# ASYMMETRY IN SEMI-INCLUSIVE ANNIHILATION* 

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## ABSTRACT

Interference between one- and two-photon processes for $\mathrm{e}^{+} \mathrm{e}^{-}$annihilation into hadrons in a two-jet parton model leads to a charge asymmetry of detected final state hadrons along the directions of the incident leptons. The asymmetry near the lepton axis grows as $2 \ln ^{2} \frac{\theta}{2}-4 \ln \frac{\theta}{2} \ln \frac{\mathrm{~s}}{\Delta \mathrm{E}^{2}}$, so that in spite of the $0(\alpha)$ suppression relative to the Born cross section, the asymmetry can amount to a - $2 \%$ effect at $\theta=2^{0}$ for $\pi^{ \pm}$or $K^{ \pm}$inclusive measurements in a typical experiment. The precise size of the asymmetry depends on the parton charges.
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## I. Introduction

As the available energy of electron colliding beams increases, so too does the range of interesting experiments. In this paper we wish to study higher order electromagnetic effects in $\mathrm{e}^{+} \mathrm{e}^{-}$annihilation. In particular, application of the naive parton model ${ }^{1}$ will enable us to discuss enhanced asymmetries in hadronic inclusive processes such as $\mathrm{e}^{+} \mathrm{e}^{-} \rightarrow \pi^{ \pm}+\mathrm{X}$, in which, due to electromagnetic interfence effects, the charge preferentially "maintains its direction of motion" so that pions of opposite charge tend to align in opposite directions measured along the colliding beam axis. The effects we study are distinct from the other well-known two-photon processes ${ }^{2}$; the latter effects are distinguishable from ours by the presence of the initial leptons in the final state and will in any case not contribute to asymmetries.

While these asymmetries are suppressed by $\mathrm{O}(\alpha)$ compared to the ordinary $\mathrm{O}\left(\alpha^{2}\right)$ lowest order Born contributions to annihilation, we find that for spin- $\frac{1}{2}$ partons, directions along or against the direction of the incoming lepton beams have asymmetries which are enhanced by squared logarithmic factors of the CM half-angle $\frac{\theta}{2}$. This limit is equivalent to the limit $-\mathrm{u} \approx \mathrm{s} \gg-t \gg \mu^{2}$ (or, in the backward direction, $-\mathrm{t} \approx \mathrm{s} \gg-\mathrm{u} \gg \mu^{2}$ ), where the first inequality puts us near the lepton axis (here the $\log$-squared terms dominate the single $\log$ terms); $\mu$ is some representative mass scale which we take to be typically hadronic. Asymmetries on the order of $2-5 \%$ are expected for $\theta \approx 2^{\circ}$ in a typical experiment providing $s$ is large enough that the above limits are satisfied. Such effects should be within reach of present experimental techniques. We shall discuss further numerical estimates later.

We might also note here that of course such logarithmic factors represent a breakdown of Bjorken scaling ${ }^{3}$ for fixed experimental resolution $\Delta \mathrm{E}$ even if such scaling were seen in the lowest order terms. In general this is hardly surprising, since it is a consequence of the existence of a renormalizable ultraviolet divergent theory in nature which is not asymptotically free, namely Quantum Electrodynamics (QED). Since the non-scaling corrections grow with the kinematic variables, we might reasonably expect such corrections to confuse the analysis of future experiments on scaling. In particular, such corrections cannot be extracted from the data in a completely model independent way.

What can observation (or non-observation) of hadronic asymmetries teach us? (i) The interference effects which we discuss are direct analogues to electromagnetic corrections which lead to differences ${ }^{4}$ between $\mathrm{e}^{+}$and $\mathrm{e}^{-}$ deep inelastic scattering. To our knowledge such correction terms have never been experimentally observed and would therefore constitute a check of QED.
(ii) Observation of the asymmetries in the hadronic state requires, as we shall see, that such states be disjoint decay products (hadrons occur in "jets") of a parton (or antiparton) which is created by the $\mathrm{e}^{+} \mathrm{e}^{-}$annihilation. In other words, observation of the asymmetry means that the hadron charge follows the parton charge, and that the parton charge will appear in the final state. One of the most important questions which the parton model raises revolves around just this point. Since an inclusive experiment is far simpler than an exclusive one, the test we propose can be most important for the parton model. Another possible class ${ }^{5}$ of parton models has the quark quantum numbers annihilating, with quark quantum numbers appearing only on the average in the parton fragmentation region. It has been shown ${ }^{6,7}$ that this type of model
has consistency problems; moreover, the actual dynamics of such models is not well understood, so even though it is conceivable that a remnant asymmetry can survive in them, we are unable to make any unambiguous calculation for them at this time. Our results do not refer to these models, and we refer the interested reader to Ref. 6 for more details. (iii) The size of the asymmetry is a function of both the parton mass and parton charges. Therefore when more careful estimates are appropriate, bounds can be put on a suitable combination of these quantities, which cannot be derived from the Born terms. (iv) Such asymmetries in $\mathrm{e}^{+} \mathrm{e}^{-}$annihilation are also predicted in gauge theories ${ }^{8}$, where they can arise simply from parity violation at an order which is a priori lower than the purely QED effect we are discussing. Asymmetry data may therefore provide us with a handle on gauge theories as well as being useful in refining parton models or checking QED, but at the same time it is necessary that the purely QED contribution be carefully sorted out. This is easily done if $\triangle E / E$ is fixed since the QED effect we discuss is then energy independent while the gauge theory effect continues to grow with energy.

In computing ${ }^{4}$ the differences between $e^{+}$and $\mathrm{e}^{-}$deep inelastic scattering, similar logarithmic enhancements occur in suitable kinematic regimes involving $q^{2}$, the spacelike continuation of $s$. We would only like to comment here that in some respects the annihilation version is simpler experimentally because only one type of experimental beam is involved, whereas in the deep inelastic experiment two separate beams are involved with a consequent possibility of systematic error.

The organization of this paper is as follows: In Section II we discuss the general features and kinematics of the annihilation process and describe the general features of the interference terms. In Section III we discuss the parton
model description of the Born term and the relevance of the parton model for the higher order terms. The relation between the parton model and $\mu$-pair production by $\mathrm{e}^{+} \mathrm{e}^{-}$annihilation is studied in Section IV, which also contains a discussion of the form of $\mu$-pair production to $\mathrm{O}\left(\alpha^{3}\right)$; this discussion employs the most recent work ${ }^{9}$ on this process. Finally in Section $V$ we apply the parton model to the inclusive hadronic process in final form, make more detailed numerical estimates, and discuss the asymmetries for different detected hadrons when partons are quarks; the difficulty in sorting purely QED from gauge theory effects will be touched upon here also.

## II. General Features

Figure 1 describes the kinematic quantities of interest for the annihilation. We are interested in the analogue to deep inelastic scattering, for which the single outgoing hadron h is detected, with variables as follows:

$$
\begin{align*}
& s=q^{2}=\left(l^{-}+l^{+}\right)^{2}>0  \tag{2.1}\\
& \nu=-p \cdot\left(l^{-}+l^{+}\right) / m_{h}<0,
\end{align*}
$$

in the region

$$
\begin{align*}
& q^{2},-\nu \gg \text { rest masses } \\
& 0<\omega=\frac{-2 m_{h}^{\nu}}{q^{2}}<1 . \tag{2.2}
\end{align*}
$$

The lowest order (one-photon) approximation to the amplitude for this process is shown in Fig. 2. In this approximation the cross section is related to a certain discontinuity of the Compton amplitude and can be given a
decomposition in terms of the usual structure functions. This decomposition as well as the relations of this process to deep inelastic scattering and three-body annihilation are well known. ${ }^{10}$

The (differential) cross section ${ }^{11}$ to $\mathrm{O}\left(\alpha^{3}\right)$ can be written as interference terms of two types, type (a) and type (b), according to the unitarity cut. Type (a) interference terms arise from interference of an $O(\alpha)$ amplitude with an amplitude of $\mathrm{O}\left(\alpha^{2}\right)$, while the type (b) terms represent the interference of two $\mathrm{O}\left(\alpha^{3 / 2}\right)$ terms. This distinction, shown in Fig. 3, is of no intrinsic worth for the computation of the cross section, but is convenient for our exposition.

Once this distinction has been made, a further breakdown of the two types is possible. For example, type (a) graphs can be uniquely written in the two terms shown in Fig. 4. Figure 4(a) shows the kind of term which will be of interest to us, since it contributes to asymmetries. Figure 4 (b), which may for example represent internal form factor corrections, is ignored in the remainder of this paper, because even though these contributions are often enhanced by logarithmic factors ${ }^{12}$ they cannot contribute to any asymmetry.

Of the type (b) terms, a single real photon must be emitted either from an external or an internal line in the individual amplitudes. In particular, we classify the photon according to whether it is emitted from a lepton line, from an external hadron line, or from an internal line, as in Fig. 5(a), (b) and (c). (The external hadron line can be either the detected hadron or part of the undetected group.) This leaves us all in all with nine terms of type (b) to be considered. This classification will prove useful when we investigate the relevance of a parton approach in Section III; there we will find it convenient to subdivide even further these contributions.

We should remark here that when higher order electromagnetic effects are taken into account, as in this work, then the usual structure function analysis ${ }^{10}$, which rests on the one-photon approximation, must break down. In other words, Rosenbluth-type formulas are no longer appropriate. It might still be interesting to ask whether the cross section scales; i. e., whether up to necessary kinematic factors the cross section depends only upon the ratios of large kinematic variables themselves rather than upon ratios of these variables to the rest masses of the problem; and as we have stated in the introduction, the answer is that scaling is indeed violated.

However we might comment that in the limit we are studying, if experimental circumstances are such that $\Delta E / E$ is constant, then scaling is recovered.

## III. The Parton Model

In the naive parton model, the one-photon term is dominated at large $s$ by the summation of pair production of partons ${ }^{1}$, since partons are at least approximately pointlike compared to observed hadronic systems. In the usual approach, partons have finite mass, an assumption we also follow. A produced parton of type i and charge $Q_{i}$ then decays with probability $D_{i}^{h}(\omega)$ into an (observed) hadron h which carries fraction $\omega$ of the (asymptotic) magnitude $\frac{1}{2} \sqrt{\mathrm{~s}}$ of the parton three-momentum. Moreover, the transverse momentum of the hadrons with respect to the parton line is limited to the usual $\sim 150 \mathrm{MeV} / \mathrm{c}$. This is pictured in Fig. 6. While the physical picture is therefore basically a two-jet model, appropriate decay characteristics of the parton can "fill in the rapidity gap" and give rising multiplicity as well as the more intuitive finite multiplicity. We refer the reader elsewhere ${ }^{1}$ for more details of this picture.

One common and crucial feature of (calculable) parton models such as that used in this paper is that the partons of the produced pair do not interact after production. When interaction ${ }^{4-6}$ (particularly with charged particle exchange) is not allowed, then the parton charge must ineluctably appear in the final state. The appropriateness of this parton model to $\mathrm{e}^{+} \mathrm{e}^{-}$ annihilation can be probed by making an exclusive examination of the final state or, as we shall hopefully convince the reader, by observing higherorder effects in the much simpler inclusive experiment.

Lct us turn now to the type (a) amplitude yielding the asymmetry, as in Fig. 4(a). Presumably one of the relevant questions concerns the size of $q_{1}^{2}$ and $q_{2}^{2}$, where (timelike) $q=\left(q_{1}+q_{2}\right)$. In particular, it is possible to imagine $q_{1}^{2}$ and $q_{2}^{2}$ both to be finite and still have asymptotically large $q^{2} \approx 2 q_{2} \cdot q_{2}$. [This is to be contrasted ${ }^{4}$ with the case of large spacelike $q^{2}$, where elementary kinematic considerations dictate that at least one of the $q_{i}^{2}$ be large and spacelike.] It is the dynamics alone which determines the relative sizes of $q_{1}^{2}, q_{2}^{2}$, and $q_{1} \cdot q_{2}$ when $q^{2}>0$. The importance of this question lies in the realization that if $q_{i}^{2}$ is finite, it need not dominantly interact with a pointlike (parton) line.

As an example, consider a case we shall employ at more length later; namely, $\mu$-pair production. The relevant two-photon terms are shown in Fig. 7. We investigated these graphs and found that in fact the quantities $q_{1}^{2}$ and $q_{2}^{2}$ both remain finite as far as the contribution of the leading logarithmic behavior is concerned. Whether this would remain true in more complicated graphs depends in detail on the particular graphs in question.

We shall now argue that in spite of this result, the leading contribution to the two-photon exchange amplitude in the interference term for hadron production is the one in which the two photons interact with a single parton line; i. e., the same as Fig. 7 with the muons replaced by partons. Rather than writing elaborate dynamical schemes for the necessary quantities, an approach which would suffer from too great a degree of particularity, we base our arguments on an assumed lack of long-range correlations, on quantum number considerations, on the assumed non-overlap of parton states and individual hadronic states, and on the necessity of overlap with the Born term, Fig. 6, for non-zero interference. The argument is then not that we are computing the entire two-photon contribution but only the most significant part of it. It does not seem to us that the assumptions used here are particularly novel or unreasonable.

In general, the two-photon interference terms are of two classes, as in Fig. 8(a) and (b). (While a variety of other more complicated diagrams involving form-factor-type corrections on the two-photon side can be drawn, we dismiss these from consideration since they are not expected to cancel the behaviors of Figs. 8(a) and (b), which will be seen to dominate for $\theta \ll 1$.) The states s in Fig. 8(a) and (b) are at this point either parton states of finite mass or coherent hadronic systems of any mass. As far as the single-state term of Fig。8(a) goes, it suffices to say that since the state $\bar{s}$ must significantly overlap with the state $\bar{p}$ from the Born term, $\bar{s}$ must itself be a (fast) $\bar{p}$ state, and hence $s$ is a parton state. We reach this conclusion independent of the size of $q_{1}^{2}$ and $q_{2}^{2}$. To compute this contribution to the cross section it suffices to use results for $\mu$-pair production (as in Fig. 7) along with familiar properties of the parton model. While we shall do this in further detail in Section IV, we remark here than in certain directions the asymmetry from this graph is enhanced by squared logarithmic factors.

Since the diagram in Fig. 8(b) is quite hard to calculate, it would be comforting to know that it could not contribute such factors to the asymmetry so that in fact the only "type (a)" graph we would need to consider is Fig. 8(a). We shall now argue that Fig. 8(b) can indeed be ignored. Consider first the kinematic limit $q_{1}^{2} \approx q_{2}^{2} \approx \frac{1}{4} q^{2}$. (We imagine this region is not suppressed by the dynamical details.) Then the lines $p_{1}, \ldots, p_{4}$ represent partons. Let us go to the center-of-mass system so that if a four vector is labeled $\mathrm{v}=\left(\mathrm{v}_{0}, \overrightarrow{\mathrm{v}}\right)$,

$$
\begin{align*}
& q=(\sqrt{\mathrm{s}}, 0) ; \\
& \mathrm{p}_{\mathrm{a}} \approx\left(\frac{1}{2} \sqrt{\mathrm{~s}}, \frac{1}{2} \sqrt{\mathrm{~s}}\right), \mathrm{p}_{\mathrm{b}} \approx\left(\frac{1}{2} \sqrt{\mathrm{~s}},-\frac{1}{2} \sqrt{\mathrm{~s}}\right) ;  \tag{3.1}\\
& \mathrm{q}_{1} \approx\left(\frac{1}{2} \sqrt{\mathrm{~s}}, 0\right) \approx \mathrm{q}_{2} ; \\
& \mathrm{p}_{1} \approx\left(\frac{1}{4} \sqrt{\mathrm{~s}}, \frac{1}{4} \sqrt{\mathrm{~s}}\right) \approx \mathrm{p}_{3}, \quad \mathrm{p}_{2} \approx\left(\frac{1}{4} \sqrt{\mathrm{~s}},-\frac{1}{4} \sqrt{\mathrm{~s}}\right) \approx \mathrm{p}_{4} .
\end{align*}
$$

The momenta going across the graph must now match; e.g. we must have $\mathrm{p}_{\mathrm{a}}=\mathrm{p}_{1}+\mathrm{p}_{3}$ and $\mathrm{p}_{\mathrm{b}}=\mathrm{p}_{2}+\mathrm{p}_{4}$. This is satisfied by the configuration of Eq. (3.1) and so we conclude that this region can indeed contribute to the interference term from a kinematic point of view. However, the quantum numbers must also match, and this is in general far more difficult. For example, the lines represented by $p_{2}$ and $p_{4}$ must have the quantum numbers of a $\bar{p}$, while $p_{1}$ and $p_{3}$ must have the quantum numbers of a $p$. At the same time $p_{3}+p_{4}$ (and $p_{2}+p_{1}$ ) must be able to annihilate. In standard quark-parton pictures the simultaneous satisfaction of these requisites is not possible. Now let us consider the other extreme kinematic limit $q_{1}^{2} \approx q_{2}^{2} \approx 0$, wherein the states represented by $p_{1}, \ldots, p_{4}$ can be general hadronic states. In this limit it is impossible to simultaneously satisfy the requirements of momentum conservation
$\left(\mathrm{p}_{\mathrm{a}}=\mathrm{p}_{1}+\mathrm{p}_{3}, \mathrm{p}_{\mathrm{b}}=\mathrm{p}_{2}+\mathrm{p}_{4}, \mathrm{q}_{1}-\mathrm{p}_{1}+\mathrm{p}_{2}, \mathrm{q}_{2}=\mathrm{p}_{3}+\mathrm{p}_{4}\right.$, and $\mathrm{q}=\mathrm{p}_{\mathrm{a}}+\mathrm{p}_{\mathrm{b}}$ ) and short-range order (in particular $\vec{p}_{1} \approx \vec{p}_{2}$ and $\vec{p}_{3} \approx \vec{p}_{4}$ ) and still keep the parton masses $p_{a}^{2}$ and $p_{b}^{2}$ finite. For example, with $q, p_{a}$, and $p_{b}$ still given by Eq. (3.1), we could write

$$
\begin{align*}
& q_{1} \approx\left(\frac{1}{2} \sqrt{s}, \frac{1}{2} \sqrt{s}\right), \quad q_{2} \approx\left(\frac{1}{2} \sqrt{s},-\frac{1}{2} \sqrt{s}\right)  \tag{3.2}\\
& p_{1} \approx\left(\frac{1}{4} \sqrt{s}, \frac{1}{4} \sqrt{s}\right) \approx p_{2}, \quad p_{3} \approx\left(\frac{1}{4} \sqrt{s},-\frac{1}{4} \sqrt{s}\right) \approx p_{4}
\end{align*}
$$

if the states $p_{1}, \ldots, p_{4}$ are of finite mass. $\quad\left(\vec{p}_{1} \approx \ldots \approx \vec{p}_{4} \approx 0\right.$ does not satisfy $q_{1}=p_{1}+p_{2}$ and $q_{2}=p_{3}+p_{4}$.) In this case, although the requirements of short-range order are met, momentum conservation is not satisfied $\left.\overrightarrow{\mathrm{p}}_{\mathrm{b}} \neq \overrightarrow{\mathrm{p}}_{2}+\overrightarrow{\mathrm{p}}_{4}, \overrightarrow{\mathrm{p}}_{\mathrm{a}} \neq \overrightarrow{\mathrm{p}}_{1}+\overrightarrow{\mathrm{p}}_{3}\right)$. Another possibility instead of Eq. (3.2) is to have $p_{1} \approx p_{a} \approx\left(\frac{1}{2} \sqrt{\mathrm{~s}}, \frac{1}{2} \sqrt{\mathrm{~s}}\right), \mathrm{p}_{4} \approx \mathrm{p}_{\mathrm{b}} \approx\left(\frac{1}{2} \sqrt{\mathrm{~s}},-\frac{1}{2} \sqrt{\mathrm{~s}}\right)$, and $\mathrm{p}_{2} \approx \mathrm{p}_{3} \approx 0$; here momentum is conserved, but the requirement of short-range order is not met $\left.\overrightarrow{\mathrm{p}}_{1} \not \approx \overrightarrow{\mathrm{p}}_{2}, \overrightarrow{\mathrm{p}}_{3} \not \approx \overrightarrow{\mathrm{p}}_{4}\right)$. (Of course the quantum number considerations discussed above also continue to apply.) Finally, no matter which kinematic region we choose to look at, there is no reason to expect Fig. 8(b) terms to have the squared logarithmic enhancements of Fig. 8(a) terms without rather special behavior of the " $2 \mathrm{~s} \rightarrow$ hadrons" vertices in Fig. 8(b). Thus Fig. 8(b) is eliminated, and Fig. 8(a) furnishes all the "type (a)" interference terms we need consider.

Next we turn to the generic "type (b)" interference diagram of Fig. 3(b), whose separate amplitudes have previously been classified in Fig. 5. Evaluation of "type (b)" diagrams is necessary to insure that the part of the "type (a)" contribution which is infrared (IR) divergent can be properly cancelled by soft photon emission processes. In the context of the parton model, the relevant
classification of amplitudes analogous to that in Fig. 5 is shown in Fig. 9, with a corresponding set of 16 interference terms contributing to the type (b) cross section. We should like to show now that the particular type (b) diagrams which need to be considered to accomplish the IR divergence cancellation are precisely those which we can easily evaluate by appealing to the $\mathrm{O}\left(\alpha^{3}\right)$ calculation of $\mathrm{e}^{+} \mathrm{e}^{-} \rightarrow \mu^{+} \mu^{-}$in Ref. 9; namely, those amplitudes of Figs. 9(a) and (b), lepton or parton (muon) bremsstrahlung. (The translation of $\mathrm{e}^{+} \mathrm{e}^{-} \rightarrow \mu^{+} \mu^{-}$considerations into parton language for our process is done in Section IV.) To show this we shall argue that cross terms between the 9(a), (b) graphs and the 9 (c), (d) graphs are small. We shall see in Section IV that interferences between the graphs of Fig. 9(a) and (b) give squared logarithmic enhancements like the type (a) graph behavior we have discussed earlier; since these enhancements would not in general be nullified by the contributions of the 9 (c) and (d) graphs, a knowledge of 9 (a) and (b) behavior is sufficient to enable us to extract that (useful) part of the asymmetry cross section (from type (a) diagrams) which is not infrared divergent.

Our argument now depends on the dominance of soft photon emission over hard emission, a well-known (and numerically sound ${ }^{9}$ ) characteristic of bremsstrahlung phenomena. Since none of the emitted hadrons in Fig. 9(d) and none of the internal lines in the photon-emitting blob in Fig. 9(c) carry the full momentum of the parton line (or equivalently the lepton line), the cross terms will then vanish. This follows for $9(\mathrm{~d})$ because ${ }^{13} \mathrm{D}(\omega) \xrightarrow[\omega \rightarrow 1]{ } 0$ and for 9(c) because the region where an internal line carries the parton (or lepton) momenta is a vanishingly small region of phase space.

## VI. Parton Pair-Production Amplitudes

The type (a) terms corresponding to the one-photon-two-photon interference as in Fig. 8 (a) and the type (b) terms corresponding to the bremsstrahlung amplitudes of Figs. $9(a)$ and (b) can be completely calculated in terms of the corresponding results for $\mu$-pair production by $\mathrm{e}^{-} \mathrm{e}^{+}$annihilation. In this section we use the results of recent work ${ }^{9}$ on this problem to write down such a cross section. The asymmetry which eventually appears in the semi-inclusive final state first appears as an asymmetry in (unit charge) parton pair production. Since the $\mu$-pair paper of Ref. 9 gives rather complete detail, we shall be content here with quoting results.

We study the differential cross section $d \sigma / d \Omega$ for pair production of partons of unit charge up to order $\alpha^{3}$, where $\Omega$ measures the center-of-mass angle of the $\mathrm{p}^{-}$with respect to the incoming $\mathrm{e}^{-}$. The (asymptotic) kinematic variables are defined in the usual fashion in Fig. 7 , with $u \approx-t-s$. We sum and average over spins as well. For $\vec{p}^{+}$and $\vec{p}^{-}$along the collision axis $d \sigma / d \Omega$ contains log-squared terms from both the type (a) and (b) terms, and we shall analytically extract only these $\log$-squared terms in the variables $s, t$, and $u$ which contribute to the asymmetry. It is most important to note that in doing so we lose the ability to determine the scale of the logarithmic terms, i. e., according to this approximation

$$
\begin{equation*}
\log \frac{s}{m_{1}^{2}}=\log \frac{s}{m_{2}^{2}}+\log \frac{m_{2}^{2}}{m_{1}^{2}} \approx \log \frac{s}{m_{2}^{2}} \tag{4.1}
\end{equation*}
$$

Therefore we employ a generic mass term $\mu^{2}$ for a scale; we shall discuss its size below.

We find that the contribution of the type (a) terms (for $\theta \approx 0$ or $\pi$ ) is in order $\alpha^{3}$

$$
\begin{aligned}
\left.\frac{d \sigma}{d \Omega}\right|_{\alpha^{3}, \text { type (a) }} & =\frac{d \sigma^{0}}{d \Omega} \frac{\alpha}{2 \pi} \frac{1}{\left(u^{2}+t^{2}\right)}\left[u^{2}+4 u t+7 t^{2}\right] \ln ^{2} \frac{s}{-t} \\
& +\frac{d \sigma}{d \Omega} \frac{-\alpha}{\pi}\left[2 \ln \frac{-t}{\mu^{2}} \ln \frac{s}{\lambda^{2}}\right] \\
& -(t \leftrightarrow u) .
\end{aligned}
$$

In this expression $\mathrm{d} \sigma^{\circ} / \mathrm{d} \Omega$ is the Born term result,

$$
\begin{equation*}
\frac{\mathrm{d} \sigma^{\mathrm{o}}}{\mathrm{~d} \Omega} \approx \frac{\alpha^{2}}{2 \mathrm{~s}^{3}}\left(\mathrm{u}^{2}+\mathrm{t}^{2}\right) \tag{4.3}
\end{equation*}
$$

and we have kept a term involving the photon mass $\lambda$ as a check that the bremsstrahlung terms will cancel these infrared divergences.

The contribution of the bremsstrahlung terms, with corresponding amplitudes shown in Fig. 10, contains double logarithmic terms in a softphoton approximation. (For the numerical effect of "hard" photon emission we refer the reader to Ref. 9.) These are the type (b) contributions (see Fig. 9(a) and (b)) which we include in our calculation; in particular we are interested as above only in that part of the type (b) contribution which leads to an asymmetry. We find

$$
\begin{align*}
\left.\frac{\mathrm{d} \sigma}{\mathrm{~d} \Omega}\right|_{\alpha^{3}, \text { type }(\mathrm{b})} & =\frac{\mathrm{d} \sigma^{\circ}}{\mathrm{d} \Omega} \frac{\alpha}{\pi}\left[2 \ln \frac{-\mathrm{t}}{\mu^{2}} \ln \left(\frac{\Delta \mathrm{E}}{\lambda}\right)^{2}+\mathrm{Z}(\mathrm{~s}, \mathrm{t}, \mathrm{u})\right]  \tag{4.4}\\
& -(\mathrm{t} \leftrightarrow \mathrm{u})
\end{align*}
$$

where $\Delta \mathrm{E}$ is the energy resolution of the experiment and $\mathrm{Z}(\mathrm{s}, \mathrm{t}, \mathrm{u})$ is a function containing no double logarithmic terms. We therefore have the appropriate infrared cancellation in the combination of Eqs. (4.2) and (4.4).

Let us now consider these terms at various angles. If the scattering angle is large, so that $\mathrm{O}(\mathrm{s})=\mathrm{O}(-\mathrm{t})=\mathrm{O}(-\mathrm{u})$ then the arguments of all the logarithms become the same, say $+\mathrm{s} / \mu^{2}$. It is then straightforward that no double logarithms at all remain, and the asymmetry (reassuringly) vanishes in this logarithmic order. Suppose however that we study the region of forward scattering, where $-\mathrm{u} \approx \mathrm{s} \gg-\mathrm{t}$. Then the sum of Eqs. (4.2) and (4.4) give

$$
\begin{equation*}
\left.\frac{\mathrm{d} \sigma}{\mathrm{~d} \Omega}\right|_{\alpha} ^{3}, \theta=\epsilon \ll 1<\frac{\mathrm{d} \sigma^{\mathrm{o}}}{\mathrm{~d} \Omega} \frac{\alpha}{\pi}\left[2 \ln ^{2} \frac{\theta}{2}-4 \ln \frac{\theta}{2} \ln \frac{\mathrm{~s}}{\Delta \mathrm{E}^{2}}\right] \tag{4.5}
\end{equation*}
$$

Similarly, in the backward direction

$$
\begin{equation*}
\left.\frac{\mathrm{d} \sigma}{\mathrm{~d} \Omega}\right|_{\alpha^{3}, \theta=\pi-\epsilon}=-\left.\frac{\mathrm{d} \sigma}{\mathrm{~d} \Omega}\right|_{\alpha^{3}, \theta=\epsilon} \tag{4.6}
\end{equation*}
$$

Thus there is a tendency for a $\mathrm{p}^{+}$to line up along the direction of the initial $\mathrm{e}^{+}$, enhanced over the intuitive $\mathrm{O}(\alpha / \pi)$ effect. In particular a convenient experimental quantity (assuming the direct parton-pair experiment were possible) would be

$$
\begin{equation*}
\mathrm{A}(\theta)=\frac{\frac{\mathrm{d} \sigma}{\mathrm{~d} \Omega}(\theta)-\frac{\mathrm{d} \sigma}{\mathrm{~d} \Omega}(\pi-\theta)}{\frac{\mathrm{d} \sigma}{\mathrm{~d} \Omega}(\theta)+\frac{\mathrm{d} \sigma}{\mathrm{~d} \Omega}(\pi-\theta)} \tag{4.7}
\end{equation*}
$$

From Eqs. (4.5) and (4.6) we find that

$$
\begin{equation*}
\mathrm{A}(\theta=0) \approx \frac{\alpha}{\pi}\left[2 \ln ^{2} \frac{\theta}{2}-4 \ln \frac{\theta}{2} \ln \frac{\mathrm{~S}}{\Delta \mathrm{E}^{2}}\right] \tag{4.8}
\end{equation*}
$$

As discussed earlier, there are two kinematical assumptions made in obtaining this result. The first is the scaling assumption, $s,|t|,|u| \gg \mu^{2}$, where $\mu$ is some characteristic mass (say the mass of the produced parton and antiparton); satisfying this limit insures that our results will be energy-independent (if $\Delta E / E$ is). The second assumption is that the observed particle comes out close to the beam axis ( $s \gg-\mathrm{t}$ or $\mathrm{s} \gg-\mathrm{u}$ ) so that squared logarithmic terms, easily separated from linear logarithmic terms in Ref. (9), are sure to dominate the asymmetry: for example, Eq 。(4.8) gives $\mathrm{A}\left(5^{\circ}\right) \approx 22 \%$ for $\Delta \mathrm{E} / \mathrm{E}=10 \%$; the exact result from Ref. (9) is $16.5 \%$. (At $5^{\circ}$ for $\Delta E / E=10 \%$ the second term in Eq. (4.8) contributes more to $A(\theta)$ than the first; this situation is reversed if the resolution is poor or if we look at smaller angles. For example, if the resolution is so poor that the second term in Eq. (4.8) is negligible, then $\mathrm{A}\left(1^{\mathrm{O}}\right) \approx 11 \%$ 。 ) It is clear from the results of Ref. (9) that a considerable asymmetry from QED effects alone may continue to exist at angles comfortably away from the beam axis, so in trying to untangle these from strictly gauge theory asymmetries ${ }^{8}$ of lower order, one must be rather careful; the expected energy dependence of the gauge theory effects at present energies should be useful here.

## V. Asymmetries in the Hadronic System

It remains for us to append the parton decay functions and derive expressions for the inclusive experiment. The additional variable $\omega$ now gives us a double differential cross section $\mathrm{d} \sigma^{\mathrm{h}} / \mathrm{d} \omega \mathrm{d} \Omega$. $\quad \Omega$ now refers to the CM angle of the detected hadron $h$; by assumption we take the parton decomposition to produce hadrons aligned with the parton. A spread of this decay then smears our results by an amount $\Delta \theta \sim p_{T} / \mathrm{p}_{\mathrm{L}} \sim 150 /(\sqrt{\mathrm{s}} / 2), \mathrm{s}$ in $(\mathrm{MeV})^{2}$, so small angle searches require high energy for well-collimated jets. A more convenient experimental cross section may be $d \sigma^{h} / d E_{h} d \Omega$. Since $E_{h} \approx \omega \frac{\sqrt{s}}{2}$,

$$
\begin{equation*}
\frac{\mathrm{d} \sigma^{\mathrm{h}}}{\mathrm{~d} \mathrm{E}_{\mathrm{h}} \mathrm{~d} \Omega} \approx \frac{2}{\sqrt{\mathrm{~s}}} \frac{\mathrm{~d} \sigma^{\mathrm{h}}}{\mathrm{~d} \omega \mathrm{~d} \Omega} \tag{5.1}
\end{equation*}
$$

To quantify the asymmetry, one might measure

$$
\begin{equation*}
A^{h}\left(E_{h}, \Omega\right)=\frac{\frac{d \sigma^{h}}{d E_{h} d \Omega}(\theta)-\frac{d \sigma^{h}}{d E_{h} d \Omega}(\pi-\theta)}{\frac{d \sigma^{h}}{d E_{h} \mathrm{~d} \Omega}(\Omega)+\frac{d \sigma^{h}}{d E_{h} d \Omega}(\pi-\theta)} \tag{5.2}
\end{equation*}
$$

In general, if parton $i$ has charge $Q_{i}$ measured in units of the electron charge, then if we denote the sum of $O\left(\alpha^{3}\right)$ type (a) and (b) interference contributions for partons of unit charge as

$$
\begin{equation*}
\frac{\mathrm{d} \sigma \text { int }}{\mathrm{d} \Omega}=\alpha \frac{\mathrm{d} \sigma^{\mathrm{o}}}{\mathrm{~d} \Omega} \mathrm{f}^{\text {int }}(\theta) \tag{5.3}
\end{equation*}
$$

then ${ }^{13}$

$$
\begin{equation*}
A^{h}\left(E_{h}, \Omega\right) \approx \alpha f^{\text {int }}(\theta) \frac{\sum_{i} Q_{i}^{3} D_{i}^{h}(\omega)}{\sum_{i} Q_{i}^{2} D_{i}^{h}(\omega)} \tag{5.4}
\end{equation*}
$$

In particular, if $\theta \approx 0$, then from Eq. (4.8)

$$
\begin{equation*}
A^{h}\left(E_{h}, \theta \approx 0\right)=A(\theta) \frac{\sum_{i} Q_{i}^{3} D_{i}^{h}(\omega)}{\sum_{i} Q_{i}^{2} D_{i}^{h}(\omega)} \tag{5.5}
\end{equation*}
$$

We pause to note that Eq. (5.4) allows one to make numerical estimates of $A^{h}$ at arbitrary angle with knowledge of the parton mass (all partons are assumed to have the same mass in (5.4), although this is easily generalized) and the full results of Ref. (9). Note also that in the portion of the asymmetry we have studied only $D_{i}^{h}(\omega)$, derivable from the lowest order results for a
definite parton model, is required. The precise size of the asymmetry depends upon details, but we might remark on an approximate linearity in $\mathbf{Q}_{\mathbf{i}}$ and on possible cancellations in the numerator.

As an illustration, we shall work out some details for the quark parton model $^{13}$, in which partons are the usual $u$, $d$, and $s$ states. From isospin and C-invariance we have

$$
\begin{align*}
& \mathrm{D}_{\mathrm{u}}^{\pi^{+}}=\mathrm{D}_{\mathrm{d}}^{\pi^{-}}=\mathrm{D} \frac{\pi^{-}}{\mathbf{u}}=\mathrm{D} \frac{\pi^{+}}{\mathrm{d}} \\
& \mathrm{D}_{\mathrm{d}}^{\pi^{+}}=\mathrm{D}_{\mathrm{u}}^{\pi^{-}}=\mathrm{D} \frac{\pi^{-}}{\mathrm{d}}=\mathrm{D} \frac{\pi^{+}}{\mathbf{u}}  \tag{5.6}\\
& \mathrm{D}_{\mathrm{s}}^{\pi^{+}}=\mathrm{D}_{\mathrm{s}}^{\pi^{-}}=\mathrm{D} \frac{\pi^{+}}{\mathrm{s}}=\mathrm{D} \frac{\pi^{-}}{\mathrm{s}}
\end{align*}
$$

and

$$
\begin{align*}
& \mathrm{D} \frac{\mathrm{~K}}{\mathrm{~S}}^{+}=\mathrm{D}{\frac{\mathrm{~K}^{0}}{}}^{0}=\mathrm{D}_{\mathrm{S}}^{\mathrm{K}^{-}}=\mathrm{D}_{\mathrm{S}}^{\overline{\mathrm{K}}^{0}} \\
& D_{d}^{K^{+}}=D_{u}^{K^{0}}=D \frac{K^{-}}{d}=D{\frac{\bar{K}^{0}}{u}}^{0} \\
& D \frac{K}{d}^{+}=D{\frac{K^{0}}{}}^{0}=D_{d}^{K^{-}}=D_{u}^{\bar{K}^{O}}  \tag{5.7}\\
& \mathrm{D} \frac{\mathrm{~K}}{}^{+}=\mathrm{D}{\frac{\mathrm{~K}^{0}}{\mathrm{O}}}^{\mathrm{O}}=\mathrm{D}_{\mathrm{u}}^{\mathrm{K}^{-}}=\mathrm{D}_{\mathrm{d}}^{\overline{\mathrm{K}}^{0}} \\
& D_{S}^{K^{+}}=D_{S}^{K^{0}}=D{\frac{K^{-}}{}}^{-}=D{\frac{\bar{K}^{0}}{S}} .
\end{align*}
$$

From SU(3) invariance we have

$$
\begin{align*}
& \mathrm{D}_{\mathrm{u}}^{\mathrm{K}^{+}}=\mathrm{D} \frac{\mathrm{~K}}{\mathrm{~s}}_{+}=\mathrm{D}_{\mathrm{u}}^{\pi^{+}} \\
& \mathrm{D}_{\mathrm{d}}^{\mathrm{K}^{+}}=\mathrm{D} \frac{\mathrm{~K}^{+}}{\mathrm{d}}=\mathrm{D}_{\mathrm{s}}^{\pi^{+}}  \tag{5.8}\\
& \mathrm{D}_{\mathrm{u}}^{\mathrm{K}^{+}}=\mathrm{D}_{\mathrm{s}}^{\mathrm{K}^{+}}=\mathrm{D}_{\mathrm{d}}^{\pi^{+}}
\end{align*}
$$

A convenient (and measured ${ }^{13}$ ) quantity is

$$
\begin{equation*}
\eta \equiv \mathrm{D}_{\mathrm{u}}^{\pi^{+}} / \mathrm{D}_{\mathrm{d}}^{\pi^{+}} \approx 3 \tag{5.9}
\end{equation*}
$$

We then find, using only Eq. (5.6)

$$
\begin{equation*}
\mathrm{A}^{\pi^{+}}(\mathrm{E}, \theta \sim 0)=\mathrm{A}(\theta) \frac{\left.\Gamma\left(\frac{2}{3}\right)^{3}+\left(\frac{1}{3}\right)^{3}\right] \mathrm{D}_{\mathrm{d}}^{\pi^{+}}(\eta-1)}{\left[\left(\frac{2}{3}\right)^{2}+\left(\frac{1}{3}\right)^{2}\right] \mathrm{D}_{\mathrm{d}}^{\pi^{+}}(\eta+1)+\left(\frac{1}{3}\right)^{2} \cdot 2 \mathrm{D}_{\mathrm{S}}^{\pi^{+}}} \tag{5.10}
\end{equation*}
$$

and

$$
\mathrm{A}^{\pi^{-}}(\mathrm{E}, \theta \approx 0)=-\mathrm{A}^{\pi^{+}}(\mathrm{E}, \theta \approx 0)
$$

In particular if ${ }^{13} \mathrm{D}_{\mathrm{S}}^{\pi^{+}}(\omega) \approx \mathrm{D}_{\mathrm{d}}^{\pi^{+}}$, then

$$
\begin{equation*}
A^{\pi^{+}}(E, \theta \approx 0)=\frac{3}{11} A(\theta) \tag{5.11}
\end{equation*}
$$

which for $\theta=2^{\circ}$ is $\mathrm{a} \approx 2 \%$ effect.
For kaons the results become most interesting when we apply Eq. (5.8).
Then we find

$$
\begin{equation*}
A^{K^{+}}(E, \cos \theta)=A^{\pi^{+}}(E, \cos \theta)=-A^{K^{-}}(E, \cos \theta) \tag{5.12}
\end{equation*}
$$

and

$$
\begin{equation*}
A^{K^{0}}(E, \cos \theta)=0 \tag{5.13}
\end{equation*}
$$

The predicted equality of $K^{ \pm}$and $\pi^{ \pm}$asymmetries and disappearance of $\mathrm{K}^{\mathrm{O}}-\overline{\mathrm{K}}^{\mathrm{o}}$ asymmetries are consequences of assumed $\mathrm{SU}(3)$ invariance in Eq. (5.8). This assumption can easily be modified within the context of the quark parton model we have used. If also for some reason the intermediate vector boson in a hypothesized gauge theory does not couple to the same kinds of partons as does the photon, then deviations from the above may be observed in data from which gauge theory effects have not been separated out.

Complete absence of asymmetry will further compound well-known paradoxes of the parton model, in that it indicates the quark charge never leaves the interaction region.

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## Figure Captions

Figure 1: Kinematics of inclusive hadron production from $\mathrm{e}^{+} \mathrm{e}^{-}$annihilation.
Figure 2: Lowest order (one-photon) approximation to the amplitude of Fig. 1.
Figure 3: The two types of interference terms contributing in $\mathrm{O}\left(\alpha^{3}\right)$ to the hadron production cross section in $\mathrm{e}^{+} \mathrm{e}^{-}$annihilation. "Type (a)" terms arise from interference of an $\mathrm{O}(\alpha)$ amplitude with an $\mathrm{O}\left(\alpha^{2}\right)$ amplitude; "type (b)" terms arise from interference of two $\mathrm{O}\left(\alpha^{3 / 2}\right)$ amplitudes and contain a real photon in the final state.

Figure 4: A further breakdown of "type (a)" interference terms of Fig. 3(a). Only the kind in Fig. 4(a) gives asymmetries.

Figure 5: A classification of amplitudes leading to "type (b)" interference terms of Fig. 3(b) is made here according to whether the final real photon is emitted from (a) a lepton line, (b) an external hadron line, or (c) an internal line. There are nine permutations of these amplitudes with their complex conjugates representing the "type (b)" interference terms.

Figure 6: The two-jet parton picture considered in the text; the heavy lines represent the parton (p) and antiparton ( $\overline{\mathrm{p}}$ ). In this model there are no interactions between the pair after they are produced.

Figure 7: The two two-photon amplitudes in $\mathrm{e}^{+} \mathrm{e}^{-} \rightarrow \mu^{+} \mu^{-}$. Interference between these and the one-photon amplitude (not shown) leads to an asymmetry in which the $\mu^{+}\left(\mu^{-}\right)$preferentially maintains the direction of motion of the $e^{+}\left(e^{-}\right)$.

Figure 8: The two classes of "type (a)" processes contributing to asymmetries in a two-jet parton picture (summation of crossed photon lines is understood). These are just the two-jet categorizations of Fig. 4(a). There is no trading of hadrons between jets.

Figure 9: This is the same as Fig. 5, but in the context of a two-jet parton picture. There are 16 permutations of these amplitudes with their complex conjugates to form the "type (b)" interference terms of Fig. 3 (b).

Figure 10: The remaining "type (b)" amplitudes left to consider after we have ruled out various contributions of amplitudes in Fig. 10. The soft photon limit of these bremsstrahlung diagrams cancels the infrared divergence of the "type (a)" interference terms.


Fig. 1


Fig. 2
(a)

(b)


Fig. 3


Fig. 4


Fig. 5


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Fig. 6

(a)


Fig. 7


Fig. 8


Fig. 9






Fig. 10

