:

$$F(K_L^o d \to K_S^o d) = F(K^o d \to K^o d) - F(\overline{K}^o d \to \overline{K}^o d) . \tag{3.5}$$

Performing the isospin sum in the above amplitude gives for the regeneration amplitude:

$$\begin{aligned} F^{reg}(p^{lab},\Delta) = {} & g^{e\ell}(|\Delta|/2) < M' \mid f_{(1)}^{\omega^o}(p^{lab},\Delta) + f_{(2)}^{\omega^o}(p^{lab},\Delta) \mid M > \\ & + \frac{i}{2\pi p^{lab}} \int d^2x\, g^{e\ell}(|x|)\, \tfrac{1}{2} < M' \mid f_{(1)}^{\omega^o}(p^{lab}, \tfrac{\Delta}{2}+x) f_{(2)}^{I\!P}(p^{lab}, \tfrac{\Delta}{2}-x) \\ & + f_{(1)}^{I\!P}(p^{lab}, \tfrac{\Delta}{2}+x)\, f_{(2)}^{\omega^o}(p^{lab}, \tfrac{\Delta}{2}-x) - 3 f_{(1)}^{\rho}(p^{lab}, \tfrac{\Delta}{2}+x) \\ & \times f_{(2)}^{A_2}(p^{lab}, \tfrac{\Delta}{2}-x) - 3 f_{(1)}^{A_2}(p^{lab}, \tfrac{\Delta}{2}+x)\, f_{(2)}^{\rho}(p^{lab}, \tfrac{\Delta}{2}-x) \mid M > . \end{aligned} \tag{3.6}$$

The spin sum is straightforward for the single scattering term. For the double scattering term in the small angle region where the amplitudes are large, one may safely neglect the rotation required to put the intermediate kaon along the z axis. This said, the prescription for calculating F^{reg} is complete; however, the full expression for F although easily obtained is somewhat lengthy and unenlightening. Using the amplitudes from Section II, and the "Hulthen wave function" for the deuteron:[5]

$$\phi(r) = C\left(e^{-a_1 r} - e^{-a_2 r}\right)/r$$

$$a_1 = 0.232\ F^{-1} \qquad a_2 = 1.202\ F^{-1} \tag{3.7}$$

the integrations for the double scattering term have been performed numerically for values of p^{lab} between 2 and 12 GeV and the cross sections calculated from:

$$\left.\frac{d\sigma}{d\Omega}\right)_{lab} = \frac{(\hbar c)^2}{3} \sum_{M'M} |\, F^{reg}(p^{lab}, \Delta)\,|^2 \quad . \tag{3.8}$$

The results of this calculation are presented inFig. 2 for incident kaon momenta of 5 and 10 GeV. There are two distinct features appearing in the cross section. The first is a shoulder beginning at $\sim 0.2\ GeV^2$ which is due to the vanishing of the imaginary part of the non-flip ω^o amplitude. The second is a broad dip at $\sim 0.9\ GeV^2$ due to the interference of the single and double scattering terms. It is interesting to note that the double scattering term is suppressed for the inelastic process relative to an elastic process due to the more rapidly varying phase of the inelastic amplitudes. This pushes the dip out to $0.9\ GeV^2$ from its usual location at $0.4\ GeV^2$ for elastic processes. As K^o regeneration provides the only available inelastic process which does not destroy the deuteron itself, this is an interesting prediction.

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References

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5. See e.g. V. Franco and E. Coleman, Phys. Rev. Letters 17, 827 (1966).

Figure Captions

Figure 1 Lab frame kinematics for K-Nucleon system.

Figure 2 Differential cross section for $K_L^o d \rightarrow K_S^o d$ at 5 and 10 GeV lab momentum.

TABLE I

MODEL PARAMETERS

$\alpha(t) = \frac{1}{2} + 0.9\,t$	$R = 5.13\ (\mathrm{GeV})^{-1}$
$g_{++}^{\omega^o} = -29.4$	$g_{+-}^{\omega^o} = -\ 4.8$
$g_{++}^{\rho} = -10.4$	$g_{+-}^{\rho} = -21.8$
$g_{++}^{A_2} = -\ 8.88$	$g_{+-}^{A_2} = -16.3$
$g_{++}^{IP} = 94.0$	
$A_V = -\ 0.88\ (\mathrm{GeV})^{-2}$	$A_T = -0.70\ (\mathrm{GeV})^{-2}$
$A_{IP} = 3.5\ (\mathrm{GeV})^{-2}$	
$B_V = 2.0\ (\mathrm{GeV})^{-2}$	$B_T = 2.0\ (\mathrm{GeV})^{-2}$
$a_V = 2.97\ (\mathrm{GeV})^{2}$	$a_T = 7.28\ (\mathrm{GeV})^{-2}$
$b_V = 8.79\ (\mathrm{GeV})^{-4}$	$b_T = 4.88\ (\mathrm{GeV})^{-2}$

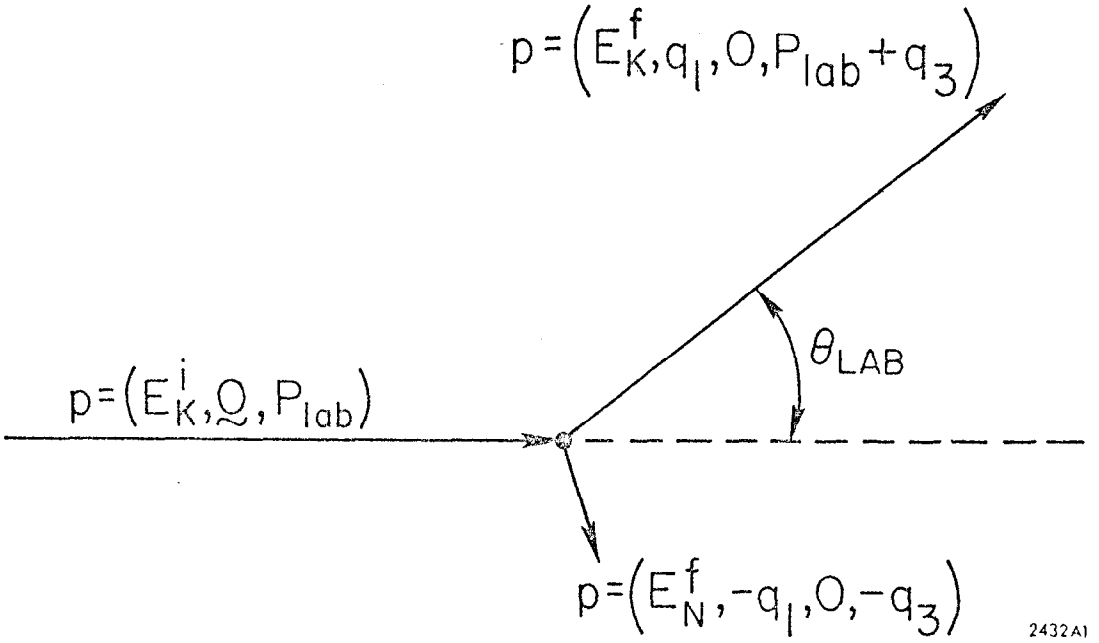
$p=\left(E_K^f, q_1, 0, P_{lab}+q_3\right)$
θ_{LAB}
$p=\left(E_K^i, \underset{\sim}{0}, P_{lab}\right)$
$p=\left(E_N^f, -q_1, 0, -q_3\right)$
2432A1

Fig. 1

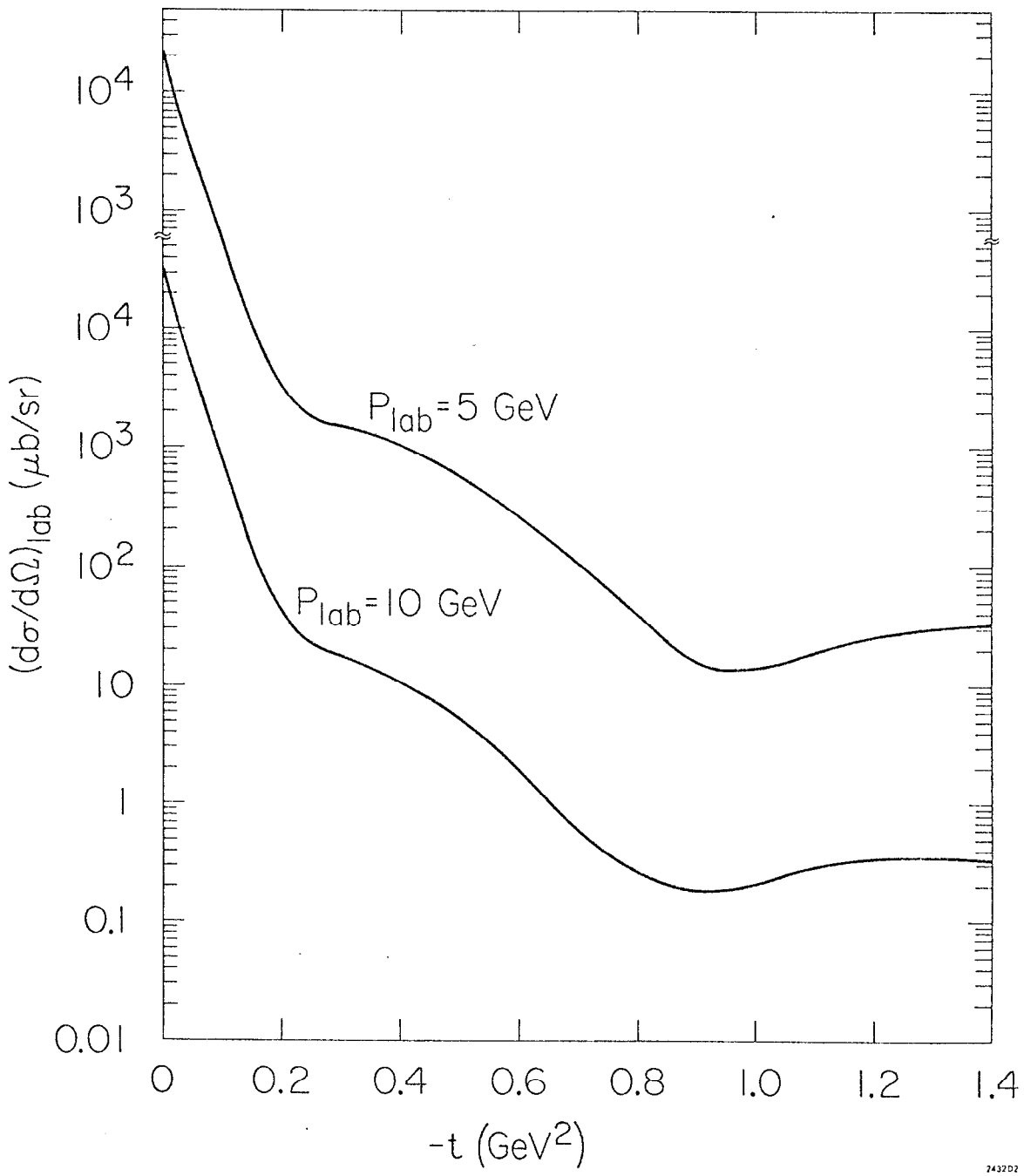
10^4
10^3
10^4
10^3
10^2
10
1
0.1
0.01
$(d\sigma/d\Omega)_{lab}$ (μb/sr)
P_{lab}=5 GeV
P_{lab}=10 GeV
0
0.2
0.4
0.6
0.8
1.0
1.2
1.4
-t (GeV^2)
7432D2

Fig. 2